Solution to Quiz 1

U20 Math 559 Spring 2021

3/13/2021

Problem 1.

Answer: This is a Frequentist approach

Reasoning: The approach here takes the expected value of observed frequency to measure the unknown, and its conclusion does not include any prior belief

Problem 2.

Guide to answer:

Key point to construct a Bayesian approach: probabaility assigned to the asnwer with the 'keyword' should receive more weight based on our subjective belief

Problem 3.

Answer:

Let EQ = Earthquake and LS = Landslide. We know the following:

- $p(EQ) = 0.25 \implies p(EQ^c) = 0.75$
- $p(LS|EQ) = 0.9$
- $p(LS|EQ^c) = 0.15$

We would like to find $p(LS|EQ)$. Now, since $p(LS|EQ) = p(EQ, LS)/p(LS)$, and

$$p(LS) = \underbrace{p(LS|EQ)p(EQ) + p(LS|EQ^c)p(EQ^c)}_{\text{total probability law}} \implies p(LS) = 0.225 + 0.1125 = 0.3375 = 2/3$$

Problem 4.

Answer

We are given the mathematical model for one observation:

$$p(x|\sigma) = \frac{x}{\sigma}\exp\{-\frac{x^2}{2\sigma^2}\}$$

where $x > 0$ and $\sigma > 0$, and σ is the unknown variable we would like to estimate. Now, let $D = \{X_1, X_2, \ldots, X_n\}$ be the i.i.d. variables representing the data, we can construct the likelihood function as

$$p(D|\sigma) = \Pi_{i=1}^n p(x_i|\sigma) = \frac{\Pi_{i=1}^n x_i}{\sigma^{2n}}\exp\{\frac{\sum_{i=1}^n x_i^2}{2\sigma^2}\}$$

To calculate the posterior, notice that $p(\sigma|D) \propto p(D|\sigma)p(\sigma)$ is a conjugate of its prior, so they have the same form, but different parameter values:

$$p(D|\sigma)p(\sigma) \propto \sigma^{-(2b-2n+1)} \exp\{-\frac{a+\sum_{i=1}^{n} x_i^2}{2\sigma^2}\} = \sigma^{-(2(b+n)+1)} \exp\{-\frac{a+\sum_{i=1}^{n} x_i^2}{2\sigma^2}\}$$

Therefore, $p(\sigma|D) \sim SqIG(a+\sum_{i=1}^{n} x_i^2, b+n)$