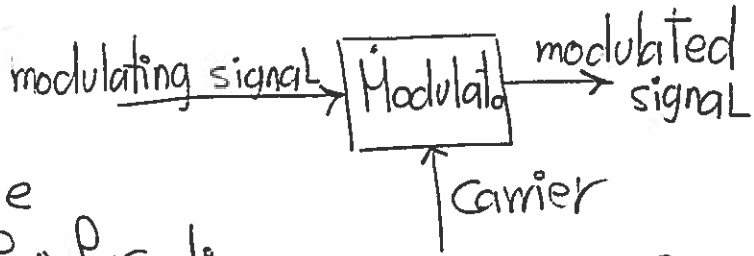
## ** Chapter « 4 »

محمد عبد الحميد

## * Angle Modulation.




- When the ampulitude of carrier signal is change according to the amp. of information signal. this modulation is denotes by « Amplitude modulation ».

$$X_c(t) = A_c \cos[2\pi f_c t + \phi_c]$$
$$= A_c \cos \theta_i(t)$$

- When the the angle of carrier signal is changes according to the amplitude of information signal. this modulation is denotes by « Angle modulation ».
- Angle modulation has two types: Frequency Modulation [FM] and phase Modulation [PM].

### 1. Phase Modulation [PM]:

The Carrier signal: $X_c(t) = A_c \cos[2\pi f_c t + \phi_c] = A_c \cos \theta_i(t)$

Thus, the instantaneous phase of the phase modulated signal is given by:

$$\theta_i(t) = 2\pi f_c t + \phi_c$$
$$= 2\pi f_c t + K_p m(t)$$

where $K_p$ (in rad/volt) is a Constant Known as the « phase sensitivity ». There fore, the phase modulated [PM] signal Can be expressed as:

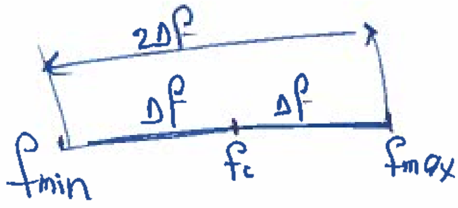
$$\boxed{S_{PM}(t)} = A_c \cos \theta_i(t)$$
$$= \boxed{A_c \cos[2\pi f_c t + K_p m(t)]} \rightarrow (*)$$



Hence, PM signal is defined as « The phase of the Carrier signal is changed according to the amplitude of information signal.

- The instantanuous frequency of the PM signal is given by: $\boxed{f_i(t)} = \frac{1}{2\pi}\cdot\frac{d\theta_i(t)}{dt} = \boxed{f_c + \frac{K_p}{2\pi} m(t)}$

Hence, the frequency of the PM signal increases linearly with $m(t)$.




$$f_{max}(t) = f_c + \frac{1}{2\pi} K_p\, m(t)_{max}$$
$$f_{min}(t) = f_c + \frac{1}{2\pi} K_p\, m(t)_{min}$$

therefore, the total frequency deviation can be given by $2\Delta f$ where $\Delta f$ is given by:

$$\Delta f = \frac{1}{2}\left[f_{max}(t) - f_{min}(t)\right]$$
$$= \frac{1}{2}\left[f_c + \frac{K_p}{2\pi} m(t)_{max} - f_c - \frac{K_p}{2\pi} m(t)_{min}\right]$$

Hence, $\boxed{\Delta f = \frac{K_p}{4\pi}\left[m(t)_{max} - m(t)_{min}\right]}$ & $\boxed{\Delta f = \frac{K_p}{2\pi} m_p(t)}$ → symmetric signal

## 2. Frequency Modulation [FM]:

The instantaneous frequency of (FM) signal is given by

$$f_i(t) = f_c + K_f\, m(t)$$

where $K_f$ (in Hz/V) known as the frequency senstivity.

- The instantaneous phase of the FM signal is given by

$$\theta_i(t) = \int_{-\infty}^{t} 2\pi f_i(t)\,dt = 2\pi \int_{-\infty}^{t} f_i(t)\,dt$$

$f_i = \frac{1}{2\pi}\frac{d}{dt}\theta_i$

$\int f_i = \frac{1}{2\pi}\cdot\theta_i$

$\boxed{\theta_i = 2\pi \int f_i}$

$$= 2\pi \left[f_c t + K_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

$$= 2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(\alpha)\,d\alpha$$

since, $S_{FM}(t) = A_c \cos\theta_i(t)$

Thus, $\boxed{S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]} \rightarrow (*)$

Hence, FM signal is defined as « the frequency of the carrier signal is changed according to the amplitude of modulating signal ».

$$\left.\begin{aligned} f_{max}(t) &= f_c + K_f\, m(t)_{max} \\ f_{min}(t) &= f_c + K_f\, m(t)_{min} \end{aligned}\right\} ; \quad f_i(t) = f_c + K_f\, m(t)$$

Therefore, $\Delta f = \frac{1}{2}\left[f_{max}(t) - f_{min}(t)\right]$

$$\boxed{\Delta f = \frac{K_f}{2}\left[m(t)_{max} - m(t)_{min}\right]} \quad \& \quad \boxed{\Delta f = K_f\, m_p} \rightarrow \text{symmet signal}$$

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Example 4.2 :

Consider the signal $m(t) = 5\cos(8\pi \times 10^4 t)$ angle modulates a carrier signal $x(t) = 5\cos(10^6 \pi t)$. Find and sketch the modulated signal and calculate the total frequency deviation in the following cases:

a) FM with $K_f = 25\ \text{KHz/V}$.

b) PM with $K_p = \frac{\pi}{4}\ \text{rad/V}$.

« Soln »

a) $S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$

$$= 5\cos\left[2\pi * \frac{10^6}{2} t + 2\pi * 25 \times 10^3 \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

$$= 5\cos\left[10^6 \pi t + 50 \times 10^3 \pi \int_{-\infty}^{t} 5\cos(8\pi \times 10^4 t)\, dt\right]$$

$$= 5\cos\left[10^6 \pi t + 50 \times 10^3 \pi * \frac{5}{8\pi \times 10^4} \sin(8\pi \times 10^4 t)\right]$$

$$\boxed{\text{Hence, } S_{FM}(t) = 5\cos\left[10^6 \pi t + \frac{25}{8} \sin(8\pi \times 10^4 t)\right] \#}$$

* The instantaneous frequency:

$f_i(t) = f_c + K_f\, m(t)$, when $f_c = 500\,KHz$.

$f_i(t)_{max} = f_c + K_f\, m(t)_{max}$

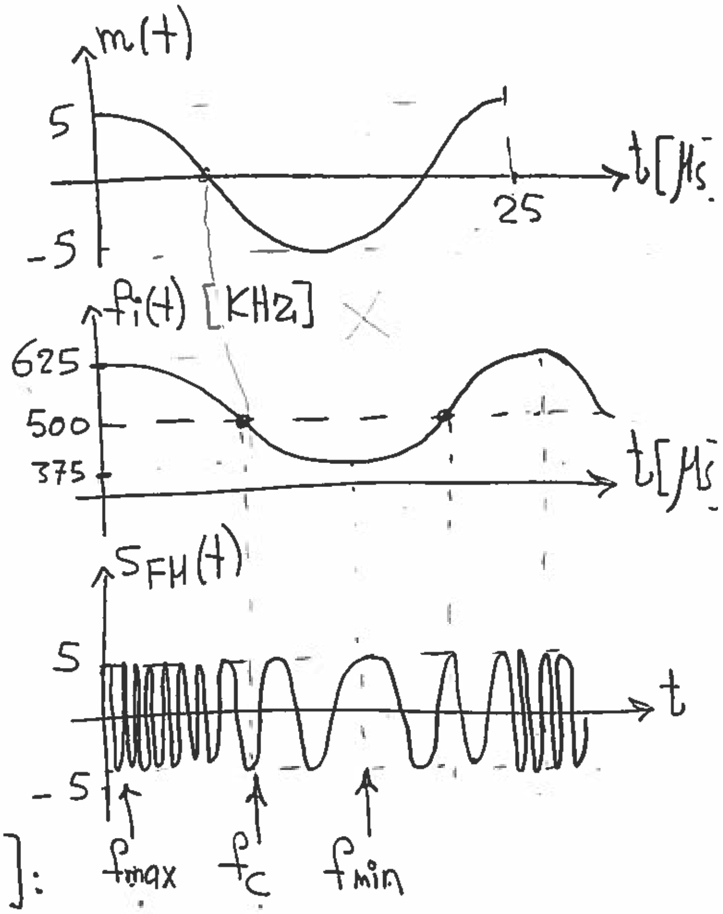
$= 500\times10^3 + 25\times10^3(5)$

$= 625\,KHz$.

$f_i(t)_{min} = f_c + K_f\, m(t)_{min}$

$= 500\times10^3 + 25\times10^3(-5)$

$= 375\,KHz$.




since, $f_m = 40\times10^3\,Hz$

Hence, $T_m = \frac{1}{40\times10^3} = 25\,\mu sec$.

* The frequency deviation $[\Delta f]$:

$\Delta f = \frac{K_f}{2}[m(t)_{max} - m(t)_{min}]$

$\boxed{\therefore \Delta f} = \frac{25\times10^3}{2}[5-(-5)] = \boxed{125\,KHz}$.

$2\Delta f = 2*125\,K$

b] PM with $K_p = \frac{\pi}{4}$ [rad/V]

$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p\, m(t)]$

$= 5\cos[2\pi * \frac{10^6}{2} t + \frac{\pi}{4}\cdot 5\cos(8\pi\times10^4 t)]$

$$\boxed{\therefore S_{PM}(t) = 5\cos[10^6\pi t + \frac{5\pi}{4}\cos(8\pi\times10^4 t)]}$$

* The instantaneous frequency:

$f_i(t) = f_c + \frac{K_p}{2\pi} \dot{m}(t)$

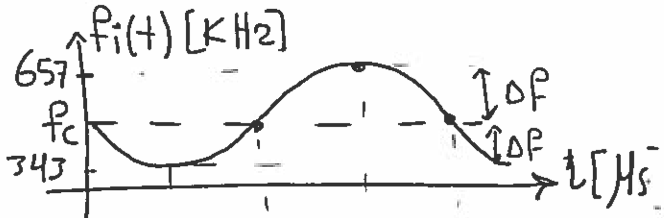
since, $m(t) = 5\cos(8\pi\times10^4 t)$

Hence, $\dot{m}(t) = -5\times8\pi\times10^4 \sin(8\pi\times10^4 t)$

$$f_i(t)_{max} = f_c + \frac{K_p}{2\pi} m'(t)_{max}$$

$$= 500\times10^3 + \frac{\pi}{4}\cdot\frac{1}{2\pi} * 5\times8\pi\times10^4 \cancel{\sin(8\pi\times10^4 t)}$$

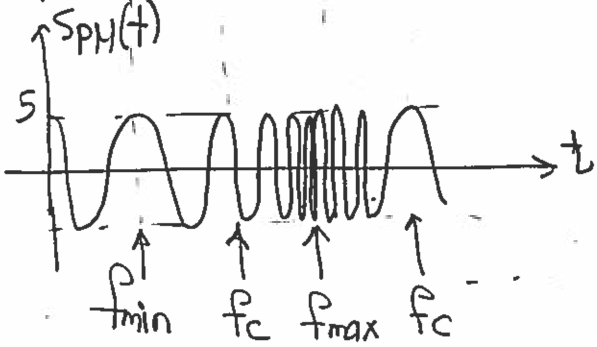
$$= 657 \text{ KHz}.$$




$$f_i(t)_{min} = f_c + \frac{K_p}{2\pi} m'(t)_{min}$$

$$= 500\times10^3 + \frac{\pi}{4}\cdot\frac{1}{2\pi}(-5\times8\pi\times10^4)$$

$$= 343 \text{ KHz}.$$




$$\Delta f = \frac{K_p}{4\pi}\left[m'(t)_{max} - m'(t)_{min}\right]$$

$$\boxed{\therefore \Delta f = 157 \text{ KHz}}$$

* Power in both FM & PM is given by:

$$P = \frac{A_c^2}{2} = \frac{5^2}{2} = \frac{25}{2} \text{ [watt]}$$

Example (exam)

Consider the signal m(t) is shown in figure. is angle modulates with carrier frequency of $10^8$ Hz. Find and sketch the modulated signal and calculate the total frequency deviation in the following cases:

a) FM with $K_f = 10^5$ Hz/V.

b) PM with $K_p = \frac{\pi}{2}$ rad/V. assuming $A_c = 10$ V.

Soln

a) $S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$

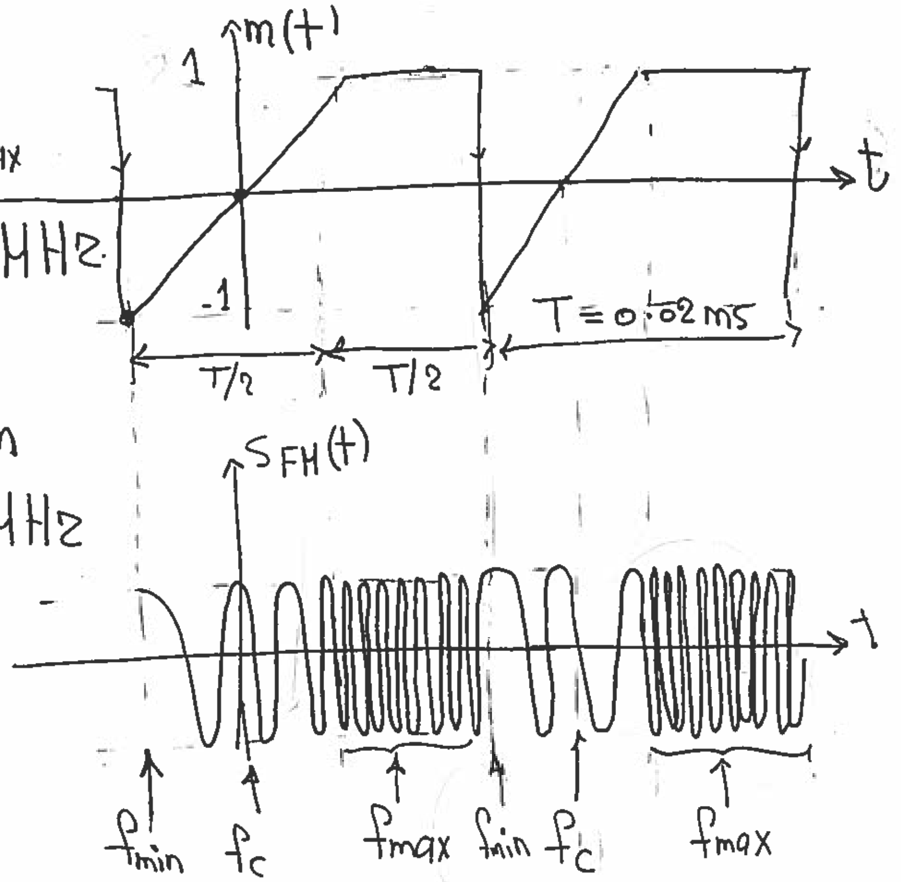
$$= A_c \cos\left[10^8\pi t + 2\times10^5\pi \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

* at $m(t) = 1$

$$f_i(t)_{max} = f_c + K_f\, m(t)_{max}$$
$$= 10^8 + 10^5(1) = 100.1\ MHz$$

* at $m(t) = -1$

$$f_i(t)_{min} = f_c + K_f\, m(t)_{min}$$
$$= 10^8 + 10^5(-1) = 99.9\ MHz$$

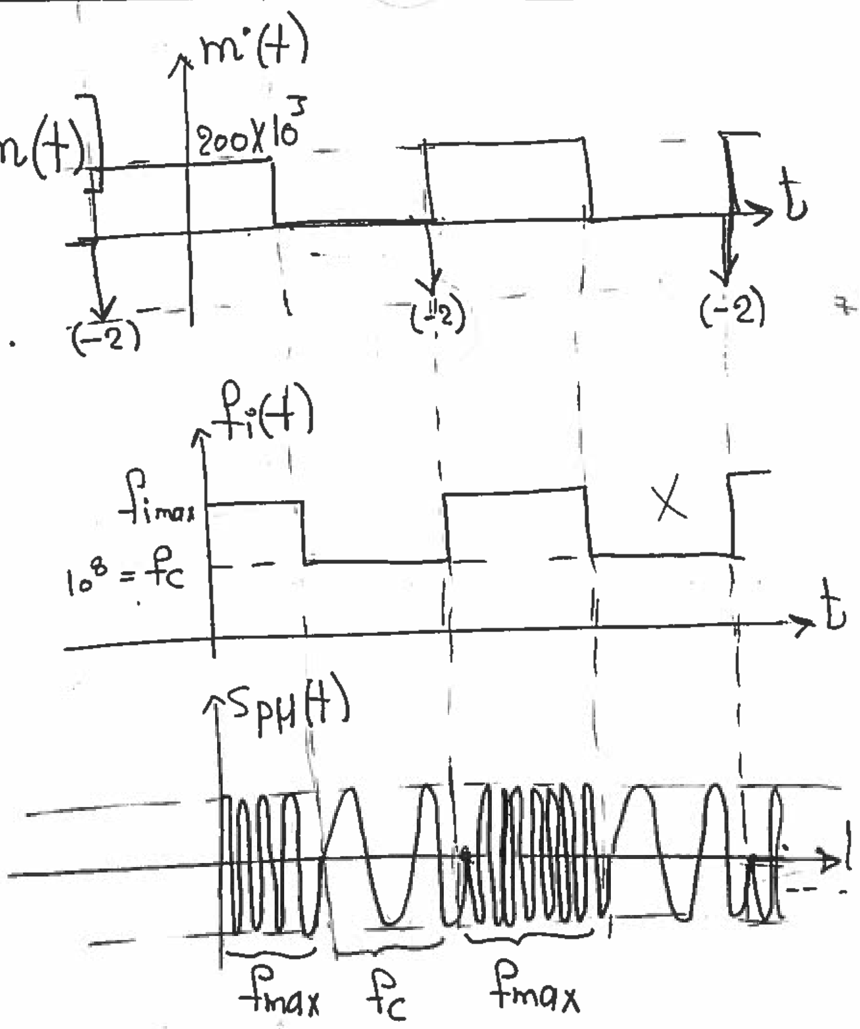



* at $m(t) = 0$

$$f_i = f_c + K_f\, m(t)$$
$$\Rightarrow f_i(t) = f_c .$$

* PM with $K_p = \pi/2$

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$
$$f_i(t) = f_c + \frac{K_p}{4\pi}\, m'(t)$$
$$\rightarrow slope = \frac{2}{T/2} = \frac{4}{T} = \frac{4}{0.02\times10^{-3}}$$
$$= 200\times10^3 .$$




at $m'(t) = 200\times10^3$:

$$f_i(t)_{max} = f_c + \frac{K_p}{4\pi}\, m'(t)_{max}$$
$$= 10^8 + \frac{\pi}{2}\cdot\frac{1}{4\pi}\times 200\times10^3$$
$$= 100.025\ MHz.$$

at $m'(t) = 0$:

$$f_i(t) = f_c$$
$$\Rightarrow \Delta\phi_1 = K_p\, \Delta m(t) = \frac{\pi}{2} * (-2) = -\pi \quad \& \quad \Delta\phi_2 = \frac{\pi}{2}(-2) = -\pi$$

$$S_{PM}(t) = 10 \cos[10^8 \pi t + K_p\, m(t)]$$

## * Narrowband FM [NBFM] signals :

Consider the FM signal given by:

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi K_f\, a(t)\right] \quad , \text{where } \boxed{a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha}$$

$$\therefore S_{FM}(t) = A_c \cos 2\pi f_c t \cos[2\pi K_f a(t)] - A_c \sin 2\pi f_c t \sin[2\pi K_f a(t)] \rightarrow$$

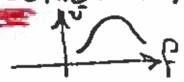
This equation can be approximated as « NBFM » if the maximum frequency deviation is small, that is $K_f m_p \ll 1$

or $|2\pi K_f a(t)| \ll \pi$

note:

If the frequency deviation is small → the voltage at the demodulator o/p will be small, Consequently « power is small ». (as in Lab)

Hence, In FM we can exchange between power and bandwidth at expense of application.

