

(1) Using Figure 8.2 as a model, illustrate the operation of COUNTING-SORT on the array $A = \langle 6; 0; 2; 0; 1; 3; 4; 6; 1; 3; 2 \rangle$.

We have $C = \langle 2, 4, 6, 8, 9, 9, 11 \rangle$

Running iterations on B

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| B | | | | | | 2 | | | | | |
| B | | | | | | 2 | | 3 | | | |
| B | | | | 1 | | 2 | | 3 | | | |
| B | | | | 1 | | 2 | | 3 | | | 6 |
| B | | | | 1 | | 2 | | 3 | 4 | | 6 |
| B | | | | 1 | | 2 | 3 | 3 | 4 | | 6 |
| B | | | 1 | 1 | | 2 | 3 | 3 | 4 | | 6 |
| B | | 0 | 1 | 1 | | 2 | 3 | 3 | 4 | | 6 |
| B | | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | | 6 |
| B | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | | 6 |
| B | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 6 | 6 |

(2) Suppose that we were to rewrite the for-loop header in line 10 of the COUNTINGSORT

As 10 for $j \leftarrow 1$ to $A.length$. Show that the algorithm still works properly. Is the modified algorithm stable?

For example, there are two elements a_1 and a_2 where $a_1 = a_2$ and a_1 is before a_2 in the input array A . In the original algorithm, if a_2 is put into the output array B at position i , i.e., put a_2 into $B[i]$, then a_1 will be put before that i.e. into $B[i - 1]$. But in the modified algorithm, a_2 is in $B[i - 1]$ and a_1 is in $B[i]$. They are reversed in the output. For the reversing is ok because $a_1 = a_2$. So, the modified algorithm works properly.

From the above example, we know that the modified algorithm will not be stable. It will work properly, but they are not stable. Equal elements will appear in reverse order in the sorted array

(3) Show how to sort n integers in the range 0 to $n^3 - 1$ in $O(n)$ time.

Using Radix Sort with base n . Now each digit will be in $\log n^3$ and it will require 3 epochs. Further we only need to run 3 epochs and so for n possible values total time will be $O(n)$.

(3) Show that the second smallest of n elements can be found with $n + \lceil \lg n \rceil - 2$ comparisons in the worst case.

The second smallest can be found in $(n-1)$ comparisons where at least one out of two elements move to next comparison. After $\log n$ rounds only the smallest elements remains. So, for second smallest element we have to do $\log n - 1$ comparisons

For Example,

1 2 5 6 7 9
1 5 7
1

So for 6 elements we get 3 elements as second smallest.

No of comparisons is $(6-1) + (3-1) = 7$

So for 6 elements smallest element is 1

Reference:

<https://sites.math.rutgers.edu>
<https://walkccc.github.io/CLRS/>
<https://github.com/gzc/CLRS>

7

No of calculations is $(6-1) + (1-1) = 6$

(4) In the algorithm SELECT, the input elements are divided into groups of 5. Will the algorithm work in linear time if they are divided into groups of 7? Argue that SELECT does not run in linear time if groups of 3 are used.

Yes, it will work in linear time as median of median is less than less than 4 elements from the group. So it is around $4n/14$ elements. So, we can say that it is

$T(n) \leq T(n/7) \leq T(10n/14) \leq O(n)$. Thus, it will work in linear time.

Now if the group of 3 is used, we have

$T(n) = T(\lceil n/3 \rceil) + T(4n/6) + O(n) \geq T(n/3) + T(2n/3) + O(n)$. so we have $\geq cn \lg n$. Thus it will grow faster than linear.

(5) Exercise 9.3-8 on Page 223 [20 points]

if $n = 1$ then

 return min($X[1]$, $Y[1]$)

end if

if $X[n/2] < Y[n/2]$ then

 return Median($X[n/2 + 1 \dots n]$, $Y[1 \dots n/2]$, $n/2$)

else

 if $X[n/2] \geq Y[n/2]$ then

 return Median($X[1 \dots n/2]$, $Y[n/2 + 1 \dots n]$, $n/2$)

 end if

end if