Design and Analysis of Algorithms

CSE 5311

Lecture 1 Administration & Introduction

Song Jiang, Ph.D.

Department of Computer Science and Engineering

Administration

- **Course CSE 5311**
  - What: Design and Analysis of Algorithms
  - When: M/W 1:00pm – 2:20pm
  - Where: WH 311
  - Who: Song Jiang (song.jiang@uta.edu)
  - Office Hour: Mon. & Wed. 10:30 ~ 11:30am at SIER319
  - or by appointments
  - TA: Mr. Zongyao Lyu, ERB413, office hour is Tu/Th 3:00pm-4:00pm
  - Homepage: http://ranger.uta.edu/~sjiang/CSE5311-fall-19/index.htm (Please visit this website regularly)

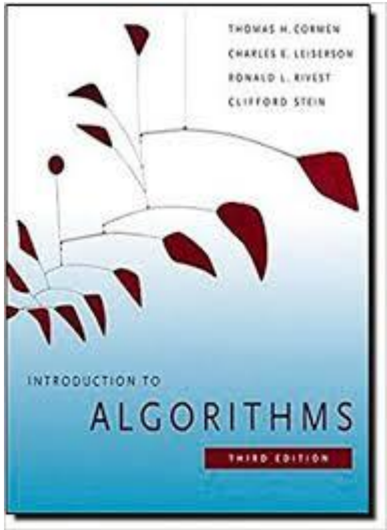
- **About your instructor**
  - Research areas: file and storage system, operating system, parallel and distributed computing, and high performance computing,

Study Materials

- **Prerequisites**
  - CSE 2320 Algorithms and Data Structures or its equivalents
  - Programming skills on a high-level language, such as C and Java.
  - Mathematical background on summations, sets, relations, probability, and matrix computation.

- **Textbook**

Thomas Cormen, Charles Leiserson, Ronald Rivest, and Clifford Stein. **Introduction to Algorithms**. 3rd ed. MIT Press, 2009.



Grading

- **Distribution**
  - 5% Class attendance
  - 30% Homework Assignments
  - 20% Quizzes
  - 20% Midterm Exam
  - 25% Final Exam

  100%

- Bonus credits may be offered for voluntarily and correctly answering in-class questions.
- **Attention**
  - Homework is as important as any other aspects of your grade.
  - Attendance is strongly encouraged.
  - The university makeup policy will be strictly adhered to. Generally, no make-up exams/quizzes except for university sanctioned reasons.
  - When missing an exam/quiz due to unavoidable circumstances, PLEASE notify the instructor and request a makeup approval **ahead of time**.

Grading

- **Late Assignments**

Late assignments will be accepted with a 20% penalty applied for each day late up to 2 days. Assignments submitted later than 2 days after the original due time will not be accepted.

- **Collaboration Policy**

Students are allowed and encouraged to collaborate on homework assignments. However, **You must write up each problem solution by yourself without assistance**, even if you collaborate with others to solve the problem. If you obtain a solution through research (e.g., on the Web), acknowledge your source, and write up the solution in your own words. **It is a violation of this policy to submit a problem solution that you cannot orally explain to the instructor or GTA.**

Final Grade

- **Final Letter Grade**
  - [90 100] --- A
  - [80 90) --- B
  - [70 80) --- C
  - [60 70) --- D
  - [00 60) --- F
- **Note**
  - [ ] denotes inclusion and ( ) denotes exclusion.
  - Your final weighted scores may be curved before assignment of your letter grade.

What's the Course About?

- **The theoretical study of analysis and design of computer algorithms**
  - **Analysis:** predict the cost of an algorithm in terms of resources and performance
  - **Design:** design algorithms which minimize the cost
- **Basic goals for an algorithm**
  - Always correct
  - Always terminates
- **Our class: performance**

Algorithms

- An ***algorithm*** is any well-defined computational procedure that takes some value, or set of values, as ***input*** and produces some value, or set of values, as ***output***.

- An example problem: sorting

  Input: A sequence of n numbers $\langle a_1, a_2, \ldots, a_n \rangle$.

  Output: A permutation (reordering) $\langle a'_1, a'_2, \ldots, a'_n \rangle$ of the input sequence such that $a'_1 \le a'_2 \le \cdots \le a'_n$.

  An instance of a problem: <31; 41; 59; 26; 41; 58>

- Design algorithms for a problem:
  - Find **a** longest common subsequence of $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$

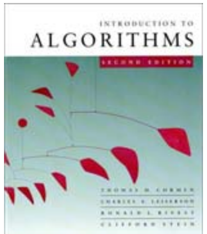
Why study algorithms and performance?

- Algorithms help us to understand ***scalability***.
- Performance often draws the line between what is feasible and what is impossible.
- Algorithmic mathematics provides a ***language*** for talking about program behavior.
- Performance is the ***currency*** of computing.
- The lessons of program performance generalize to other computing resources.
- Speed is fun!

Machine Model

- **Generic Random Access Machine (RAM)**
  - Executes operations sequentially
  - Set of primitive operations: Arithmetic. Logical, Comparisons, Function calls

- **Simplifying assumption**
  - All operations cost one unit
  - Eliminates dependence on the speed of our computer
  - Otherwise impossible to verify and to compare

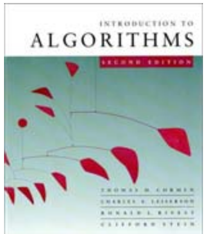


Analysis of algorithms

The theoretical study of computer-program performance and resource usage.

What's more important than performance?

- modularity
- correctness
- maintainability
- functionality
- robustness
- user-friendliness
- programmer time
- simplicity
- extensibility
- reliability



The problem of sorting

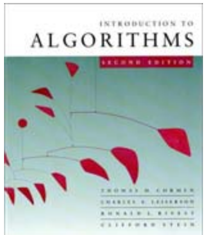
Input: sequence $\langle a_1, a_2, \ldots, a_n \rangle$ of numbers.

Output: permutation $\langle a'_1, a'_2, \ldots, a'_n \rangle$ such that $a'_1 \le a'_2 \le \cdots \le a'_n$.

Example:

Input: 8 2 4 9 3 6

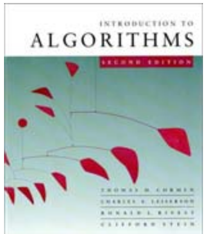
Output: 2 3 4 6 8 9



Insertion sort

"pseudocode"

```
INSERTION-SORT (A, n)        ▷ A[1 . . n]
    for j ← 2 to n
        do key ← A[ j]
           i ← j – 1
           while i > 0 and A[i] > key
               do A[i+1] ← A[i]
                  i ← i – 1
           A[i+1] = key
```



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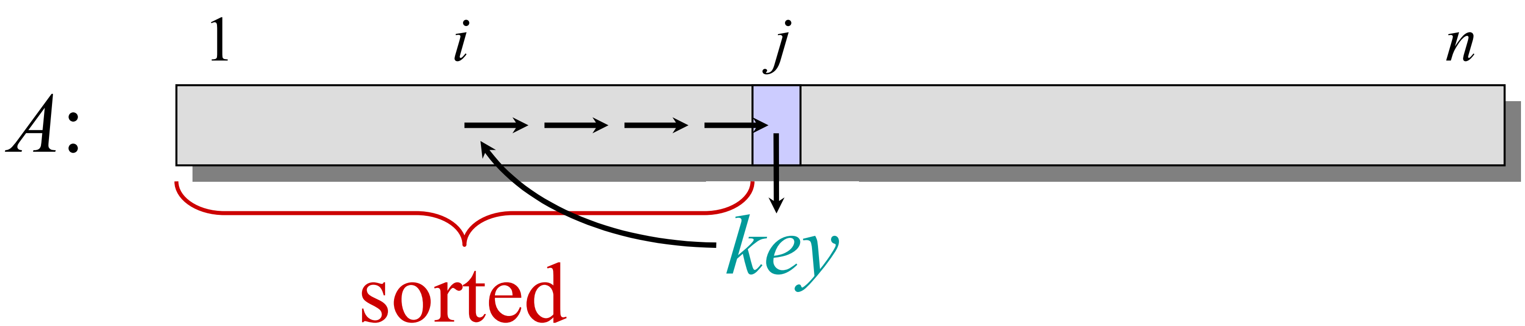
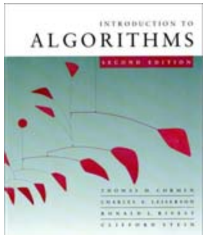




