Topic 4 - Advanced diversification methods

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***Abstract*—**

I. Introduction

The concept of diversification is a key principle in modern portfolio theory, and is often used by investors as a means of reducing risk and increasing returns. In the financial world, there are two main approaches to diversification: the most diversified portfolio and the equally risk-weighted portfolio. The most diversified portfolio is characterized by a wide range of assets, with the aim of maximizing diversification and minimizing risk. On the other hand, the equally risk-weighted portfolio is designed to allocate equal amounts of risk to each asset in the portfolio, with the goal of achieving a more balanced risk profile. In this research paper, we will compare these two approaches to diversification, examining the pros and cons of each and exploring the potential benefits and drawbacks of each approach. Through an in-depth analysis of the available literature and data, we will aim to determine the most effective approach to diversification for investors seeking to maximize returns while minimizing risk.
We will start this paper by providing a breakdown of the methods we used, then section 3 aims at presenting the data we used to conduct this project and analyse the periods of stress our portfolios might go through. In section 4, we will present the results of our research and how they can help us answer our big question of which portfolio is the most effective for investors who seek a strong return to volatility ratio. And finally, section 5 will conclude our project.

II. Methods

The capital asset pricing model (CAPM) has faced significant criticism, particularly regarding the efficiency of market capitalization weighted indices. As a result, various alternatives to these indices have been proposed by academics and practitioners. Putting theory in practice, Haugen and Baken introduced the minimum-variance portfolio in 1972.
Many literature work aim at presenting the many alternatives and comparing them. One of the best alternatives that emerged is the risk parity. A central assumption of risk-based strategies like risk parity is that all asset classes have the same expected risk-adjusted returns. If this assumption is true, then the optimal portfolio would be the one that maximizes diversification. However, the question remains whether equalizing risk contribution leads to maximum diversification.
Therefore, our goal might be to measure diversification to see which portfolio present the best returns. There has been significant progress in recent years on refining methods for more accurately measuring portfolio diversification. Previous research has largely focused on the development of equity portfolios designed to be more diversified, with the aim of identifying alternatives to traditional, capitalization-weighted benchmarks. Choueifaty (2006) introduced the concept of maximum diversification through the use of a metric called the diversification ratio (DR). He also described the most diversified portfolio (MDP) as an efficient alternative to the market capitalization weighted index, which is characterized by its ability to maximize the DR.
A different definition of the most diversified portfolio (MDP), known as the "core property," offers a clear understanding of its characteristics. According to this definition, any stock not included in the MDP is more correlated to the MDP than any of the stocks that are part of it. Additionally, all stocks belonging to the MDP have the same correlation to it. This property is the most significant characteristic of the MDP and demonstrates that all assets in the universe under consideration are effectively represented in the MDP, even if the portfolio does not physically hold them.
The parity family can be deconstructed in three different parts. The first one is volatility parity, where the portfolio equalizes the weighted volatilities of the assets. But, the two sections of parity strategies that we will focus on are the Risk parity, with the equal risk contribution portfolio, and the correlation parity, with the most diversified portfolio. Risk parity is another approach in the family of diversification strategies. It equalizes the risk contribution of each asset, taking into account the correlation structure of the assets. If an asset has a much lower average correlation than the other assets, risk parity will allocate more to that asset than volatility parity would. However, it is worth noting that risk parity only achieves maximum diversification when all pairwise correlations are equal, and does not fully utilize the differential correlation structure to maximize diversification. The most diversified portfolio (MDP) can be considered as a form of correlation parity strategy. If the covariance matrix is known perfectly (must be definite), correlation parity will produce the most diversified portfolio possible for a given set of asset classes. When all Sharpe ratios are equal, this will also be the optimal portfolio solution. Correlation parity involves equalizing the correlation-weighted risk contribution of each asset in the portfolio. When the asset correlation matrix is multiplied by the risk contributions, it results in a vector of equal values. Within the framework of risk contributions, correlation

parity takes correlations into account twice, while risk parity considers them once and volatility parity does not consider them at all.

