Lecture 3.0

Reinforcement Learning: Policy Gradient Methods

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Illustration example: Left or right?

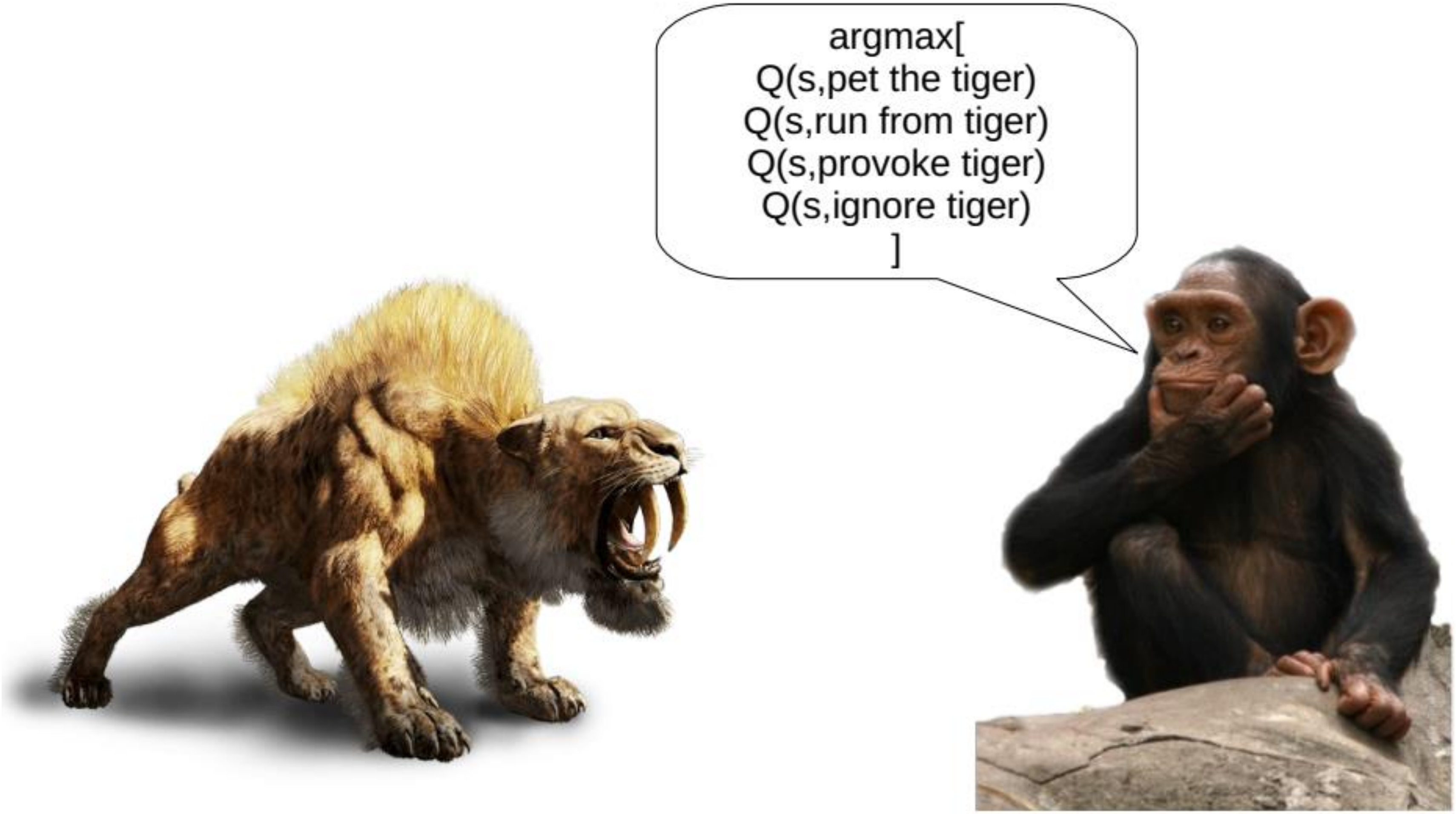
Why not using Q-learning everywhere?

$$L \approx E[Q(s_t, a_t) - (r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a'))]^2$$

