

Deep Learning 2026: Hand-in Assignment 1

Exercise 1:

Initialization:

```
def initialize_network(layer_sizes):
    K = len(layer_sizes) - 1 # K is the number of layers
    all_weights = [None] * K
    all_biases = [None] * K

    for k in range(K):
        n_in = layer_sizes[k]
        n_out = layer_sizes[k+1]

        # Randomly pick weights; scale them based on input size
        all_weights[k] = np.random.normal(0, np.sqrt(2.0/n_in), size=(n_out, n_in)) #He initialization as per instruction
        all_biases[k] = np.zeros((n_out, 1)) + 0.01

    return all_weights, all_biases
```

In my initialization function, weights are defined as He initialization method to prevent the gradients from vanishing in deep layers. Weights matrices are also initialized randomly because that will help neurons in layers to learn different features rather than calculate identical values. Biases are initialized as zero.

Activation functions:

I have implemented both Relu and Sigmoid Activation functions with their derivatives using mathematical expression.

```
def ReLU(x):
    return np.maximum(0, x)

def d_ReLU(x):
    return (x > 0).astype(float)

def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def d_sigmoid(x):
    s = sigmoid(x)
    return s * (1 - s)
```

Forward propagation:

```
def forward_pass(X, all_weights, all_biases, activation_type='ReLU'):
    K = len(all_weights)
    all_f = [None] * K
    all_h = [None] * (K + 1)

    act_fn = ReLU if activation_type == 'ReLU' else sigmoid
    all_h[0] = X

    for k in range(K):
        # Matrix multiplication for the layer
        all_f[k] = np.matmul(all_h[k], all_weights[k].T) + all_biases[k].T

        # Applying activation function
        if k < K - 1:
            all_h[k+1] = act_fn(all_f[k])
        else:
            all_h[k+1] = all_f[k]

    return all_f[K-1], all_f, all_h
```

This function performs forward propagation for my neural network. Here we applied linear transformation, activation function , only linear transformation for the final layer. We transpose the weight matrix for dimension match. The output layer produce logits which later gone through the softmax function and convert into probabilities.

Softmax and cost:

We used a typical softmax function to convert logit into probabilities. Then implemented the compute_cost function, which computes the numerically stable softmax cross-entropy loss by using the log-sum-exp trick and averaging the loss over all training samples.

```
def compute_cost(f_K, y_oh):
    max_f = np.max(f_K, axis=1, keepdims=True)

    #Log-Sum-Exp term
    log_sum_exp = max_f + np.log(np.sum(np.exp(f_K - max_f), axis=1, keepdims=True))

    #Full Cross-Entropy Loss
    loss_i = log_sum_exp - np.sum(y_oh * f_K, axis=1, keepdims=True)

    # 4. Return mean over the batch
    return np.mean(loss_i)
```

Backward propagation:

In our backward_pass function we calculate the error between the output layer values with the true labels which gives us the gradients of loss with respect to the logits. Then the error is propagated backward and in each layer weight and biases are computed and error is adjusted later by using derivative Activation functions.

```

D def backward_pass(all_weights, all_biases, all_f, all_h, y_oh, activation_type='ReLU'):
    "Works backward from the error to see how to adjust each weight"
    K = len(all_weights) - 1
    all_dl_dweights = [None] * (K + 1)
    all_dl_dbaises = [None] * (K + 1)
    all_dl_df = [None] * (K + 1)
    all_dl_dh = [None] * (K + 1)

    all_dl_df[K] = d_cost_d_output(all_f[K], y_oh)

    for layer in range(K, -1, -1):
        all_dl_dweights[layer] = np.matmul(all_dl_df[layer].T, all_h[layer])
        all_dl_dbaises[layer] = np.sum(all_dl_df[layer], axis=0, keepdims=True).T

        if layer > 0:
            all_dl_dh[layer] = np.matmul(all_dl_df[layer], all_weights[layer])
            if activation_type == 'ReLU':
                all_dl_df[layer-1] = all_dl_dh[layer] * d_ReLU(all_f[layer-1])
            else:
                all_dl_df[layer-1] = all_dl_dh[layer] * d_sigmoid(all_f[layer-1])

    return all_dl_dweights, all_dl_dbaises

```

Take a step: This update_parameter function updates the network's weights and biases using gradient descent.