These portfolios are related to cap-weighted indexes, unlevered, long-only and provide comparable access to the equity risk premium. In all the previous research, we see that the most diversified portfolio produces the best sharpe ratio and it yields that the MDP is the best candidate for tangency portfolio.

We will thus conduct the research in two parts, first focusing on in-sample data, but we know that in-sample are not the most realistic simulations of actual investing, we will then reduce the number of stocks in our portfolio and conduct an analysis out-of-sample.

III. Data

The data used for this research paper was sourced from MSCI Europe and covers the period from 1995-1-1 to 2021-12-1. MSCI Europe is a leading provider of investment indices and market intelligence, and the data used in this study was collected from Refinitiv Eikon database.

To ensure the reliability and validity of the data, several steps were taken to ensure its quality and accuracy. First, only data from reputable sources was used. Second, the data was carefully reviewed and checked for errors or inconsistencies. Finally, statistical tests were conducted to ensure the data was free from bias and accurately reflected the underlying trends and patterns.

The original data set was composed of the 427 companies that constituted the MSCI Europe index on the 15th of November 2022. I then went to retrieve the price of these stocks from the 1st of January 1994 to the the 31st of December 2021, because I wanted the data to reflect the 3 big periods of stress (dotcom bubble index, subprimes crisis and COVID 19 crash) the markets went though to see how our portfolios were affected by them. Then, some of the companies in my data set didn't have data at the beginning of the universe, so I decided to get rid of them, so that our final portfolios can really reflect these periods of stress. The final data set is hence, consituted of 223 companies that compose the MSCI Europe.

I also resampled the portfolio to have monthly returns . Using monthly return data instead of daily return data offers several benefits, one of which is that returns are generally more normally distributed on a monthly basis (or, at the very least, the assumption of normality is much less extreme). This is especially important because it allows for the use of simplifying assumptions that are not as unrealistic for monthly returns as they are for daily returns.

The benchmark we use is a price index of the MSCI Europe. It is important for me to use this index, so that it can give us the most realistic comparaison tool for our portfolios.

Then in the second part of the research, I chose to focus on 15 companies to conduct the out-of-sample analysis of the portfolios. These companies are: *"NESTLE 'N'"*, *"ROCHE HOLDINGS 'B'"*, *"LVMH"*, *"ASTRAZENECA"*, *"NOVO NORDISK 'B'"*, *"NOVARTIS 'R'"*, *"TOTALENERGIES"*, *"UNILEVER (UK)"*, *"HSBC HOLDINGS"*, *"L'OREAL"*, *"ALLIANZ (XET)"*, *"BMW (XET)"*, *"BNP PARIBAS"*, *"NOKIA"*, *"ZURICH INSURANCE GROUP"*, and the reason why I chose to go with them if that they are the companies that had the biggest market capitalization in the index. As covariance and correlation between stocks are the important metrics when trying to compute the diversification ratio of our portfolios, here are a plot of these metrics for our fifteen stocks. As we can see, these fifteen stocks exhibit a strong correlation between them, which is a very interesting propoerty for our most diversified portfolio.