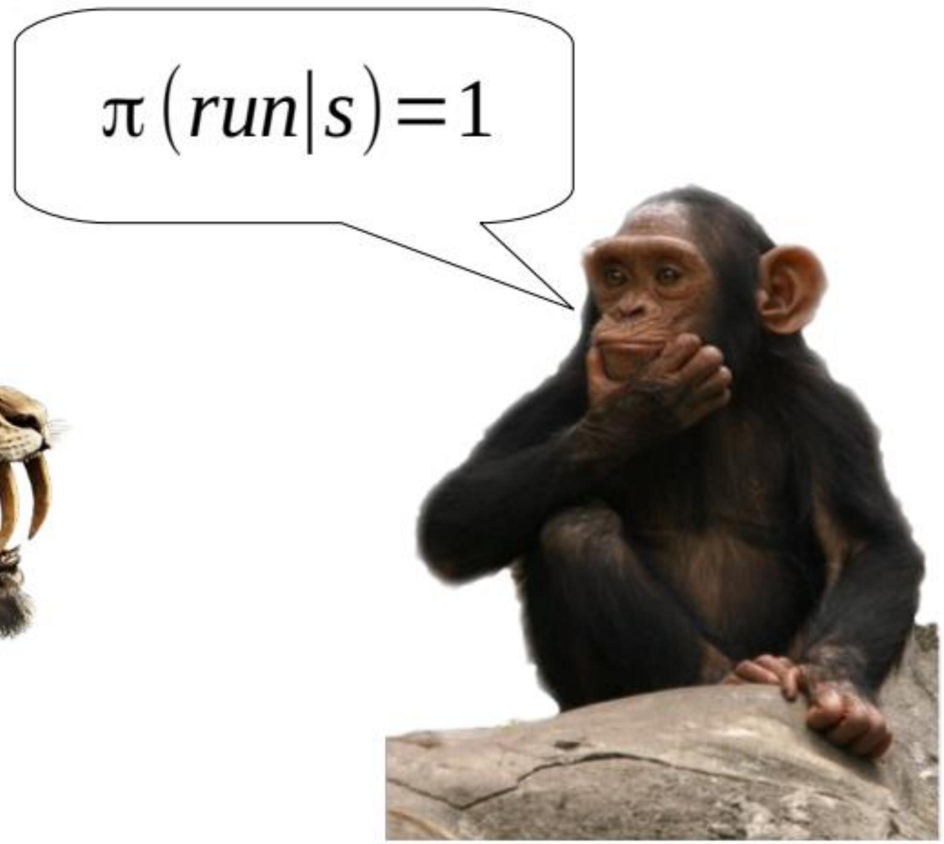
Simple 2-state world

	True	(A)	(B)
Q(s0,a0)	*1*	*1*	*2*
Q(s0,a1)	*2*	*2*	*1*
Q(s1,a0)	*3*	*3*	*3*
Q(s1,a1)	*100*	*50*	*100*

Evolution: how humans not survived



Evolution: how humans survived



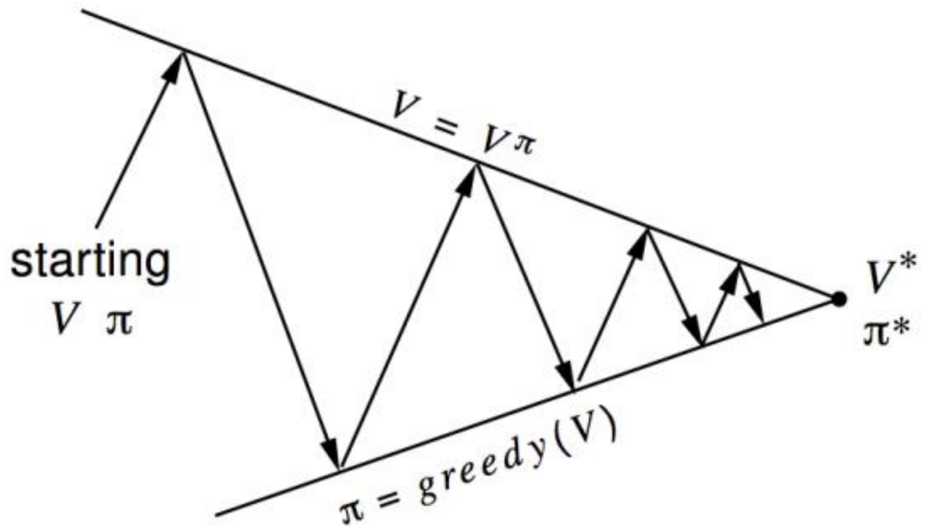
Outline

- Often computing q-values is harder than picking optimal actions!

- We could avoid learning value functions by directly learning agent's policy

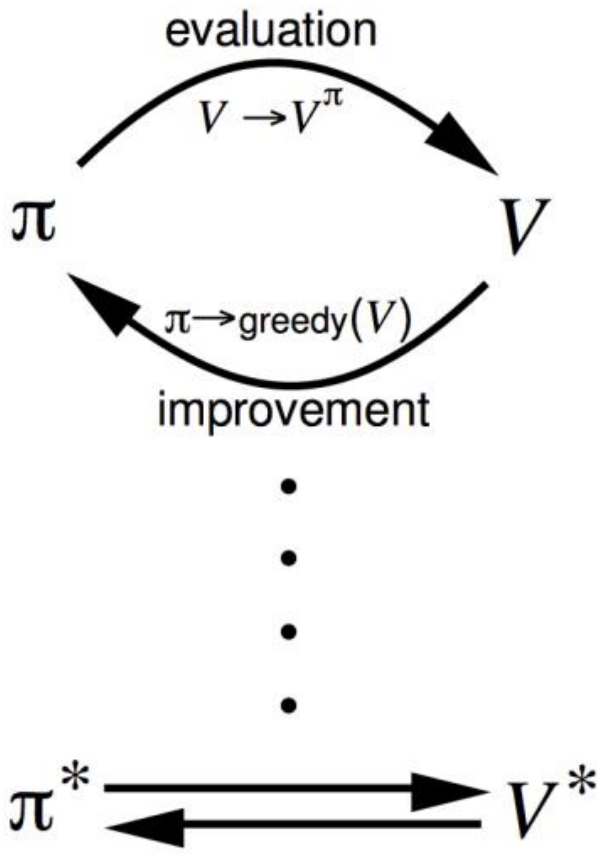
Q: what algorithm works that way?