Example of insertion sort

8 2 4 9 3 6

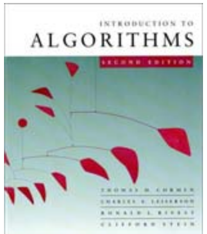
Example of insertion sort

8 2 4 9 3 6

Example of insertion sort

8 2 4 9 3 6

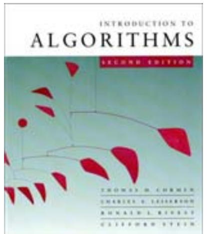
2 8 4 9 3 6



Example of insertion sort

8 **2** 4 9 3 6

2 8 **4** 9 3 6

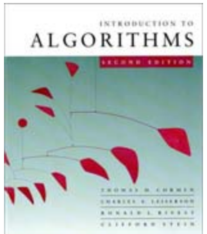


Example of insertion sort

8 2 4 9 3 6

2 8 4 9 3 6

2 4 8 9 3 6



Example of insertion sort

8 2 4 9 3 6

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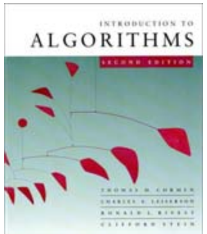

Example of insertion sort

8 2 4 9 3 6

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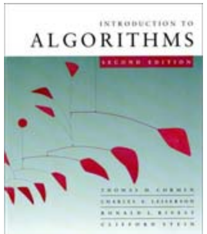
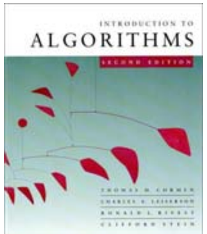


Example of insertion sort

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6



Example of insertion sort

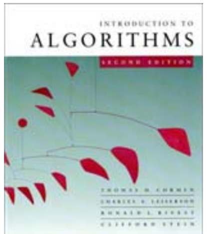
8 **2** 4 9 3 6

2 8 **4** 9 3 6

2 4 8 **9** 3 6

2 4 8 9 **3** 6

2 3 4 8 9 6



Example of insertion sort

8 2 4 9 3 6

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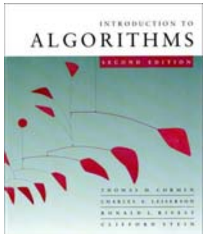
2 3 4 8 9 6



Example of insertion sort

8 2 4 9 3 6

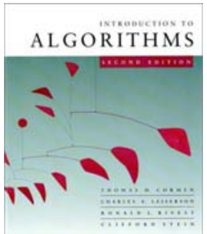
2 8 4 9 3 6

2 4 8 9 3 6

2 4 8 9 3 6

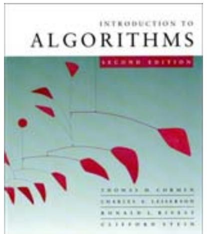
2 3 4 8 9 6

2 3 4 6 8 9 *done*



Running time

- The running time depends on the input: an already sorted sequence is easier to sort.
- Parameterize the running time by the size of the input, since short sequences are easier to sort than long ones.
- Generally, we seek upper bounds on the running time, because everybody likes a guarantee.



Kinds of analyses

Worst-case: (usually)

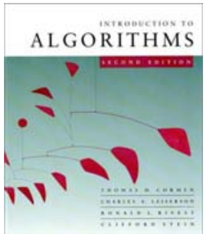
- $T(n)$ = maximum time of algorithm on any input of size n.

Average-case: (sometimes)

- $T(n)$ = expected time of algorithm over all inputs of size n.
- Need assumption of statistical distribution of inputs.

Best-case: (bogus)

- Cheat with a slow algorithm that works fast on *some* input.



Machine-independent time

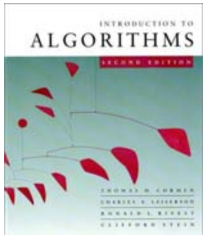
What is insertion sort's worst-case time?

- It depends on the speed of our computer:
  - relative speed (on the same machine),
  - absolute speed (on different machines).

BIG IDEA:

- Ignore machine-dependent constants.
- Look at ***growth*** of $T(n)$ as $n \rightarrow \infty$.

"Asymptotic Analysis"



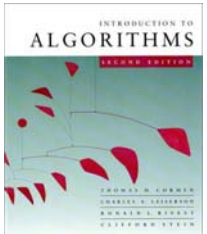
Θ-notation

Math:

$\Theta(g(n)) = \{ f(n) :$ there exist positive constants c_1, c_2, and n_0 such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0 \}$

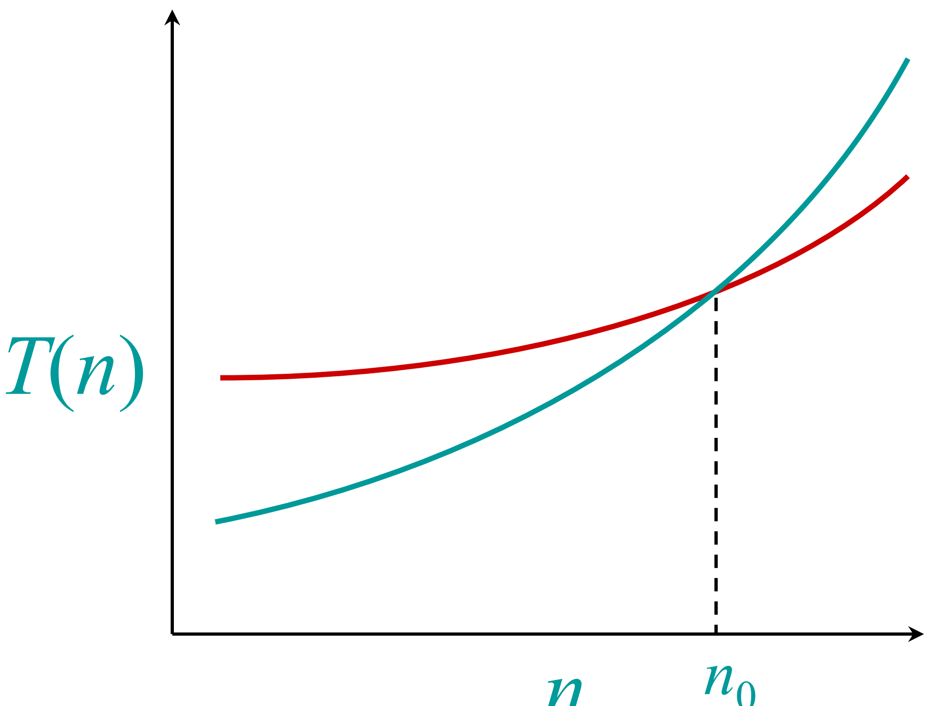
Engineering:

- Drop low-order terms; ignore leading constants.
- Example: $3n^3 + 90n^2 - 5n + 6046 = \Theta(n^3)$

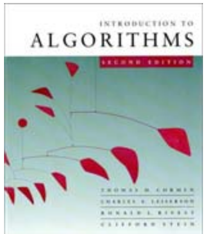


Asymptotic performance

When n gets large enough, a $\Theta(n^2)$ algorithm *always* beats a $\Theta(n^3)$ algorithm.



- We shouldn't ignore asymptotically slower algorithms, however.
- Real-world design situations often call for a careful balancing of engineering objectives.
- Asymptotic analysis is a useful tool to help to structure our thinking.



Insertion sort analysis

Worst case: Input reverse sorted.

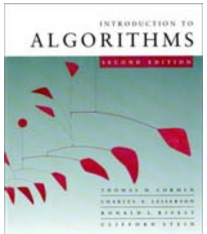
$$T(n) = \sum_{j=2}^{n} \Theta(j) = \Theta(n^2) \qquad \text{[arithmetic series]}$$

Average case: All permutations equally likely.

$$T(n) = \sum_{j=2}^{n} \Theta(j/2) = \Theta(n^2)$$

Is insertion sort a fast sorting algorithm?

- Moderately so, for small n.
- Not at all, for large n.