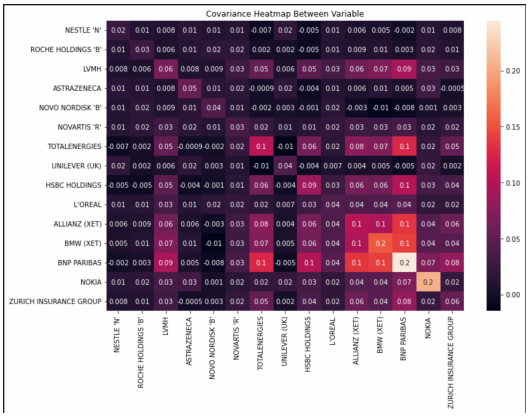

Fig. 1. Covariance heatmap between stocks

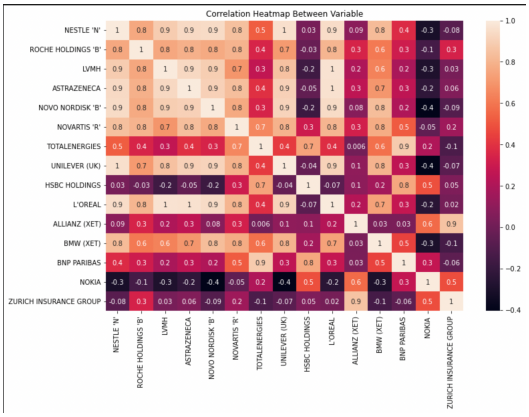

Fig. 2. Correlation heatmap between stocks

IV. Results

Let's first start by looking at the in-sample performance of our portfolios. The portfolio's empirical performances are represented in figure 3. Both portfolios strongly outperformed the benchmark which is consistent with our expectations that risk parity portfolio are a very good approach to maximizes our returns. The ERC and MDP portfolios deliver significantly higher returns, the equal risk contribution portfolio has a lower volatility that the benchmarks wheras the MDP has a slightly higher one which might be due to the fact that the maximum diversification portfolio includes less assets than the two others (The MDP includes 129 assets compared to

the ERC which includes 223). The MDP also appears to give stronger weights to the assets than the ERC portfolio (Figure 2)

Fig. 3. Comparaison of quatitative portfolio performances in-sample

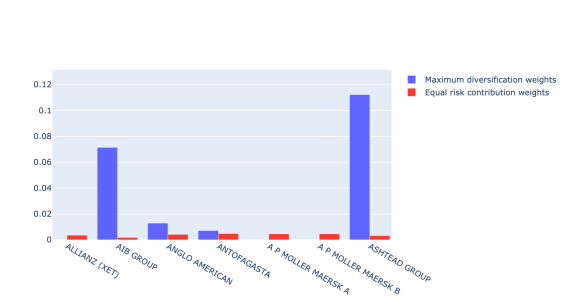

Fig. 4. Comparaison of weights in quantitative portfolios

	MDP	ERC	MSCI Europe
CAGR	11.85%	11.18%	NaN
Volatility	19.27%	15.80%	18.43%
Excess Return	9.37%	8.19%	2.39%
Sharpe Ratio	0.49	0.52	0.13
Tracking Error	12.84%	9.09%	0%
Information Ratio	0.55	0.64	NaN

The MDP presents the highest excess return and volatility which leads to its sharpe ratio being a little smaller that the sharpe ratio of the ERC portolio. As we can see, both portfolios exhibit a significant tracking error which shows that they don't follow the market. The information ratio is a measure that compares the active return of an investment to the amount of risk taken on. It indicates the additional return that an investor receives for each unit of risk they take on. As we can see, the ERC portfolio provides a higher active return in-sample, meaning that it is the best managed portfolio in-sample.

If we have a look at the periods of stress forecasted in figure 1, we can see that both portfolio suffered substanially less that the index during the dotcom bubble crisis. The biggest drawdown for both portfolios was recorded during the financial crisis of 2008, where the maximum diversification portfolio lost more than 60% of its value while the ERC portfolio lost more than 50%. During the covid 19 crisis, we saw both portfolios suffering a bit and losing around 20% of their values. But the interesting point to note is that both

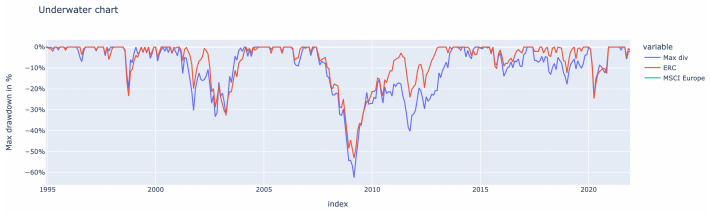

Fig. 5. Drawdowns of our portfolios

quantitative portfolios experience smaller drawdowns than the benchmark which shows that they also perform a little bit better in periods of stress.