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* Since, frequency deviation is small, in this case Eq (1) becomes

$$S_{FM}(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carier}} - \underbrace{2\pi K_f A_c a(t) \sin 2\pi f_c t}_{DSB}$$

$\sin 0 = 0$

$\sin(0.1) = 0.1$

الـ قيمة صغيرة

$$\therefore \boxed{S_{NBFM}(t) = A_c \cos 2\pi f_c t - 2 A_c \pi K_f a(t) \sin 2\pi f_c t}$$

$\cos 0 = 1$

قيمة صغيرة

This eqn is similar to an AM signal. where in AM, the signal $S_{AM}(t) = A_c \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$.

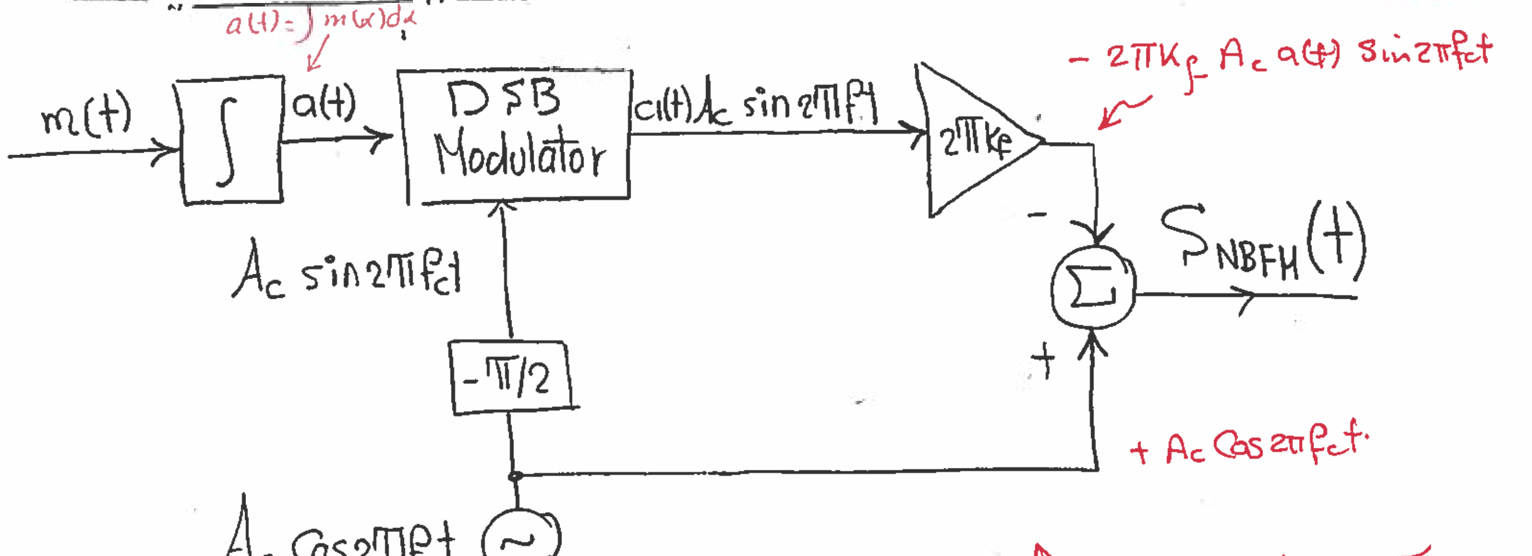
Hence, NBFM is an AM modulation for the modulating signal $a(t)$. therefore, the bandwidth of the NBFM signal is twice the bandwidth of $a(t)$.

AM $m(t) \rightarrow BW = B$

$2BW$ of $m(t)$

$a(t)$ modulating sig

* Generation of NBFM signal indirectly Using «DSB»




↑ $S_{NBFM}(t)$ أهم معادلة

## Example : (Special Case : Narrow band tone FM modulation)

### Tone NBFM :

Consider the modulating signal is sinsoidal, that is

$$m(t) = A_m \cos 2\pi f_m t$$

Then, $a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha = \frac{A_m}{2\pi f_m} \sin 2\pi f_m t$

since, $S_{NBFH}(t) = A_c \cos 2\pi f_c t - 2 A_c \pi K_f\, a(t) \sin 2\pi f_c t$

Hence, $S_{NBFH}(t) = A_c \cos 2\pi f_c t - 2\pi A_c K_f \cdot \frac{A_m}{2\pi f_m} \sin 2\pi f_c t \sin 2\pi f_m t$

$$= A_c \cos 2\pi f_c t - A_c \boxed{\frac{K_f A_m}{f_m}} \sin 2\pi f_c t \sin 2\pi f_m t$$

Hence, $S_{NBFM}(t) = A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \sin 2\pi f_m t$

where $\boxed{\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}}$,

NBFM → Δf↓↓
$\beta = \frac{\Delta f}{f_m} \ll 1$

is called the «the modulation index». the Condition for narrow band modulation is ($\boxed{\beta \ll 1}$) ∴ Δf↓↓

$$S_{NBFH}(t) = A_c \cos 2\pi f_c t - \left[\frac{\beta A_c}{2} \cos[2\pi(f_c - f_m)] - \frac{\beta A_c}{2} \cos[2\pi(f_c + f_m)]\right]$$

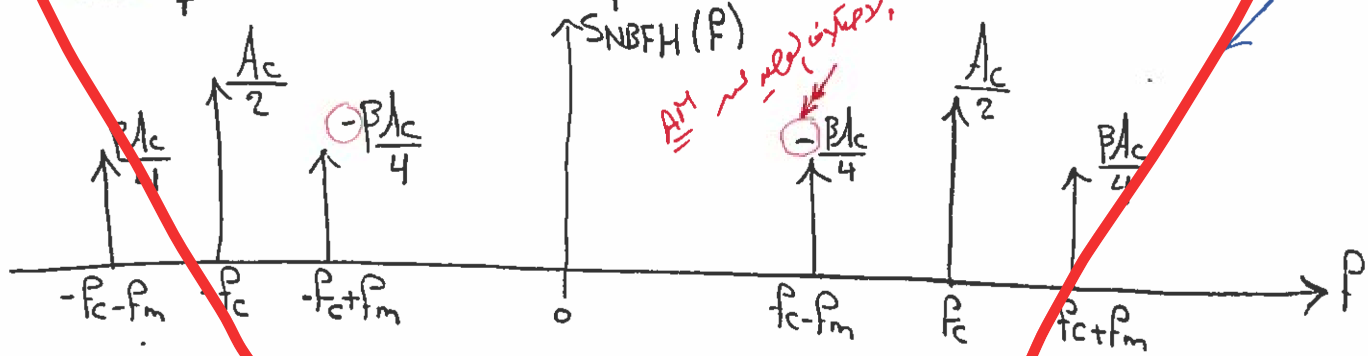
Carrier, USB, LSB

$$\therefore S_{NBFH}(t) = A_c \cos 2\pi f_c t + \frac{\beta A_c}{2} \cos[2\pi(f_c + f_m)] - \frac{\beta A_c}{2} \cos[2\pi(f_c - f_m)]$$

Hence, the spectrum of this signal is,

$$S_{NBFM}(f) = \frac{A_c}{2}\delta(f-f_c) + \frac{A_c}{2}\delta(f+f_c) + \frac{\beta A_c}{4}\delta(f-f_c-f_m) + \frac{\beta A_c}{4}\delta(f+f_c+f_m)$$
$$- \frac{\beta A_c}{4}\delta(f-f_c+f_m) - \frac{\beta A_c}{4}\delta(f+f_c-f_m)$$

in tone NBFM




In case of AM tone modulation, the modulated signal is

$$S_{AM}(t) = A_c\cos 2\pi f_c t + m(t)\cos 2\pi f_c t \quad , m(t) = A_m\cos 2\pi f_m t$$

$$\therefore S_{AM}(t) = A_c\cos 2\pi f_c t + A_m\cos 2\pi f_c t\cos 2\pi f_m t$$

$$= A_c\cos 2\pi f_c t + \frac{A_m}{2}\left[\cos 2\pi(f_c-f_m) + \cos 2\pi(f_c+f_m)\right] \quad , A_m = \mu A_c$$

$\mu = \frac{A_m}{A_c}$ (mp)

$$\therefore S_{AM}(t) = \underbrace{A_c\cos 2\pi f_c t}_{\text{Carrier}} + \frac{\mu A_c}{2}\underbrace{\cos 2\pi(f_c+f_m)}_{\text{LSB}} + \frac{\mu A_c}{2}\underbrace{\cos 2\pi(f_c-f_m)}_{\text{LSB}}$$

** The basic difference between an AM signal and NBFM signal that is « algebraic sign » of the Lowerside band in NBFM. Thus, NBFM requires essentially the same transmission bandwidth as AM signal, that is $\boxed{B_{NBFM} = 2B}$

as $B_{AM}$)

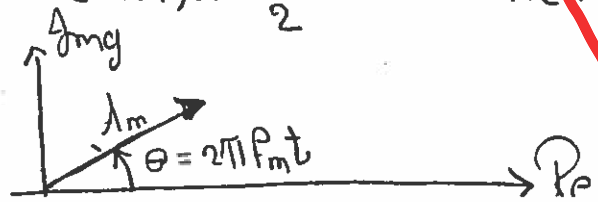
## * Phasor diagram of AM signals:

$$S_{AM}(t) = [A_c + A_m\cos 2\pi f_m t]\cos 2\pi f_c t$$

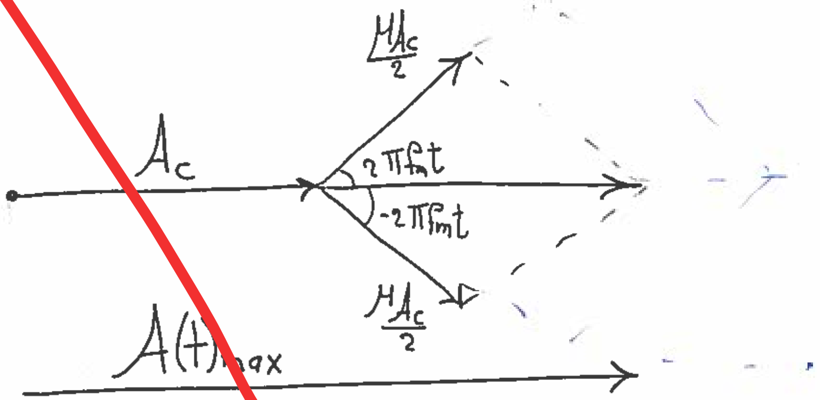
$$= A_c\cos 2\pi f_c t + \frac{\mu A_c}{2}\cos 2\pi(f_c-f_m)t + \frac{\mu A_c}{2}\cos 2\pi(f_c+f_m)t \rightarrow (1)$$

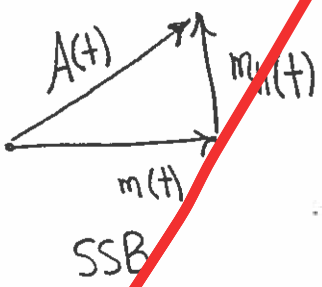
note:

we can represents:

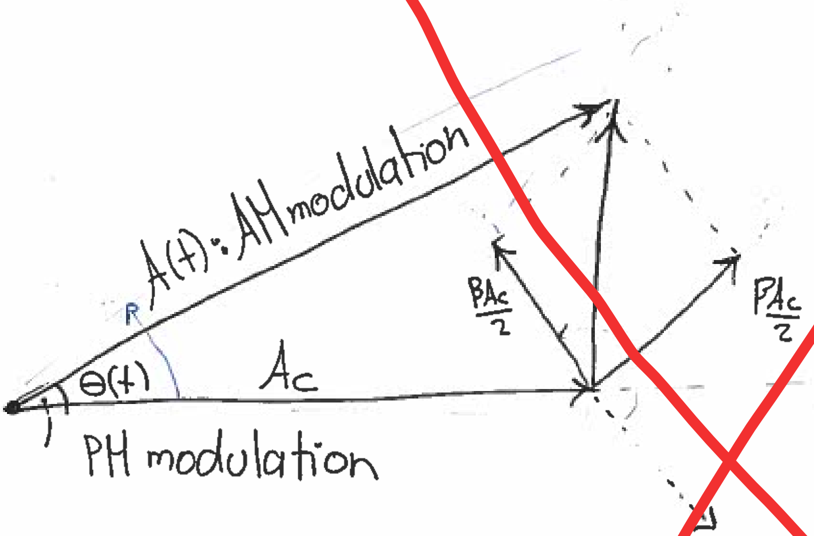


Hence eqn (1) Can be represents as:




## * Phasor diagram of NBFM:




- AM provides only amplitude modulation, but NBFM provides phase Modulation and amplitude modulation.

## * Narrow Band PM (NBPM) Signals:

$$S_{PM}(t) = A_c \cos(w_c t + K_p m(t))$$

$$= A_c \cos w_c t \cos(K_p m(t)) - A_c \sin w_c t \sin(K_p m(t))$$

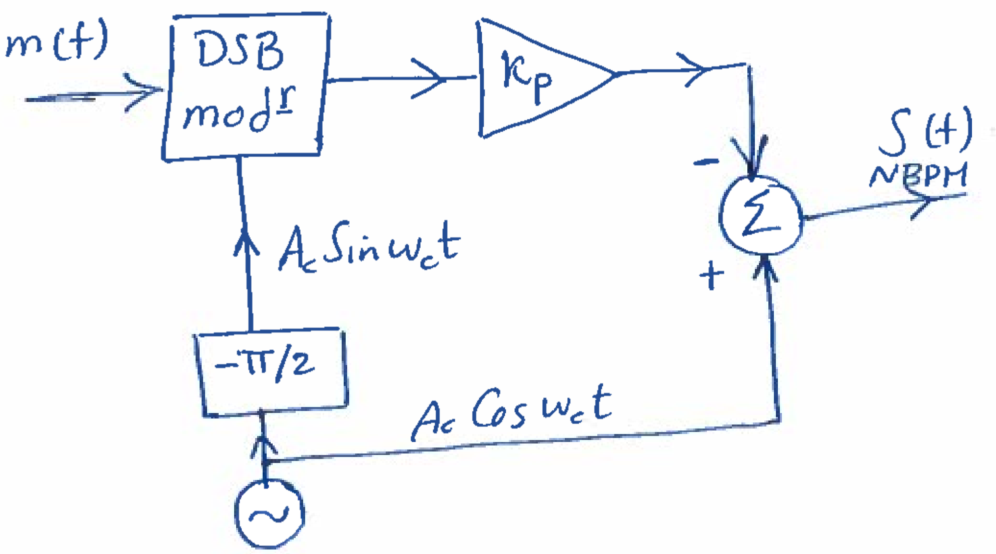
PM Signal Can be approximated as NBPM signal if $K_p m_p \ll 1$.

$$S_{NBPM}(t) = A_c \cos w_c t - A_c K_p m(t) \sin w_c t$$

$$S_{NBPM}(f) = A_c \delta(f - f_c) + j A_c K_p M(f - f_c)$$

$$B_{NBPM} = 2B$$

Generation of NBPM Signal



# * Tone Wide band Modulation :

Tone Wide band FM :

Consider FM technique where the FM signal is

$$S_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha] \quad , m(t) = A_m \cos 2\pi f_m t$$

$$= A_c \cos[2\pi f_c t + 2\pi k_f A_m \frac{\sin 2\pi f_m t}{2\pi f_m}]$$

$$= A_c \cos[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t].$$

$$= A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \rightarrow$$

$$= \mathrm{Re}.A_c [e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}]$$

since, the Complex function $e^{j\beta \sin 2\pi f_m t}$ is periodic with period $T = \frac{1}{f_m}$.

Hence, it can be represented in a fourier series form ~~fro~~ as

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} X(n)\, e^{j2\pi n f_m t}$$

where, $X(n)$ is the spectral Coefficient at frequency $nf_m$ given by

$$X(n) = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-j2\pi n f_m t}\, dt$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j\beta \sin 2\pi f_m t} \cdot e^{-j2\pi n f_m t} \cdot dt$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j[\beta \sin 2\pi f_m t - 2\pi n f_m t]} \cdot dt$$

Let $u = 2\pi f_m t$

$$\therefore X(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin u - nu)} \cdot du \equiv J_n(\beta)$$

where, $J_n(\beta)$ is a Bessel fn of the first Kind and order $n$.

$uv - \int v\,du$

$u = 2\pi f_m t \rightarrow t = \frac{u}{2\pi f_m}$

$du = 2\pi f_m dt \rightarrow dt = \frac{du}{2\pi f_m}$

$$X(n) = f_m \int_{t=-\frac{1}{2f_m}}^{1/2f_m} e^{j \dots} \cdot dt$$

$$= \frac{f_m}{2\pi f_m} \int_{-\pi}^{\pi} \dots \cdot du$$

$u = \frac{1}{2f_m} \cdot 2\pi f_m$

Since, $S_{FM}(t) = \mathcal{R}e\left\{A_c\, e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\right\}$

Since, $e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} X(n)\, e^{j2\pi n f_m t}$ & $X(n) = J_n(\beta)$

$$= \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{-j2\pi n f_m t}$$

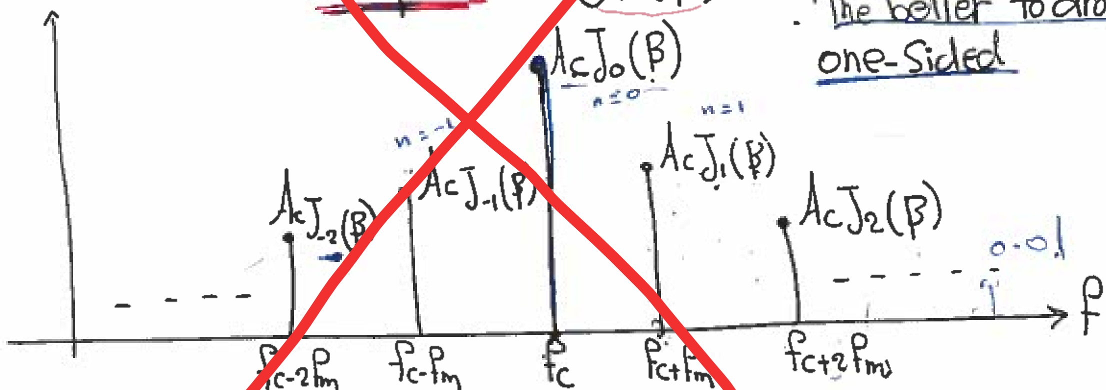
Hence, $S_{FM}(t) = \mathcal{R}e\left\{A_c\, e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t}\right\}$

$$= \sum_{n=-\infty}^{\infty} J_n(\beta) A_c\, \mathcal{R}e\left\{ e^{j2\pi (f_c + n f_m) t} \right\}$$

Hence, $\boxed{S_{FM}(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) A_c \cos 2\pi (f_c + n f_m) t}$

Hence, FM, represented by ⟹ infinite sum of harmonic with amplitude $J_n(\beta)$.

. The better to draw one-Sided




. The upper side frequency has an amplitude $A_cJ_n(\beta)$, while the Lower side frequency has an amplitude $A_cJ_{-n}(\beta)$

Since, $\boxed{J_{-n}(\beta) = (-1)^n J_n(\beta)}$

Hence, Two side frequencies has the Same amplitude but they have opposite polarity (phase shift $\pi$) for odd $n$

$J_n(\beta)$ is oscillatory with its peak value decaying with increasing $\beta$. ; $\beta\uparrow \rightarrow$ peak of $J_n(\beta)\downarrow \rightarrow$ $\Delta f\uparrow$ ; $BW\uparrow \rightarrow \Delta f\uparrow$ ; Spectral line $\uparrow \rightarrow BW\uparrow$

. Large value of $\beta$ implies a Large bandwidth

Large $\beta$ means Large of frequency deviation of the carrier, Large no. of spectral lines are included within the deviation range $2\Delta F$. Hence, provides « Large B.W of FM signal ».