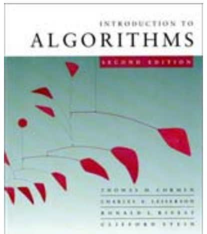


Merge sort

MERGE-SORT $A[1 \,.\,.\, n]$

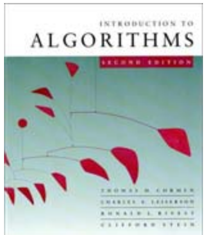
1. If $n = 1$, done.
2. Recursively sort $A[\,1 \,.\,.\, \lceil n/2 \rceil\,]$ and $A[\,\lceil n/2 \rceil + 1 \,.\,.\, n\,]$.
3. "***Merge***" the 2 sorted lists.

Key subroutine: **MERGE**



Merging two sorted arrays

20	12
13	11
7	9
2	1



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20	12
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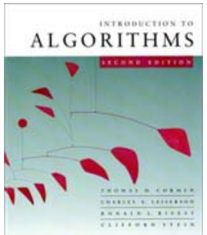
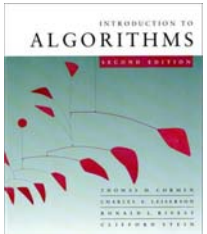


Merging two sorted arrays

20	12	20	12
13	11	13	11
7	9	7	9
2	1	2	

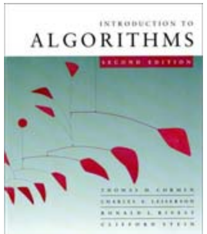
1



Merging two sorted arrays

20	12	20	12
13	11	13	11
7	9	7	9
2	1	2	

1 2

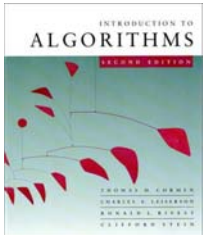


Merging two sorted arrays

20	12	20	12	20	12
13	11	13	11	13	11
7	9	7	9	7	9
2	1	2			
1		2			

Merging two sorted arrays

20	12	20	12	20	12
13	11	13	11	13	11
7	9	7	9	7	9
2	1	2			
1		2		7	



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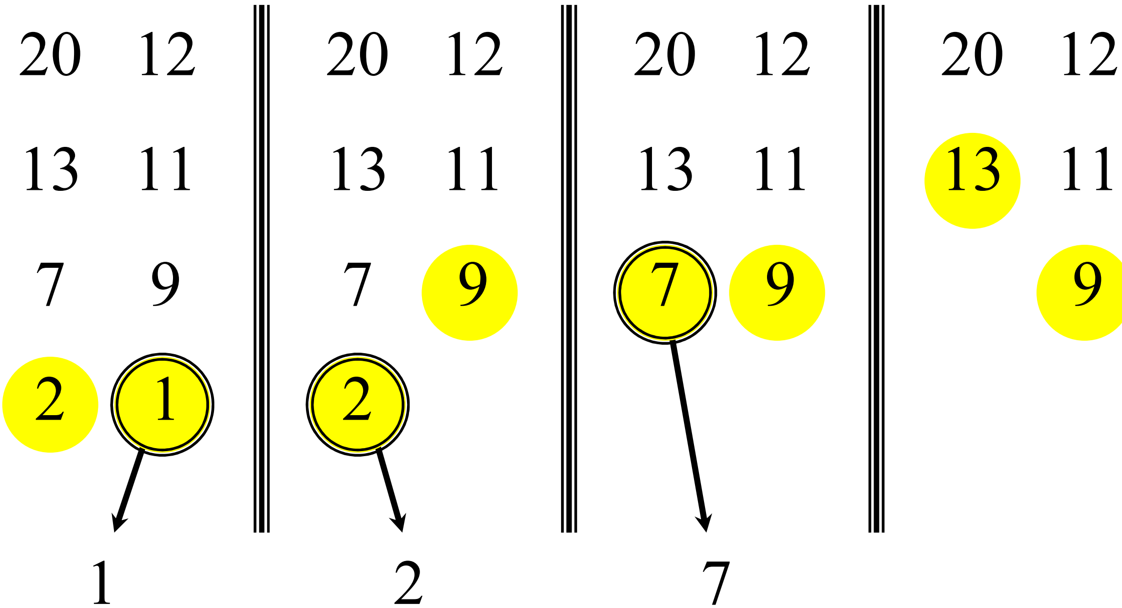

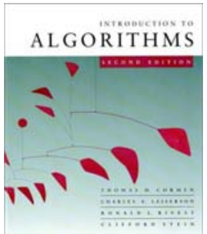

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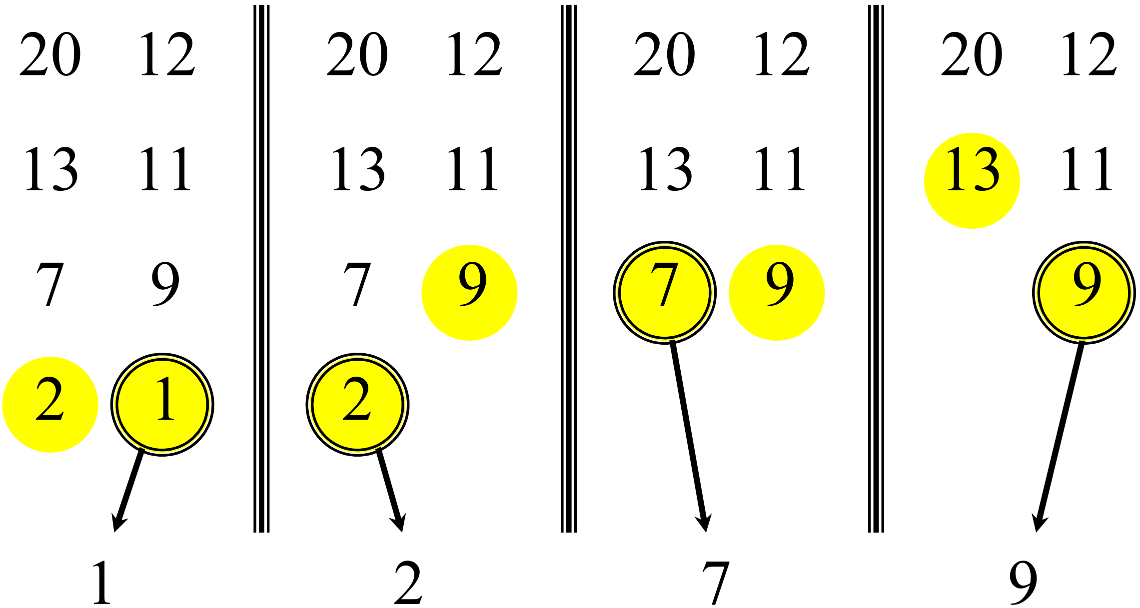