Generalized Policy Iteration



Policy evaluation Estimate v_π
Any policy evaluation algorithm

Policy improvement Generate $\pi' \geq \pi$
Any policy improvement algorithm



Policy-based RL

- In previous lectures we approximated the value or action-value function using parameters:

$$V_{\Theta}(s) \approx V_{\pi}(s)$$
$$Q_{\Theta}(s, a) \approx Q_{\pi}(s, a)$$

- A policy was generated directly from the value function e.g. using ε-greedy
- In this lecture we will directly parametrize the policy

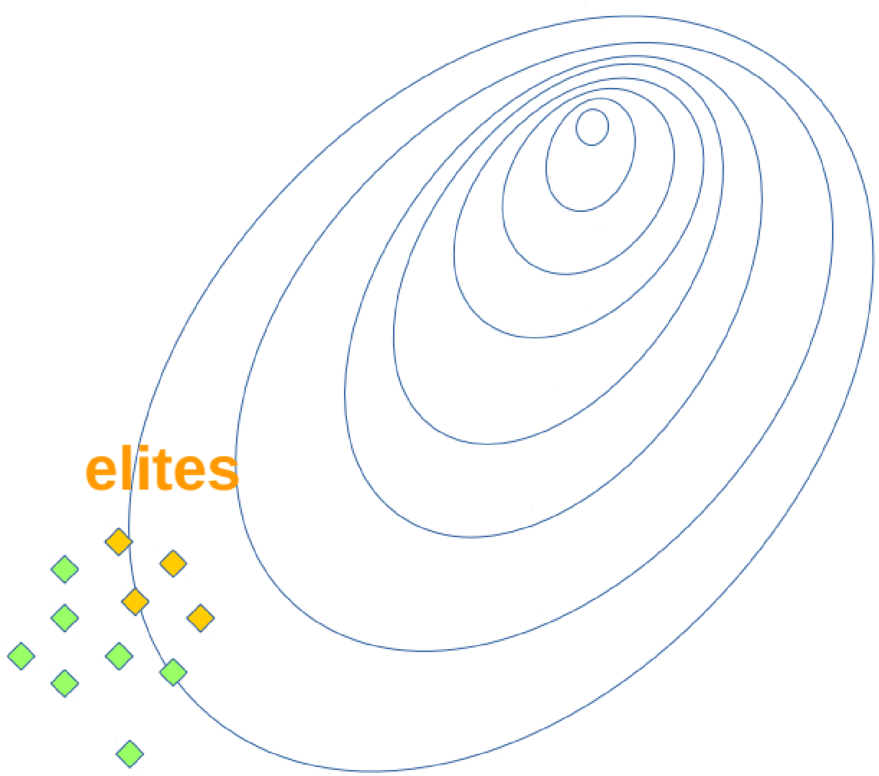
$$\pi_{\Theta}(s, a) = \mathrm{P}\,[a \mid s, \Theta]$$

- We will focus again on model-free reinforcement learning

Recap: Cross Entropy Method

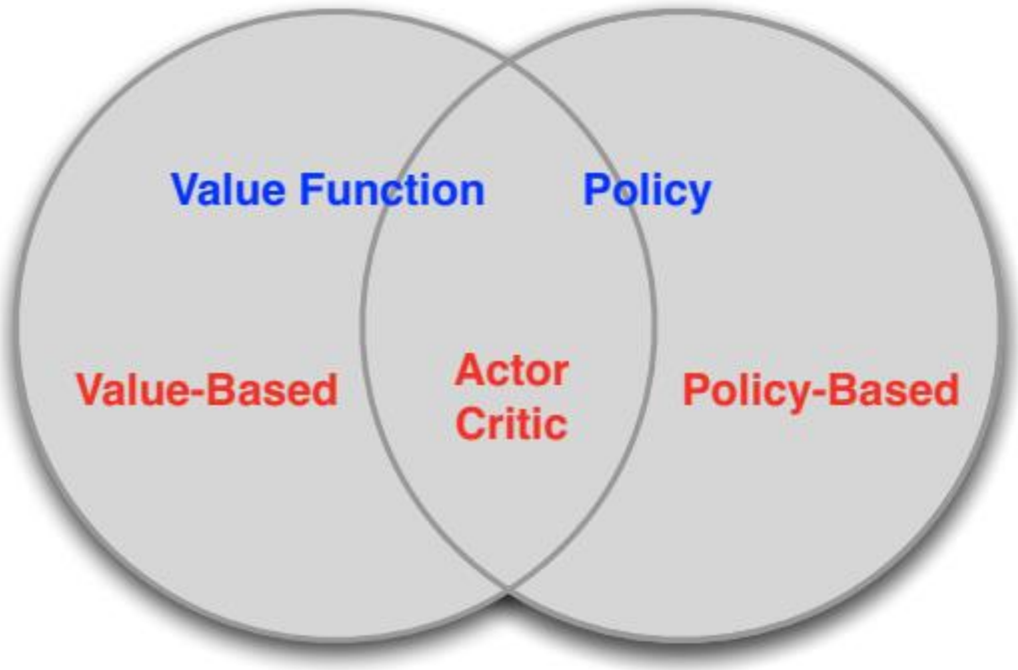
- CEM:
  - Evolutionary
  - Go in the direction where elite goes.
  - Easy to implement
  - Black Box