Table 2 shows the results of a series of Fama French (1993) three-factor regressions for each portfolio construction methodology. The factors are labeled MKT for MSCI Europe index in excess of the LIBOR, HML for the performance difference between the MSCI Europe value and growth indexes, and SMB for the performance difference between the smallest 30% and the largest 30% stocks by market capitalization.

Portfolio	MKT	SMB	HML
MDP	0.1634	0.8066	0.2499
t-statistic	2.693	5.866	2.195
ERC	0.1365	0.6515	0.1739
t-statistic	2.720	5.728	1.846

As we can see, both portfolios show significant exposure to the market and small minus big factors, and all positively loaded on all three factors. The maximum diversification portfolio is a little bit more loaded on all factors than the equal risk contribution portfolio. It can be seen that the ERC portfolio performs a little bit better than the MDP portfolio in-sample, even though the MDP portfolio delivers the highest return.

Now, we can have a look at the performance of our portfolios out-of-sample to get a better understanding of how they would perform in a real world simulation. To do so, I used a 36 months rolling window to compute the covariance matrix of assets, than rolled it by one month each time and recomputed the weights for our portfolios thanks to an optimization.

But, using 223 assets, it was impossible to compute both optimizations as we had too much data. Thus, I decided to reduce the sample to fifteen assets, which I chose looking at their market capitalization, to see how working out-of-sample with these assets will affect the performance of our quantitative portfolios. The performance of the portfolios throughout the whole sample can be see in Figure 5, and as we can see, the maximum diversification portfolio substantially outperfomed both the ERC portfolio and the benchmark.

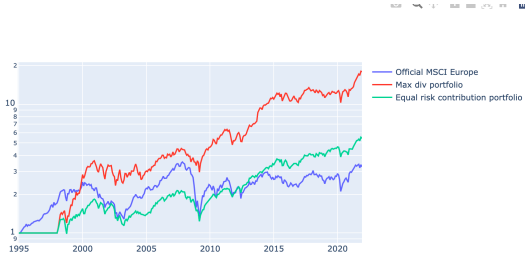

Fig. 6. Comparaison of quatitative portfolio performances in-sample

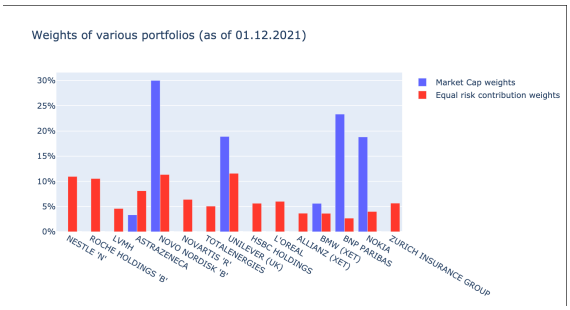

Fig. 7. Comparaison of weights in quantitative portfolios

	MDP	ERC	MSCI Europe
CAGR	11.28%	6.47%	4.48%
Volatility	18.49%	13.73%	18.43%
Excess Return	8.72%	3.51%	2.39%
Sharpe Ratio	0.47	0.26	0.13
Tracking Error	14.73%	12.21%	0%
Information Ratio	0.43	0.09	NaN

Similar to what happened in-sample, we can see in Figure 7 that the MDP allocates weights really differently from the ERC as its goal is the maximize the diversification ratio. On average, the MDP focuses on 9.6 assets each month throughout the whole sample whereas the ERC attributes weights to 13.3 assets each month. This time, the MDP strongly outperformed the ERC portfolio, having a much higher excess return, sharpe ratio and information ratio. One can note that the MDP and ERC performed less well than in-sample, but this is something that we were expecting, as in-sample simulation doesn't provide a very reliable estimation of the universe. In this approach, the MDP portfolio appears to be the best approach to risk diversification and the strongest candidate to be the tangency portfolio.

