1. Prove of a recursive definition (Combination of r items from n items)

Answer: As we know,

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$^{3}C_{2} = \frac{3!}{2!\,(3-2)!}$$

$$= \frac{3 \times 2 \times 1}{2 \times 1 \times 1}$$

$$= 3$$

To evaluate this we have base cases:

(i) $r = 0$ then return 1.

(ii) $r = 1$ then return n

Otherwise the function will call recursively,

Now considering, $\left(^{n-1}C_{r-1}\right) + \left(^{n-1}C_{r}\right)$

$$= \frac{(n-1)!}{(r-1)!\,(n-1-r+1)!} + \frac{(n-1)!}{r!\,(n-1-r)!}$$

$$= \frac{(n-1)! \times r}{(r-1)!\,(n-r)! \times r} + \frac{(n-1)! \times (n-r)}{r!\,(n-r-1)!\,(n-r)}$$

$$= \frac{(n-1)! \times r}{r!\,(n-r)!} + \frac{(n-1)! \times (n-r)}{r!\,(n-r)!}$$

$$= \frac{(n-1) \cdot r + (n-1)! \, (n-r)}{(n-r)! \cdot r!}$$

$$= \frac{(n-1)! \, r + (n-1)! \cdot n - (n-1)! \, r}{r! \, (n-r)!}$$

$$= \frac{(n-1)! \, n}{r! \, (n-r)!}$$

$$= \frac{n!}{r! \, (n-r)!}$$ which is equal to ${}^{n}C_{r}$

[Proved]