  - Need to play full episode to start learning



Value-based & Policy-based RL

- Value Based:
  - Learnt Value Function
  - Implicit policy
    (e.g. ε-greedy)
- Policy Based:
  - No Value Function
  - Learnt Policy
- Actor-Critic:
  - Learnt Value Function
  - Learnt Policy



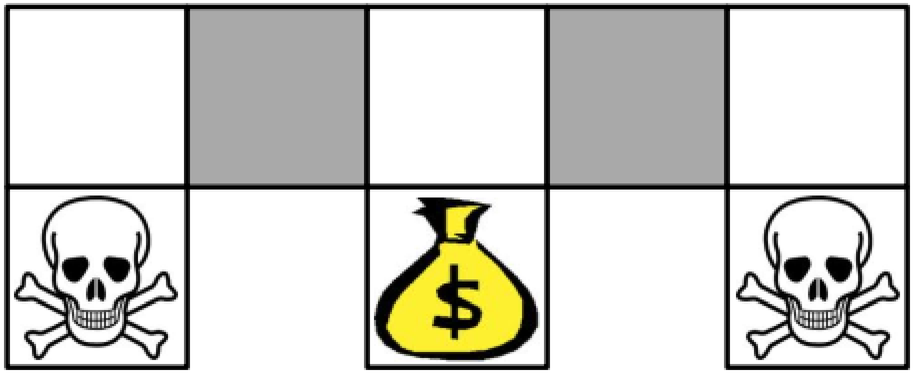
Advantages of Policy-Based RL

- Advantages:
  - Better convergence properties
  - Effective in high-dimensional or continuous action spaces
  - Can learn stochastic policies
- Disadvantages:
  - Typically converge to a local rather than global optimum
  - Evaluating a policy is typically inefficient and high variance

Example: Rock-Paper-Scissors

- Two-player game of rock-paper-scissors
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Consider policies for *iterated* rock-paper-scissors
  - A deterministic policy is easily exploited
  - A uniform random policy is optimal

Example #2: Aliased Grid world



- The agent cannot differentiate the grey states
- Consider features of the following form (for all N, E, S, W)

$$\varphi(s; a) = \mathbf{1}(\text{wall to N}, a = \text{move E})$$

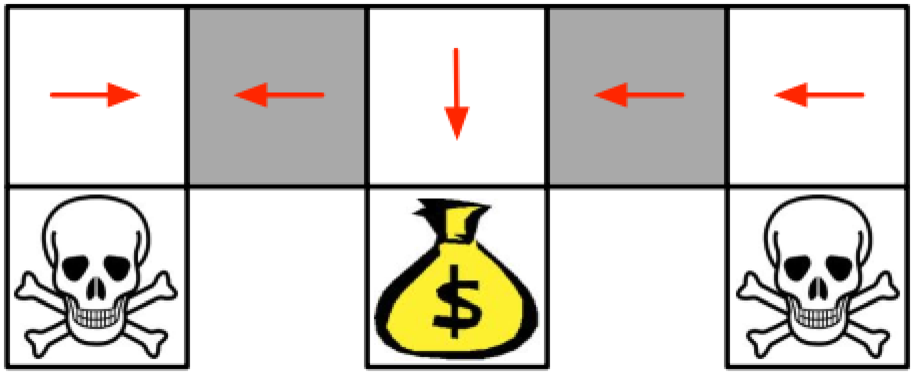
- Compare value-based RL, using an approximate value function

$$Q_{\Theta}(s; a) = f(\varphi(s, a), \Theta)$$

- To policy-based RL, using a parametrized policy

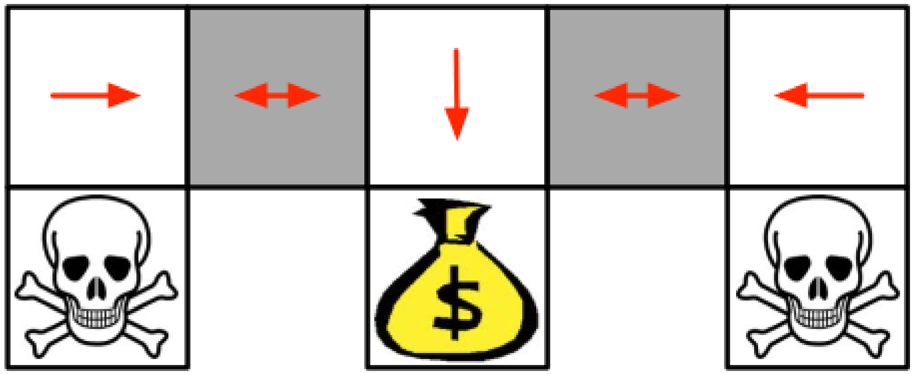
$$\pi_{\Theta}(s, a) = g(\varphi(s, a), \Theta)$$