Predic: This function **evaluates the trained neural model** by calculating its predictions, accuracy, and cost on a given dataset. Here the input data are passed through forward pass and it calculates the logits then softmax are used to convert it into probabilities and highest probability are chosen for the comparison. Later cross-entropy loss measure how well it match with the actual label.

Mini-batch generation: This function randomly generates mini training data batches. Mini batches are good for faster, efficient code implementation as it reduce the memory usages and stabilizes the learning.

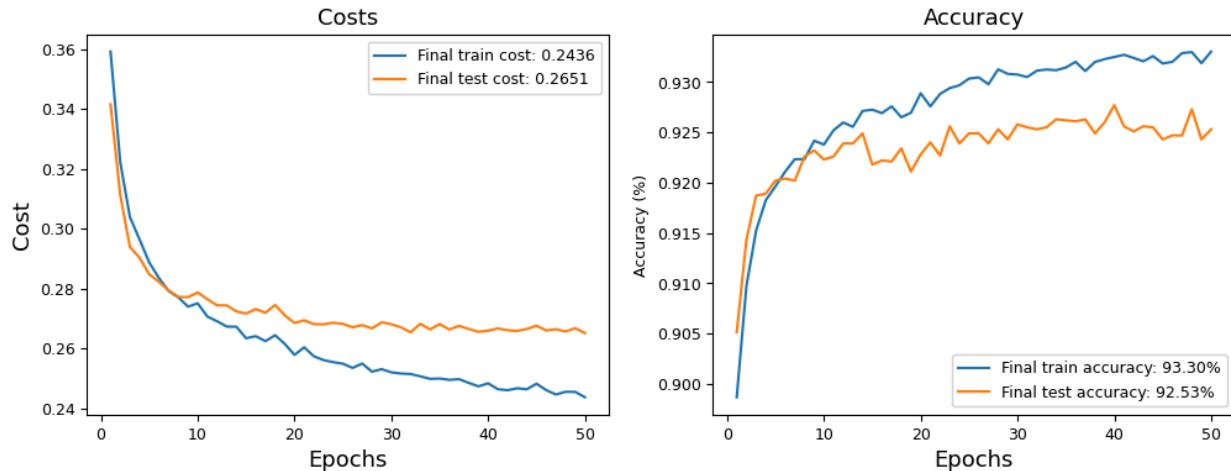
Model training: We have implemented everything as instructed for the model training. We used number of epochs as 50, 0.1 learning rate and batch size as 64. I have tried different parameter but as my assumption 50 epochs, 0.1 and 64 batch size are good for achieving almost 98% accuracy for a multi-layer neural network.

Exercise 2:

a)

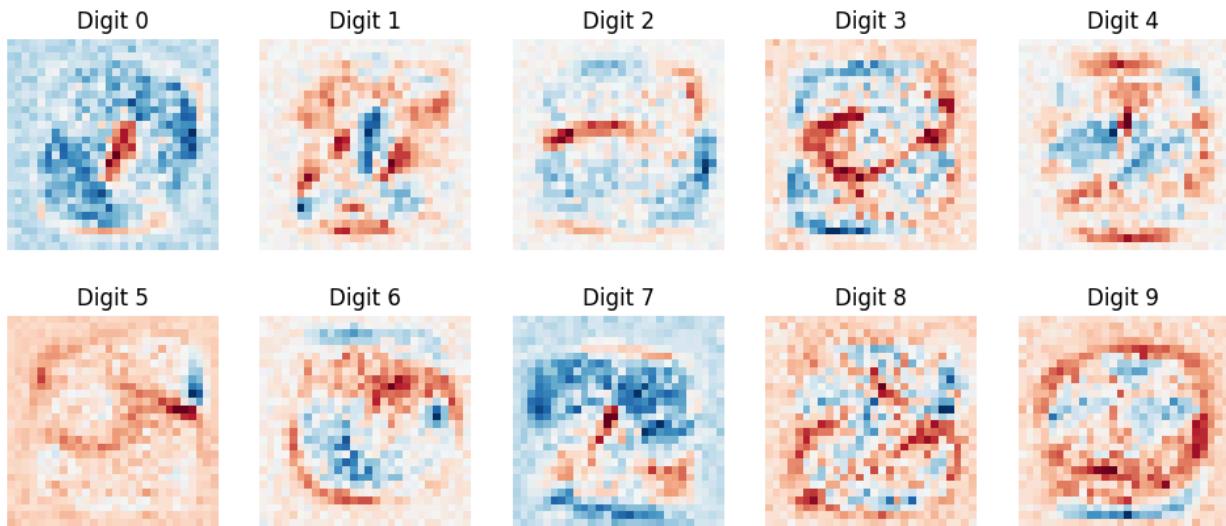
Exercise 2: Linear Model Evaluation

| Batch size:64 | Learning rate:0.1 | Number of Epochs:50 | Training time: 0 min 32.4 sec |



