A Generalized Multi-Objective Framework for UAV Mission Planning

Babak Salamat
Institute of Networked and Embedded Systems
Alpen-Adria-Universität Klagenfurt
9020, Austria
babaksa@edu.aau.at

Andrea M. Tonello
Institute of Networked and Embedded Systems
Alpen-Adria-Universität Klagenfurt
9020, Austria
andrea.tonello@aau.at

Abstract—**We present a novel framework for mission/trajectory planning of electrically powered UAVs by formulating a multi-objective multi-constraints optimization problem. The problem considers a number of objectives such as, flight time, harshness and aging under maximum available energy, peak power, maximum allowed time and initial-final target state. In particular, we show that the power demand is related to the angular velocity, as well as we introduce the concept of harshness (inverse of trajectory smoothness), and aging of the mechanical structure induced by the use or by environmental effects. The multi-objective optimization problem (MOP) is solved by introducing a new decomposition approach where a number of scalar optimization sub-problems are jointly solved using an evolutionary algorithm. Numerical results are reported to both verify performance and mission feasibility as well as to determine computation time which is a key element to be considered for real time applications.**

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1. INTRODUCTION

Electrically powered autonomous aerial vehicles are gaining high attention for several diverse applications. In particular, quadrotor helicopters offer mechanical simplicity and maneuverability. To increase performance beyond what is offered today, advanced mission/trajectory planning techniques should be developed. An important aspect about mission/trajectory planning is the description and characterization of the quadrotor helicopter dynamics including internal and external physical parameters (torque, velocity, acceleration, vibration, center of gravity, e.g.). A large effort has been dedicated to implement mission planning with acceptable computational complexity. There are three main approaches. In the first approach, some waypoints are located in the search space by the user [1], or they are generated by a stochastic approach [2, 3].

In the second approach, the waypoint path (geometric path) is converted into time parameterized trajectories [4, 5]. A remarkable study includes [6], in which the authors present the LQG-Obstacle algorithm for collision-free mission. However, the proposed algorithm is not fast enough for real-time use due to the high computational complexity. The approach in [3, 5] uses the flatness-based property of a quadrotor, to convert predefined waypoints into polynomial trajectories using quadratic programming. In [7, 8] the trajectory planning is extended as a constrained optimization problem and the parametrization of the trajectory is modeled as a composition of a parametric function $P(\lambda)$ defining the path and a monotonically increasing function $\lambda(t)$ specifying the motion on this path. $P(\lambda)$ and $\lambda(t)$ are modeled using B-spline functions. The tuning parameters of $P(\lambda)$ and $\lambda(t)$ are obtained using the sequential quadratic programming technique. In [9], the authors exploit the Bézier polynomial function to solve the constrained optimization problem.

The third approach eliminates the concept of waypoints and uses a virtual corridor to represent position constraints of a quadrotor helicopter [10, 11]. The algorithm in [12] generates a safe corridor in real-time by using the standard A* algorithm and then tries to fit trajectories that stay within the corridor.

As described above, most of the existing work focuses on path planning with simple objectives, e.g., collision avoidance. In this paper, we formulate a more general framework for mission planning by introducing a multi-objective optimization problem (MOP). We follow a bottom-up approach that considers a number of crucial objectives such as energy consumption, flight time, harshness of the mission and aging under maximum available energy, peak power, maximum allowed time and start-stop target location. The solution to the MOP is found through an efficient method to be implemented in real time applications. This is rendered possible by introducing a new decomposition approach where a number of scalar optimization sub-problems are jointly solved using an evolutionary algorithm. Feasible trajectories are found in the search space by firstly setting some way points. A simple expression of a given trajectory is obtained via parametrization exploiting a machine learning approach, and in particular non-linear support vector regression (SVR) [13, 14]. As Fig. 1 exemplifies, our approach eliminates the concept of polynomial functions and uses SVR to parameterize the trajectory. This reduces the complexity in the search of trajectories in the multi-objective optimization problem.

