

1 Introduction

2 Literature Review

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This chapter presents a systematic, two-stage methodology for sizing and validating a rotary-actuated Stewart platform, starting with a one-degree-of-freedom analogue and ending with an optimized six-degree-of-freedom motion simulator.

In the first stage, we isolate the most demanding motion from the dataset, vertical translation along the z -axis, using a single-axis slider-crank model

We derive its forward and inverse kinematics to verify that the required workspace can be reached, then formulate the inverse dynamics to assess whether off-the-shelf servomotors can meet the dynamic performance requirements.

This analysis serves two purposes: it identifies which servos are capable of the task, and it establishes whether advancing to a full Stewart-platform architecture is justified.

In the second stage, we develop the complete Stewart-platform model. We derive its inverse kinematics to determine the geometric parameters required to span the entire 6-DOF workspace. Building on that, we formulate the inverse dynamics and optimize the platform for maximum dynamic performance.

3.1 Operational Envelope:

From the processed torso-motion data-set, we identified the key performance targets that the platform must satisfy. Table below summarizes the required translation, acceleration, and rotational limits in each axis along with fundamental associated frequencies.

Feature	Axis	Value	Fundamental Frequency (Hz)
Translation	Z	± 4 cm	3
Rotation	Z (transverse)	$\pm 15.5^\circ$	1.5
Rotation	Y (sagittal)	$\pm 2.5^\circ$	3
Rotation	X (frontal)	$\pm 3.0^\circ$	1.5
Linear Acceleration	Z	$-1g$ to $+6g$	3

Table 1: Torso Motion Dataset

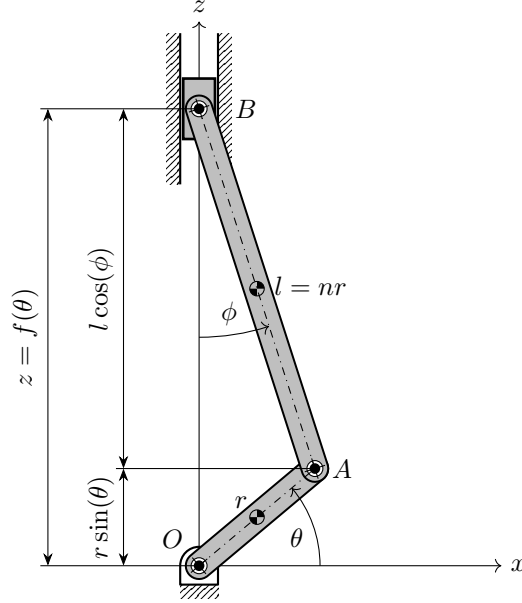


Figure 1: Slider-crank mechanism

From the table above, we use the translation and rotation features to define the kinematic workspace requirements and the linear acceleration feature for determining the dynamic requirements.

3.2 1-DOF Analysis(Pick a good name):

A slider-crank mechanism is a four-link system composed of three revolute joints and one prismatic joint. The kinematics and dynamics of a Stewart platform operating strictly along the vertical z -axis can be approximated using 6 identical slider-crank mechanisms, each contributing equally to the motion and load distribution(proof in appendix).

3.2.1 Forward Kinematics:

We consider a vertical slider-crank mechanism in which a crank of length r drives a slider B via a connecting rod of length l . The vertical displacement of the slider, z , can be written as:

$$z = r \sin(\theta) + l \cos(\phi) \quad (1)$$

Here, θ is the angle between the crank arm and the horizontal x -axis, and ϕ is the angle between the connecting rod and the vertical z -axis.