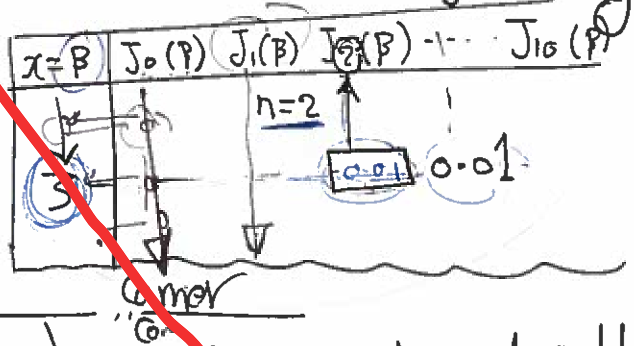
To determine $B_T$, you must find max number of significant side frequency « $n_{max}$ » whose All amplitude are greater than some value that is selected to be « 0.01 » of the carrier amplitude.

$n_{max}$, is the value of $n$ such that $|A_c J_n(\beta)| \geq 0.01 A_c$

that is, $\boxed{|J_n(\beta)| \geq 0.01}$

- In this case, the transmission bandwidth given by:

$$\boxed{B_{FM} = 2\, n_{max} f_m} \rightarrow \text{Exact BW}$$

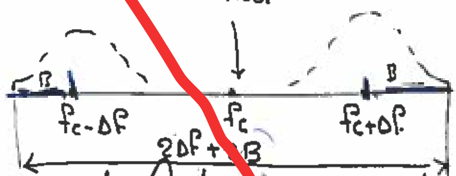



$\Rightarrow J_{n_{max}}(\beta) \geq 0.01$

$J_{n_{max}+1}(\beta) < 0.01$

- An approximate rule for the transmission bandwidth whether it is NBFM or WBFM, is known as « Carson's rule » given by,

$$\boxed{B_T = 2(\Delta f + f_m)}, \text{ for tone Mod.}$$




$$\boxed{B_T = 2(\Delta f + B)}, \text{ B: bandwidth of modulating signal.}$$

* The power of FM signal is $A_c^2/2$, which should equal to the power in all the sidebands, that is, $P_x = \sum_{n=-\infty}^{\infty} \frac{A_c^2 J_n^2(\beta)}{2} = \frac{A_c^2}{2}$

: Consider a tone FM signal is passed through an ideal band pass filter with center frequency « $f_c$ », and bandwidth $B_{FM} = 2N f_m$. Thus, the first $N$ sidebands are passed through the filter.

Hence, the power at the filter o/p is:

$$P_{out} = \frac{A_c^2}{2} \sum_{n=-N}^{N} J_n^2(\beta)$$

$$= \frac{A_c^2}{2} \left[ J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + \dots + 2J_N^2(\beta) \right].$$

** ~~PM~~ tone Wideband PM [WBPM]

$\Delta\Phi = \frac{\Delta f}{f_m}$, $\Delta f = \frac{K_p}{2\pi} \dot{m}_p$

$m(t) = A_m \cos 2\pi f_m t$

$\Rightarrow \dot{m}_p = 2\pi f_m A_m$

$\Rightarrow \Delta f = \frac{K_p}{2\pi} \cdot 2\pi f_m A_m$

$\Delta\Phi = \frac{K_p f_m A_m}{f_m} = K_p A_m$

since, $S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

$$= A_c \cos[2\pi f_c t + K_p A_m \cos 2\pi f_m t]$$

$$= A_c \cos[2\pi f_c t + \Delta\Phi \sin(2\pi f_m t + \tfrac{\pi}{2})] \rightarrow (1)$$

where, $\Delta\Phi$, is the modulation index in PM

Since, $S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \rightarrow (2)$

Hence, tone WBPM signal has the same formula as tone WBFM signal, but mod index in PM, $\boxed{\Delta\Phi = K_p \dot{m}_p}$

Hence, $\boxed{S_{WBPM}(t) = \sum_{n=-\infty}^{\infty} J_n(\Delta\Phi) A_c \cos\left[2\pi (f_c + n f_m) t + \frac{n\pi}{2}\right]}$

Also, the spectral component at freq $f_c + n f_m$ in the PM signal has an added phase shift $\frac{n\pi}{2}$. #

## PM For Arbitrary Modulating signals:

Consider the modulating signal "m(t)", is modulated as PM signal, that is:

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

$$= A_c \, Re\{e^{j2\pi f_c t} \cdot e^{jK_p m(t)}\}$$, and Taking Telor exp.

$$\Rightarrow S_{PM}(t) = Re\left\{A_c e^{j2\pi f_c t}\left[1 + jK_p m(t) + \frac{[jK_p m(t)]^2}{2!} + \frac{[jK_p m(t)]^3}{3!} + \cdots\right]\right\}$$

$$= A_c \cos 2\pi f_c t - A_c K_p m(t) \sin 2\pi f_c t - \frac{A_c}{2} K_p^2 m^2(t) \cos 2\pi f_c t + \frac{A_c}{3!} K_p^3 m^3(t) \sin 2\pi f_c t + \cdots \longrightarrow 1)$$

Hence, Generally, the PM signal can be express as:

$$S_{PM}(t) = \sum_{n=0}^{\infty} (-1)^n \frac{A_c}{n!} K_p^n m^n(t) \cos\left(2\pi f_c t + \frac{n\pi}{2}\right)$$

For $n=0$: the unmodulated carrier $A_c \cos 2\pi f_c t$ is obtain

- For Narrow band modulation, the condition $|K_p m(t)| \ll 1$ hence, ~~the~~ only the first two term are considerable, that is,

$$\boxed{S_{NBPM}(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} - \underbrace{A_c K_p}_{\Delta\phi} \underbrace{m(t) \sin 2\pi f_c t}_{\text{DSB}} .}$$

Notes:

eqn 1) show that, PM modulation is a nonlinear process. It is equivalent to DSB modulation using the same carrier freq, $x(t) = A_c \cos(2\pi f_c t)$ but different modulating signals $m(t), m^2(t), m^3(t), \ldots$ which are all centered at $f_c$. Therefore, ~~the spe~~ ctra of all DSB modulated signals overlap and the PM modulated signal

Eng. Sameh Fathy
Chapter 3
3rd year : Comm.

# CHAPTER 3

# Angle Modulation

let us Consider a generalized Carrier Signal $x_c(t)$ given by

$$x_c(t) = A_c \cos(\omega_c t + \phi(t))$$

$$= A_c \cos \theta_i(t)$$

Angle

frequency     phase

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$\theta_i(t) = 2\pi \int_{-\infty}^{t} f_i(t)\, dt$$

## • Frequency Modulation (FM):

the frequency $f_i(t)$ is varied linearly with $m(t)$

$$f_i(t) \propto m(t)$$

Frequency Sensitivity (Hz/V)

$$f_i(t) = f_c + K_f\, m(t)$$

$$\theta_i(t) = 2\pi \int_{-\infty}^{t} f_i(t)\, dt$$

$$= \omega_c t + 2\pi K_f \int_{-\infty}^{t} m(t)\, dt$$

$$S_{FM}(t) = A_c \cos(\omega_c t + 2\pi K_f \int_{-\infty}^{t} m(t)\, dt)$$

$$f_i(t) = f_c + K_f\, m(t)$$

$$f_i(t)\Big|_{max} = f_c + K_f\, m_{max}(t)$$

$$f_i(t)\Big|_{min} = f_c + K_f\, m_{min}(t)$$

if $m(t)$ is Symmetric Signal $(-m_p, m_p)$

## • Phase Modulation (PM):

the phase $\phi(t)$ is varied linearly with $m(t)$

$$\phi_i(t) \propto m(t)$$

Phase Sensitivity (rad/V)

$$\phi_i(t) = K_p\, m(t)$$

$$\theta_i(t) = \omega_c t + \phi_i(t)$$

$$= \omega_c t + K_p\, m(t)$$

$$S_{PM}(t) = A_c \cos(\omega_c t + K_p\, m(t))$$

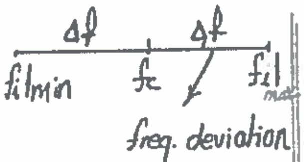
$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = f_c + \frac{K_p}{2\pi} \dot{m}(t)$$

$$f_i(t)\Big|_{max} = f_c + \frac{K_p}{2\pi} \dot{m}_{max}(t)$$

$$f_i(t)\Big|_{min} = f_c + \frac{K_p}{2\pi} \dot{m}_{min}(t)$$

if $\dot{m}(t)$ is Symmetric Signal $(-\dot{m}_p, \dot{m}_p)$

$f_i(t)\big|_{max} = f_c + K_f m_p$




$f_i(t)\big|_{min} = f_c - K_f m_p$

$\Delta f = \frac{f_i|_{max} - f_i|_{min}}{2}$

$\Delta f = K_f m_p$

$P_{FM} = A_c^2/2$

$B_{FM} = 2(B + \Delta f)$ Carson's rule

$f_i(t)\big|_{max} = f_c + \frac{K_p}{2\pi} m_p$

$f_i(t)\big|_{min} = f_c - \frac{K_p}{2\pi} m_p$

$\Delta f = \frac{f_i|_{max} - f_i|_{min}}{2}$

$\Delta f = \frac{K_p}{2\pi} m_p$

$P_{PM} = A_c^2/2$

$B_{PM} = 2(B + \Delta f)$ Carson's rule

where $B$: bandwidth of modulating Signal $m(t)$

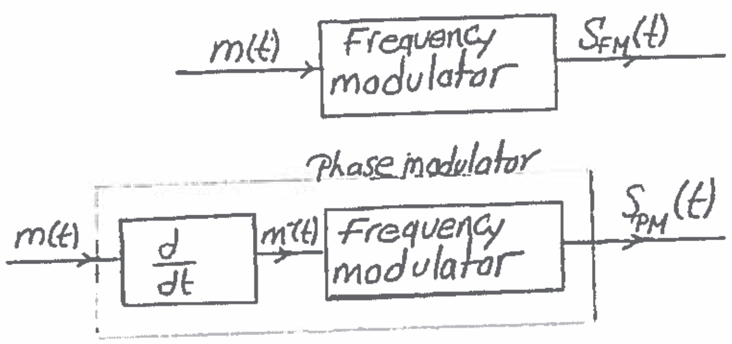
the modulation index or the deviation ratio $D$:

$D = \frac{\Delta f}{B}$

. Generalized Concept of Angle Modulation:

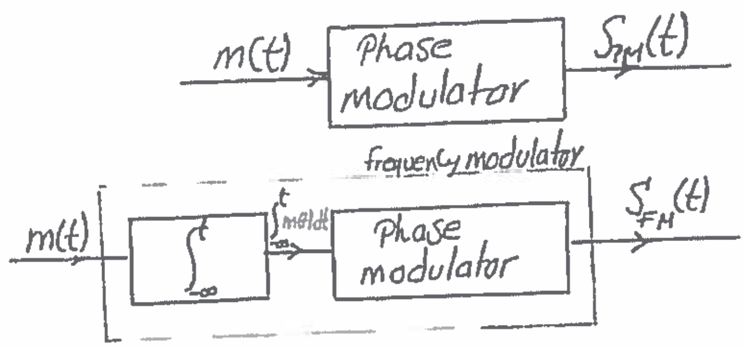
FM:

$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$

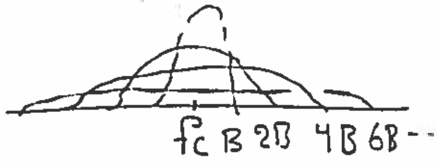



PM:

$\theta_i(t) = 2\pi \int_{-\infty}^{t} f_i(t)\,dt$



case infinite B.W, since, bandwidth of DSB signals are $2B, 4B, 6B, \ldots$




⇒ when $K_p$ is small, we can neglect the higher order of $K_p$ ⇒ i.e ⇒ NBPM.

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Example [4.6]

in north America, the maximum value of the frequency deviation is fixed at 75 KHz for commercial FM broadcasting by radio. Find the transmission bandwidth if the maximum freq of the audio frequency of interest in FM transmission is 15 KHz. also find the deviation ratio

<< Soln >>

$\Delta f = 75\,KHz$, $f_m = 15\,KHz$, $B_{FM} = ?$, $D = ?$

: Deviation ratio $[D] = \frac{\Delta f}{B} = \frac{\Delta f}{f_m} = \frac{75}{15} = 5 \equiv \beta$

B.W by using carson's role :

$$B_{FM} = 2(\Delta f + B) = 2(75 + 15) = 180\,KHz.$$

~~: [B.W] by using Bessel fn :~~

~~$B_{FM} = 2\, n_{max} f_m$, since $\beta = D = \frac{\Delta f}{B} = 5$~~

~~$\beta = 5$ ⇒ From table ⇒ $n_{max} = 8$~~

~~hence, $B_{FM} = 2 \times 8 \times 15 \times 10^3 = 240\,KHz$.~~

---

Example [4.7] :

a carrier signal of frequency 20 MHz ($f_c$) is frequency modulated by a sinsoidal signal of amplitude 5 volts and frequency 20 KHz. the frequency senstivity of the modulator is ~~20~~ 10 KHz/V.

1) Determine the approximate band width of FM signal using Carson's role.

2) Determine the band width ~~of~~ by transmitting only those side ~~ba~~ frequencies whose amplitudes exceed $\boxed{\text{1 percent}}$ of the unmodulated carrier amplitude using the universal curve of Bessel fn.

3) Repeat your calculation assume that the amplitude of the modulating signal is doubled.

4) Repeat your calculation assume the modulation frequency is doubled.

$f_m$

<< Soln >>

$f_c = 20\,MHz$, sin signal, $A_m = 5\,V$, $f_m = 20\,KHz$, $K_f = 10\,KHz/V$

a) $B_{FM} = 2(B + \Delta f)$ ← Carson's role

$= 2(\Delta f + B)$, $\Delta f = K_f\, m_p = 10\times10^{3}\times5 = 50\,KHz$.

$\Rightarrow B_{FM} = 2(50\times10^{3} + 20\times10^{3}) = 140\,KHz$.

b) select (calculate) $n_{max} = ?$

since, $\beta = \dfrac{\Delta f}{f_m} = \dfrac{50}{20} = 2.5 \Rightarrow n_{max} = 5$

$\Rightarrow B_{FM} = 2\, n_{max} f_m = 2(5)(20\times10^{3}) = 200\,KHz$.

c) $A_m$ is doubled $\Rightarrow \acute{A}_m = 10\,V$

$\Rightarrow \Delta f = K_f \acute{A}_m = 10\times10^{3}\times10 = 100\,KHz$

* using Carson's role:

$B_{FM} = 2(\Delta f + f_m) = 2(100 + 20\times10^{3}) = 240\,KHz$

→ Using Bessel fn table:

$$\beta = \frac{\Delta f}{f_m} = \frac{100}{20} = 5 \Longrightarrow n_{max} = 8$$

Hence, $B_{FM} = 2 n_{max} f_m = 2(8)(20 \times 10^3) = 320\, KHz.$

Notes
. as amplitude is doubled, the frequency deviation is doubled, β also is doubled (total freq deviation, includes more spectral lines).

1) $f_m$ is doubled ⇒ $f_m' = 40\,KHz.$

⇒ Using Carson's rule:

$$B_{FM} = 2(\Delta f + f_m') = 2(50+40) = 180\, KHz.$$

→ From Bessel table:

$$\beta = \frac{\Delta f}{f_m'} = \frac{50}{40} = 1.25 \Rightarrow n_{max} = 3$$

$$B_{FM} = 2 n_{max} f_m' = 2(3)(40) = 240\, KHz.$$

notes:
. as $f_m$ is changes → Δf is kept unchange,
→ $f_m$ is doubled ⇒ Hence, β is decrease to half
(no of spectral lines within (2Δf) decreased).

Example $\boxed{4.8}$:

A carrier signal of frequency ($f_c$) 2 MHz in phase modulated by a sinsoidal signal of amplitude 5 volts ($m_p$) and frequency ($f_m$) 4 KHz. the phase senstivity of the modulator ($k_p$) is $0.25\pi$ rad/V.

a) Determine the approximate bandwidth of the PM sig.

b) Repeat your calculation assume the amplitude of of the modulating signal is doubled.

c) Repeat your calculations assuming the modulation frequency is doubled.

Soln

$f_c = 2\ MHz$, $A_m = 5V$, $f_m = 4\ KHz$, $K_p = 0.25\pi\ rad/V$

a) $B_{PM} = ?$

→ Using Carson's rule:

$B_{PM} = 2(\Delta f + B)$ , $\Delta f)_{sym.} = \frac{K_p}{2\pi} m_p = \frac{K_p}{2\pi} * A_m 2\pi f_m$

$= K_p A_m f_m = \frac{\pi}{4}(5)(4\times10^3) = 5\pi\times10^3$

$= 15.7\ KHz$

$\Rightarrow B_{PM} = 2(15.7 + 4)$

$= 39.4\ KHz.$

→ Using Bessel fn table:

$B_{PM} = 2 n_{max} f_m$ , $\Delta\phi = K_p A_m = \frac{\pi}{4} * 5 = 3.9 \rightsquigarrow n = 6$

$B_T = 2 n_{max} f_m = 2(6)(4\times10^3) = 48\ KHz.$

---

B) amp. is doubled ⇒ $A_m = 10\ V$

→ If the amplitude of the modulating signal is doubled, the frequency deviation is doubled

$\Rightarrow \cancel{\Delta f = 2 (\frac{\pi}{24})(5) = 15.7/2 = 7.85 KHz}$

$\Rightarrow \Delta f = 2(15.7\times10^3) = 31.4\ KHz.$

→ Using Carson's rule:

$B_{PM} = 2(\Delta f + B) = 2(31.4 + 4) = 70.8\ KHz.$

⇒ Using Bessel fn:

since, amp. is doubled → $\Delta\phi$ is also doubled ⇒ $\Delta\phi = 7.85$

since, $\Delta\phi = 7.85 \rightarrow n_{max} = 9$.

Hence $B_{FM} = 2 n_{max} f_m = 2(9)(4\times10^3) = 72\ KHz$.

c) assume $f_m$ is doubled $\Rightarrow f_m' = 8\ KHz$.

since, $\Delta f)_{symmetric} = K_p A_m f_m' = \frac{\pi}{4}(5)(8) = 10\pi\times10^3\ Hz$
$= 31.4\ KHz$

$\Rightarrow B_{PM} = 2(\Delta f + B)$

$= 2(31.4 + 8) = 78.8\ KHz$..

$\rightarrow$ Using Bessel fn :

$\Delta\phi = K_p A_m = 7.85 \leftarrow$ no change $\Rightarrow n_{max} = 9$

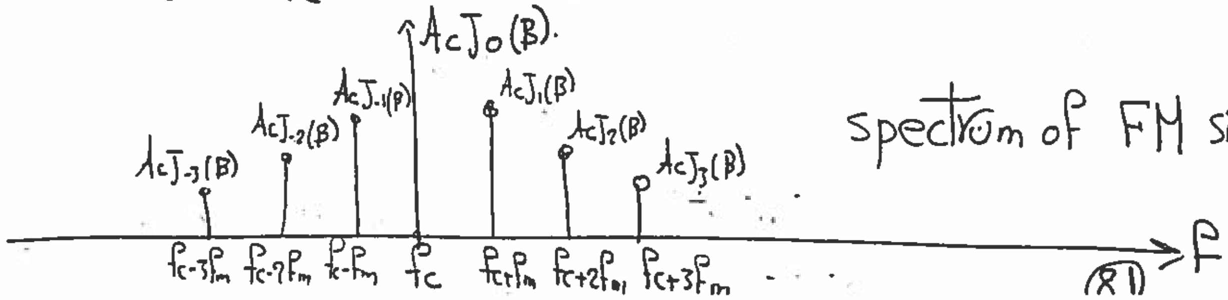
Hence, $B_{FM} = 2 n_{max} f_m' = 2(9)(8\times10^3) = 144\ KHz$. #

Example $\boxed{4.9}$

Consider a Sinsoidal signal of frequency 2KHz ($f_m$) that frequency modulates a carrier of frequency 500KHz ($f_c$) and amplitude $A_c = 2V$. the modulation index is $\beta = 5$. The FM signal is transmitted through a filter with magnitude transfer fn $H(f)$. Determine the magnitude spectrum and the power of the signal at the filter o/p if the filter transfer fn is

a) $H(f) = rect\left(\frac{f-f_c}{9f_m}\right)$ b) $H(f) = \frac{1}{\sqrt{1+\left(\frac{f-f_c}{4f_m}\right)^2}}$.