Here we plotted the training and testing loss of our linear model. Here we got almost 92.5% accuracy on test data and loss is showing very high as it was 0.26. Training accuracy are also showing in the graph.

b)



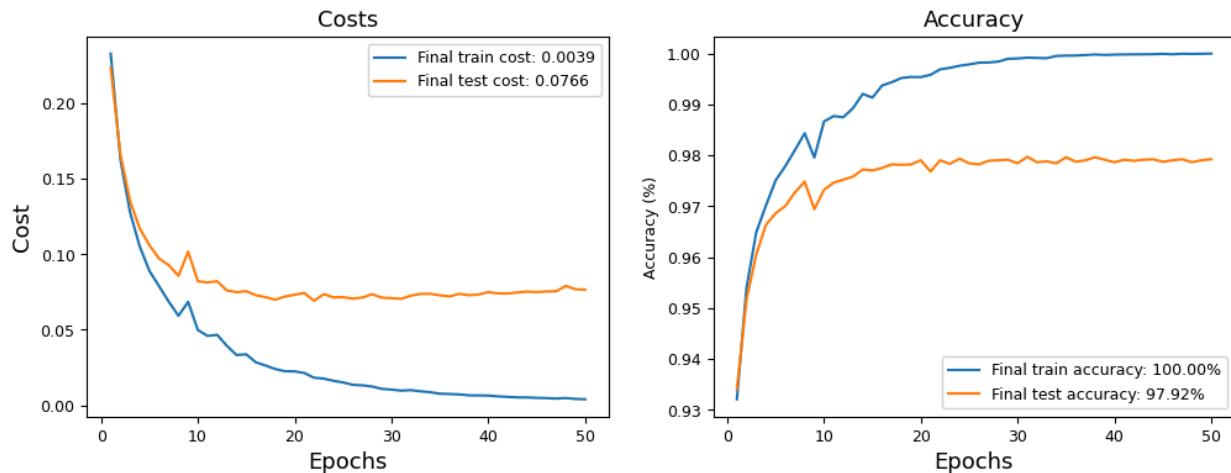
Some of the digit catch the digit correctly but most of them couldn't identify the digit as I can 0, 3 and 7 with my eyes but others do not. Perhaps it just learns the pattern but not enough to classify more confidently.

Exercise 3. Evaluate your multi-layer neural network:

Here we trained the multi layered model with two activation functions one with Relu and another one with Sigmoid. And we get almost 98% accuracy,

