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IT-24635

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a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$

1. Start with first two congruences:

Let $x = 3a + 1$. Substitute into the second congruence,

$3a + 1 \equiv 2 \pmod 5$

$\Rightarrow 3a = 1 \pmod 5$

The inverse of 3 module 5 is 2, so:

$a \equiv 2 \pmod 5$

$\Rightarrow a = 5b + 2$

Then $x = 3(5b+2) + 1 = 15b + 7$

2. Now use third congruence

$15b + 7 \equiv 3 \pmod 7$

Since $15 \equiv 1 \pmod 7$ & $7 \equiv 0 \pmod 7$ then simplifies to, $b \equiv 3 \pmod 7 \Rightarrow b = 7c + 3$

Then, $x = 15(7c+3) + 7 = 105c + 52$

$x \equiv 52 \pmod{15} \Rightarrow 52$ Ans:-

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b) $x \equiv 5 (\text{mod } 11)$, $x \equiv 14 (\text{mod } 29)$, $x \equiv 15 (\text{mod } 31)$

1. Combine the first two congruence,

$x = 11a + 5$ Substitute into the second ingruence

$11a + 5 \equiv 14 \ (\text{mod } 29)$

$11a \equiv 9 \ (\text{mod } (29))$

The inverse of 11 module 29

That means we want $11x = 1 \text{ mod } 29$

$\therefore 11 \cdot 8 = 1 \text{ mod } 29$

So the inverse is 8. Multiply both sides

$a = 9 \times 8 = 72 \equiv 14 \ (\text{mod } 29)$

$a = 29b + 14$

Substitute back :

$x = 11(29b + 14) + 5 = 319b + 159$

So $x = 159 \ (\text{mod } 319)$

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2. Combine with third congruence,

We have, $319b + 159 \equiv 15 \pmod{31}$

$319 \equiv 9 \pmod{31}$ & $159 \equiv 4 \pmod{31}$

$\therefore$ so : $9b + 4 \equiv 15 \pmod{31} \Rightarrow 9b \equiv 11 \pmod{31}$

Find the inverse of 9 module 31 :

$9 \times 7 = 63 \equiv 1 \pmod{31}$

So the inverse is 7. Multiply both sides :

$b \equiv 11 \times 7 = 77 \equiv 15 \pmod{31} \Rightarrow b = 31c + 15$

Substitute back, $x = 319(31c + 15) + 159$

$= 9889c + 4944$

$\therefore x = 4944 \pmod{9889} \Rightarrow 4944$ Ans:-

c) $x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$

1. Combine the first two ingruence,

$x = 6a + 5$. Substitute into the second congruence

(P.T.O)

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$6a+5 = 4 \pmod{11}$

$6a \equiv -1 \equiv 10 \pmod{11}$

The inverse of 6 module 11 is 2

Since $6 \times 2 = 12 \equiv 1 \pmod{11}$:

$a = 2 \times 10 = 20 = 9 \pmod{11}$

$\Rightarrow a = 11b+9$

Then $x = 6(11b+9)+5 = 66b+59$

2. Now use the 3rd congruence, $66b+59 \equiv 3 \pmod{17}$

Reduce mod 17, $66b = 15 \pmod{17}$, $59 \equiv 8 \pmod{17}$

So, $15b+8 \equiv 3 \pmod{17} \Rightarrow 15b = -5 \equiv 12 \pmod{17}$

The inverse of 15 mod 17 is 8 (since $15 \times 8 = 120$)

$\therefore 15 \times 8 = 120 \equiv 1 \pmod{17}$:

$b = 8 \times 12 = 96 \equiv 11 \pmod{7} \Rightarrow b = 17c+11$

Then, $x = 66(17c+11)+59 = 1122c+785$

$\therefore x = 785 \pmod{1122}$

$\Rightarrow 785$ Ans:-