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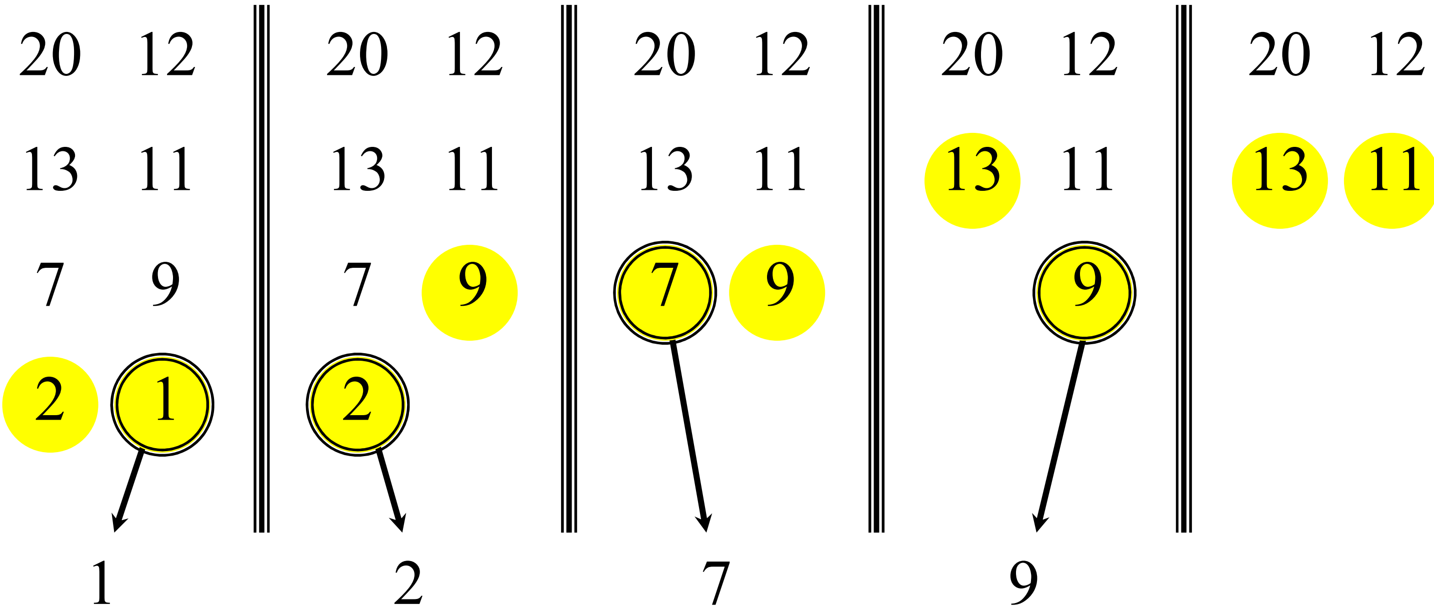
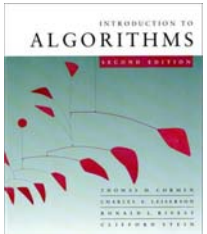
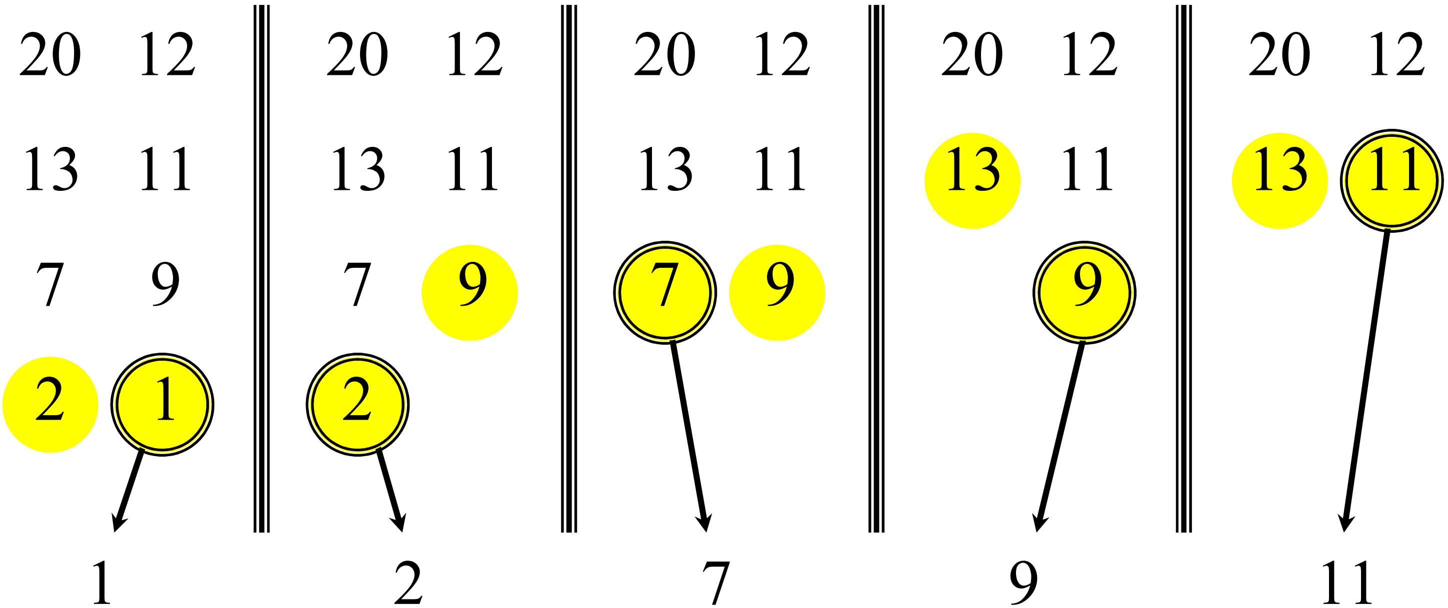
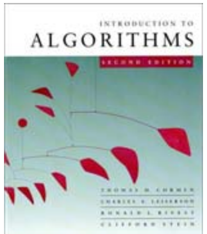