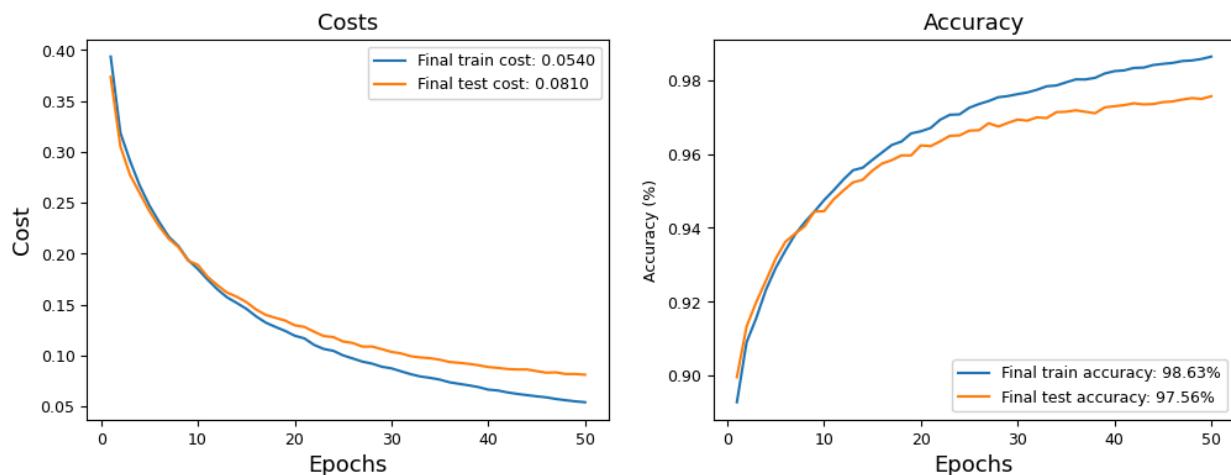
Exercise 3: MLP with ReLU Activation

| Batch size:64 | Learning rate:0.1 | Number of Epochs:50 | Training time: 2 min 5.8 sec |



Exercise 3: MLP with Sigmoid Activation

| Batch size:64 | Learning rate:0.1 | Number of Epochs:50 | Training time: 2 min 33.3 sec |



```
import numpy as np
import matplotlib.pyplot as plt
from utils import load_mnist

!unzip -q MNIST.zip  #unzip the file

x_train, y_train, x_test, y_test = load_mnist()

print(f"Loaded {x_train.shape[0]} training images.") #loaded all images
Loaded 60000 training images.
```

```
def one_hot_encode(y, num_classes=10):
    #convert a single number into a vector of one and zeros( one for the correct label)
    y_int = np.array(y, dtype=int).flatten()
    return np.eye(num_classes)[y_int]
```

✓ initialize the network

```
def initialize_network(layer_sizes):
    K = len(layer_sizes) - 1 # K is the number of layers
    all_weights = [None] * K
    all_biases = [None] * K

    for k in range(K):
        n_in = layer_sizes[k]
        n_out = layer_sizes[k+1]

        # Randomly pick weights; scale them based on input size
        all_weights[k] = np.random.normal(0, np.sqrt(2.0/n_in), size=(n_out, n_in)) #He initialization as per instruction
        all_biases[k] = np.zeros((n_out, 1)) + 0.01

    return all_weights, all_biases
```

✓ Activation Function and their derivatives

```
def ReLU(x):
    return np.maximum(0, x)

def d_ReLU(x):
    return (x > 0).astype(float)
```

```
def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def d_sigmoid(x):
    s = sigmoid(x)
    return s * (1 - s)
```

- Forward Pass Function, use the equationn $F = X\Omega^T + \beta^T$. to calculate the output

```
def forward_pass(X, all_weights, all_biases, activation_type='ReLU'):
    K = len(all_weights)
    all_f = [None] * K
    all_h = [None] * (K + 1)

    act_fn = ReLU if activation_type == 'ReLU' else sigmoid
    all_h[0] = X

    for k in range(K):
        # Matrix multiplication for the layer
        all_f[k] = np.matmul(all_h[k], all_weights[k].T) + all_biases[k].T

        # Applying activation function
        if k < K - 1:
            all_h[k+1] = act_fn(all_f[k])
        else:
            all_h[k+1] = all_f[k]

    return all_f[K-1], all_f, all_h
```

- softmax function

```
def softmax(f):
    #turn the final layer output into probabilities
    f_shifted = f - np.max(f, axis=1, keepdims=True)
    exp_f = np.exp(f_shifted)
    return exp_f / np.sum(exp_f, axis=1, keepdims=True)
```

- Cost Funtion

```
def compute_cost(f_K, y_oh):
    max_f = np.max(f_K, axis=1, keepdims=True)

    #Log-Sum-Exp term
```

```

log_sum_exp = max_f + np.log(np.sum(np.exp(f_K - max_f), axis=1, keepdims=True))

#Full Cross-Entropy Loss
loss_i = log_sum_exp - np.sum(y_oh * f_K, axis=1, keepdims=True)

# 4. Return mean over the batch
return np.mean(loss_i)

```

✓ d_cost_d_output

```

def d_cost_d_output(f_K, y_oh):
    I = f_K.shape[0]
    return (softmax(f_K) - y_oh) / I