Example #2: Aliased Grid world



- Under aliasing, an optimal deterministic policy will either
  - move W in both grey states (shown by red arrows)
  - move E in both grey states
- Either way, it can get stuck and *never* reach the money
- Value-based RL learns a near-deterministic policy
  - e.g. greedy or ϵ-greedy
- So it will traverse the corridor for a long time

Example #2: Aliased Grid world



- An optimal stochastic policy will randomly move E or W in grey states:

$$\pi_\theta(\text{wall to N and S, move E}) = 0.5$$
$$\pi_\theta(\text{wall to N and S, move W}) = 0.5$$

- It will reach the goal state in a few steps with high probability
- Policy-based RL can learn the optimal stochastic policy

Policy Objective Functions

- Goal: given policy $\pi_\Theta(s, a)$ with parameters Θ, find best Θ
  But how do we measure the quality of a policy π_Θ?

- In episodic environments we can use the start value:

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta}[v_1]$$

- In continuing environments we can use the average value:

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

- Or the average reward per time-step:

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) \mathcal{R}_s^a$$

- where $d\pi_\Theta(s)$ is stationary distribution of Markov chain for π_Θ

Policy Optimization

- Policy based reinforcement learning is an optimization problem
- Find Θ that maximizes $J(\Theta)$

- Some approaches do not use gradient
  - Hill climbing
  - Genetic algorithms
  - Simplex

- Greater efficiency often possible using gradient
  - Gradient descent
  - Conjugate gradient
  - Quasi-newton

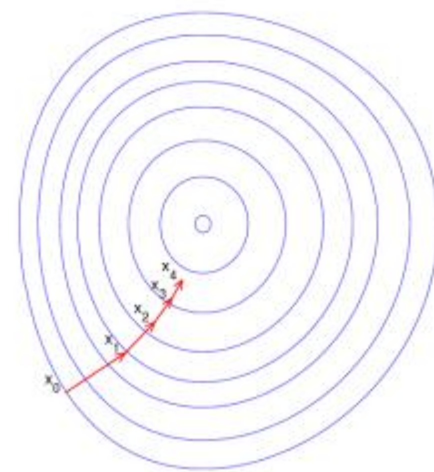
- We focus on gradient descent, many extensions possible

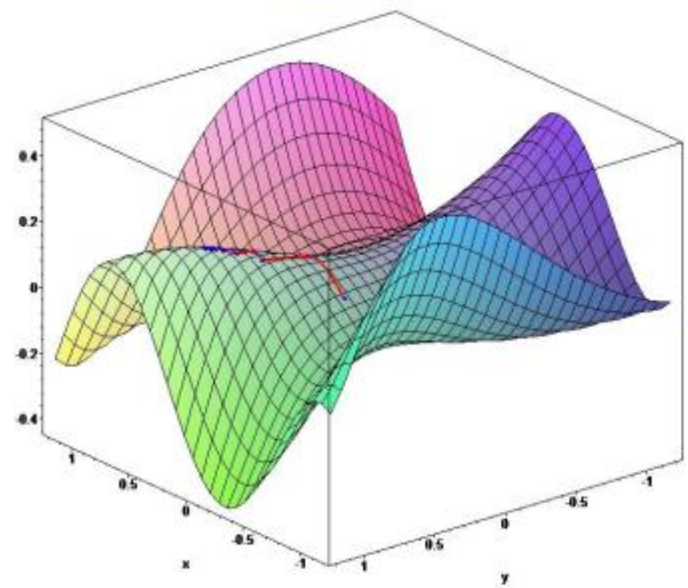
Policy Gradient

- Let *J*(*Θ*) be any policy objective function

- Policy gradient algorithms search for a *local* maximum in *J*(*Θ*) by ascending the gradient of the policy, w.r.t. parameters *Θ*

$$\nabla_\theta J(\theta) = \begin{pmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{pmatrix}$$





Finite Differences approach

- To evaluate policy gradient of $\pi_\Theta(s, a)$
- For each dimension $k \in [1, n]$
  - Estimate kth partial derivative of objective function w.r.t. Θ
  - By perturbing Θ by small amount ε in kth dimension

$$\frac{\partial J(\theta)}{\partial \theta_k} \approx \frac{J(\theta + \epsilon u_k) - J(\theta)}{\epsilon}$$

  - where u_k is unit vector with 1 in kth component, 0 elsewhere
- Uses n evaluations to compute policy gradient in n dimensions
- Simple, noisy, inefficient - but sometimes effective
- Works for arbitrary policies, even if policy is not differentiable

Analytic Gradient Computation

- We now compute the policy gradient *analytically*
- Assume policy π_Θ is differentiable whenever it is non-zero
- <u>Log Derivative trick:</u>

$$\nabla_\theta \pi_\theta(s,a) = \pi_\theta(s,a)\frac{\nabla_\theta \pi_\theta(s,a)}{\pi_\theta(s,a)}$$
$$= \pi_\theta(s,a)\nabla_\theta \log \pi_\theta(s,a)$$

- We can express gradient of π_Θ as function of π_Θ.