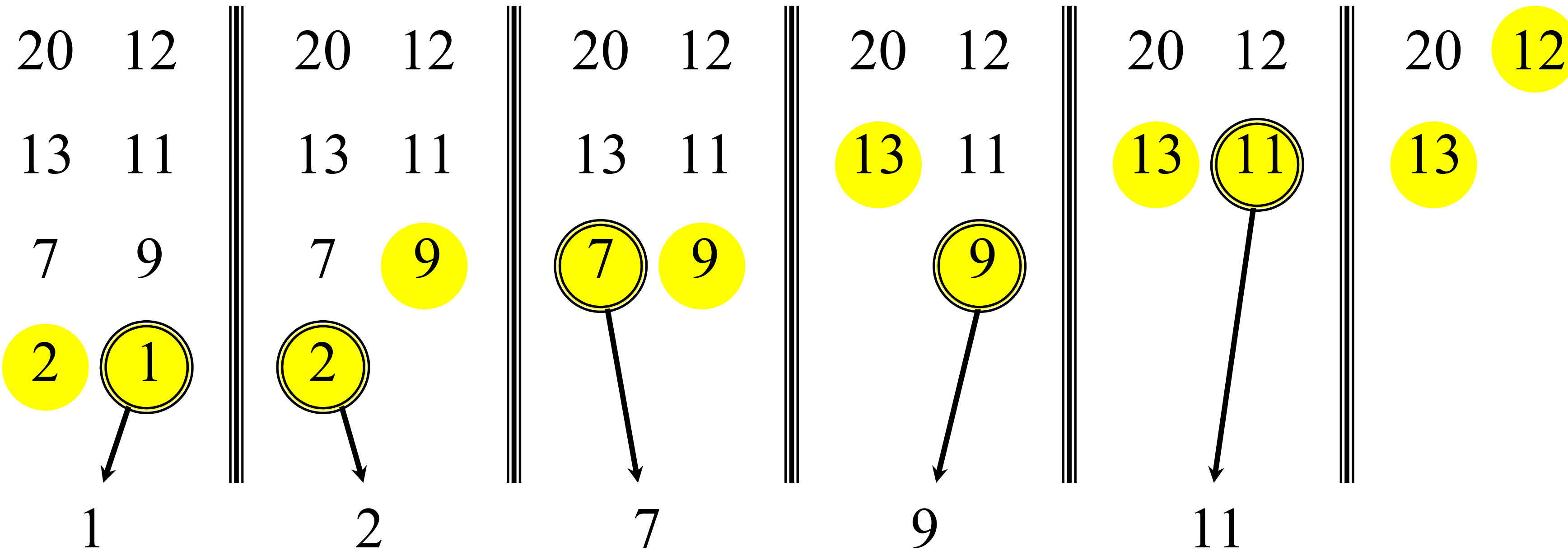


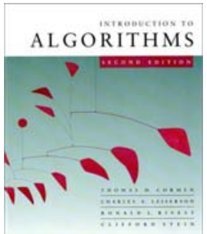
Merging two sorted arrays





Merging two sorted arrays





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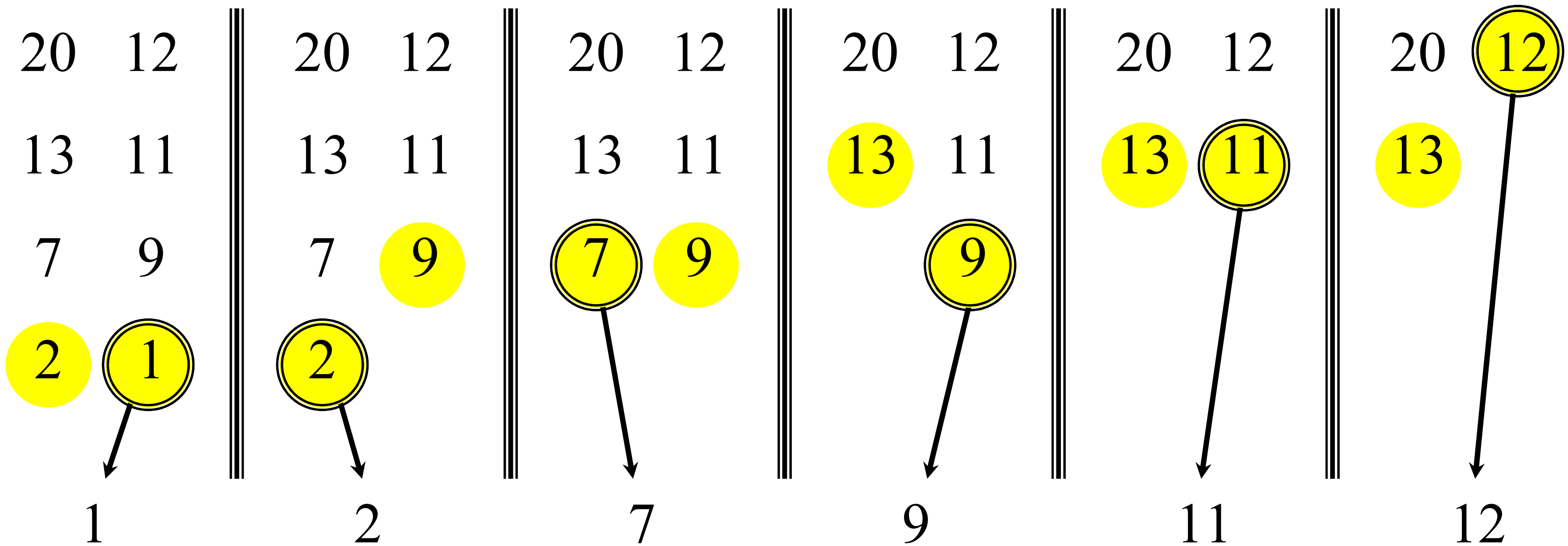
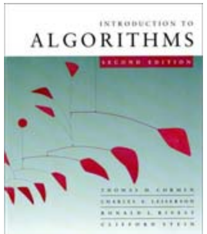
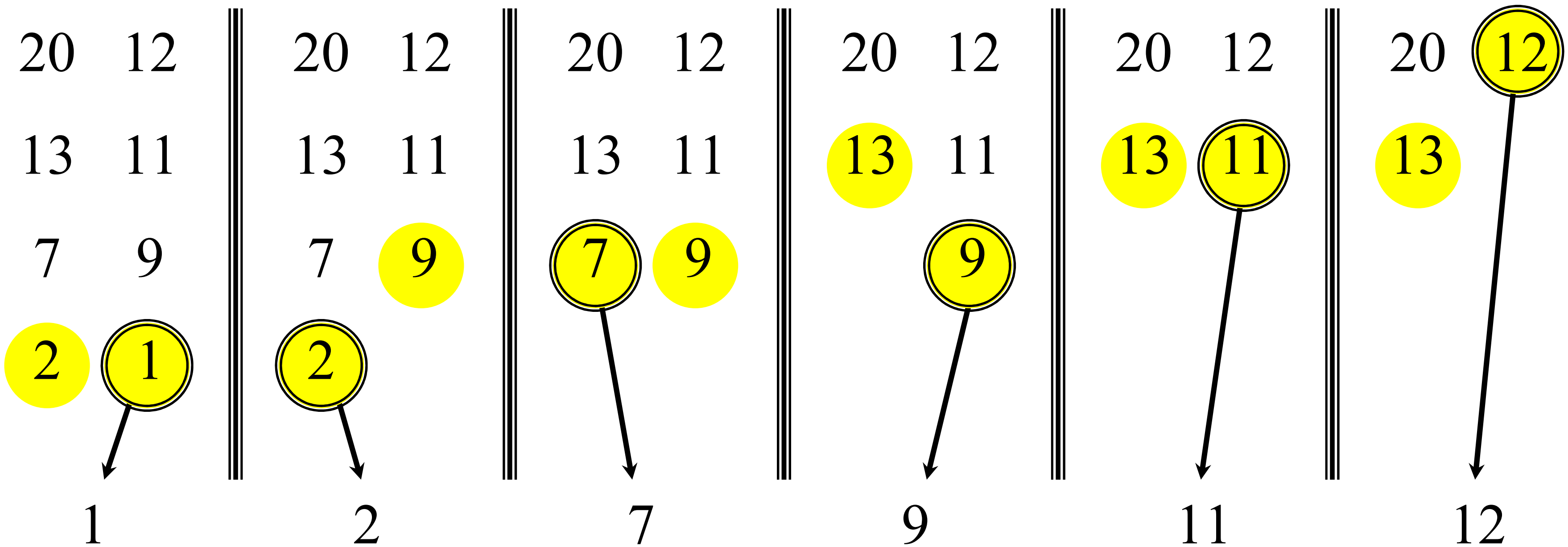
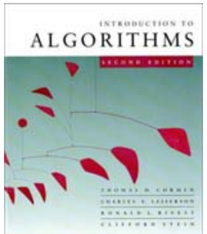




Merging two sorted arrays



Time = $\Theta(n)$ to merge a total of n elements (linear time).

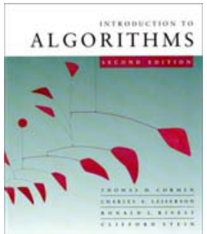


Analyzing merge sort

$T(n)$	MERGE-SORT $A[1\,..\,n]$
$\Theta(1)$	1. If $n = 1$, done.
$2T(n/2)$	2. Recursively sort $A[\,1\,..\,\lceil n/2\rceil\,]$ and $A[\,\lceil n/2\rceil+1\,..\,n\,]$.
$\Theta(n)$	***3. “Merge”*** the 2 sorted lists

Abuse

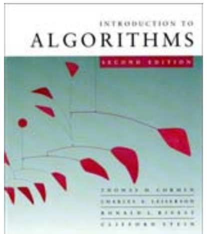
Sloppiness: Should be $T(\lceil n/2\rceil) + T(\lfloor n/2\rfloor)$, but it turns out not to matter asymptotically.



Recurrence for merge sort

$$T(n) = \begin{cases} \Theta(1) \text{ if } n = 1; \\ 2T(n/2) + \Theta(n) \text{ if } n > 1. \end{cases}$$

- We shall usually omit stating the base case when $T(n) = \Theta(1)$ for sufficiently small n, but only when it has no effect on the asymptotic solution to the recurrence.
- CLRS and Lecture 2 provide several ways to find a good upper bound on $T(n)$.



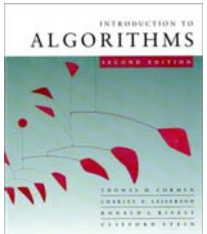
Recursion tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.

Recursion tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.

$$T(n)$$

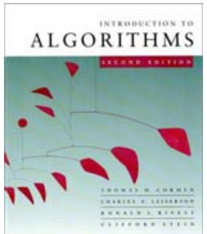


Recursion tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.

$$cn$$

$$T(n/2) \qquad\qquad T(n/2)$$



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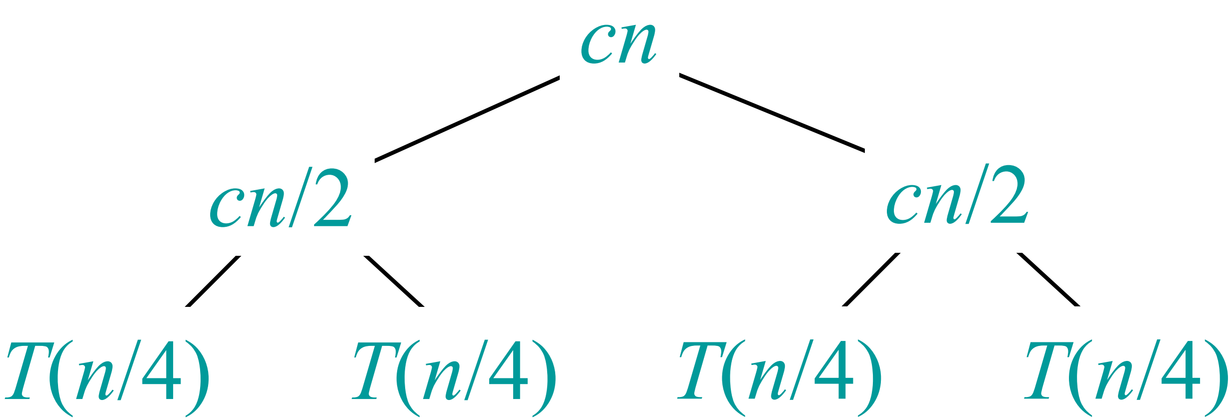