<<Soln>>




spectrum of FM signal

$f_m = 2KHz$, $f_c = 500KHz$, $A_c = 2V$, $\beta = 5$.

7] $H(f) = rect\left(\frac{f-f_c}{9f_m}\right)$.

$\Rightarrow$ This filter is bandpass filter with $\underline{B = 9f_m}$, Centered at $f_c$.

Since, $\beta = 5 \Longrightarrow n_{max} = 8$

<u>notes</u>: $B = 2Nf_m \Rightarrow 9\cancel{f_m} = 2N\cancel{f_m} \Rightarrow \underline{N = 9/2 \simeq 4}$

Hence, This filter passes Completly the carrier and the 1st four pairs of sidebands with amplitudes:

$J_0(5) = -0.18$, $\pm J_1(5) = \mp 0.33$, $\pm J_2(5) = 0.05$

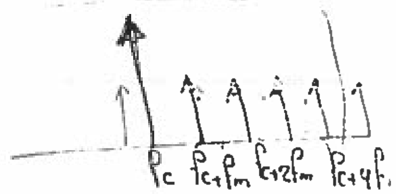
$\pm J_3(5) = \pm 0.36$, $\pm J_4(5) = 0.39$

$\Rightarrow$ The one sided spectrum of the signal $y(t)$ at the filter o/p is:

8-sideband Comp $\rightarrow$ $\boxed{H(f)}$ $\rightarrow$ 4-sideband Comp.

$$Y(f) = S(f)\,H(f)$$

$$= rect\left(\frac{f-f_c}{9f_m}\right)\sum_{n=-\infty}^{\infty} A_c J_n(\beta)\,\delta(f-f_c-nf_m)$$




$$\begin{cases} = A_c J_0(\beta)\delta(f-f_c) \\ + A_c J_1(\beta)\delta(f-f_c-f_m) + A_c J_{-1}(\beta)\delta(f-f_c+f_m) \\ + A_c J_2(\beta)\delta(f-f_c-2f_m) + A_c J_{-2}(\beta)\delta(f-f_c+2f_m) \\ + A_c J_3(\beta)\delta(f-f_c-3f_m) + A_c J_{-3}(\beta)\delta(f-f_c+3f_m). \\ + A_c J_4(\beta)\delta(f-f_c-4f_m) + A_c J_{-4}(\beta)\delta(f-f_c+4f_m). \end{cases}$$

Thus, The power of the signal at the filter o/p is

$$P_{out} = \frac{A_c^2}{2}\left[J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + 2J_3^2(\beta) + 2J_4^2(\beta)\right]$$

$$= \frac{4}{2}\left[(0.18)^2 + 2(0.33)^2 + 2(0.05)^2 + 2(0.36)^2 + 2(0.39)^2\right]$$

$$= 1.6372\ W$$

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- The power at the filter i/p is $\frac{A_c^2}{2} = 2W$.
- The decrease in the power at the filter output is due to the suppression of all frequency components for which $|f| \geq f + 5f_m$.

b] $H(f) = \dfrac{1}{\sqrt{1+\left(\frac{f-f_c}{4f_m}\right)^2}}$. $= \frac{1}{\sqrt{2}}$ $f = f_c + n f_m$


- The filter is not ideal and its transfer fn is depends on the frequency.

Hence, The filter o/p is :

$$Y(f) = S(f)\,H(f)$$

$$= \frac{1}{\sqrt{1+\left(\frac{f-f_c}{4f_m}\right)^2}} \cdot \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\,\delta(f-f_c-nf_m)$$

at $f = f_c + nf_m \Rightarrow H(f) = \dfrac{1}{\sqrt{1+\left(\frac{f_c+nf_m-f_c}{4f_m}\right)}} = \dfrac{1}{\sqrt{1+\left(\frac{n}{4}\right)^2}}$

ence, $Y(f) = \sum_{n=-\infty}^{\infty} \dfrac{1}{\sqrt{1+\left(\frac{n}{4}\right)^2}} \cdot A_c J_n(\beta)\,\delta(f-f_c-nf_m)$

$$\simeq A_c J_0(\beta)\,\delta(f-f_c)$$

$$+\frac{4}{\sqrt{17}} A_c J_1(\beta)\,\delta(f-f_c-f_m) + \frac{4}{\sqrt{17}} A_c J_{-1}(\beta)\,\delta(f-f_c \pm f_m)$$

$$+\frac{2}{\sqrt{5}} A_c J_2(\beta)\,\delta(f-f_c-2f_m) + \frac{2}{\sqrt{5}} A_c J_2(\beta)\,\delta(f-f_c+2f_m)$$

$$+\frac{4}{5} A_c J_3(\beta)\,\delta(f-f_c-3f_m) + \frac{4}{5} A_c J_{-3}(\beta)\,\delta(f-f_c+3f_m)$$

$$+\frac{1}{\sqrt{2}} A_c J_4(\beta)\,\delta(f-f_c-4f_m) + \frac{1}{\sqrt{2}} A_c J_{-4}(\beta)\,\delta(f-f_c+4f_m)$$

$$P_{out} = \frac{A_c^2}{2}\left[J_0^2(5) + 2\left(\frac{16}{17}\right)J_1^2(5) + 2\left(\frac{4}{5}\right)J_2^2(5) + 2\left(\frac{16}{25}\right)J_3^2(5) + 2\left(\frac{1}{2}\right)J_4^2(5)\right]$$

$$\therefore P_{out} = \frac{4}{2}\left[(0.18)^2 + 2\left(\frac{16}{17}\right)(0.33)^2 + 2\left(\frac{4}{5}\right)(0.05)^2 + 2\left(\frac{16}{25}\right)(0.36)^2 + 2\left(\frac{1}{2}\right)(0.39)^2\right] = 1.1188 \text{ W} \ \#.$$

---

**Example 4.10:**

A carrier signal is frequency modulated using a sinsoidal signal of frequency $f_m$ and amplitude $A_m$.

a) determine the values of the modulation index $\beta$ for which the carrier component of the FM signal is reduced to zero.

b) If $f_m = 1$ KHz and $A_m$ is increased starting from zero volts and the carrier component is reduced to zero for the first time at $A_m = 2$ volts, find the frequency senstivity of the modulator. What is the value of $A_m$ for which the carrier component reduced to zero for the second time?

«Soln»

a) $\beta = ? \Rightarrow$ The carrier component is $A_c J_0(\beta)\cos 2\pi f_c t$.

$\Rightarrow$ This component reduces to zero when $J_0(\beta) = 0$.

$\rightarrow$ From the Bessel fn table, $J_0(\beta) = 0$ for $\beta = 2.4, 5.6, 8.6$

---

b) $f_m = 1$ KHz, $A_m = 2 \rightarrow$ (carrier component reduce to zero for 1st time) $\Rightarrow \beta = 2.4$

since, $\beta = \frac{K_f m_p}{f_m} \Rightarrow K_f = \beta \cdot \frac{f_m}{m_p} = (2.4)\frac{(1\times10^3)}{2} = 1.2$ KHz/V.

$\Rightarrow$ For 2nd time $\Rightarrow \beta = 5.6$

since, $\beta = \frac{\Delta f}{f_m} = \frac{K_f m_p}{f_m} \Rightarrow A_m = m_p = \beta \cdot \frac{f_m}{K_f} = (5.6)\frac{(1\times10^3)}{(1.2\times10^3)} = 4.7$ V.

---

** If the 1st sideband is suppressed, find the value of $A_m$

«Soln»



$J_1(\beta) = 0 \rightarrow \beta = 7$

Hence $A_m = \beta \cdot \frac{f_m}{K_f} = (7)(1) \cdot = 5.8$ V

Example:

For the FM signal $s(t) = 100\cos[10^4\pi t + 0.05\sin(400\pi t)]$ is transmitted.

a) Find the instanteneous frequency of the modulated signal, the modulating signal, the carrier frequency and the frequency senstivity, the modulation index and the power of modulated signal.

b) Express the signal as a NBFM signal, find its spectrom, and find its power.

—« Soln »—

Since, $S_{FM}(t) = A_c\cos[2\pi f_c t + 2\pi K_f \int m(\alpha)d\alpha]$

$$= 100\cos[10^4\pi t + 0.05\sin(400\pi t)]$$

Since, $f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)$

$$= \frac{1}{2\pi}\frac{d}{dt}[10^4\pi t + 0.05\sin(400\pi t)]$$

$$= \frac{1}{2\pi}[10^4\pi + 0.05 \times 400\pi\cos(400\pi t)]$$

$$= \frac{10^4}{2} + \frac{0.05(400\pi)}{2\pi}\cos(400\pi t)$$

$$= 5\times10^3 + 10\cos(400\pi t)$$

Since, $f_i(t) = f_c + K_f m(t)$

Let $K_f = 10$ Hz/V , $m(t) = \cos(400\pi t)$ , $f_c = 5\times10^3$ Hz

$\beta = \frac{\Delta f}{f_m}$ , $\Delta f = K_f m_p = 10(1) = 10$

$2\pi f_m = 400\pi \Rightarrow f_m = \frac{400\pi}{2\pi} = 200$ Hz.

Hence, $\beta = \frac{10}{200} = 0.05$



$P = \frac{A_c^2}{2} = \frac{(100)^2}{2} = 5$ Kw.

⇒ Since, $S_{NBFM}(t) = A_c \cos 2\pi f_c t - 2\pi K_f A_c\, a(t) \sin 2\pi f_c t$

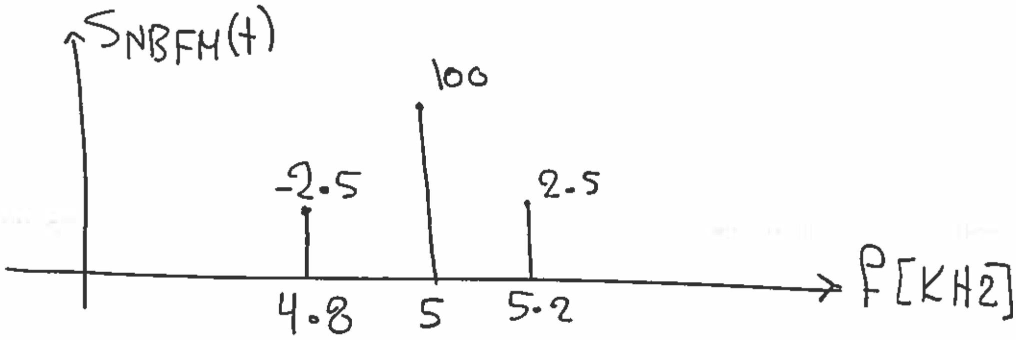
$$\Rightarrow S_{NBFM}(t) = 100 \cos 2\pi \times 5 \times 10^3 t - 2\pi (10)(100)\, a(t) \sin 2\pi \times 5 \times 10^3 t$$

Since, $a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha = \int_{-\infty}^{t} \cos(400\pi t) = \frac{1}{400\pi} \cdot \sin(400\pi t)$

Hence, $S_{NBFM}(t) = 100 \cos 10^4 \pi t - \frac{2000\pi}{400\pi} \sin(400\pi t) \sin(10^4 \pi t)$.

$$= 100 \cos 10^4 \pi t - \frac{5}{2}\left[\cos(10^4 - 400)\pi t - \cos(10^4 + 400)\pi t\right]$$

$$= 100 \cos 10^4 \pi t - \frac{5}{2}\cos(2\pi \times 4.8 \times 10^3 t) + \frac{5}{2}\cos(2\pi \times 5.2 \times 10^3 t)$$




«one-sided representation».

**Notes:**

NBFM provides amplitude modulation and phase Mod.

$$\Rightarrow P_{NBFM} = \frac{(100)^2}{2} + \frac{(2.5)^2}{2} + \frac{(2.5)^2}{2} = 5.00625 \text{ KW } \#.$$

## FM Generation :

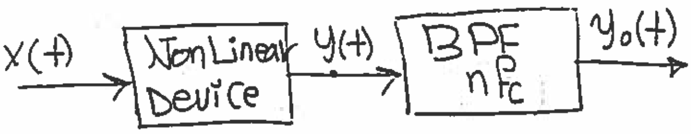
### Indirect [Armstrong] Method :

In this Method, NBFM signal is generated, this modulation can be implement using a DSB Modulator. The bandwidth of NBFM, is the same as in DSB which is 2B. This bandwidth can be increased by a factor n if the NBFM signal is passed through a frequency multiplier of order n. However the frequency multiplier increases the carrier [center freq] by the same factor, which may be undesirable Large. In this case a frequency converter is used to decrease carrier (center freq) without reduction for signal B.W.

### * Frequency Multiplier :

Freq Multiplier by order n can be realized by a nonLinear device of order n followed by BPF.

$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \cdots + a_n x^n(t)$




since, $x(t) = \cos 2\pi f_c t$.

hence, $y_{out}(t) = a_0 + a_1 \cos 2\pi f_c t + a_2 \cos^2 2\pi f_c t + \cdots + a_n \cos^n (2\pi f_c t)$

using BPF centered at $nf_c$ :

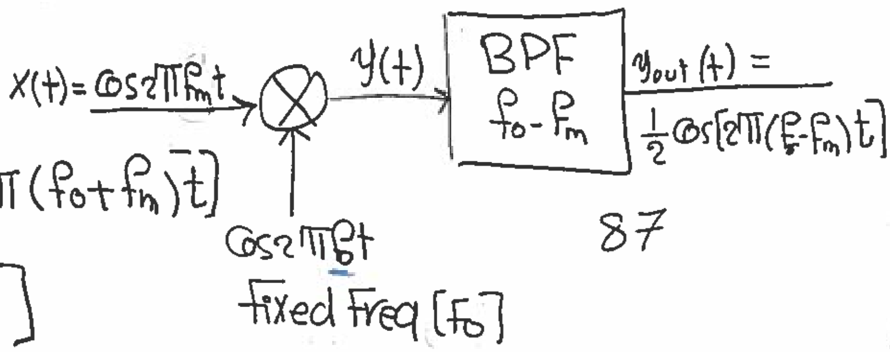
$$y_{out}(t) \doteq K \cos(2\pi n f_c t).$$

### * Frequency Converter :

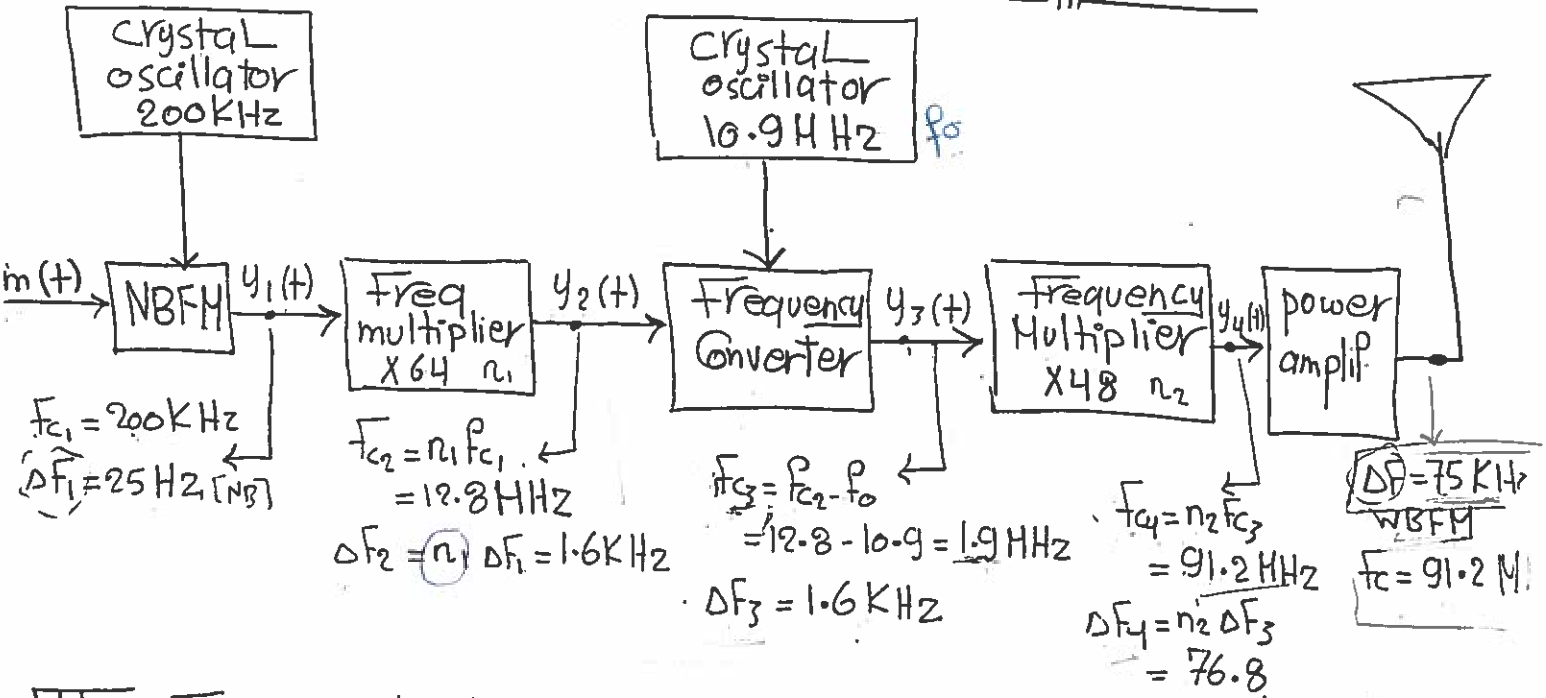
$y(t) = \cos(2\pi f_m t)\cos(2\pi f_0 t)$

$= \frac{1}{2}\cos[2\pi(f_0 - f_m)t] + \frac{1}{2}\cos[2\pi(f_0 + f_m)t]$

$y_{out}(t) = \frac{1}{2}\cos[2\pi(f_0 - f_m)t]$



# Simplified block diagram of a Commercial FM transmitter using Armstrong's method.




- The final output is FM signal with carrier frequency of 91.2 MHz and peak frequency deviation $\Delta f = 75$ KHz.

- NBFM provides a NBFM signal $y_1(t)$ that has:
  $f_{c_1} = 200$ KHz , $\Delta f_1 = 25$ Hz .

- To acheive $\Delta f = 75$ KHz, the total multiplication factor should be $n = \frac{\Delta f}{\Delta f_1} = \frac{75 \times 10^3}{25} = 3000$ → This can be implemented by two multiplier stage, since $\boxed{n = n_1 n_2 = 3000}$

⇒ $n_1 = 64$ and $n_2 = 48$, which provides a total multiplication factor of 3072.

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3008

$y_2(t)$: $f_{c_2} = n_1 f_{c_1} = 64 \times 200 \times 10^3 = 12.8$ MHz

$\Delta f_2 = n_1 \Delta f_1 = 64 \times 25 = 1.6$ KHz.

⇒ Frequency Converter [down Conversion] $f_{c_2}$ to $f_{c_3}$ and Keeping peak deviation Unchange.