```

✓ Backward Propagation

```

def backward_pass(all_weights, all_biases, all_f, all_h, y_oh, activation_type='ReLU'):
    "Works backward from the error to see how to adjust each weight"
    K = len(all_weights) - 1
    all_dl_dweights = [None] * (K + 1)
    all_dl_dbiases = [None] * (K + 1)
    all_dl_df = [None] * (K + 1)
    all_dl_dh = [None] * (K + 1)

    all_dl_df[K] = d_cost_d_output(all_f[K], y_oh)

    for layer in range(K, -1, -1):
        all_dl_dweights[layer] = np.matmul(all_dl_df[layer].T, all_h[layer])
        all_dl_dbiases[layer] = np.sum(all_dl_df[layer], axis=0, keepdims=True).T

        if layer > 0:
            all_dl_dh[layer] = np.matmul(all_dl_df[layer], all_weights[layer])
            if activation_type == 'ReLU':
                all_dl_df[layer-1] = all_dl_dh[layer] * d_ReLU(all_f[layer-1])
            else:
                all_dl_df[layer-1] = all_dl_dh[layer] * d_sigmoid(all_f[layer-1])

    return all_dl_dweights, all_dl_dbiases

```

✓ Update Parameter (Take a step)

```

def update_parameters(all_weights, all_biases, grads_w, grads_b, alpha):
    for k in range(len(all_weights)):

```

```

# Subtract the gradient scaled by alpha
all_weights[k] = all_weights[k] - alpha * grads_w[k]
all_biases[k] = all_biases[k] - alpha * grads_b[k]
return all_weights, all_biases

def predict(X, y, all_weights, all_biases, activation_type='ReLU'):
    # forward_pass returns: f_K, all_f, all_h
    f_K, _, _ = forward_pass(X, all_weights, all_biases, activation_type)

    # Compute probabilities for accuracy
    probs = softmax(f_K)
    predictions = np.argmax(probs, axis=1)

    # Compute cost for the dataset
    y_oh = one_hot_encode(y)
    cost = compute_cost(f_K, y_oh)

    # Calculate accuracy
    accuracy = np.mean(predictions == y.flatten())
    return predictions, accuracy, cost

```

▼ Mini-batch generation

```

def random_mini_batches(X, y_oh, batch_size):
    I = X.shape[0]
    mini_batches = []

    indices = np.random.permutation(I)
    X_shuffled = X[indices]
    y_shuffled = y_oh[indices]

    for i in range(0, I, batch_size):
        X_mini = X_shuffled[i : i + batch_size]
        y_mini = y_shuffled[i : i + batch_size]
        mini_batches.append((X_mini, y_mini))

    return mini_batches

```

▼ Finally Model Training

```

def train_model(x_train, y_train_oh, x_test, y_test_oh, model_architecture,
                num_epochs=50, learning_rate=0.1, batch_size=64):

    all_weights, all_biases = initialize_network(model_architecture)

```

```

# Get 1D labels for accuracy checks
y_train_labels = np.argmax(y_train_oh, axis=1)
y_test_labels = np.argmax(y_test_oh, axis=1)

# Lists to store metrics for plotting
train_costs, test_costs = [], []
train_accs, test_accs = [], []

print(f"Training {model_architecture} for {num_epochs} epochs...")

for epoch in range(num_epochs):
    batches = random_mini_batches(x_train, y_train_oh, batch_size)

    for x_mini, y_mini_oh in batches:
        # Forward and Backward pass
        _, all_f, all_h = forward_pass(x_mini, all_weights, all_biases)
        grads_w, grads_b = backward_pass(all_weights, all_biases, all_f, all_h, y_mini_oh)

        # Update
        all_weights, all_biases = update_parameters(all_weights, all_biases, grads_w, grads_b, learning_rate)

    # Evaluation at the end of each epoch
    # Use the predict function to get metrics on full datasets
    _, t_acc, t_cost = predict(x_train, y_train_labels, all_weights, all_biases)
    _, v_acc, v_cost = predict(x_test, y_test_labels, all_weights, all_biases)

    train_costs.append(t_cost)
    test_costs.append(v_cost)
    train_accs.append(t_acc)
    test_accs.append(v_acc)

    print(f"Epoch {epoch}: Loss {t_cost:.4f}, Test Acc {v_acc:.4f}")

return all_weights, all_biases, train_costs, test_costs, train_accs, test_accs

```

```

import time
import matplotlib.pyplot as plt
from utils import training_curve_plot

# Setup Parameters
arch_linear = [784, 10]
lr = 0.1
bs = 64
epochs = 50

start_time = time.time() #Train and track time
weights, biases, t_costs, v_costs, t_accs, v_accs = train_model(
    x_train, y_train, x_test, y_test,
    model_architecture=arch_linear,
    num_epochs=epochs,
)