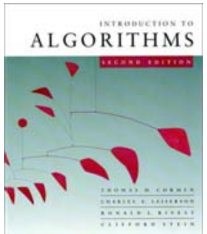

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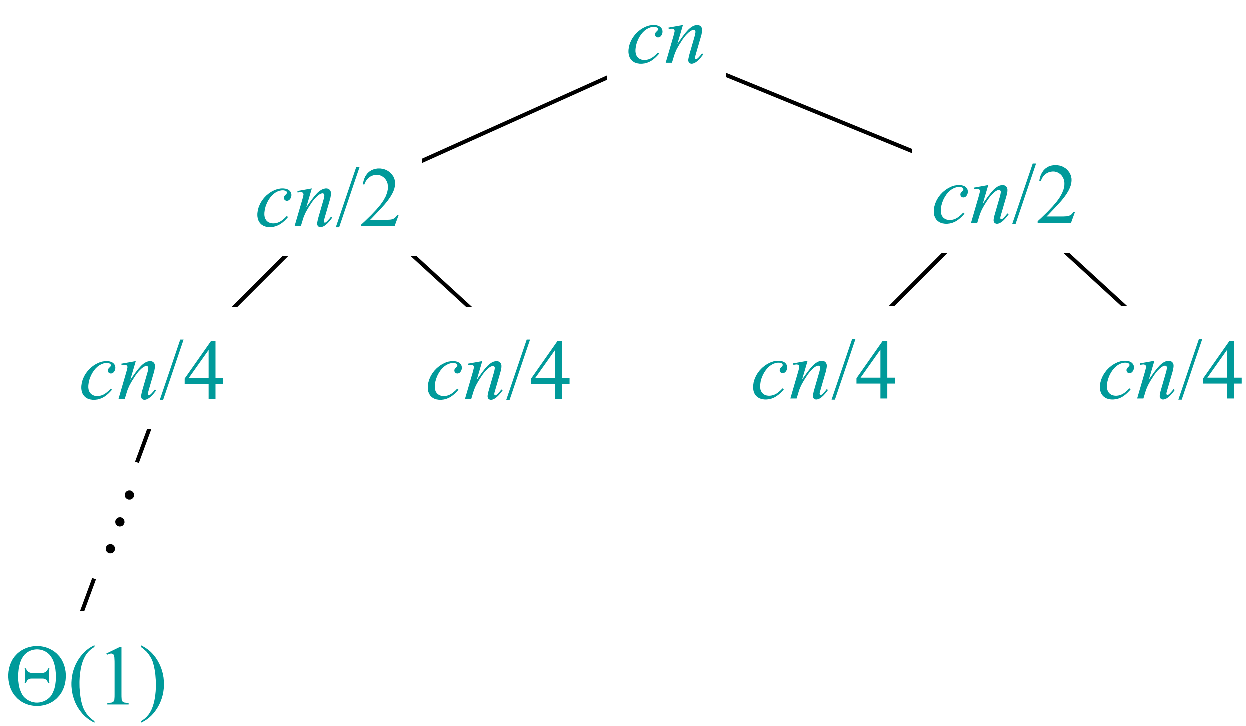

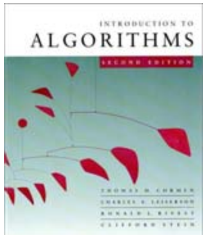

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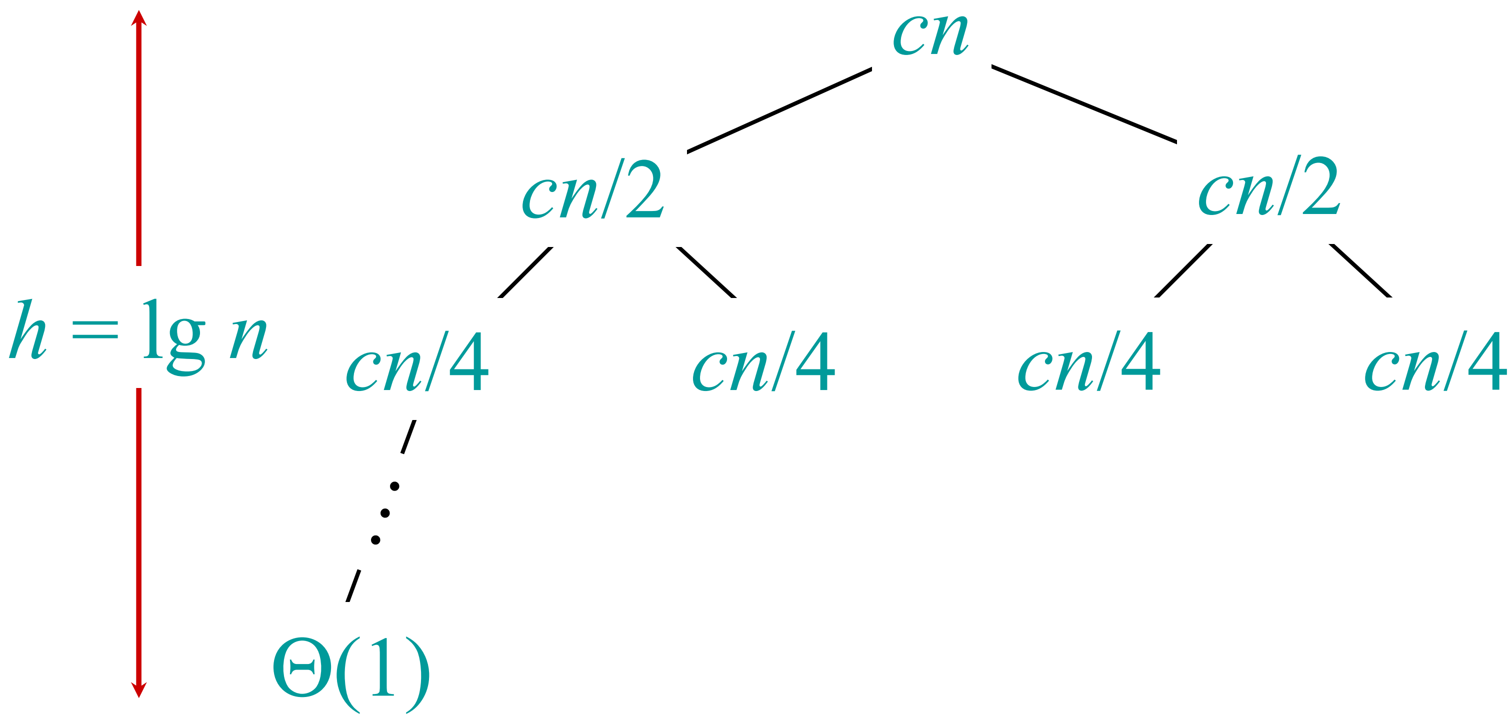