One-step MDPs

- Consider a simple class of one-step MDPs Starting in state $s \sim d(s)$
- Terminating after one time-step with reward $r = R_{s,a}$
- Use log-derivative trick to compute the policy gradient

$$
\begin{aligned}
J(\theta) &= \mathbb{E}_{\pi_\theta}[r] \\
&= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a) \mathcal{R}_{s,a} \\
\nabla_\theta J(\theta) &= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a) \nabla_\theta \log \pi_\theta(s,a) \mathcal{R}_{s,a} \\
&= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) r]
\end{aligned}
$$

Policy Gradient Theorem

- The policy gradient theorem generalizes the likelihood ratio approach to multi-step MDPs
- Replaces instantaneous reward *r* with long-term value $Q_\pi(s, a)$ ☺
- Policy gradient theorem applies to start state objective, average reward and average value objective

Theorem

For any differentiable policy $\pi_\theta(s, a)$,
for any of the policy objective functions $J = J_1, J_{avR},$ *or* $\frac{1}{1-\gamma} J_{avV}$,
the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s, a)\; Q^{\pi_\theta}(s, a)\right]$$

REINFORCE algorithm

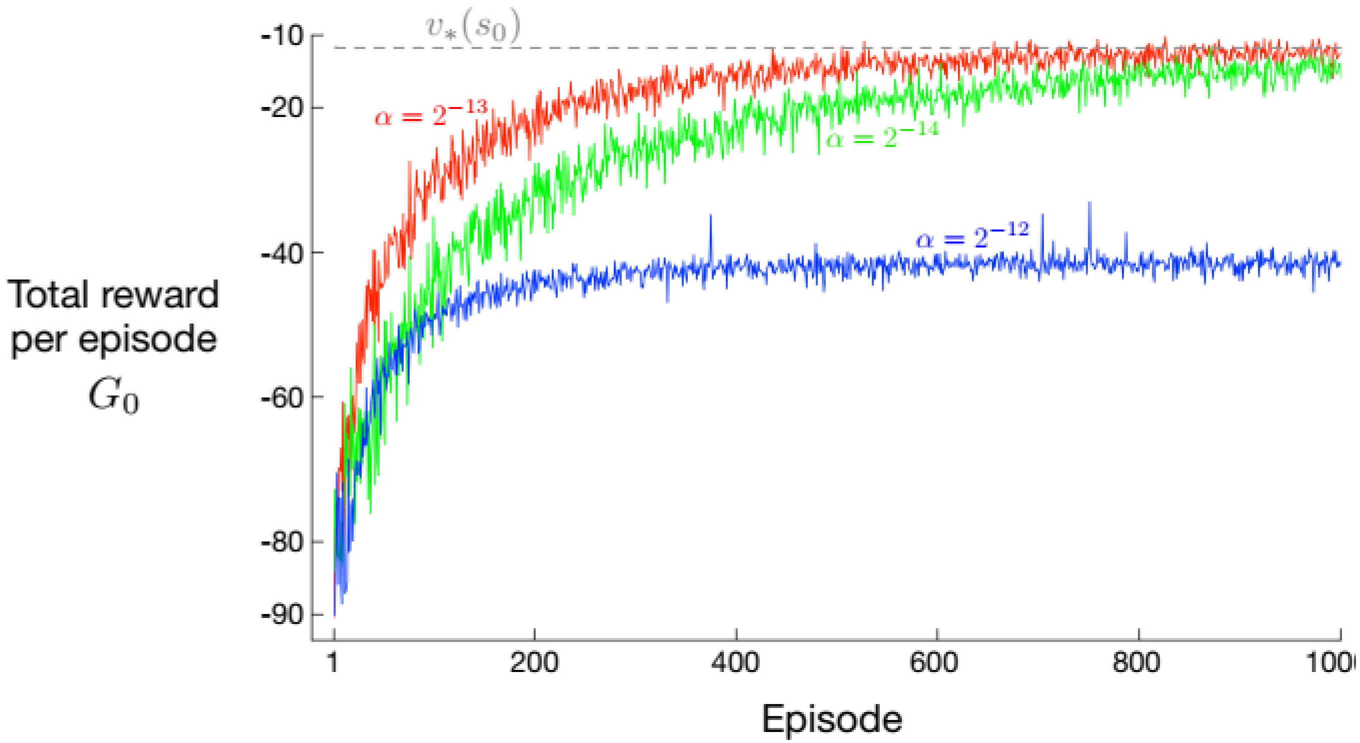
- Update parameters by stochastic gradient ascent (!!)
- Using policy gradient theorem
- Using return G_t as an unbiased sample of $Q\pi_\Theta(s_t, a_t)$

REINFORCE, A Monte-Carlo Policy-Gradient Method (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

