Stochastic Processes Homework 2

Due Date: October 24, 2020

Instruction: Please type or write your answers clearly and show your work. You are encouraged to use the Rmarkdown version of this assignment as a template to submit your work. Unless stated otherwise, all programming references in the assignment will be in `R`. For this assignment, problems roughly covers content from the lectures 4 - 5.

1. A stochastic matrix is called *doubly stochastic* if its rows and columns sum to 1. Show that a Markov chain whose transition matrix is doubly stochastic has a stationary distribution, which is uniform on the state space.

2. Consider a Markov chain with transition matrix

$$\begin{bmatrix} 1-a & a & 0 \\ 0 & 1-b & b \\ c & 0 & 1-c \end{bmatrix}$$

where $0 < a, b, c < 1$. Find the stationary distribution.

3. Let $\mathbf{P}$ be a stochastic matrix.

a) If $\mathbf{P}$ is regular, is $\mathbf{P}^2$ regular?
b) If $\mathbf{P}$ is the transition matrix of an irreducible Markov chain, is $\mathbf{P}^2$ the transition matrix of an irreducible Markov chain?

4. The California Air resources Board warns the public when smog levels are above certain thresholds. Days when the board issues warnings are called *episode days*. A model (Lin, 1981) of the daily sequence of episode and nonepisode days is presented below as a Markov chain with transition matrix

	nonepisode	episode
nonepisode	0.77	0.23
episode	0.24	0.76

Use `R` to answer the following:

a) What is the long-term probability that a given day will be an episode day?
b) Over a year's time about how many days are expected to be episode days?
c) In th long-term, what is the average number of days that will transpire between episode days?

5. Consider a Markov chain with transition matrix

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$$

Obtain a closed form expression for $\mathbf{P}^n$. Exhibit the matrix $\sum_{n=0}^{\infty} \mathbf{P}^n$ (some entries may be $+\infty$). Explain what this shows about the recurrence and transience of the states.