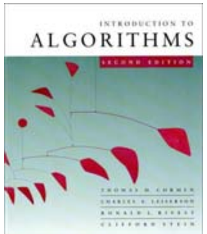

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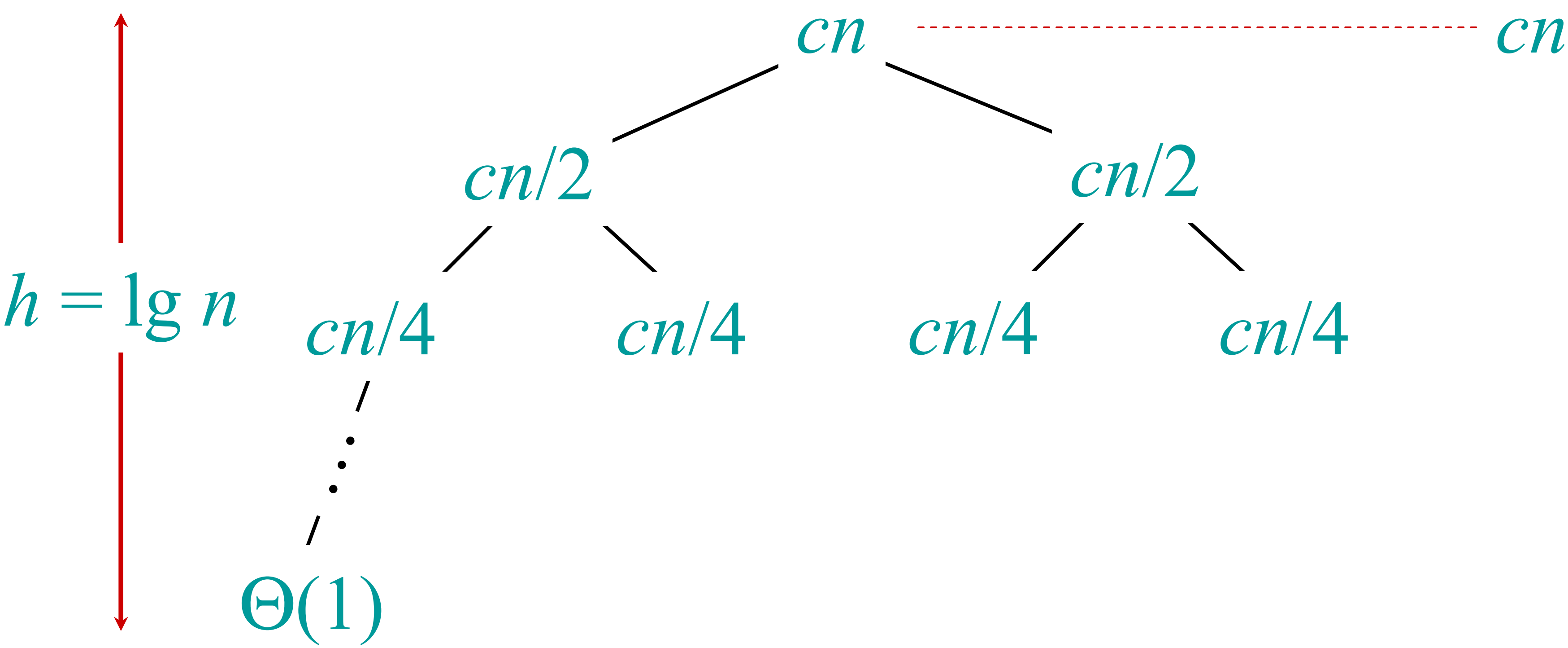

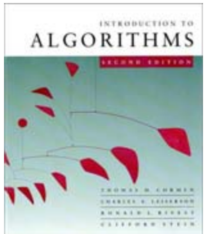

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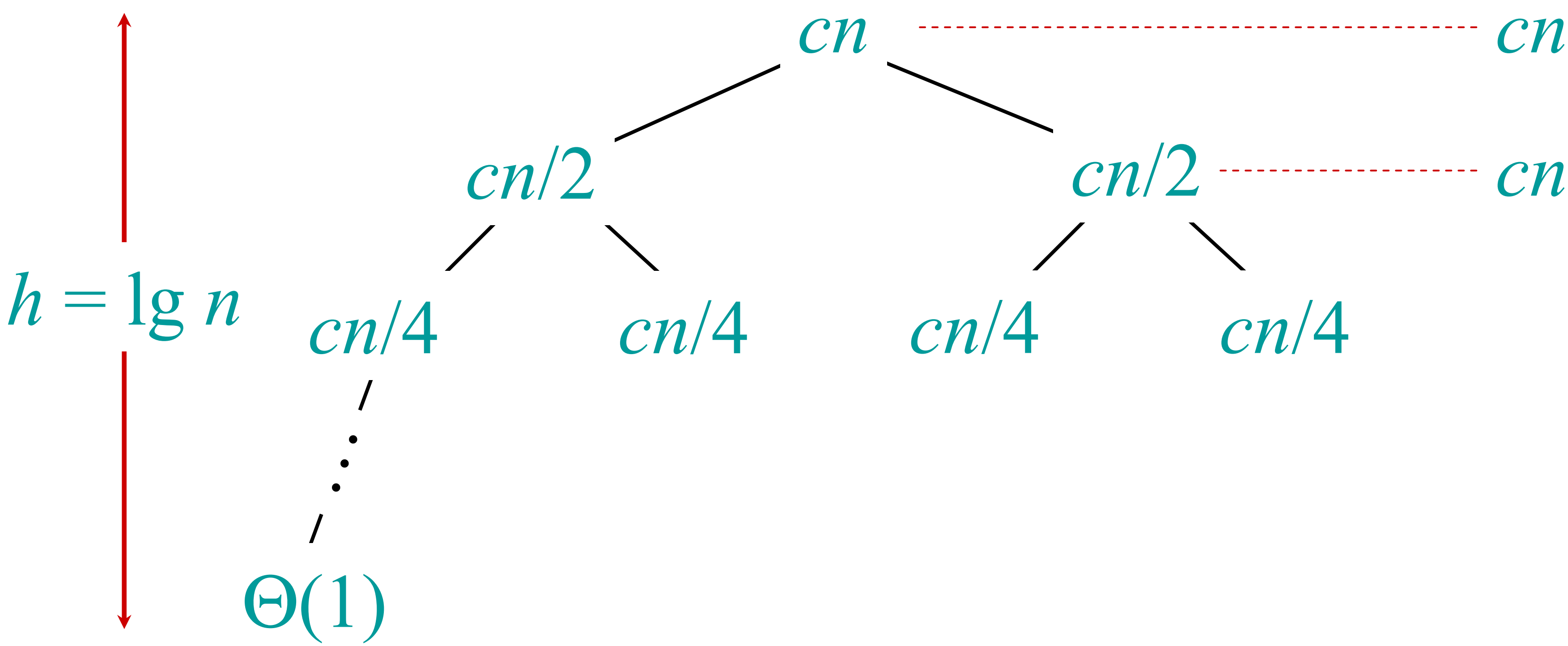

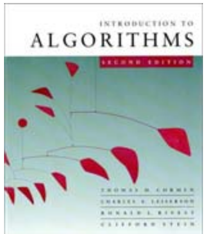

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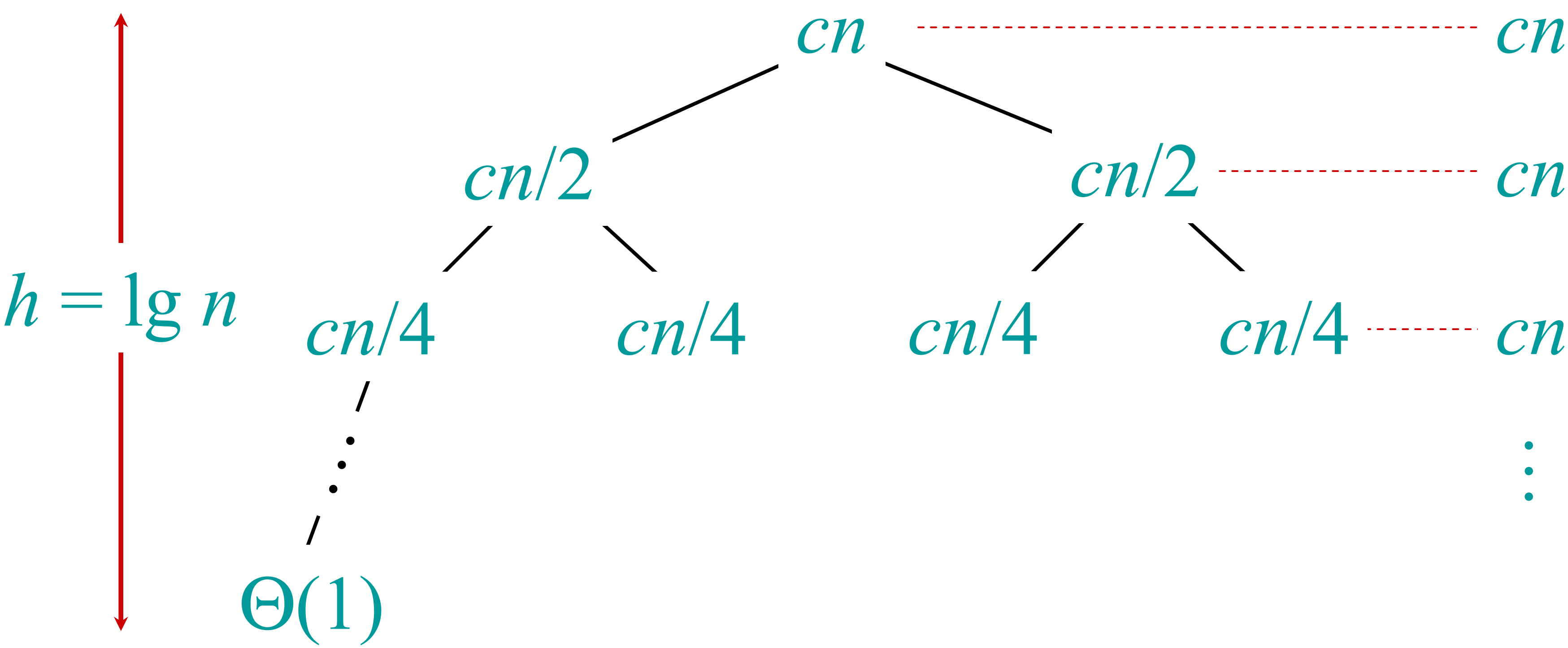