```

```
        learning_rate=lr,
        batch_size=bs
    )
elapsed_time = time.time() - start_time

#Part (a): Generate the Training Curves

training_curve_plot(
    "Exercise 2: Linear Model Evaluation",
    t_costs,
    v_costs,
    t_accs,
    v_accs,
    bs,
    lr,
    epochs,
    elapsed_time
)

# Part (b): Visualization (The 28x28 Weight Templates)
plt.figure(figsize=(12, 5))
for i in range(10):
    # weights[0] is the weight matrix for the first layer
    w_image = weights[0][i, :].reshape(28, 28)
    plt.subplot(2, 5, i+1)
    plt.imshow(w_image, cmap='RdBu')
    plt.title(f"Digit {i}")
    plt.axis('off')
plt.show()
```



```
Training [784, 10] for 50 epochs...
Epoch 0: Loss 0.3591, Test Acc 0.9051
Epoch 1: Loss 0.3222, Test Acc 0.9143
Epoch 2: Loss 0.3038, Test Acc 0.9187
Epoch 3: Loss 0.2963, Test Acc 0.9189
Epoch 4: Loss 0.2886, Test Acc 0.9202
Epoch 5: Loss 0.2834, Test Acc 0.9204
Epoch 6: Loss 0.2792, Test Acc 0.9202
Epoch 7: Loss 0.2770, Test Acc 0.9226
Epoch 8: Loss 0.2739, Test Acc 0.9232
Epoch 9: Loss 0.2750, Test Acc 0.9223
Epoch 10: Loss 0.2706, Test Acc 0.9226
Epoch 11: Loss 0.2690, Test Acc 0.9239
Epoch 12: Loss 0.2672, Test Acc 0.9239
Epoch 13: Loss 0.2672, Test Acc 0.9249
Epoch 14: Loss 0.2634, Test Acc 0.9218
Epoch 15: Loss 0.2640, Test Acc 0.9222
Epoch 16: Loss 0.2624, Test Acc 0.9221
Epoch 17: Loss 0.2644, Test Acc 0.9234
Epoch 18: Loss 0.2614, Test Acc 0.9211
Epoch 19: Loss 0.2578, Test Acc 0.9228
Epoch 20: Loss 0.2603, Test Acc 0.9240
Epoch 21: Loss 0.2573, Test Acc 0.9227
Epoch 22: Loss 0.2560, Test Acc 0.9256
Epoch 23: Loss 0.2554, Test Acc 0.9239
Epoch 24: Loss 0.2548, Test Acc 0.9249
Epoch 25: Loss 0.2547, Test Acc 0.9250
Epoch 26: Loss 0.2549, Test Acc 0.9239
Epoch 27: Loss 0.2522, Test Acc 0.9253
```

Exercise 3: Multi-layer Neural Network

```
def train_model(x_train, y_train_oh, x_test, y_test_oh, model_architecture,
                num_epochs=10, learning_rate=0.1, batch_size=64, activation_type='ReLU'):

    all_weights, all_biases = initialize_network(model_architecture)

    y_train_labels = np.argmax(y_train_oh, axis=1)
    y_test_labels = np.argmax(y_test_oh, axis=1)

    train_costs, test_costs = [], []
    train_accs, test_accs = [], []

    print(f"Training {model_architecture} using {activation_type} for {num_epochs} epochs...")

    for epoch in range(num_epochs):
        batches = random_mini_batches(x_train, y_train_oh, batch_size)

        for x_mini, y_mini_oh in batches:

            _, all_f, all_h = forward_pass(x_mini, all_weights, all_biases, activation_type)
            grads_w, grads_b = backward_pass(all_weights, all_biases, all_f, all_h, y_mini_oh, activation_type)

            all_weights, all_biases = update_parameters(all_weights, all_biases, grads_w, grads_b, learning_rate)

    return all_weights, all_biases
```

```

    _, t_acc, t_cost = predict_eval(x_train, y_train_labels, all_weights, all_biases, activation_type)
    _, v_acc, v_cost = predict_eval(x_test, y_test_labels, all_weights, all_biases, activation_type)

    train_costs.append(t_cost)
    test_costs.append(v_cost)
    train_accs.append(t_acc)
    test_accs.append(v_acc)

    print(f"Epoch {epoch}: Loss {t_cost:.4f}, Test Acc {v_acc:.4f}")

return all_weights, all_biases, train_costs, test_costs, train_accs, test_accs

def predict_eval(X, y_labels, all_weights, all_biases, activation_type='ReLU'):
    f_K, _, _ = forward_pass(X, all_weights, all_biases, activation_type)
    predictions = np.argmax(f_K, axis=1)
    accuracy = np.mean(predictions == y_labels.flatten())
    y_oh = one_hot_encode(y_labels)
    cost = compute_cost(f_K, y_oh)
    return predictions, accuracy, cost