This paper is organized as follows. The framework formulation, in particular, the definition of the objectives is given in Section II. A new decomposition formulation where a number of scalar optimization are jointly solved using an evolutionary algorithm is given in Section III. The next section presents the trajectory generation methodology. Simulation results are given in Section V to verify performance and path feasibility as well as to determine computation time. Finally, the

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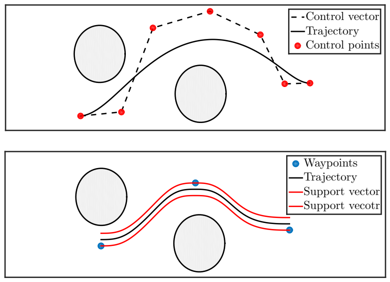

Figure 1. Comparison between our previous approach that use smooth B-spline trajectory as in [2] (top) and our new approach that uses SVR (bottom).

conclusion is given in Section VI.

2. Framework formulation

We herein propose a general framework for generating trajectories to be followed by a quadrotor helicopter mission. We first report the fundamental objectives and constraints. Then, we formulate a general multi-objective multi-constraints optimization problem.

Objectives

The quadrotor helicopter is commanded to fly from an initial to a final state. The mission/trajectory planning deals with obtaining the trajectory as a function of time $\boldsymbol{F}(t) = \big(x(t), y(t), z(t), \phi(t), \theta(t), \psi(t)\big)$, for $\boldsymbol{t} \in [t_0 \quad t_f]$ and its time derivatives. $\boldsymbol{F}(t)$ comprises the position $\boldsymbol{\xi} = [x,\, y,\, z]^T$ and the attitude $\boldsymbol{\Theta} = [\phi,\, \theta,\, \psi]^T$ of the quadrotor helicopter. In general, the trajectory evolution vector (TEV) is defined by a set of variables in the joint space and they are $\boldsymbol{\Xi} = \big[\boldsymbol{F}(t)^T,\, \dot{\boldsymbol{F}}(t)^T,\, ...,\, \boldsymbol{F}^{(n-1)}(t)^T\big]^T$. Let $\boldsymbol{\Xi} \in \boldsymbol{\chi} \subset \mathbb{R}^{6n}$ be a system state, consisting of the position, attitude and its derivatives (velocity, acceleration, jerk, jounce, etc.). Therefore, the trajectory is the path and its evolution over time containing all the derivatives up to order $n-1$ required to follow the path, from the mission start to the mission end. Let $\boldsymbol{\chi}^{obs-free} \in \boldsymbol{\chi}$ represent the free region of the search space including the obstacle-free positions $\boldsymbol{P}_{obs}^{free}$ and constraints $\boldsymbol{C}_d$ related to the system's dynamics, i.e., maximum and minimum value of the time derivatives of $\boldsymbol{F}(t)$.

Remark 1: Note that $\boldsymbol{P}_{obs}^{free}$ is bounded by the size of the search space (map).

Therefore, we can define

$$\boldsymbol{\chi}^{obs-free} := \boldsymbol{P}_{obs}^{free} \times \boldsymbol{C}_d, \tag{1}$$

and the obstacle region is defined as

$$\boldsymbol{\chi}^{obs} := \boldsymbol{\chi} \setminus \boldsymbol{\chi}^{obs-free}. \tag{2}$$

Now, we can proceed by defining the objectives and constraints in our generalized framework.

Minimum length—The first objective in mission/trajectory planning is determining the minimum path length to guide the quadrotor helicopter from an initial to a final state. Mathematically, this can be expressed as a minimization problem of the function

$$f_1 = \int_{t_0}^{t_f} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt, \tag{3}$$

where, t_0 and t_f are the initial and final time instant of the flight, respectively.

Minimum harshness—We are interested in finding state trajectories that are smooth (minimum harshness), and that can respect the constraints on the dynamics of the quadrotor helicopter. Therefore, the second objective that minimizes harshness can be written as:

$$f_2 = \int_{t_0}^{t_f} \|\frac{d^3\boldsymbol{F}(t)}{dt^3}\| dt, \tag{4}$$

where, $\|.\|$ is the norm two.

Minimum time—In the mission/trajectory planning, we may want to find the minimum flight time to drive the system from an initial to a final state. Therefore, this third objective targets the minimization of

$$f_3 = t_f - t_0. \tag{5}$$

The decision variable is the final time t_f.