$y_3(t)$: $f_{c_3} = f_{c_2} - f_o = 12.8$ MHz $- 10.9$ MHz $= 1.9$ MHz 88

$\Delta f_3 = 1.6$ KHz

## $2^{nd}$ Multiplier:

$y_4(t)$: $f_{c4} = n_2 f_{c3} = 48 \times 1.9 \times 10^6 = 91.2$ MHz

$\Delta F_4 = n_2 \Delta F_3 = 48 \times 1.6 \times 10^3 = 76.8$ KHz.

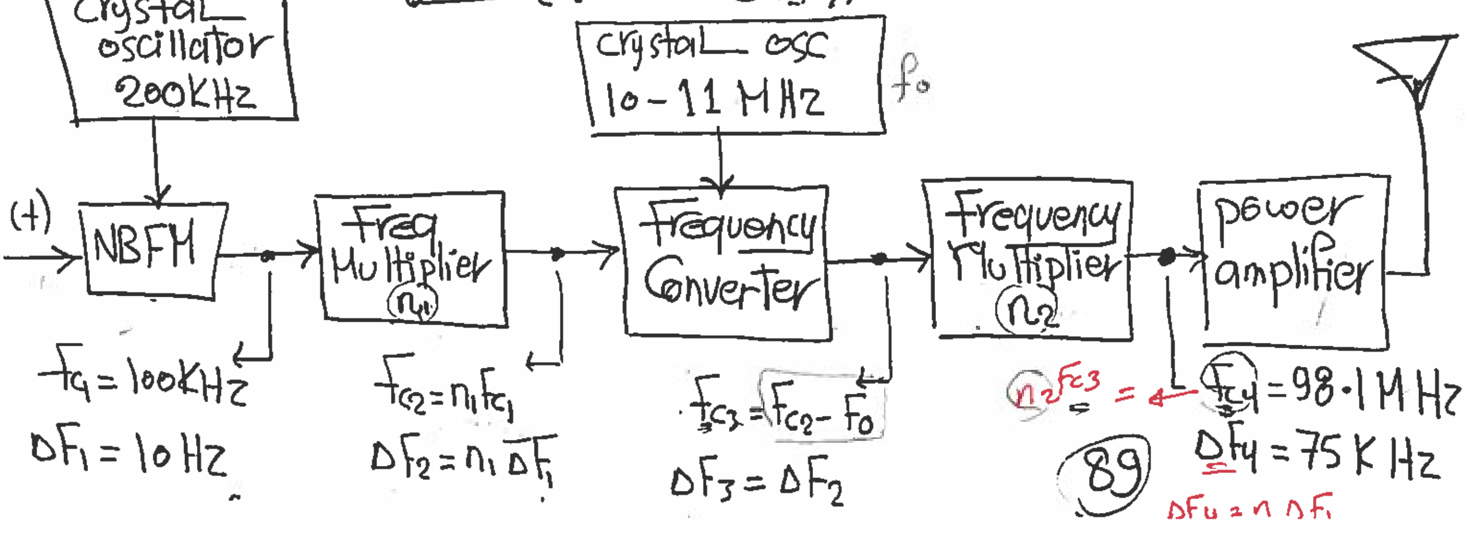
⇒ Then, FM signal is applied to an RF power amplifier that increase signal power to level enough for the message to be recovered properly at the RX.

- The Indirect FM generation has the advantage of frequency stability, but it is suffer from noise caused by the excessive multiplication and distortion at Lower modulating frequency, where $\beta$ is not small enough

## Example:

Design (only the block diagram) an Armstronge indirect FM modulation to generate an FM signal with carrier frequency of 98.1 MHz and peak deviation $\Delta F = 75$ KHz. A Narrowband FM generator is available at carrier frequency 100 KHz and frequency deviation 10 Hz. The stock room has also an oscillator with an adjustable frequency in the range of 10 to 11 MHz. There are also plenty of frequency doublers, triplers.

### Soln



$$\boxed{\text{since } f_{c3} = f_{c2} - f_o = \frac{f_{c4}}{n_2}}$$ VIP

$$\rightarrow f_{c3} = n_1 f_{c1} - f_o = \frac{f_{c4}}{n_2} \quad , \quad n = n_1 n_2 \rightarrow n_2 = \frac{n}{n_1}$$

$$\rightarrow f_{c3} = n_1 f_{c1} - f_o = \frac{f_{c4}}{n/n_1} = \frac{n_1 f_{c4}}{n}$$

$$\Rightarrow n_1 f_{c1} - f_o = \frac{n_1 f_{c4}}{n} \Rightarrow n_1 f_{c1} - \frac{n_1 f_{c4}}{n} = f_o \Rightarrow n_1 \left(f_{c1} - \frac{f_{c4}}{n}\right) = f_o$$

Hence, $$\boxed{n_1 = \frac{1}{\left[f_{c1} - \frac{f_{c4}}{n}\right]} f_o}.$$

since, total freq Multiplication factor $[n] = \frac{\Delta f_4}{\Delta f_1} = \frac{75\,KHz}{10\,Hz} = 7500$

Hence, $$n_1 = \frac{1}{\left(100 \times 10^3 - \frac{98.1 \times 10^6}{7500}\right)} \cdot (10 \rightarrow 11 \times 10^6)$$

$$n_1 = \frac{1}{86920} (10 \rightarrow 11 \times 10^6) = 115 \rightarrow 126.5$$

Select $n_1$ by number accept the dividing on 7500

Let $n_1 = 125 \Rightarrow n_2 = \frac{n}{n_1} = \frac{7500}{125} = 60.$

Hence:

$f_{c2} = n_1 f_{c1} = 125(100 \times 10^3) = 12.5\,MHz$

$\Delta f_2 = n_1 \Delta f_1 = 125(10) = 1.25\,KHz.$

$f_{c3} = f_{c2} - f_o = \frac{f_{c4}}{n_2} = \frac{98.1\,MHz}{60} = 1.635\,MHz.$

$\Delta f_3 = \Delta f_2 = 1.25\,KHz.$

#.

---

Example [4.14]:

Solve as previous example. $n_1 = \frac{1}{\left[f_{c1} - \frac{f_{c4}}{n}\right]} \cdot f_o$

go

Example 4.15 :

in Armstrong's FM transmitter, the audio signal has frequencies in the range of 100 Hz to 15 KHz. the NBPM is supplied with Carrier frequency of $f_c$ 100 KHz. The converter has an oscillator of frequency 9.5 MHz. The carrier frequency at the transmitter o/p is 100 MHz, the min freq deviation $\Delta f = 75$ KHz, and the maximum modulation index in the phase modulator = 0.3 radian

a) Calculate the multiplication ratios $n, n_1, n_2$

b) Specify the values of the carrier frequency and frequency deviation at the various points in the modulator.

« Soln »

$f_m = 100\,Hz \rightarrow 15\,KHz$, NBPM, $f_{c1} = 100\,KHz$, $f_o = 9.5\,MHz$

$f_{c4} = 100\,MHz$, $\Delta f_4 = 75\,KHz$, $\beta_{max} = 0.3$ rad.

Since, $\beta_{max} \uparrow = \frac{\Delta f_1}{\downarrow f_{min}} \Longrightarrow \Delta f_1 = \beta_{max} f_{min} = (0.3)(100) = 30\,Hz.$

Since, $n = \frac{\Delta f_4}{\Delta f_1} = \frac{75 \times 10^3}{30} = 2500 \rightarrow \cancel{n = \frac{f_{c4}}{f_{c1}}}$ X

since, $f_{c3} = f_{c2} - f_o = \frac{f_{c4}}{n} = \frac{n_1 f_{c4}}{n}$

$\Rightarrow n_1 = \frac{f_o}{\left[f_{c1} - \frac{f_{c4}}{n}\right]} = \frac{9.5 \times 10^6}{\left[100 \times 10^3 - \frac{100 \times 10^6}{2500}\right]} \simeq 158$

Since, $n_1 \simeq 158 \rightarrow n_2 = \frac{n}{n_1} = \frac{2500}{158} \simeq 16$

$f_{c2} = n_1 f_{c1}$

$\Delta f_2 = n_1 \Delta f_1$

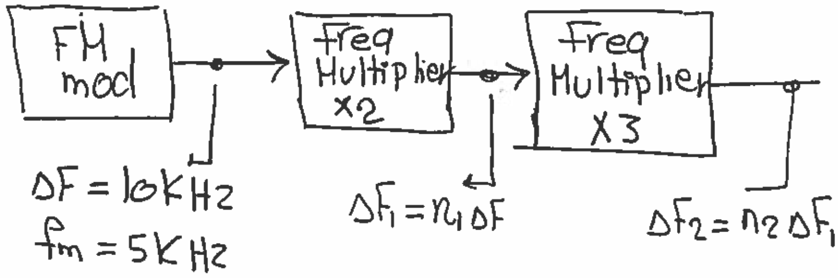
$f_{c3} = f_{c4}/n_2 = f_{c2} - f_o$

$\Delta f_3 = \Delta f_2$

Example 4.16 :

A FM signal with frequency deviation of 10 KHz at a modulation frequency of 5 KHz is applied to two frequency multipliers connected in cascade. The first multiplier doubles the frequency and the second multiplier triples the freq. Determine the freq deviation and the modulation index of the FM signal obtained at the second multiplier output.

<< Soln >>




1st Multiplier:

$$\Delta F_1 = n_1 \Delta F = 2\Delta F = 20\,KHz.$$

$$\beta_1 = \frac{\Delta F_1}{f_m} = \frac{20\,KHz}{5\,KHz} = 4$$

2nd Multiplier:

$$\Delta F_2 = n_2 \Delta F_1 = 3\Delta F_1 = 3(20) = 60\,KHz.$$

$$\beta_2 = \frac{\Delta F_2}{f_m} = \frac{60\,KHz}{5\,KHz} = 12 \quad \#$$

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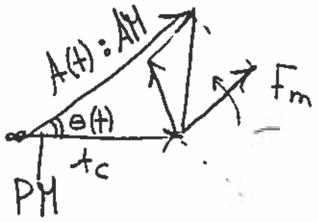
## * Distortion in Armstrong's method:

The NBFM signal is given by

$$S_{NBFM}(t) = A_c \cos(2\pi f_c t) - 2\pi k_f A_c\, a(t) \sin 2\pi f_c t.$$

$$= E(t)\cos[2\pi f_c t + \theta(t)]$$




where, $E(t) = A_c\sqrt{1+(2\pi k_f)^2 a^2(t)}$

and $\theta(t) = \tan^{-1}[2\pi k_f\, a(t)]$.

## * There are two kind of distortion:

A] Amplitude distortion: it takes places, when the envelope $E(t)$ varies with time. This amplitude variatio Can be eleminate by using Limiter.

b] Frequency distortion:

the instantaneous phase, $\theta_i(t) = 2\pi f_c t + \theta(t)$

and, the inst frequency, $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$.

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[2\pi f_c t + \theta(t)]$$

$$= \frac{1}{\cancel{2\pi}}\cdot \cancel{2\pi} f_c + \frac{1}{2\pi}\frac{d}{dt}\theta(t), \quad \theta(t) = \tan^{-1}[2\pi k_f\, a(t)]$$

$$= f_c + \boxed{\frac{1}{2\pi}\cdot\frac{2\pi k_f m(t)}{1+[2\pi k_f a(t)]^2}} \equiv \dot\theta(t)$$

$\tan^{-1} x = \frac{1}{1+x^2}$

$$\dot\theta(t) = \frac{2\pi k_f m(t)}{1+[2\pi k_f]^2 a^2(t)}, \quad \frac{1}{1+x} = 1 - x + x^2 - \dots$$

$$= 2\pi k_f m(t)\left[1 - [2\pi k_f]^2 a^2(t) + [2\pi k_f]^4 a^4(t) + \dots\right]$$

Since, $f(t) = f_c + k_f m(t) \Rightarrow f_c + \Delta f(t)$

hence, $\Delta f(t) = \frac{1}{\cancel{2\pi}}\cancel{2\pi} k_f m(t)\left[1 - [2\pi k_f]^2 a^2(t) + [2\pi k_f]^4 a^4(t) + \dots\right]$

ence, $\Delta F(t) = K_f m(t)\left[1 - [2\pi K_f]^2 a^2(t) + [2\pi K_f]^4 a^4(t) + \cdots\right]$

Since, The frequency deviation should be change linearly with the modulating signal.

Hence, $\boxed{\ }$ Only First term is desired while the other terms are undesired, this causes that signal is distorted at the receiver o/p.

* Third Harmonic Distortion [THD] $\equiv [D_3]$

Defined as, the ratio of the third harmonic level to the desired Level, that is:

$$D_3 = \frac{V_3}{V_1} = \frac{\beta^2}{4-\beta^2} \leftarrow \text{assuming tone Modulation.}$$

Example

Calculate the percentage of the third harmonic distortion assuming the modulating signal is sinsoidal with 5V amplitude and 5KHz freq. the frequency senstivity is 1000 Hz/V.

« Soln »

$D = ?$, $m_p = 5V$, $f_m = 5\,KHz$, $K_f = 1000\,Hz/V$.

Since, $\beta = \frac{\Delta F}{f_m} = \frac{K_f m_p}{f_m} = \frac{(1000)(5)}{5\times 10^3} = 1$.

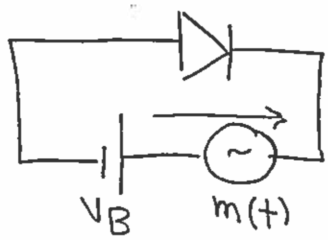
Hence $D_3 = \frac{\beta^2}{4-\beta^2} = \frac{1}{4-1} = \frac{1}{3} * 100\% = 33\%$



Note D: given →

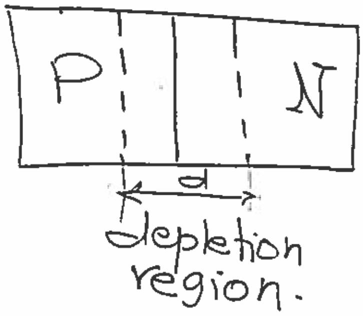
## * Direct FM Generation :

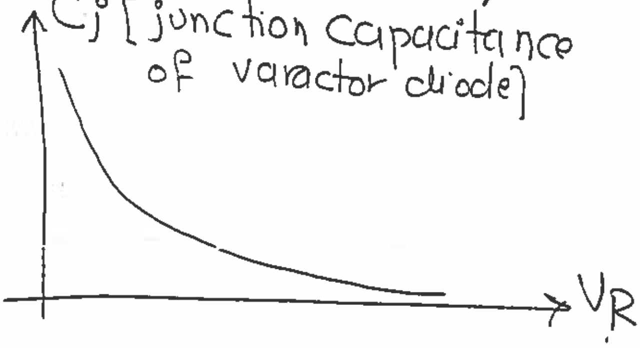
Direct FM generation is [VCO].
The oscillation frequency depends Linearly on the applied voltage.

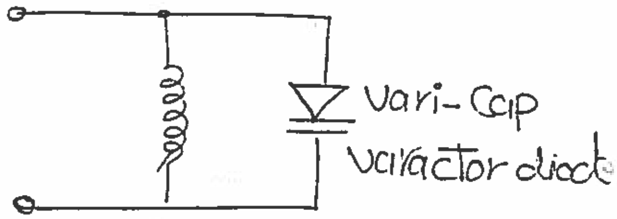



- A reverse bias semiconductor diods acts as a Capacitor whose Capacitance varies Linearly with the bias voltage. [varicap]

- The DC reverse bias using battery $V_B$ such that it remains reverse biased for all values of $m(t)$ such that, $V_B + m(t) > 0$




$$\cancel{f_o} = \frac{1}{2\pi\sqrt{L\cancel{C}}}$$

Hence, as $V_R = m(t) \uparrow \rightarrow C_j \downarrow \rightarrow f_o \uparrow$

- The Capacitance of depelation region of [varactor] diode is given by :

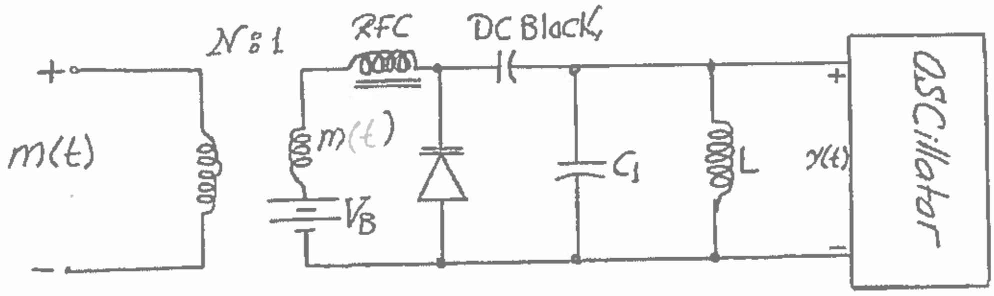
$$C_d = \frac{\epsilon A}{d} = \frac{\epsilon A}{d_o + K' m(t)}$$

where, A : area of pn junction.
$\epsilon$ : the premitivity of the junction.
d : depletion region width.
$d_o$ : is the spacing (depletion region) when $m(t) = 0$

$$\text{Hence, } C_d = \frac{\epsilon A}{d_o + K' m(t)} = \frac{\epsilon A}{d_o\left[1 + \frac{K' m(t)}{d_o}\right]}$$

$$= \frac{\epsilon A}{d_o}\left[1 + \frac{K' m(t)}{d_o}\right]^{-1} = C_o \left[1 + \frac{K' m(t)}{d_o}\right]^{-1}$$

## Direct Generation of FM Signals:




VCO Circuit with varactor diode for variable reactance

- VCO : Voltage Controlled Oscillator.
- A reverse-biased Semiconductor diode acts as a Capacitor (varicap or varactor) whose Capacitance varies linearly with the bias voltage.
- The modulating Signal m(t) is applied to (VCO) which provides an oscillation frequency that varies linearly with the applied Signal.
- The varactor is dc reverse biased using the battery voltage $V_B$ such that it remains reverse biased for all values of m(t) $[m(t) + V_B > 0]$.
- The I/P transformer, RF Choke (RFC), and dc block serve to isolate the low frequency, the high frequency, and the dc voltages.
- The major disadvantage with this type of Circuit is that the Carrier frequency tends to shift and it must be stabilized by feedback frequency Control. For this reason many older FM transmitters are of the indirect type (Armstrong method).

$$f_i(t) = f_c + K_f\, m(t)$$

where, $C_o' = \frac{\epsilon A}{d_o}$ is the capacitance of the diode when $m(t) = 0$.

assuming that $d_o$ is Large : $K'/d_o \ll 1$

Hence, $C_d = C_o' \left[1 - \frac{K' m(t)}{d_o} + \left(\frac{K'}{d_o}\right)^2 \frac{m^2(t)}{2!} - \cdots\right]$

$\therefore C_d \simeq C_o' - \frac{C_o' K' m(t)}{d_o}$ , $K = \frac{C_o' K'}{d_o}$

Hence, $\boxed{C_d \simeq C_o' - K m(t)}$

Hence, $C_d$ decrease by increasing $m(t)$.

- The total Capacitance of the tuned circuit becomes:

$C(t) = C_o + C_d(t)$

where, $C_o = C_o' + C_1$ is the total Capacitance when $m(t) = 0$

Hence, $\boxed{C(t) = (C_o' + C_1) - K m(t)}$

* The instantaneous frequency $f_i(t)$ of the oscillator is given by:

$$f_i(t) = \frac{1}{2\pi\sqrt{LC(t)}} = \frac{1}{2\pi\sqrt{L[C_o - Km(t)]}} = \frac{1}{2\pi\sqrt{LC_o[1 - \frac{K}{C_o}m(t)]}}$$

$$\boxed{f_i(t) = \frac{1}{2\pi\sqrt{LC_o}} \cdot \left[1 - \frac{K}{C_o} m(t)\right]^{-1/2}}$$

inf:

$$(1+x)^n \simeq 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots$$

Hence, $f_i(t) = \frac{1}{2\pi\sqrt{LC_o}} \left[1 + \left(\frac{-1}{2}\right)\left(\frac{-K}{C_o}\right)m(t) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{2!}\left(\frac{-K}{C_o}\right)^2 m^2(t) + \cdots\right]$

$$= f_c \left[1 + \left(\frac{1}{2}\right)\left(\frac{K}{C_o}\right)m(t) + \left(\frac{3}{8}\right)\left(\frac{K}{C_o}\right)^2 m^2(t) + \cdots\right]$$

$$f_i(t) = f_c + \frac{1}{2} f_c \left(\frac{K}{C_o}\right) m(t) + \frac{3}{8} f_c \left(\frac{K}{C_o}\right)^2 m^2(t) + \cdots$$

- In Ideal FM modulation, the instantaneous freq is:

$\boxed{f_i(t) = f_c + k_f\, m(t)}$ where $\boxed{k_f = \frac{K f_c}{2 C_o}}$

while the others terms are undesired.