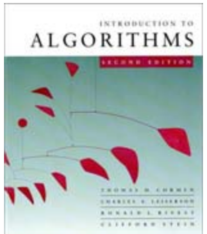

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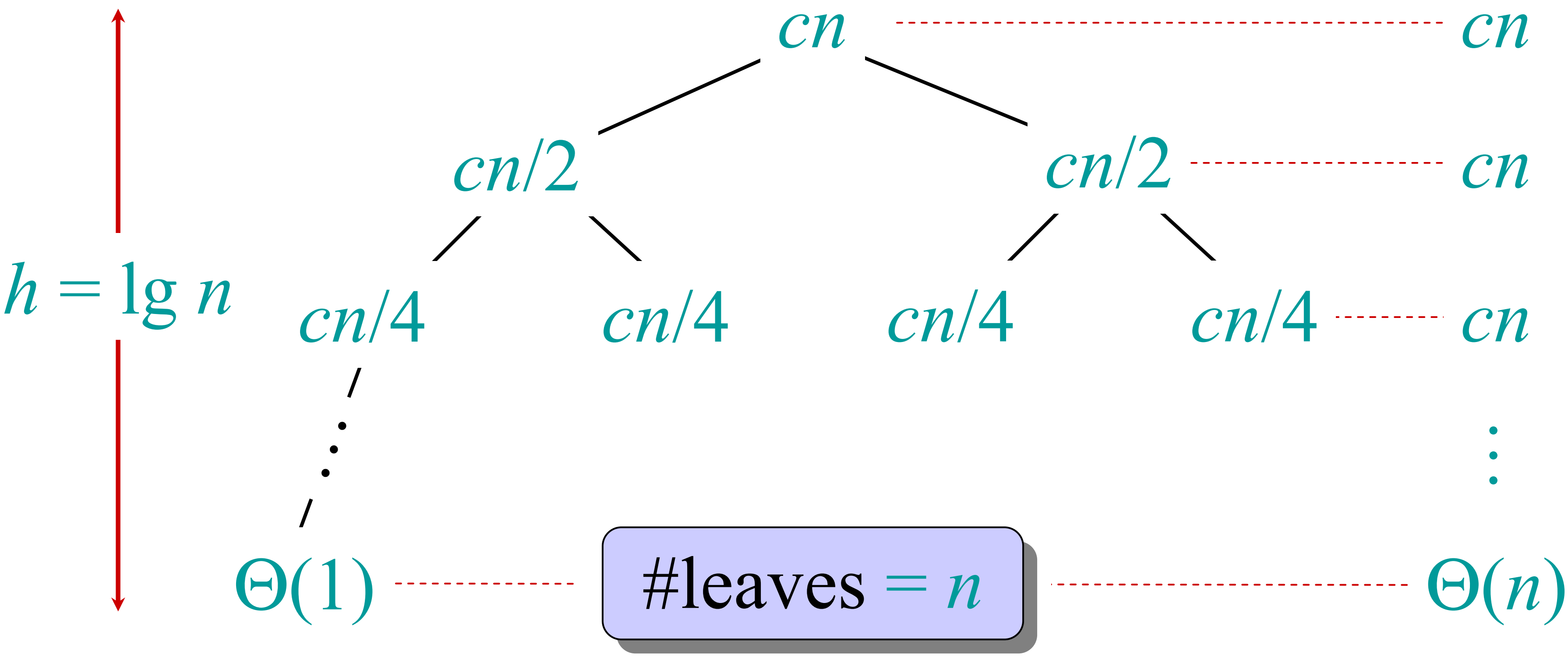

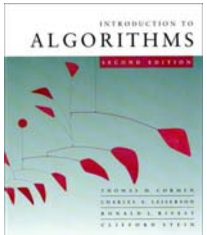

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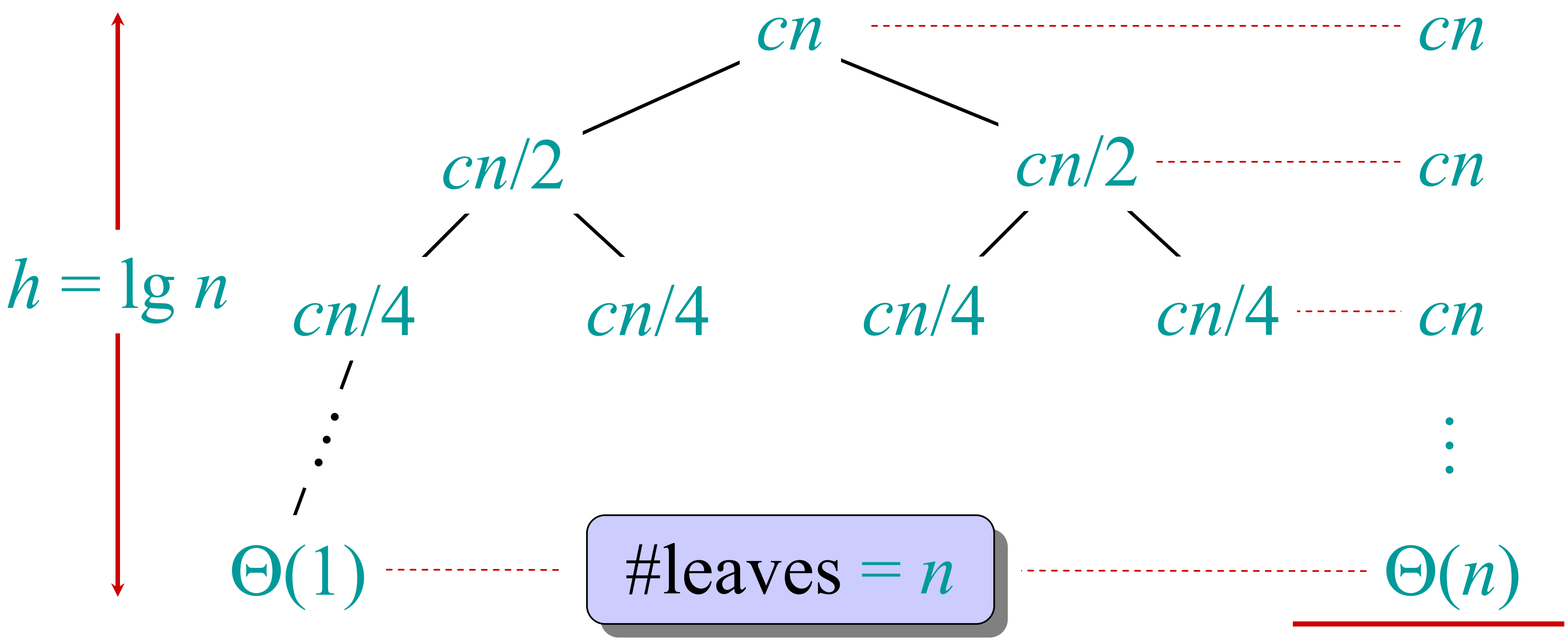

Total $= \Theta(n \lg n)$

Conclusions

- $\Theta(n \lg n)$ grows more slowly than $\Theta(n^2)$.
- Therefore, merge sort asymptotically beats insertion sort in the worst case.
- In practice, merge sort beats insertion sort for $n > 30$ or so.
- Go test it out for yourself!