Minimum energy—A fourth objective is the minimization of the energy consumption. The energy consumption depends on many factors, but it can be shown that it is heavily related to the torque demand of the four rotors, herein denoted with the vector $\boldsymbol{\tau} = [\tau_1, ..., \tau_4]$. Therefore, the cost function in our optimization problem can be defined as

$$f_4 = \int_{t_0}^{t_f} \big(\boldsymbol{\tau}^T \boldsymbol{R} \boldsymbol{\tau}\big) dt, \tag{6}$$

where $\boldsymbol{R} \in \mathbb{R}^{4\times 4}$ is the control weighting matrix. $\boldsymbol{R}$ is chosen as

$$\boldsymbol{R} = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix}. \tag{7}$$

The diagonal elements of $\boldsymbol{R}$ can be used to penalize the torque signals. Generally, by choosing large values we force the system to consume less energy. It is finally possible to write the mapping from motor torques to the control-inputs at the quadrotor helicopter as follows

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} +\frac{b}{dm} & +\frac{b}{dm} & +\frac{b}{dm} & +\frac{b}{dm} \\ 0 & -\frac{lb}{dI_x} & 0 & +\frac{lb}{dI_x} \\ -\frac{lb}{dI_y} & 0 & +\frac{lb}{dI_y} & 0 \\ +\frac{1}{I_z} & -\frac{1}{I_z} & +\frac{1}{I_z} & -\frac{1}{I_z} \end{pmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix}, \tag{8}$$

where m is the mass and b and d are coefficients related to the Mach number, the Reynolds number and the angle of attack. l is the lever length. $\boldsymbol{\tau} = [\tau_1\ \tau_2\ \tau_3\ \tau_4]^T \in \mathbb{R}^4$ is the torque vector generated by four rotors. It can be shown (see Section V relations (27)) that the dynamic system fulfills the flatness property [15]. Therefore, it is possible to express all quantities of the trajectory planning as a function of the time evolution of only four parameters $\boldsymbol{F}(t) = (x(t), y(t), z(t), \phi(t))$ and their time derivatives.

Minimum aging—Another aspect to be considered is aging to be intended as the increase of the obsolescence of the body structure and components which include mechanical actuators, engines, electronics, wiring etc. Modeling aging is very much dependent on the specific quadrotor helicopter (machine) [16]. In general, we can state that it is determined by the exposure to weather conditions, aggressiveness of the mission/trajectory, and time of flight. Since flight safety depends on aging, it is of great importance to include minimization of aging into the general optimization problem by defining it as a function of the trajectory as follows

$$f_5 = f\big(\boldsymbol{F}(t), \dot{\boldsymbol{F}}(t), \ddot{\boldsymbol{F}}(t), t_f\big). \tag{9}$$

Constraints

In the search of the optimal trajectory, we have to consider a number of constraints. The most relevant ones are defined below.

Obstacle avoidance constraint—Our objective is to find a feasible path $\boldsymbol{\chi}^{obs-free}$ from a start point $\boldsymbol{\xi}_0 = [x_0, y_0, z_0]^T$ to a final destination point $\boldsymbol{\xi}_f = [x_f, y_f, z_f]^T$, avoiding collision with obstacles. The distance from the path to each obstacle is defined by

$$d^i_{obs} = \sqrt{(x - r^i_{obs,x})^2 + (y - r^i_{obs,y})^2 + (z - r^i_{obs,z})^2}, \tag{10}$$

where, $\boldsymbol{P}_{obs} = [r^i_{obs,x}, r^i_{obs,y}, r^i_{obs,z}]^T$ is the obstacle position and the superscript i refers to the number of obstacles in the environment. To ensure a collision-free trajectory, we define a violation function to be verified for each obstacle as follows:

$$v^i_{obs} = \max\left(\frac{d^i_{obs}}{r^i_{obs}} - 1, 0\right). \tag{11}$$

Velocity constraints—During the flight mission, the velocity $\dot{\boldsymbol{\xi}} = [\dot{x}, \dot{y}, \dot{z}]^T$ of the quadrotor helicopter should remain between a lower and upper bound. This constraint can be defined through a violation function in the following form:

$$v_{\dot{x}} = \max\left(\frac{\dot{x}}{|\dot{x}_{limit}|} - 1, 0\right), \tag{12}$$

$$v_{\dot{y}} = \max\left(\frac{\dot{y}}{|\dot{y}_{limit}|} - 1, 0\right), \tag{13}$$

$$v_{\dot{z}} = \max\left(\frac{\dot{z}}{|\dot{z}_{limit}|} - 1, 0\right), \tag{14}$$

Boundary conditions—The boundary conditions related to the mission are those imposed on the trajectory:

$$\boldsymbol{F}(t_0) = \boldsymbol{F}_{init}, \tag{15}$$

$$\boldsymbol{F}(t_f) = \boldsymbol{F}_{fin}. \tag{16}$$

By taking (10)-(15) into account, we can define a violation function becomes

$$\begin{aligned} < v_t > &= \omega_1 < v^i_{obs} > + \omega_2 < v_{\dot{x}} > \\ &+ \omega_3 < v_{\dot{y}} > + \omega_4 < v_{\dot{z}} >, \end{aligned} \tag{17}$$

where $< . >$ is the mean operator and ω_i are weighting coefficients.