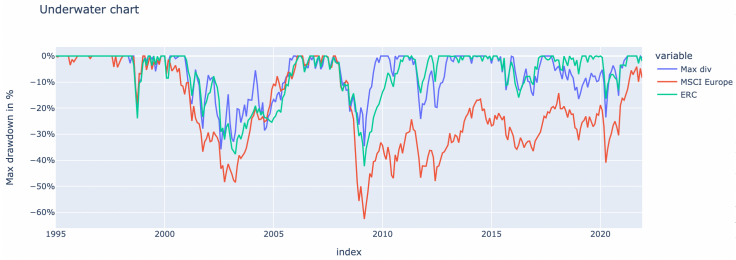

Fig. 8. Drawdowns of our portfolios

If we have a look at the drawdown series of our portfolios, we can also see that the MDP portfolio and the ERC portfolios suffered less in periods of stress compared to the benchmark, losing at most 30% of its value for the MDP and 40% for the ERC during the 2008 financial crisis.

Table 3 shows the result of the regression for each portfolio construction methodology on Fama-French 3 factors model.

Portfolio	MKT	SMB	HML
MDP	0.1513	0.5280	0.0338
t-statistic	2.514	3.872	0.229
ERC	0.0955	0.3656	0.1150
t-statistic	2.149	3.631	1.379

As we can see, both portfolios still show significant exposure on the MKT and SMB factors, even though it is smaller than in-sample, and are positively loaded on all three factors. The MDP delivers the highest alpha, indicating that the performance of the MDP is significantly higher that what its Fama-French factors exposure would predict. This aligns with the objective of the MDP to provide a balanced exposure to the effective risk factors in the universe, thereby maximizing diversification.

The MDP presents both the largest sharpe ratio and the largest diversification ratio. As such it is the strongest candidate to being the tangency portfolio. Figure 9 exhibits the Markowitz efficient frontier with 223 assets and shows the position of the MDP (black dot).

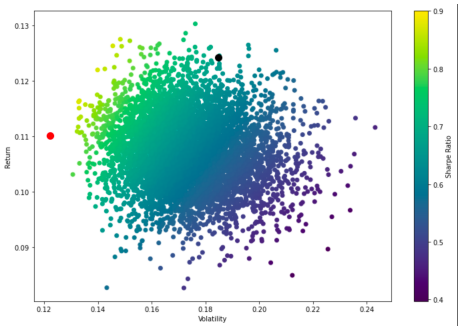

Fig. 9. Efficient frontier

V. Conclusion

In conclusion, this research paper aimed to compare the two main approaches to diversification in modern portfolio theory: the most diversified portfolio and the equally risk-weighted portfolio. Through an in-depth analysis of the available literature and data, we aimed to determine the most effective approach to diversification for investors seeking to maximize returns while minimizing risk. We found that both the most diversified portfolio and the equally risk-weighted portfolio can be effective approaches to diversification, but the MDP out-of-sample stands out both in terms of relative performance and exposure to Fama-French factors. The most diversified portfolio can be an efficient alternative to

traditional, capitalization-weighted benchmarks, and has the advantage of maximizing diversification. However, the equally risk-weighted portfolio may be more appropriate for investors seeking a more balanced risk profile, as it allocates equal amounts of risk to each asset in the portfolio. Ultimately, the choice between these two approaches to diversification will depend on the individual needs and preferences of the investor. The goal of the MDP is to deliver full benefit of the equity risk premium to investors, and I believe that this research forecasts that the maximum diversification portfolio is a string candidate for being the undiversifiable portfolio.

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