

1. Show how OS-SELECT($T, \text{root}, 10$) operates on the red-black tree T of Figure 14.1.

The sequence to call on red black tree is:

OS-SELECT ($T.\text{root}$, 10)

OS-SELECT ($T.\text{root}.\text{left}$, 10)

OS-SELECT ($T.\text{root}.\text{left}.\text{right}$, 2)

OS-SELECT ($T.\text{root}.\text{left}.\text{right}.\text{left}$, 2)

OS-SELECT ($T.\text{root}.\text{left}.\text{right}.\text{left}.\text{right}$, 1)

Then, we have that node (with key 20) that is returned is $T.\text{root}.\text{left}.\text{right}.\text{left}.\text{right}$

2. Show how OS-RANK(T, x) operates on the red-black tree T of Figure 14.1 and the node x with $x.\text{key} = 35$

r is set to 0 and y is set to x . (35: $r = 0$)

1st iteration of the while loop, y is set to the node with key 38. (38: $r = 0$)

2nd iteration, r is increased to 2 and y is set to the node with key 30. (30: $r = r + 2 = 3$)

3rd iteration, y is set to the node with key 41. (41: $r = 3$)

4th iteration r is increased to 15 and y is set to the node with key 26 (26: $r = r + 12 = 15$)

This breaks the while loop and rank 15 is returned.

3. Given an element x in an n -node order-statistic tree and a natural number i , how can we determine the i^{th} successor of x in the linear order of the tree in $O(\lg n)$ time?

OS-SELECT (T , OS-RANK(T , x) + i)

By property of black tree, run time is $O(h)$

4. Describe an efficient algorithm that, given an interval i , returns an interval overlapping i that has the minimum low endpoint, or $T.\text{nil}$ if no such interval exists.

Let us consider the usual interval search given, but, instead of coming out of the loop as soon as we have an overlap, we keep record of the most recently seen overlap, and keep going in the loop until we are at $T.\text{nil}$. We then return the most recently seen overlap. We have that this is the overlapping interval with minimum left end point because the search always goes to the left it contains an overlapping interval, and the left children are the ones with smaller left endpoint.