Applying the sine rule to $\triangle OAB$, we obtain :

$$\frac{r}{\sin(\phi)} = \frac{l}{\sin(\frac{\pi}{2} - \theta)} \quad (2)$$

We define a ratio $n = l/r$ for convenience of calculation. substituting $l = nr$ and solving for ϕ and we get-

$$\phi = \arcsin\left(\frac{\cos \theta}{n}\right) \quad (3)$$

So the displacement equation becomes -

$$z = f(\theta) = r \sin(\theta) + nr \cos(\sin^{-1}(\frac{\cos \theta}{n})) \quad (4)$$

Equation(4) gives the forward kinematics , the vertical position z of the slider as a function of crank angle θ .

3.2.2 Inverse Kinematics:

The inverse kinematics problem involves determining the crank angle θ required to achieve a given vertical slider position z . In $\triangle OAB$ applying cosine rule, we obtain :

$$\angle AOB = \frac{\pi}{2} - \theta = \cos^{-1}\left(\frac{h^2 + r^2 - l^2}{2hr}\right) \quad (5)$$

substituting $l = nr$ and solving for θ and we obtain -

$$\theta = \sin^{-1}\left(\frac{h^2 + r^2(1 - n^2)}{2hr}\right) \quad (6)$$

This expression allows us to compute the crank angle θ given the slider displacement h .

3.2.3 Inverse Dynamics:

The dynamics of the system are derived using Lagrangian mechanics. We consider the slider has a mass m and the inertia of motor driving the crank is J . At this stage of the analysis we assume the crank arm and the connecting rod has no mass. In the θ as the generalized co-ordinate system.

The total kinetic energy of the system can be written as:

$$\begin{aligned} T &= T_{\text{motor}} + T_{\text{slider}} \\ &= \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}m\dot{z}^2 \\ &= \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}m\left(\frac{dz}{d\theta}\right)^2\dot{\theta}^2 \\ &= \frac{1}{2}\left(J + m\left(\frac{dz}{d\theta}\right)^2\right)\dot{\theta}^2 \end{aligned} \quad (7)$$

The potential energy of the system :

$$V = mgz = mgr(\sin(\theta) + n \cos(\sin^{-1}(\frac{\cos \theta}{n}))) \quad (8)$$

Hence the Lagrangian of the system is :

$$\mathcal{L}(\theta, \dot{\theta}) = \frac{1}{2}\left(J + m\left(\frac{dz}{d\theta}\right)^2\right)\dot{\theta}^2 - mgz(\theta) \quad (9)$$

For the generalized co-ordinate system q , the Euler-Lagrange equations are:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = Q \quad (10)$$

Where, Q represents the generalized force. For our system, $q = \theta$, and thus required torque becomes $Q = \tau$.

Substituting we get:

$$\begin{aligned} \tau &= \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} \\ &= \frac{d}{dt} \left[(J + m(z'(\theta))^2) \dot{\theta} \right] - \left[\frac{1}{2} (2m z'(\theta) z''(\theta)) \dot{\theta}^2 - mg z'(\theta) \right] \\ &= (J + m(z'(\theta))^2) \ddot{\theta} + 2m z'(\theta) z''(\theta) \dot{\theta}^2 - (m z'(\theta) z''(\theta) \dot{\theta}^2 - mg z'(\theta)) \\ &= (J + m(z'(\theta))^2) \ddot{\theta} + m z'(\theta) z''(\theta) \dot{\theta}^2 + mg z'(\theta). \end{aligned} \quad (11)$$

Here,

$$\begin{aligned} z'(\theta) &= r \cos \theta + nr \left[-\sin(\sin^{-1}(\frac{\cos \theta}{n})) \frac{d}{d\theta} \sin^{-1}(\frac{\cos \theta}{n}) \right] = r \cos \theta + \frac{r \cos \theta \sin \theta}{\sqrt{n^2 - \cos^2 \theta}}, \\ z''(\theta) &= -r \sin \theta + r \left(\frac{\cos 2\theta}{\sqrt{n^2 - \cos^2 \theta}} - \frac{\cos^2 \theta \sin^2 \theta}{(n^2 - \cos^2 \theta)^{3/2}} \right). \end{aligned} \quad (12)$$