```

```
# Input (784) -> Hidden Layer (128) -> Output (10)
arch_mlp = [784, 128, 10]
```

```
# Training parameters
learning_rate = 0.1
batch_size = 64
num_epochs = 50
```



```

import time
start_time = time.time()

# Train with ReLU
w_relu, b_relu, t_costs_r, v_costs_r, t_accs_r, v_accs_r = train_model(
    x_train, y_train, x_test, y_test,
    model_architecture=arch_mlp,
    num_epochs=num_epochs,
    learning_rate=learning_rate,
    batch_size=batch_size,
    activation_type='ReLU'
)

training_curve_plot(
    "Exercise 3: MLP with ReLU Activation",
    t_costs_r, v_costs_r, t_accs_r, v_accs_r,
    batch_size, learning_rate, num_epochs, time.time() - start_time
)

```



```
Training [784, 128, 10] using ReLU for 50 epochs...
Epoch 0: Loss 0.2330, Test Acc 0.9340
Epoch 1: Loss 0.1630, Test Acc 0.9517
Epoch 2: Loss 0.1278, Test Acc 0.9605
Epoch 3: Loss 0.1052, Test Acc 0.9663
Epoch 4: Loss 0.0889, Test Acc 0.9686
Epoch 5: Loss 0.0788, Test Acc 0.9700
Epoch 6: Loss 0.0686, Test Acc 0.9727
Epoch 7: Loss 0.0592, Test Acc 0.9748
Epoch 8: Loss 0.0684, Test Acc 0.9694
Epoch 9: Loss 0.0498, Test Acc 0.9732
Epoch 10: Loss 0.0459, Test Acc 0.9746
Epoch 11: Loss 0.0466, Test Acc 0.9752
Epoch 12: Loss 0.0393, Test Acc 0.9758
Epoch 13: Loss 0.0332, Test Acc 0.9772
Epoch 14: Loss 0.0337, Test Acc 0.9770
Epoch 15: Loss 0.0284, Test Acc 0.9775
Epoch 16: Loss 0.0263, Test Acc 0.9782
Epoch 17: Loss 0.0239, Test Acc 0.9781
Epoch 18: Loss 0.0225, Test Acc 0.9782
Epoch 19: Loss 0.0224, Test Acc 0.9790
Epoch 20: Loss 0.0213, Test Acc 0.9768
Epoch 21: Loss 0.0182, Test Acc 0.9790
Epoch 22: Loss 0.0176, Test Acc 0.9783
Epoch 23: Loss 0.0161, Test Acc 0.9793
```

```
start_time = time.time()

# Train with Sigmoid
w_sig, b_sig, t_costs_s, v_costs_s, t_accs_s, v_accs_s = train_model(
    x_train, y_train, x_test, y_test,
    model_architecture=arch_mlp,
    num_epochs=num_epochs,
    learning_rate=learning_rate,
    batch_size=batch_size,
    activation_type='sigmoid'
)

training_curve_plot(
    "Exercise 3: MLP with Sigmoid Activation",
    t_costs_s, v_costs_s, t_accs_s, v_accs_s,
    batch_size, learning_rate, num_epochs, time.time() - start_time
)
Epoch 40: Loss 0.0051, Test Acc 0.9792
Epoch 41: Loss 0.0048, Test Acc 0.9787
Epoch 42: Loss 0.0047, Test Acc 0.9790
Epoch 43: Loss 0.0044, Test Acc 0.9792
Epoch 44: Loss 0.0047, Test Acc 0.9786
Epoch 45: Loss 0.0041, Test Acc 0.9790
Epoch 46: Loss 0.0039, Test Acc 0.9792
```

Exercise 3: MLP with ReLU Activation

| Batch size:64 | Learning rate:0.1 | Number of Epochs:50 | Training time: 2 min 5.8 sec |