The violation function (17) will be used in the relaxed optimization problem in the next section.

3. General multi-objective problem

From the discussion in the previous section, it follows that the general MOP can be stated as

$$\begin{aligned} minimize \quad & z(\Xi) = \big(f_1(\Xi), ..., f_5(\Xi)\big)^T \\ subject\ to \quad & \Xi \in \boldsymbol{\chi} \end{aligned} \tag{18}$$

where, Ξ is the decision (variable) space, $z : \Xi \rightarrow \mathbb{R}^5$ consists of five real-valued objective functions. The attainable objective set is defined as the set $\{z(\Xi) \mid \Xi \in \boldsymbol{\chi}\}$. For notational convenience, we define the vector $z(\Xi) = \big(z_1(\Xi), ..., z_5(\Xi)\big)^T := \big(f_1(\Xi), ..., f_5(\Xi)\big)^T$.

Relaxing the constraints and interpreting them as a total violation function of (18), we can formulate a related MOP with $z_1(\Xi) := f_1(\Xi)(1+\beta < v_t >)$, where β is the Lagrange multiplier.

Decomposition

There are several classical methods for solving the multi-objective optimization based on decomposition [17]. In the following, we introduce a new decomposition, which is used in our paper.

New Decomposition

Another approach for solving the multi-objective optimization based on decomposition is the worst-case design [18]. This is a reasonable approach since it grants robustness in critical applications such as UAV navigation. In this paper, we approach the problem of $\bar{m}$ scalar optimization subproblems by using the worst-case approach [18] and by introducing the following objective of the kth subproblem:

$$minimize \quad g^{new}(\Xi \mid \boldsymbol{\lambda}^K, o^\star) = \alpha\Phi_1 + (1-\alpha)\Phi_2, \tag{19}$$

with

$$\Phi_1 = \max_i \ \{\lambda^K_i \, |f_i(\Xi) - o^\star_i|\} \tag{20}$$

$$\Phi_2 = \sum_i^n \lambda^K_i \, |f_i(\Xi) - o^\star_i|^2, \tag{21}$$

where $\boldsymbol{\lambda}^K = \{\lambda^K_1, ..., \lambda^K_n\}^T$ and $\alpha \in [0,1]$ is the policy coefficient which allows a compromise between Φ_1 and Φ_2.

Basically, we propose to solve the MOP (19) by combining an evolutionary algorithm to evolve possible trajectory candidates and non-linear support vector regression (SVR) to obtain a specific trajectory given a number of waypoints $\{(t_1, f_1), ..., (t_N, f_N)\} \subset \mathbb{R} \times \mathbb{R}$ in the search space. More in detail each one of the waypoints is defined with coordinates in the search space, which are $(x_w(t), y_w(t), z_w(t), \phi_w(t))$ and each of them is generated stochastically as a uniform random variable ranging between a lower and an upper limit that are distributed over time in the mission. This configuration enables a stochastic behaviour for $\boldsymbol{F}(t)$. Then, we generate the best trajectory according to the non-linear SVR (27). Finally, we keep evolving the trajectories by generating waypoints through the particle swarm optimization (PSO) technique such that the fitness function of the PSO in (19) is minimized to yield the global best of the particles [19].

Remark 2: The steps of the PSO (the update of the velocity and position) is done according to [2].

For what matters the application of non-linear SVR for the generation of a continuous trajectory $\boldsymbol{F}(t) = (x(t), y(t), z(t), \phi(t))$ we apply non-linear SVR as described in the next section.