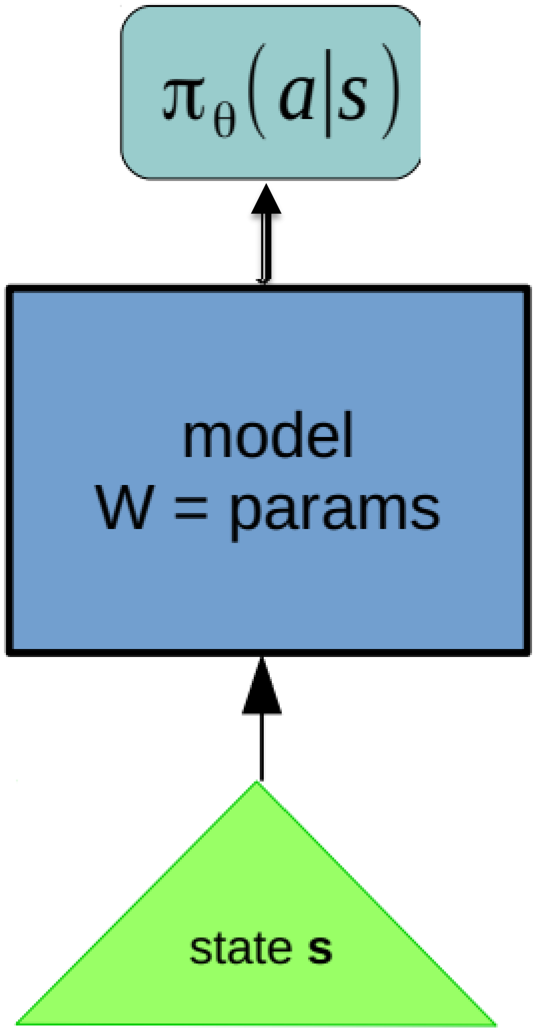
Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, \ldots, T-1$:
    $G \leftarrow$ return from step t (G_t)
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla_{\boldsymbol{\theta}} \ln \pi(A_t|S_t, \boldsymbol{\theta})$

REINFORCE training



REINFORCE as NN

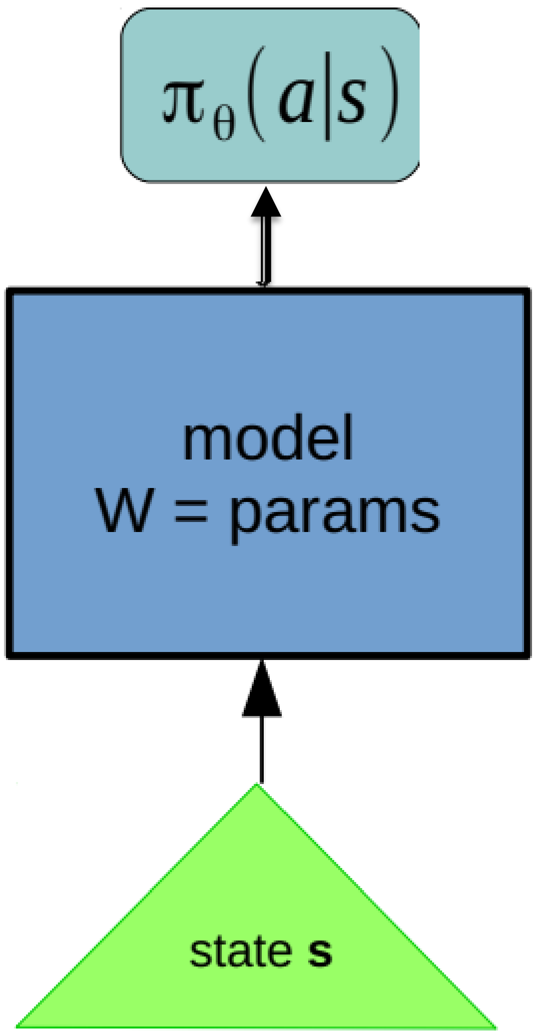


- Initialize NN weights
  - *Θ – random init*
- Cycle:
  - Sample N sessions **z** under current policy $\pi_\Theta(s_t, a_t)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot Q(s,a)$$

  - Apply gradient update

REINFORCE as NN with a baseline



- Initialize NN weights
  - *Θ – random init*
- Cycle:
  - Sample N sessions **z** under current policy $\pi_\Theta(s_t, a_t)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot (Q(s,a) - b(s))$$