- The freq of the modulated signal depends on $m(t)$, $m^2(t)$, $m^3(t)$, and so on.
- The desired signal has a bandwidth B while the undesired signal have bandwidth 2B, 3B, --

Hence, The spectra of the undesired signals overlap with the spectrum of the desired signal, cannot be Filtered. which causes « Distortion of the desired signal ».

* If Only the undesired signal that varies with $m^2(t)$ is Considered, The percentage distortion due to this term [i.e: percentage distortion due to non Linearity]

$$\boxed{D} = \frac{\text{undesired signal}}{\text{Desired signal}} = \frac{\frac{3}{8}\cancel{f_c}\left(\frac{K}{C_o}\right)^2 m^2(t)}{\frac{1}{2}\cancel{f_c}\left(\frac{K}{C_o}\right) m(t)} = \boxed{\frac{3}{4}\frac{K}{C_o} m_p}$$

Example $\boxed{4.13}$ :

In Direction FM generation, the modulating signal $m(t)$ is used a reverse voltage for the varactor. The total capacitance of the tuned circuit is $C(t) = C_o - K m(t)$ where $C_o = 10^{-7}$ F, the oscillation frequency $f_c = 1$ MHz when $m(t) = 0V$. Determine $k_f$, $\Delta f$ and the percentage distortion due to non Linearity assuming $m(t)$ have a peak voltage of 5V. Find the peak voltage such that this error is less than 0.02.

$\longleftarrow$ Soln $\longrightarrow$

$C(t) = C_o - K m(t)$ , $C_o = 10^{-7}$ F , $f_c = 1$ MHz $\longrightarrow m(t) = 0$ V

$K = 10^{-9}$ F/V . Determine $K_f$ , $\Delta F$ , $D \rightarrow m_p = 5$ V .

since, $K_f = \frac{K f_c}{2 C_o} = \frac{10^{-9} \times 1 \times 10^{6}}{2 \times 10^{-7}} = 5$ KHz/V .

since, $\Delta f = K_f m_p = 5 \times 10^{3} \times 5 = 25$ KHz .

since, $D = \frac{3}{4} \frac{K}{C_o} m_p = \frac{3}{4} \frac{10^{-9}}{10^{-7}} 5 \times 100\% = 3.75\%$.

$\Rightarrow$ Find $m_p = ?$ $\longrightarrow D \leqslant 0.02$ .

Since, $D = \frac{3}{4} \frac{K}{C_o} m_p \Rightarrow 0.02 = \frac{3}{4} \cdot \frac{10^{-9}}{10^{-7}} \cdot m_p$

Hence, peak-voltage $[m_p] = \frac{0.02 \times 4}{3 \times 10^{-2}} = 2.67$ V .

---

Example $\boxed{4.14}$ :

Consider a varactor is connected in parallel with an inductor forming a resonant circuit for an oscillator. the capacitance of the pn jonction is $C(t) = \frac{K'}{\sqrt{1 + 0.5\, v(t)}}$ where, $v(t) = V_B + m(t)$ . is the reverse bias voltage, and $m(t)$ is the modulating signal.

The parallel LC circuit is toned to the center frequenc $f_c = 2$ MHz when $V_B = 4$ volts . Determine $K_f$ and $\Delta F$ such that the distortion due to nonLinearity of the frequency-voltage chr doesnot exceed 2% .

$\longleftarrow$ Soln $\longrightarrow$

at $v(t) = V_B = 4$V $\longrightarrow m(t) = 0$V , $f_i(t) = f_c = 2$ MHz .

since, $f_i(t) = \frac{1}{2\pi \sqrt{L C(t)}} = \frac{1}{2\pi \sqrt{L \cdot \frac{K'}{\sqrt{1 + 0.5 v(t)}}}} = \frac{1}{2\pi \sqrt{L K' \left[\frac{1}{\sqrt{1 + 0.5 v(t)}}\right]}}$

$$f_i(t) = \frac{1}{2\pi\sqrt{LK'}} \cdot \frac{1}{[1+0.5v(t)]^{-1/4}}$$

Hence, $\boxed{f_i(t) = \frac{1}{2\pi\sqrt{LK'}} \cdot [1+0.5v(t)]^{1/4}} \rightarrow$

$\Rightarrow$ at $v(t) = V_B \rightarrow f_i(t) = f_c$

Hence, $\boxed{f_c} = \frac{1}{2\pi\sqrt{LK'}} \cdot [1+0.5V_B]^{1/4} = \boxed{\frac{[1+0.5V_B]^{1/4}}{2\pi\sqrt{LK'}}}$

$$\frac{f_i(t)}{f_c} = \frac{[1+0.5v(t)]^{1/4}}{[1+0.5V_B]^{1/4}} \quad , v(t) = V_B + m(t)$$

$$= \frac{[1+0.5V_B+0.5m(t)]^{1/4}}{[1+0.5V_B]^{1/4}}$$

$$\Rightarrow f_i(t) = f_c\left[\frac{1+0.5V_B+0.5m(t)}{1+0.5V_B}\right]^{1/4} = f_c\left[1+\frac{0.5m(t)}{1+0.5V_B}\right]^{1/4}$$

$$= f_c\left[1+\frac{1}{6}m(t)\right]^{1/4}.$$

Hint:

$$[1+x]^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

Hence, $f_i(t) = f_c\left[1+\left(\frac{1}{4}\right)\left(\frac{1}{6}\right)m(t) + \frac{(1/4)(-3/4)}{2!}\frac{1}{36}m^2(t) + \cdots\right]$

$$= f_c\left[1+\frac{1}{24}m(t) - \frac{1}{384}m^2(t) + \cdots\right]$$

Hence, $f_i(t) = f_c + \frac{1}{24}f_c m(t) - \frac{1}{384}f_c m^2(t) + \cdots \longrightarrow 1]$

Since $f_i(t) = f_c + K_f\, m(t) \longrightarrow 2]$

Comparing equation 1], 2]

Hence, $K_f = \frac{f_c}{24} = \frac{2\times10^6}{24} = 83.3$ KHz/V.

Since, $D = \frac{\frac{1}{384}\cancel{f_c}\, m^2(t)}{\frac{1}{24}\cancel{f_c}\, m(t)} = \frac{24\, m_p^2}{384\, \cancel{m_p}} = \frac{1}{16}m_p$

Hence, $m_p = 0.02(16) = 0.32$ volts.

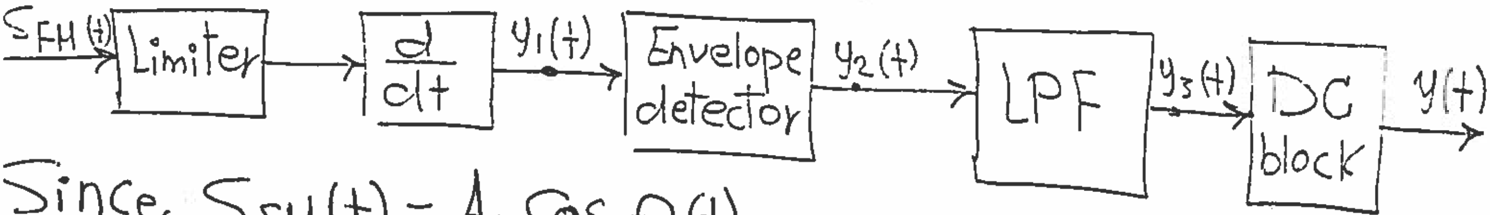
Hence, $\Delta F = K_f\, m_p$

$$= 83.3 \times 10^{3} \times 0.32 = 26.67 \text{ KHz}.$$

## ** Frequency Detection :

A frequency detector, produces an output voltage that should vary Linearly with the instantaneous frequency of the input.

### * FM-to-AM Converter [slope Detector] :




Since, $S_{FM}(t) = A_c \cos \theta_i(t)$

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int m(t)\,dt$$

$$\Rightarrow y_1(t) = \frac{d}{dt}\left[A_c \cos \theta_i(t)\right] = -A_c\, \dot{\theta}_i(t) \sin \theta_i(t).$$

Hence, $y_1(t) = -A_c\left[2\pi f_c + 2\pi K_f m(t)\right] \sin \theta_i(t)$.

Hence, the o/p of differentiator is an FM signal but it has an amplitude (envelope) changes according to modulating signal.

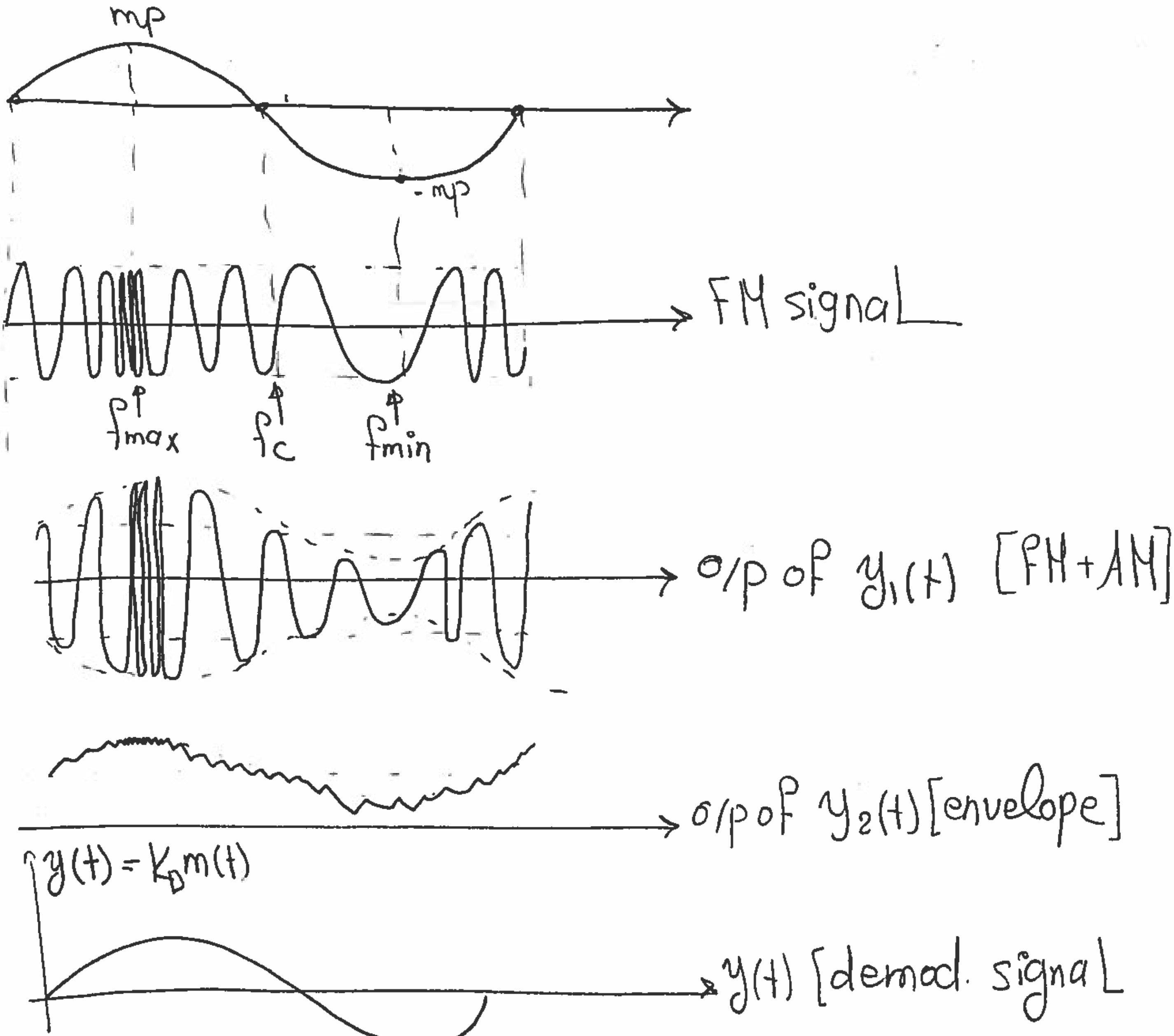
Hence, $y_2(t) = K\left[f_c + K_f\, m(t)\right] = K f_c + K K_f\, m(t)$

where, $K$, is the envelope detector constant.

- After LPF, DC block, the o/p becomes.

$$y(t) = K_D\, m(t)$$

where, $K_D$ is a Constanst that includes the frequency sensitivity.




* **adv of angle Mod. Compared to amplitude Mod:**

1] immunity against noise and interferance.

2] immunity against rapid fading.

3] in Angle mod there is found a tradeoff between power and band width.

4] Angle Mod is Less Sensitive to non Linearity.

* **disadv of angle Mod Compared by amplitude Mod:**

1] Transmission band width is Larger than amp. mod.

2] Complexity in design for Mod/Demod than amp. mod.

3] The Cost of FM is more expensive than AM.

« Good Luck »

|A|

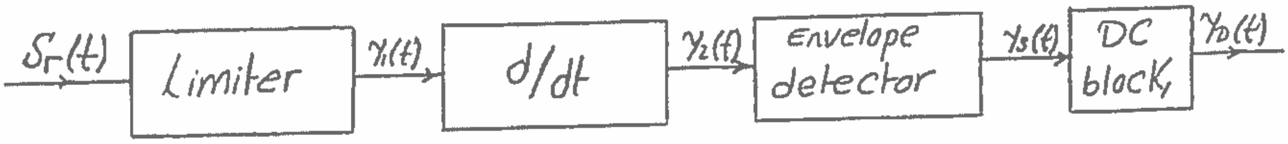
# Frequency Demodulation

## Introduction Information :

A frequency detector, often Called a discriminator, produces an output voltage that should vary linearly with the instantaneous freq. of the input.

VIP

## (A) FM_to_AM Converter : differential FM Demodulator:




$r_1(t) = S_{FM}(t)$ A limiter is a CirCuit Whose output is a Constant amplitude for all inputs above a Critical value.

$$r_2(t) = \frac{d}{dt} r_1(t) = \frac{d}{dt}\left[A_c \cos\left(2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(t)\,dt\right)\right]$$

$$r_2(t) = A_c\left(W_c + 2\pi K_f\, m(t)\right) \cos\left(2\pi f_c t + 2\pi K_f \int_{-\infty}^{t} m(t)\,dt\right)$$

$$r_3(t) = 2\pi A_c\left(f_c + K_f\, m(t)\right)$$

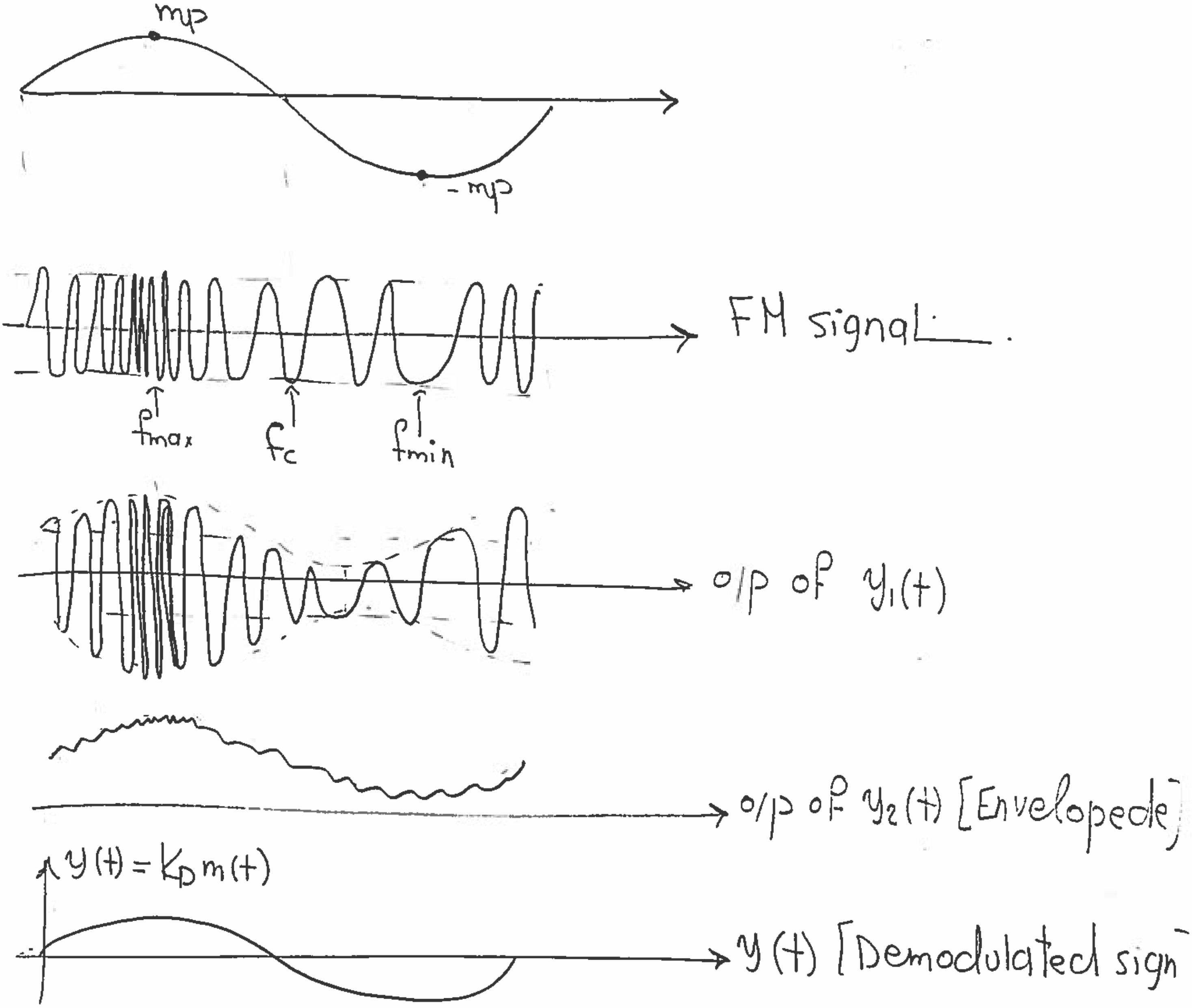
$$= K\left(f_c + K_f\, m(t)\right)$$

where $K = 2\pi A_c$

Since a dc block will remove the Constant Carrier frequency offset from the output Signal.

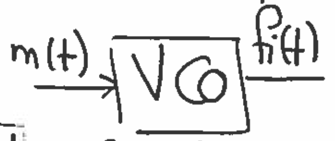
$$r_D(t) = K_D\, m(t)$$

where $K_D = K K_f = 2\pi A_c K_f$



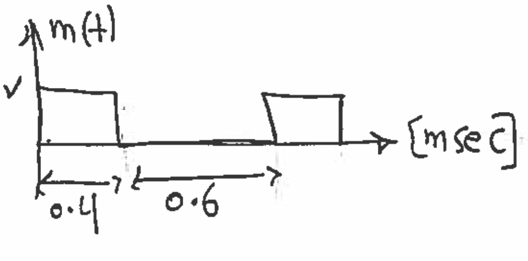

## Example:

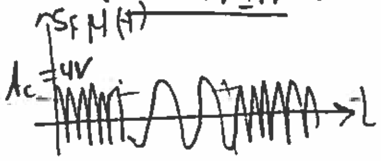
A periodic rectangular signal m(t) of a period $T = 1\,msec$, duration $\tau = 0.4\,msec$, peak amplitude of 5V, freq modulate a carrier of frequency 1MHz and amplitude of 4V with Frequency sensitivity $K_f = 2000\,Hz/V$. The signal is demodulated using the method of direct differentiator. Draw the block diagram of Mod/Demod, writ and sketch the wave forms at each block.