4. Trajectory Generation

Our objective is to develop a machine learning approach to generate the trajectory evolution $\boldsymbol{F}(t) = (x(t), y(t), z(t), \phi(t))$ in particular a non-linear support vector regression (SVR) [13, 14] is used. In more detail (Fig. 1), the trajectory that is defined by non-linear SVR is of class $\mathcal{C}^{\infty}$ and it is defined by N waypoints $\{(t_1, f_1), ..., (t_N, f_N)\}$. In our model, the first and the last waypoint are fixed and they correspond to the start and the end of the mission, respectively. In the search space between these two waypoints, additional waypoints are placed in order to generate the segments of the non-linear SVR trajectory.

Non-linear Support Vector Regression (SVR)

We now apply non-linear SVR to each component f_t of $\boldsymbol{F}(t)$, where f_t can be either x, y, z or ϕ. We set a number N of waypoints for each f_t with respect to the time t in the search space $\{(t_1, f_1), ..., (t_N, f_N)\}$ as explained in the previous section. According to Fig.1, suppose that we want to find the trajectory that passes among the N waypoints. Mathematically, this can be expressed as a convex minimization problem of a cost function as follows

$$
\begin{aligned}
minimize \quad & \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i^+ - \alpha_i^-)(\alpha_j^+ - \alpha_j^-)k(t_i, t_j) \\
& - \sum_{i=1}^{N}(\alpha_i^+ - \alpha_i^-)f_i + \epsilon\sum_{i=1}^{N}(\alpha_i^+ + \alpha_i^-)
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
subject\ to \quad & \\
& \sum_{i=1}^{N}(\alpha_i^+ - \alpha_i^-) = 0 \\
& \forall i : 0 \le \alpha_i^+ \le C \\
& \forall i : 0 \le \alpha_i^- \le C,
\end{aligned}
$$

where

$$
k(t_i, t_j) = \exp\left(-\frac{1}{2\sigma^2}\|t_i - t_j\|^2 \right), \tag{23}
$$

and α_i^+, α_i^- are Lagrange multipliers. The constant C is the box constraint, a positive numeric value that controls the penalty imposed on observations that lie outside the epsilon margin ϵ. It helps to prevent over-fitting (regularization). Solving the optimization problem (22) gives us $\{\alpha_i^+, \alpha_i^-\}$. We can construct the so-called support vector as follows:

$$
\boldsymbol{S} = \left\{ i \ \in [1, N] \ | \ 0 \le \alpha_i^+ + \alpha_i^- \le C \right\}. \tag{24}
$$

Once we get α_i^+, α_i^-, we can calculate the bias term $b(t)$ as

$$
\begin{aligned}
b(t) = \frac{1}{|\boldsymbol{S}|}\sum_{i\in\boldsymbol{S}}\bigg(& f_i - \sum_{i\in\boldsymbol{S}}(\alpha_i^+ - \alpha_i^-)k(t_i, t) \\
& -sgn(\alpha_i^+ - \alpha_i^-)\epsilon \bigg) \ i \in \boldsymbol{S},
\end{aligned} \tag{25}
$$

where $\mathrm{sgn}(.)$ is the sign function.

Remark 3: In (25) we calculate the mean of the bias.

Finally, the model (function) to be used to estimate new values is given by

$$
f(t) = \sum_{i=1}^{N}(\alpha_i^+ - \alpha_i^-)k(t_i, t) + b(t), \tag{26}
$$

where (26) guarantees that when $t = t_i$, we have $f(t) = f_i \ \forall \ i \in 1, ..., N$.

5. Simulation Results

In this section, we report simulation results to show the effectiveness and capability of the proposed approach. Finally, we compare the performance of the method in comparison with the previous work [2] where trajectories were generated using B-splines. We consider the model of a quadrotor helicopter described in [2] whose dynamics equations read as follows:

$$
\begin{aligned}
\ddot{x} &= u_{1x}, \\
\ddot{y} &= u_{1y}, \\
\ddot{z} &= u_1(\cos(\theta)\cos(\psi)) - g, \\
\ddot{\psi} &= u_2 + \dot{\phi}\dot{\theta}a_1, \\
\ddot{\theta} &= u_3 + \dot{\phi}\dot{\psi}a_2, \\
\ddot{\phi} &= u_4 + \dot{\psi}\dot{\theta}a_3,
\end{aligned} \tag{27}
$$

where $a_1 = \left(\frac{I_y - I_z}{I_x}\right)$, $a_2 = \left(\frac{I_z - I_x}{I_y}\right)$, $a_3 = \left(\frac{I_x - I_y}{I_z}\right)$ and $u_{1x} = u_1(\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi))$ and $u_{2x} = u_1(\sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi))$. $\boldsymbol{\xi} = [x, y, z]^T$ is the position of the quadrotor helicopter center of gravity in the inertial frame and $\boldsymbol{\Theta} = [\phi, \theta, \psi]^T$ is the attitude (roll, pitch and yaw). I_i $(i = x, y, z)$ are the moments of inertia along the x, y and z directions. u_1 is the total force generated by the four rotors and directly related to the altitude in the z direction. u_2, u_3 and u_4 are related to the yaw, pitch, and roll motion respectively. The quantities (u_1, u_2, u_3, u_4) are the control inputs of our dynamical system. A number of parameters is initially set as shown in Table 1. The maximum torque τ_{max} is assumed equal to $0.05Nm$. In general, the start point, end point and a number of waypoints can be defined by the user. The environment is occupied by different obstacles as shown in Fig. 2. The position of the obstacles can be located randomly or fixed by the user. The motion is defined to a fixed altitude $z = 2m$. The number of waypoints for the non-linear SVR is $N = 3$. The maximum velocity is $2m/s$. The mission/trajectory planning is performed using the model (27) and the multi-objective optimization problem (18). The objective is to drive the quadrotor helicopter from an initial to a final state while not violating constraints. The initial state is $\boldsymbol{F}(t_0) = (0, 0, 2, 0)$ m. The final state is $\boldsymbol{F}(t_f) = (6, 6, 2, 0)$ m. In this case the

Table 1. Setup parameters of the quadrotor helicopter

Parameters	Symbol	Value	Unit
Mass	m_q	0.65	Kg
Inertia around x axis	I_{xx}	0.07582	$Kg \bullet m^2$
Inertia around y axis	I_{yy}	0.07582	$Kg \bullet m^2$
Inertia around z axis	I_{zz}	0.1457924	$Kg \bullet m^2$
Gravitational acceleration	g	9.806	m/s^2
Drag coefficient	d	7.5×10^{-7}	$Nm \bullet s^2$
Trust coefficient	b	3.13×10^{-5}	$N \bullet s^2$
Lever arm	l	0.232	m

Table 2. Comparison results

Parameters	Previous work [2]	PSO+SVR
Length (m)	11.48325	**9.4748**
Violation	0.00145	**0**
Minimum harshness	**0.3318**	0.8003
Minimum energy	0.0184	**0.0147**

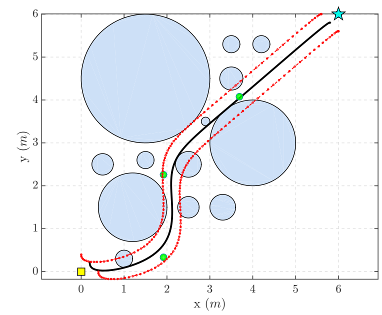

Figure 2. **2D trajectory of the quadrotor helicopter.**

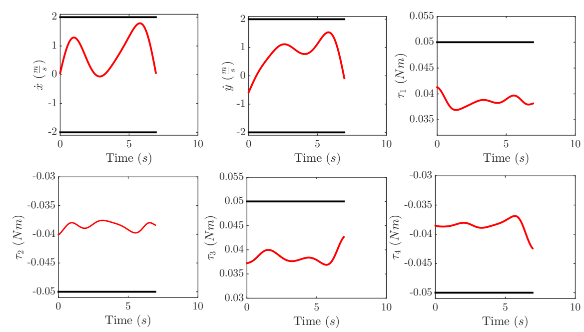

Figure 3. **Velocities and torques obtained during the flight mission.**

minimum transfer time found in simulation is $t_f = 6.5038s$. Fig. 2 shows the two-dimension trajectory of the quadrotor helicopter. Blue and green circles show the obstacles and waypoints, respectively, and the two red line-points show the support vectors. The velocities and torques obtained during the simulation flight are in the feasible domain (see Fig. 3). Black lines show the upper and lower bound. As it can be seen, our approach nicely find a feasible path in the search space. As already mentioned before, to illustrate the effectiveness of the proposed method, we consider our previous method [2] according to which we can generate trajectories using B-splines. However, that method is here used to solve the full MOP problem (18). The comparison results are summarized in Table 2. One can see that both approaches find a solution. However method [2] does not bring to a null violation. Furthermore, the new method finds a shorter path and lower energy. Finally, as shown in Fig. 4 the new approach has much faster convergence.