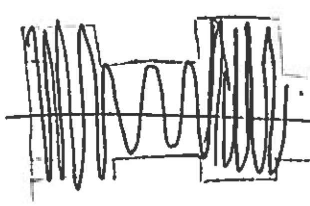



$f_i(t) = f_c + K_f\, m(t)$

Soln



Chapter 4

4-1. Consider a tone modulated FM or PM signal with $f_m = 10\,kHz$, $\beta = 2$, $A_c = 100$ and $k_f = 50\,kHz$ [illegible] instantaneous freq $f(t)$, draw [illegible] the modulated signal, and calculate the transmitted power directly and from the spectral components.

Ans.

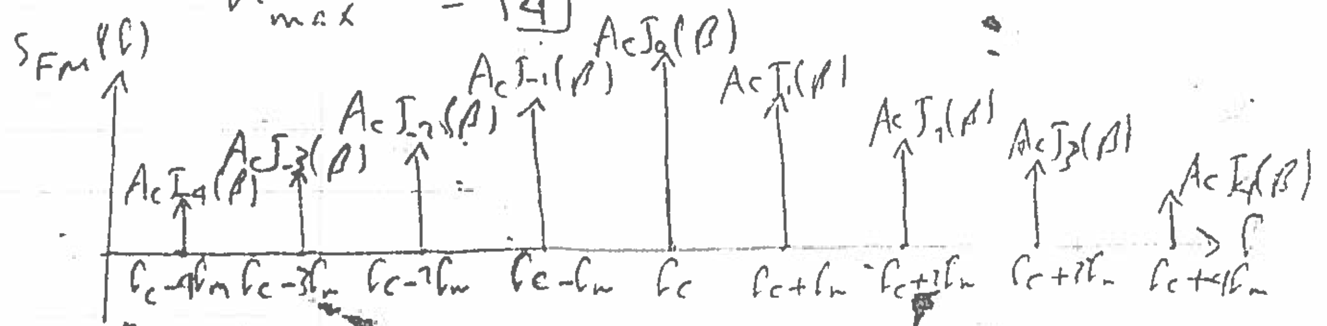
for FM: $\longrightarrow f_i(t) = f_c + K_f\, m(t)$

for PM: $\longrightarrow f_i(t) = f_c + \frac{K_p}{2\pi}\, m'(t)$

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m) t)$$

$J_n(2) \longrightarrow$ from Bessel's table

$n_{max} = \boxed{4}$




$$S_{FM}(t) = A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos(2\pi(f_c + f_m) t)$$
$$+ A_c J_{-1}(\beta) \cos(2\pi(f_c - f_m) t) + A_c J_2(\beta) \cos(2\pi(f_c + 2f_m) t)$$
$$+ A_c J_{-2}(\beta) \cos(2\pi(f_c - 2f_m) t) + A_c J_3(\beta) \cos(2\pi(f_c + 3f_m) t)$$
$$+ A_c J_{-3}(\beta) \cos(2\pi(f_c - 3f_m) t) + A_c J_4(\beta) \cos(2\pi(f_c + 4f_m) t)$$
$$+ A_c J_{-4}(\beta) \cos(2\pi(f_c - 4f_m) t)$$

b-

same BW

$(\beta) = J_n(\Delta\Phi)$

$\Delta\Phi = \beta$

$K_p A_m = \beta$

$$K_p = \frac{\beta}{A_m} = \frac{\beta}{1} = \beta$$

e- For the carrier to be suppressed

$\beta = 2.4 \longrightarrow$ from Bessel's table

$\beta = \frac{\Delta f}{B}$

$\Delta f = \beta B = 2.4 \times 2000 = 4.8\ KHZ$

$\Delta f = K_f A_m$

$$A_m = \frac{\Delta f}{K_f} = \frac{4.8 \times 10^3}{10^4 \pi} = 0.2V$$

power of 1'st sidebands $= \frac{A_c^2}{2} (2 J_1^2(\beta))$

$= A_c^2 J_1^2 (2.4) = (10)^2 . (0.52)^2$

$= 27.04\ w$

4-19) A signal $m(t) = A_m \sin(4000\pi t)$ is used to FM modulate a carrier signal of 10 Volts amplitude and 1 MHZ frequency with $K_f = 10^4\pi$.

a) Assuming $A_m = 1$, sketch the instantaneous freq of the FM signal and find its BW using both Carson's rule and Bessel function tables.

b) Assuming phase modulation, find the value of $K_p$ such that the PM signal has the same bandwidth as FM signal.

c) Repeat part (a) if the amplitude of $m(t)$ is doubled

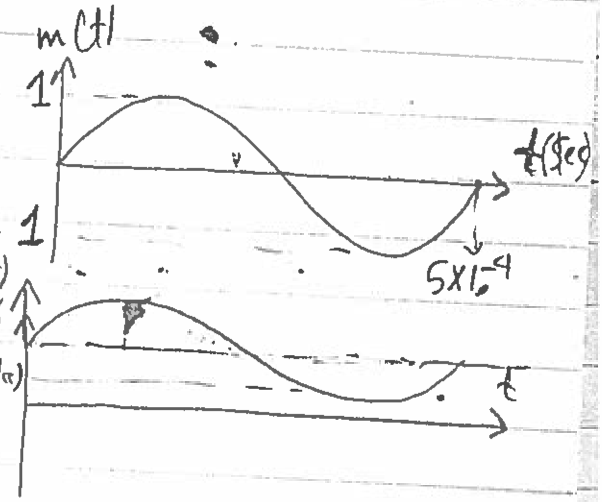
d) Repeat part (a) if the freq. of $m(t)$ is doubled

e) find the value of $A_m$ such that the carrier of the FM signal is suppressed. in this case find the power of first sidebands.

Ans:

a) $f_i(t) = f_c + K_f\, m(t)$

$= 10^6 + (10^4\pi\ m(t))$




from Carson's Rule;

$B = 2(\Delta f + f_m)$ & $\Delta f = K_f\, m_p f_i(t)$

$\Delta f = 10^4\pi \times 1 = 10^4\pi$

$\therefore B = 2(10^4\pi + 2000)$

$= 66.8$ KHZ

$\beta = \frac{\Delta f}{f_m} = \frac{10^4\pi}{2000} = 5\pi = 15.7$

$$C_V(t) = C_2 V_B \left(1 - \frac{1}{2}\frac{m(t)}{N V_B}\right)$$

$$= C_2 V_B - \frac{1}{2} C_2 \frac{m(t)}{N}$$

$$= C_0 - K\,m(t) \longrightarrow \boxed{K = \frac{C_2}{2N}} \quad \boxed{C_0 = C_2 V_B}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{L C(t)}} = \frac{1}{2\pi\sqrt{L C_0\left(1 - \frac{K m(t)}{C_0}\right)}}$$

$$= \frac{1}{2\pi\sqrt{L C_0}}\left(1 - \frac{K m(t)}{C_0}\right)^{-\frac{1}{2}}$$

$$= f_c\left[1 + \frac{K m(t)}{2 C_0} + \frac{3}{8}\left(\frac{K}{C_0}\right)^2 m^2(t) + \cdots\right]$$

$$\Delta f = f_c\,\frac{K m(t)}{2 C_0} = \frac{f_c C_2 m(t)}{4 C_0 N} = \frac{f_c \cancel{C_2} m(t)}{4 \cancel{C_2} V_B N}$$

$$= \frac{f_c\, m(t)}{4 \times 300} = f_c m(t)/300$$

divide by $m(t)$ for normalization

$$\therefore \Delta f < \frac{f_c}{300}$$

4-17) The equivalent tuning capacitance in fig 4.12 is $C(t) = C_1 + C_v(t)$ where $C_v(t) = \frac{C_2}{\sqrt{V_B + m(t)/N}}$

show that $C(t) \approx C_0 - C\,m(t)$ with 1 percent accuracy if $NV_B \geq 300/4$. Then show that the corresponding limitation on frequency deviation is $\Delta f < f_c/300$.

Ans;

$$C(t) = C_1 + C_v(t)$$

$$= C_1 + C_2 / \sqrt{V_B + m(t)/N}$$

$$C_v(t) = C_2 \left(V_B + m(t)/N\right)^{-1/2}$$

$$= C_2 V_B \left(1 + \frac{m(t)}{NV_B}\right)^{-1/2}$$

$$\left(1 + \frac{m(t)}{NV_B}\right)^{-1/2} = 1 - \frac{1}{2}\frac{m(t)}{NV_B} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) m^2(t)}{2N^2V_B^2}$$

$$= 1 - \frac{1}{2}\frac{m(t)}{NV_B} + \frac{3\,m^2(t)}{8\,N^2V_B^2} + \cdots$$

$$\frac{\frac{3}{8}\frac{m^2(t)}{N^2V_B^2}}{\frac{1}{2}\,m(t)/NV_B} \leq 0.01\, m(t)$$

$$\frac{3}{4}\frac{m(t)}{NV_B} \leq 0.01\, m(t)$$

$$\boxed{NV_B \geq \frac{300}{4}}$$

$$n = \frac{\delta f_4}{\delta f_1} = \frac{1.2\,KHZ}{100} = 120$$

number of doublers $= 7$

let $n_1 = 8$ $\quad n_2 = 16 \rightarrow n = 128$

$$f_0 = n_1 f_{c_1} - \frac{f_{c4}}{n_2} = 8 \times 10^4 - \left(\frac{10^6}{16}\right)$$

$= 17.5\,KHZ$

$f_{c_1} = 10\,KHZ$

$f_{c2} = n_1 f_{c_1} = 8 \times 10\,KHZ = 80\,KHZ$

$\delta f_2 = n_1 \delta f_1 = 8 \times 100 = 800\,HZ$

$\delta f_3 = \delta f_2 = 800\,HZ$

$f_{c3} = f_{c2} - f_0$

$= 80\,KHZ - 17.5\,KHZ = 62.5\,KHZ$

$f_{c4} = n_2 f_{c3} = 16 \times 62.5\,KHZ = 1\,MHZ$

$\delta f_4 = n_2 \delta f_3 = 16 \times 800 = 12.8\,KHZ$

5

no of tripplers = 8

$f_{c1} = 500 KHZ$

$f_{c2} = n_1 f_{c1} = 27 \times 500 KHZ = 13.5 MHZ$

$f_o = n_1 f_{c1} - \left(\frac{f_{c4}}{n_2}\right)$

$= 13.5 MHZ - \left(\frac{915 \times 10^6}{243}\right)$

$= 9.7 MHZ$

$f_{c3} = f_{c2} - f_o = 13.5 MHZ - 9.7 MHZ$

$= 3.8 MHZ$

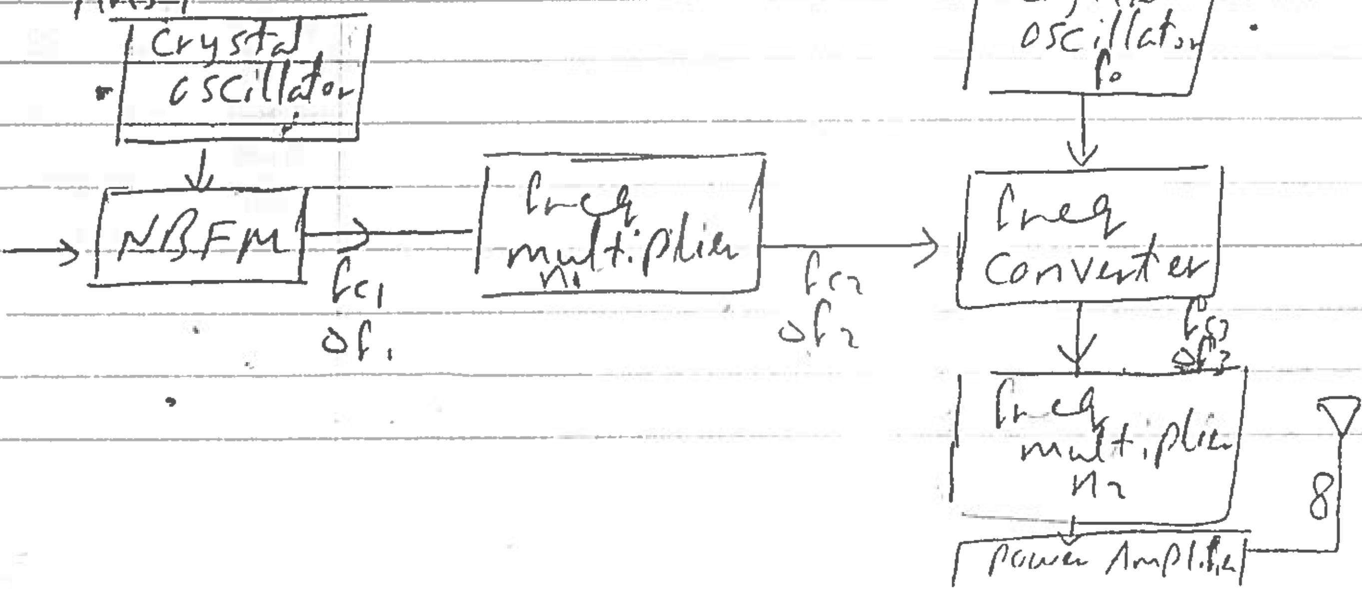
$f_{c4} = n_2 f_{c3} = 243 \times 3.8 MHZ = 923.4 MHZ$

$\Delta f_1 = 20 HZ$ $\quad \Delta f_2 = n_1 \Delta f_1 = 27 \times 20 = 540 HZ$

$\Delta f_3 = \Delta f_2 = 540 HZ$ $\quad \Delta f_4 = n_2 \Delta f_3 = 243 \times 540 = 1312 KHZ$

4-15) A signal with $B = 4 KHZ$ is transmitted using indirect FM with $f_c = 1 MHZ$ and $\Delta f = 12 KHZ$ if $\Delta f < 100$ and $f_{c1} = 10 KHZ$ find the number of doublers needed to achieve the desired output parameters. Draw the Block diagram of the system indicating the value and location of the local oscillator such that no frequency exceeds 10 MHZ.

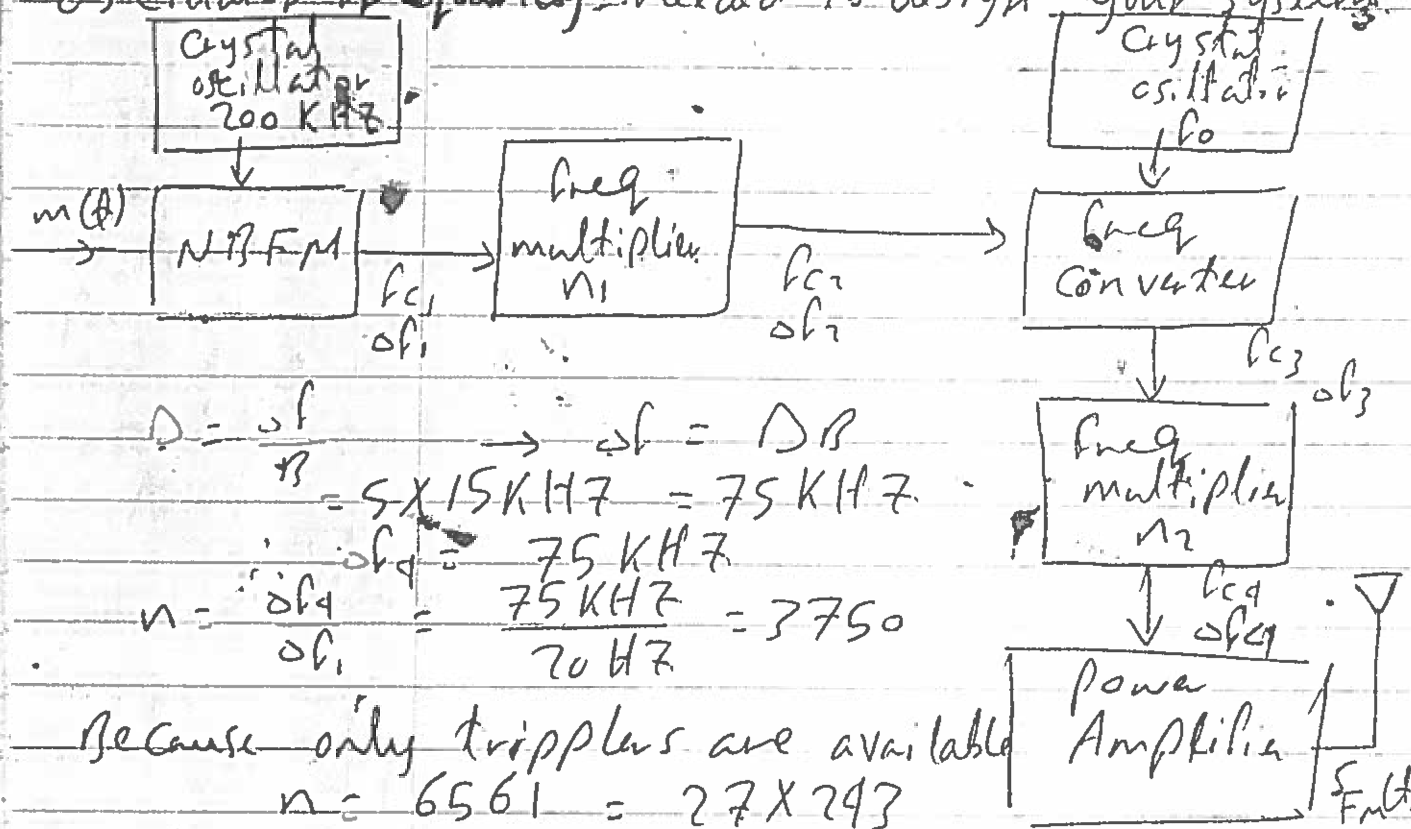
Ans;



$$y_o(t) = \frac{1}{2}\pi f_c A_c K_p^2 m^2(t) - \frac{1}{4}\pi f_c A_c K_p^{1} m^{1}(t)$$

The o/p is distorted

4-13) Design a wireless stereo speaker system using indirect FM. Assuming $B = 15\,KHZ$, $D = 5$, $f_{c1} = 500\,KHZ$, $f_c = 915\,MHZ$, $\Delta f < 20\,HZ$ determine the number of tripplers needed in your multiplier stage, and find the value of the local oscillator frequency needed to design your system.




$D = \frac{\Delta f}{B} \longrightarrow \Delta f = DB$

$= 5 \times 15\,KHZ = 75\,KHZ$

$\Delta f_4 = 75\,KHZ$

$n = \frac{\Delta f_4}{\Delta f_1} = \frac{75\,KHZ}{20\,HZ} = 3750$

Because only tripplers are available

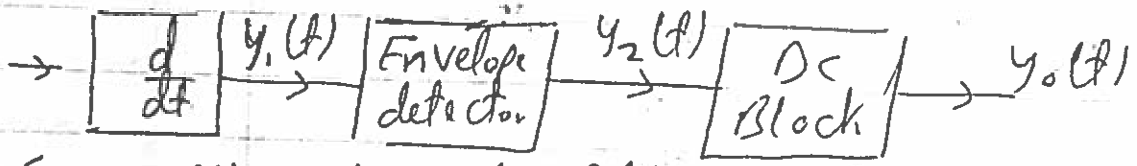
$n = 6561 = 27 \times 243$

$n_1 = 27$

$n_2 = 243$

4-17] Consider the transmitted signal is generated by an NBPM shown in Fig 4.14. Describe the distortion on the o/p message signal if it is Recieved by an FM-detector. (Hint consider the relationship between the message signal Amplitude and frequency and the modulation index.