6. Conclusion

A new framework for mission/trajectory planning of electrically powered UAVs is considered. The framework considers a number of important objectives such as energy consumption, flight time, harshness and aging under constraints. New decomposition for the multi-objective optimization problem is used to obtain the feasible trajectory. No extra waypoints are required, which is a significant advantage of this methodology. Using artificial intelligence in the trajectory planning increases the performance of the mission planning. Future work will also consider the application of the proposed approach to the multiple UAVs scenario.

Appendix

Herein, we show how to solve the optimization problems (22) with quadratic programming (QP). Let us define $k(t_i, t_j) = H = [h_{ij}] \in \mathbb{R}^{n \times n}$. Therefore, the optimization problem (22) can be written in the following form:

$$\begin{aligned} minimize \quad & \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i^+ \alpha_j^+ h_{ij} - \\ & \alpha_i^- \alpha_j^+ h_{ij} - \alpha_i^+ \alpha_j^- h_{ij} + \alpha_i^- \alpha_j^- h_{ij} \\ & - \sum_{i=1}^{N} (\alpha_i^+ - \alpha_i^-) f_i + \epsilon \sum_{i=1}^{N} (\alpha_i^+ + \alpha_i^-) \end{aligned} \tag{28}$$

Now (28) can be written as:

$$\begin{aligned} minimize \quad & \boldsymbol{\alpha}^T \boldsymbol{\mathcal{H}} \boldsymbol{\alpha} + \boldsymbol{G}^T \boldsymbol{\alpha} \\ subject\ to & \\ & \sum_{i=1}^{N} (\alpha_i^+ - \alpha_i^-) = 0 \\ & \forall i : 0 \leq \alpha_i^+ \leq C \\ & \forall i : 0 \leq \alpha_i^- \leq C, \end{aligned} \tag{29}$$

where

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} H & -H \\ -H & H \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \alpha^+ \\ \alpha^- \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} -f_i + \epsilon \\ \dots \\ f_i - \epsilon \end{bmatrix}. \tag{30}$$

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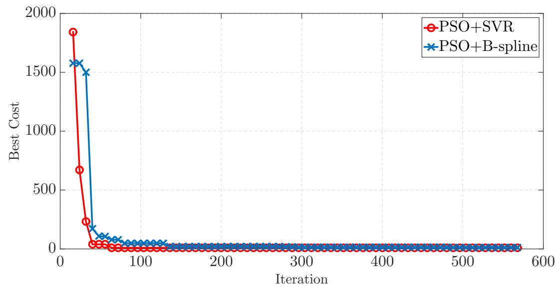

Figure 4. Cost function vs. iteration for both approaches: PSO with SVR based trajectory generation and PSO with B-spline based trajectory generation.

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BIOGRAPHY

Babak Salamat *received the B.S. degree in mechanical engineering and the M.S. degrees in aerospace engineering from the Air-force University of Shahid Sattari, Tehran-Iran, in 2012 and 2014, respectively. He is currently a Ph.D. student in information technology at the University of Klagenfurt, Austria. He received the Aerospace Journal Best Paper Award in 2017. His current research activities include navigation systems, path planning and robust control for autonomes aerial vehicles.*

Andrea M. Tonello *received the D.Eng. degree (Hons.) in electronics and the D.Res. degree in electronics and telecommunications from the University of Padova, Italy, in 1996 and 2002, respectively. From 1997 to 2002, he was with Bell Labs-Lucent Technologies, Whippany, NJ, USA, as a Member of the Technical Staff. Then, he was promoted to Technical Manager and appointed to Managing Director of the Bell Labs Italy division. In 2003, he joined the University of Udine, Udine, Italy, where he became an Aggregate Professor in 2005 and an Associate Professor in 2014. He is currently the Chair of the Embedded Communication Systems Group at University of Klagenfurt, Klagenfurt, Austria. He received several awards, including the Distinguished Visiting Fellowship from the Royal Academy of Engineering, U.K., in 2010, the IEEE VTS and COMSOC Distinguished Lecturer Awards in 2011 and 2018, and 9 best paper awards. He was/is an Associate Editor of the IEEE Trans. on Vehicular Technology, IEEE Trans. on Communications, IEEE Access, and IET Smart Grid.*