Ans:




$$s_{NBPM}(t) = A_c \cos(2\pi f_c t) - A_c K_p m(t) \sin(2\pi f_c t)$$

$$y_1(t) = -A_c 2\pi f_c \sin(2\pi f_c t) - A_c K_p m(t) 2\pi f_c \cos(2\pi f_c t)$$

$$E(t) = \sqrt{(2\pi f_c A_c)^2 + (A_c K_p m(t) 2\pi f_c)^2}$$

$$= (2\pi f_c A_c)\left(1 + (K_p m(t))^2\right)^{\frac{1}{2}}$$

$$\left(1 + (K_p m(t))^2\right)^{1/2} \doteq 1 + \frac{1}{2} K_p^2 m^2(t) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \frac{(K_p m(t))^4}{2} + \cdots$$

$$= 1 + \frac{1}{2} K_p^2 m^2(t) - \frac{1}{8} K_p^4 m^4(t) + \cdots$$

$$\therefore E(t) = 2\pi f_c A_c + \frac{1}{2} \pi f_c A_c K_p^2 m^2(t) - \frac{1}{4} \pi f_c A_c K_p^4 m^4(t) + \cdots$$

After DC Block

for talk

$\Delta f = D B = 5 \times 5 \times 10^3 = 25 \, KHZ$

$BW = 2(25 + 5) \times 10^3 = \boxed{60 \, KHZ}$

4-8) An FM system with $\Delta f = 30 \, KHZ$ has been designed for $B = 10 \, KHZ$. Find the percentage of BW occupied by when the modulating signal is a unit amplitude tone at $f_m = 0.1$, 1, or 5 KHZ.

Ans;

$m_p = 1$

$\Delta f = 30 \, KHZ$

$f_m = 0.1, 1$ or $5 \, KHZ$

$B = 10 \, KHZ$

for $f_m = 0.1 \, KHZ$

$BW = 2(\Delta f + f_m) = 2(0.1 + 30) \times 10^3 = 60.2 \, KHZ$. (Carson's Rule)

The percentage of the BW $= \dfrac{10 \, KHZ}{60.2 \, KHZ} =$

$0.17 \times 100\% = 17\%$

4-5) A message has a bandwidth $B = 15\,KHZ$. Estimate the FM transmission bandwidth for $\Delta f = 0.5, 4, 50$ and $500\,KHZ$

Ans:

$$BW = 2(\Delta f + B)$$

$$= 2(\Delta f + 15 \times 10^3) \rightarrow \text{Carson's Rule}$$

for $\Delta f = 0.5\,KHZ$

$$= 2(0.5 + 15) \times 10^3 = \boxed{31\,KHZ}$$

for $\Delta f = 4\,KHZ$

$$BW = 2(4 + 15) \times 10^3 = 38\,KHZ$$

4-7) A commercial FM radio station alternates between music and talk show formats. The Broadcast CD music is Bandlimited to $15\,KHZ$ based on Convension and the Voice signals can be bandlimited to $5\,KHZ$. Assume $D = 5$ is used for both music and voice. find the bandwidth requirements for both formats.

Ans:

$$D = \beta = \frac{\Delta f}{B}$$

for CD music

$$\Delta f = D B = 5 \times 15\,KHZ = 75\,KHZ$$

$$BW = 2(\Delta f + B) = 2(75 + 15) \times 10^3 = \boxed{180\,KHZ}$$

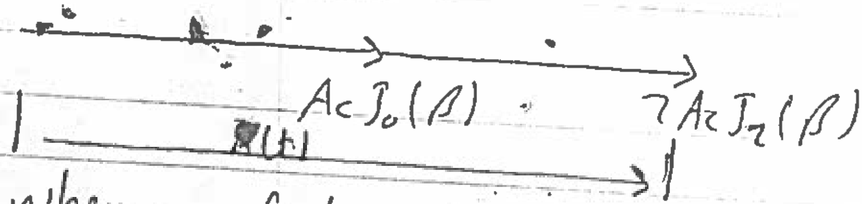
4

4-3 Construct phasor diagrams for tone modulated FM signal with $A_c = 10$ and $\beta = 0.5$ when $2\pi f_m t = 0, \pi/4$ and $\frac{\pi}{2}$ and compare with the theoretical values.

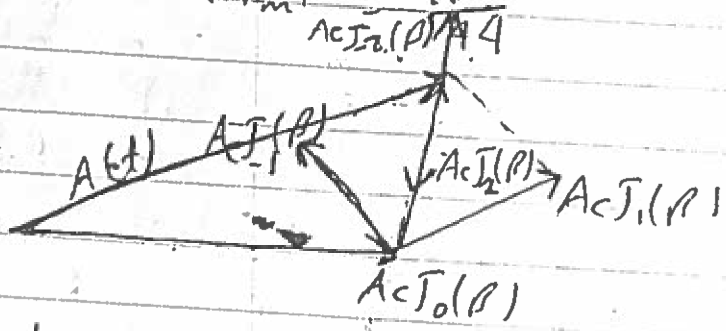
Ans: for $\beta = 0.5$ $\longrightarrow$ $n_{max} = 2$ (from table)

$$S_{FM}(t) = A_c J_0(\beta)\cos(2\pi f_c t) + A_c J_1(\beta)\cos(2\pi(f_c + f_m)t)$$
$$+ A_c J_{-1}(\beta)\cos(2\pi(f_c - f_m)t) + A_c J_2(\beta)\cos(2\pi(f_c + 2f_m)t)$$
$$+ A_c J_{-2}(\beta)\cos(2\pi(f_c - 2f_m)t)$$

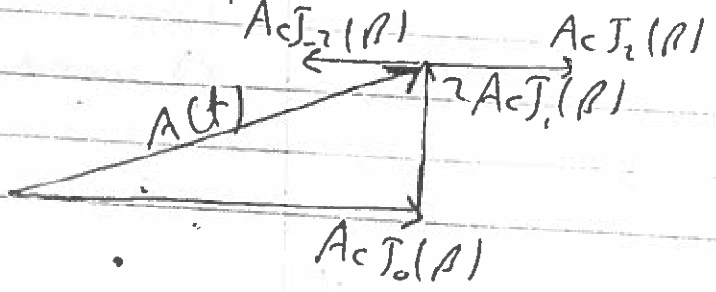
when $2\pi f_m t = 0$




when $2\pi f_m t = \frac{\pi}{4}$




when $2\pi f_m t = \pi/2$



$J_{-1}(\beta) = -J_1(\beta) = \pm 0.58 \quad J_{-2}(\beta) = J_2(\beta) = 0.35$

$J_{-3}(\beta) = -J_3(\beta) = \pm 0.13 \quad J_{-4}(\beta) = J_4(\beta) = 0.03$

$P_0$ (directly) $= \frac{A_c^2}{2} = \frac{10^4}{2}$ W

$P_0$ (from the spectral components) =

$$\frac{A_c^2}{2}\left[J_0^2(2) + 2J_1^2(2) + 2J_2^2(2) + 2J_3^2(2) + 2J_4^2(2)\right]$$

$$= \frac{10^4}{2}\left[(0.22)^2 + 2(0.58)^2 + 2(0.35)^2 + 2(0.13)^2 + 2(0.03)^2\right] \ldots$$

W.

# Ch.4 Angle Modulation

## wave form of FM, PM ①

وكنت المشكلة مش عارف أعمل Generation للمعادلات بتاعت
FM, PM Signals in T.D

لأن المعلومة موجودة في angle علشان كده رحنا إلى

## NBFM ②

$BW = 2B$

$\cos \rightarrow \cos\cos$, $\sin\sin$ مضطرين نفرض وتقليل

$\Delta f = \sin \Delta f$ نفرض انه كمان قليل $\Delta f$

$\theta = \sin \theta$

وكنت ظهرت مشكلة هي وجود distortion

فاهنا الطريقة فرحنا إلى

## Wide Band ③

$BW = 2(B + \Delta f)$

علشان أزود ال BW

مضطرين نفرض أن $\Delta f$ ليست صغيرة والمعلومة داخل angle

ومش عارف أتعامل معاها فلا نفرض أن المعلومة Sinusoidal
Sin/Cos.

وعملنا شوية عمليات رياضية (غير مهم للطلاب)

هاوصل إلى معادلة (لل amplitude of Sinusoidal signal) عبارة عن Bessel $f^n$

وبالتالي هاعرف أرسم ال Spectrum بتاع المعادلة بتاعت FM Signal

# Generation of FM, PM signals



FM

## indirect

Armstrong

يعني بس انه باخذ NB modr

ويعمله freq. multiplication

ويحوله الى wideband

لنحصل NB → WB

ولكن بس ~ NB فيه مشكلة distortion

وبالتالي WB ← distortion

لانه ما بنحكي عليه بالتفصيل

وهذا الجزء عليه ~ كل شيء

(مش مطلوب ب ٢٤)

## direct

يعني بس ~ بشكل مباشر

VCO

~~.....~~ اقدر اتحكم oscillator

من الفولتية بتاعه ~~.....~~

بطبيعة apply voltage $m(t)$

لانه تذكرة osc. بيولد اشارة

sinsoidal with تردد

بمعطيه بتحكم في L, C

ولكن التذكرة هنا انه هاشي

C دا مستعمله varactor ~~.....~~ diode

اللي تذكرته عبارة عن diode ولكن

reverse voltage $(m(t))$ اطبق عليه


مكافئ

$$C = \frac{\varepsilon A}{d}$$

A → area

d → distance

depletion region

يعني كلما زادت depletion region كلما زاد العزل

يعني diode يشتغل كأنه مكثف.

أما PM Generation فمش موجود أوي في analog لانه generation

بتاعته فيه مشاكل وتطبيقاته ليست كتير في analog

بس كتير في ال digital والأداء بتاعه أقل من FM

الا فيه performance بتاع phase أقل من freq.

<u>Wideband</u>: trade off between performance and the bandwidth.

in <u>NB</u>. $BW = 2B$ <u>As</u> AM.

but in WB. $BW = 2(\Delta f + B)$

$\boxed{1}$

$$x(t) = A\cos[\underbrace{\omega_c t + \phi(t)}_{\theta_i(t)}]$$

$$= Re\left\{A\ \cancel{\cos}\ \exp j(\omega_c t + \phi(t))\right\}$$

: Exponential Modulation

$\theta_i(t)$ instantenous phase of the Carrier

$\omega_i(t) = \dfrac{d\theta_i(t)}{dt} \equiv$ instantenous frequency of the Carrier

$\because\ \theta_i(t) = \omega_c t + \phi(t)$

$\therefore\ \omega_i(t) = \omega_c + \dfrac{d\phi(t)}{dt}$

$\phi(t)$ : instantenous phase deviation

$\dfrac{d\phi(t)}{dt}$ : frequency deviation

---

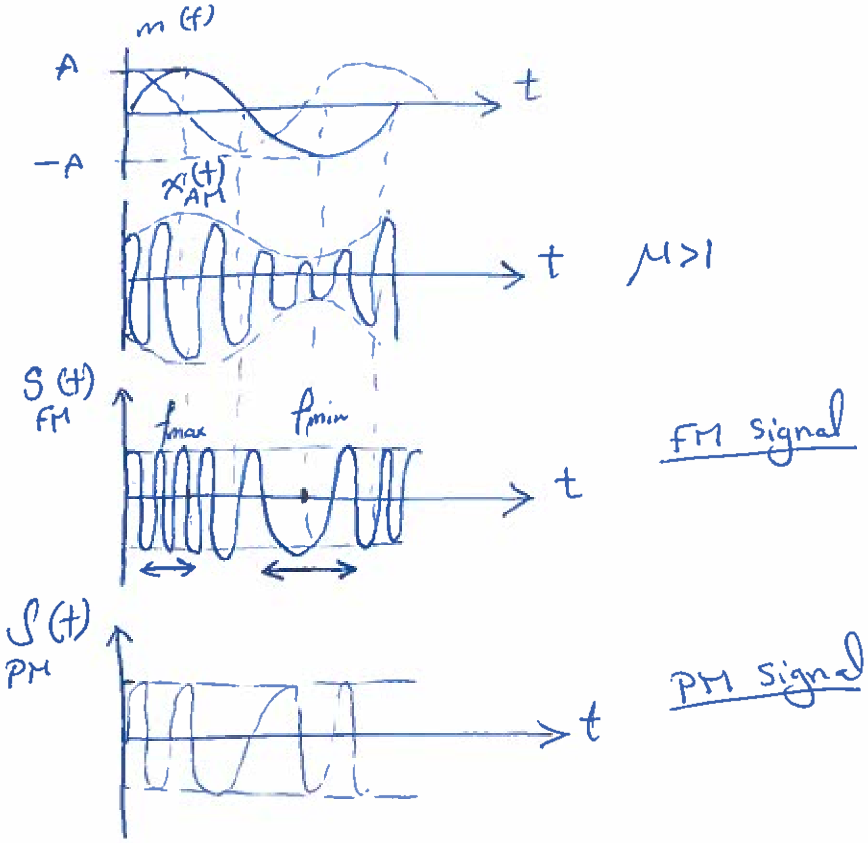
* phase Modulation : $\phi(t) = K_p\, m(t)$ ; phase Sensitivity of the modulator ; in rad/V

* freq. modulation : $\dfrac{d\phi(t)}{dt} = K_f\, m(t)$ ; freq. Sensitivity of the modulator HZ/V

$$\phi(t) = K_f \int_{-\infty}^{t} m(s)\,ds$$

PM Signal: $x(t) = A\cos[\omega_c t + K_p m(t)]$

FM Signal: $x(t) = A\cos[\omega_c t + \underbrace{K_f \int_{-\infty}^{t} m(s)\,ds}_{\phi(t)}]$




* Spectrum, Bandwidth, Power of Angle Modulated Signals:

- FM, PM are nonlinear modulation unlike Amplitude Modulation (Linear).
- Exact Spectrum Calculation rather difficult
  ⇒ for general messge signals

- possible to study the spectrum when $m(t) = A_m \cos w_m t$

PM: $\phi(t) = K_p A_m \cos w_m t$

FM: $\phi(t) = \frac{K_f A_m}{w_m} \sin w_m t$

$$\boxed{x_{FM}(t) = A \cos(w_c t + \beta \sin w_m t)}$$

$\beta = \frac{K_f A_m}{w_m}$ [for PM: $\beta = K_p A_m$

$x(t) = A \, Re \{ e^{jw_c t} \cdot e^{j\beta \sin w_m t} \}$ → Complex periodic $f^n$

Fourier Series F.S.

$$e^{j\beta \sin w_m t} : f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnw_m t}$$

$$c_n = \frac{w_m}{2\pi} \int_{-\pi/w_m}^{\pi/w_m} e^{j\beta \sin w_m t} \cdot e^{-jnw_m t} \, dt$$

$$; w_m t = x$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

$$\triangleq J_n(\beta) \quad : \text{Bessel function of order } \underline{n}$$

$$x(t) = A \, Re \left\{ e^{jw_c t} \sum_{n=-\infty}^{\infty} c_n e^{jnw_m t} \right\}$$

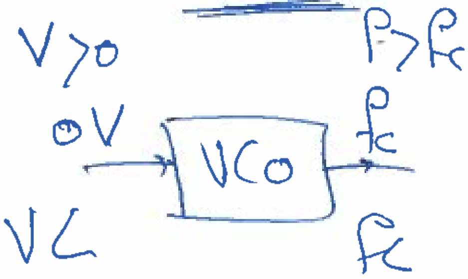
$$= A \, Re \left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(w_c + nw_m)t} \right\}$$

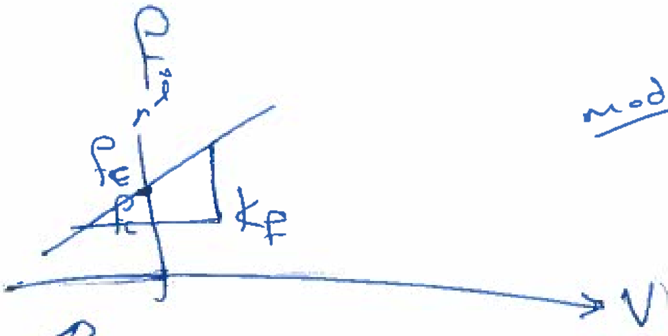
$$= A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(w_c + nw_m)t$$

; A series expontional for FM signal

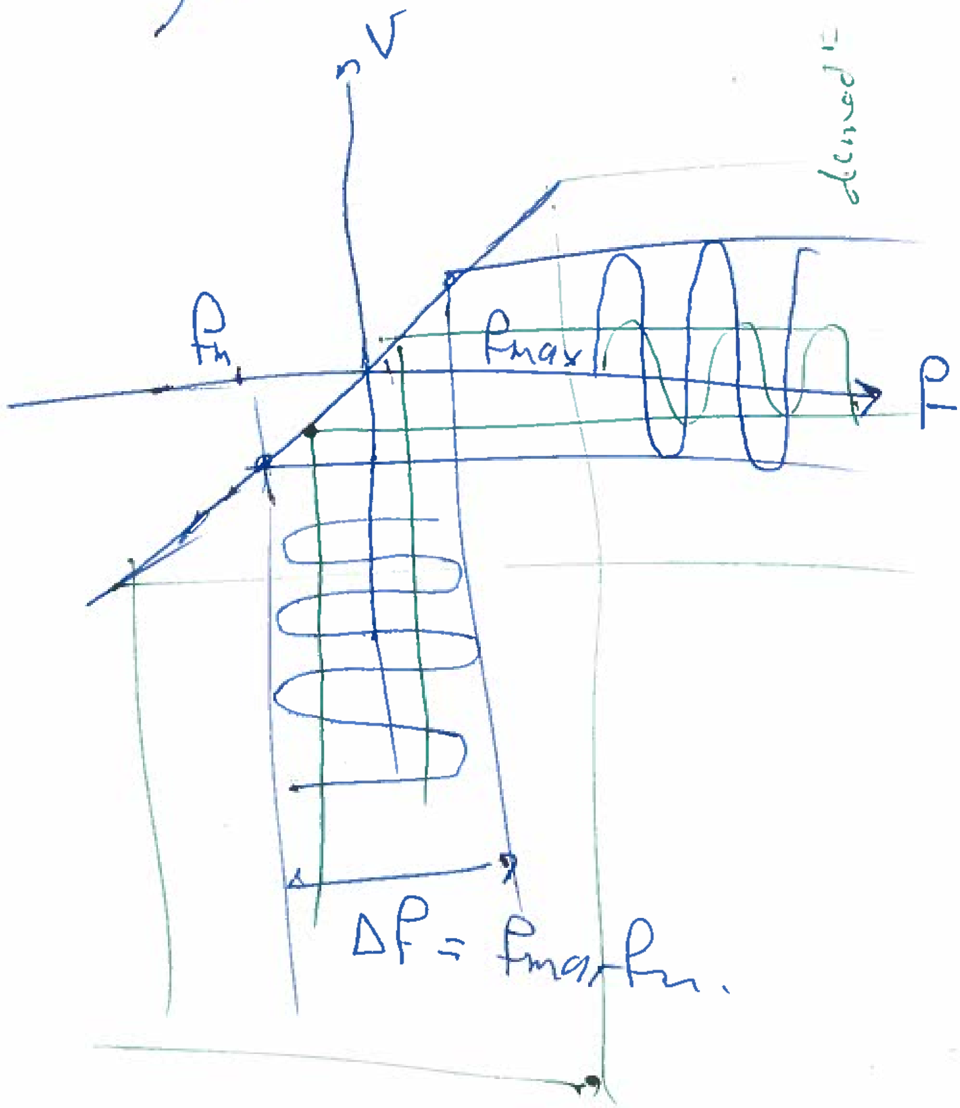
eб540

mode vco




$$f_i = f_c + k_f m(t)$$




$$\Delta f = f_{max} - f_m .$$