  - Apply gradient update

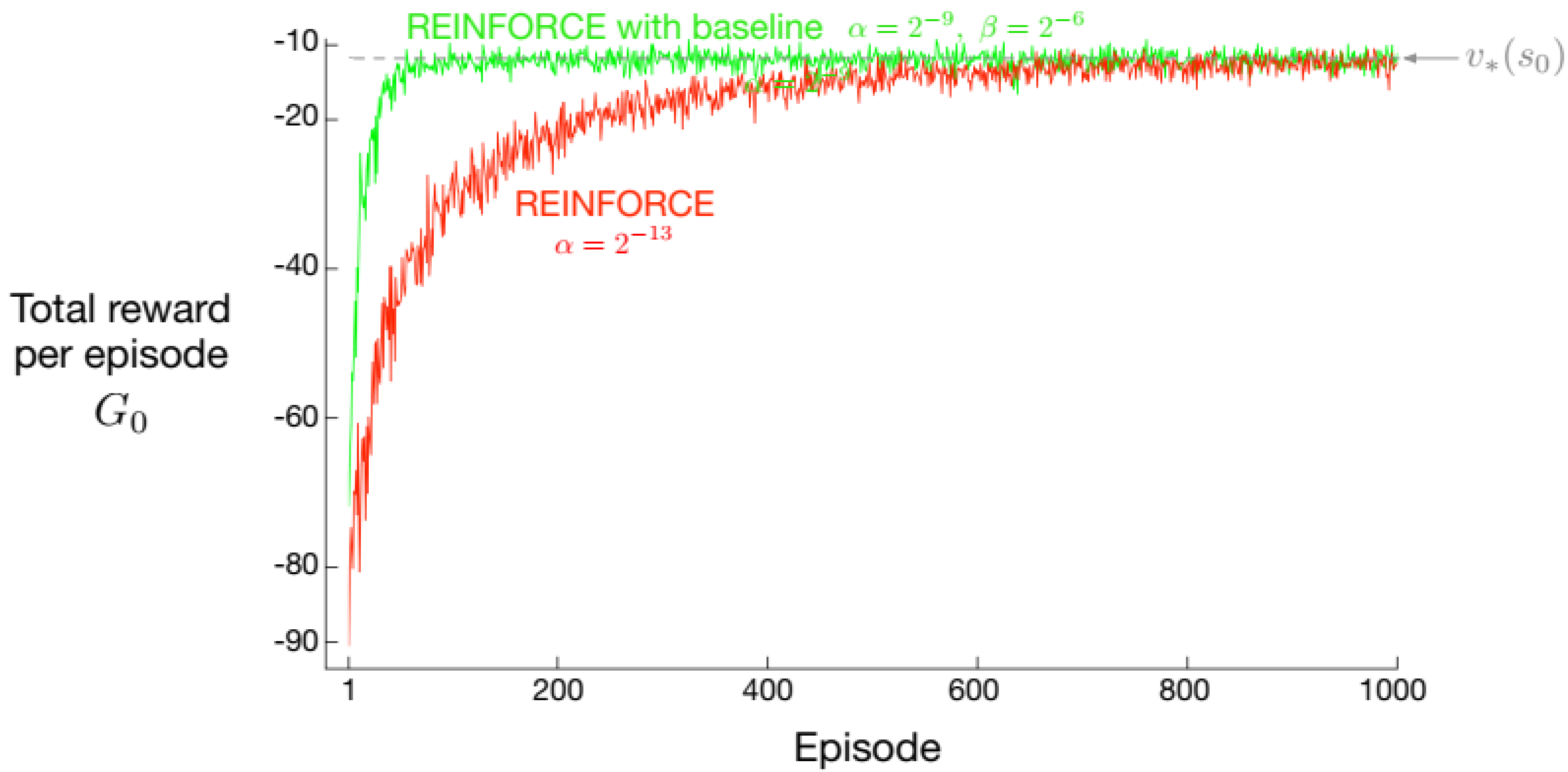
REINFORCE with a baseline

REINFORCE with Baseline (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
    Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
    Loop for each step of the episode $t = 0, \ldots, T-1$:
        $G \leftarrow$ return from step t (G_t)
        $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \gamma^t \delta \nabla_{\mathbf{w}} \hat{v}(S_t,\mathbf{w})$
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla_{\boldsymbol{\theta}} \ln \pi(A_t|S_t, \boldsymbol{\theta})$

REINFORCE with a baseline training



Next Method Name?

Actor-Critic

- Monte-Carlo policy gradient still has high variance
- We use a critic to estimate the action-value function

$$Q_w(s, a) \approx Q\pi_\Theta(s, a)$$

- Actor-critic algorithms maintain *two* sets of parameters
  - Critic: Updates action-value function parameters *w*
  - Actor: Updates policy parameters *Θ*, in direction suggested by critic

  - Actor-critic algorithms follow an *approximate* policy gradient

$$\nabla_\theta J(\theta) \approx \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s, a)\ Q_w(s, a)\right]$$
$$\Delta\theta = \alpha \nabla_\theta \log \pi_\theta(s, a)\ Q_w(s, a)$$

Estimating the action-value function

- The critic is solving a familiar problem: policy evaluation

- How good is policy π_{Θ} for current parameters Θ?

Action-Value Actor-Critic

- Simple actor-critic algorithm based on action-value critic
- Using linear value fn approx. $Q_w(s,a) = \phi(s,a)^\top w$

  Critic Updates w by linear TD(0)

  Actor Updates θ by policy gradient

function QAC
    Initialise s, θ
    Sample $a \sim \pi_\theta$
    for each step **do**
        Sample reward $r = \mathcal{R}_s^a$; sample transition $s' \sim \mathcal{P}_{s,\cdot}^a$
        Sample action $a' \sim \pi_\theta(s', a')$
        $\delta = r + \gamma Q_w(s', a') - Q_w(s, a)$
        $\theta = \theta + \alpha \nabla_\theta \log \pi_\theta(s, a) Q_w(s, a)$
        $w \leftarrow w + \beta \delta \phi(s, a)$
        $a \leftarrow a', s \leftarrow s'$
    end for
end function

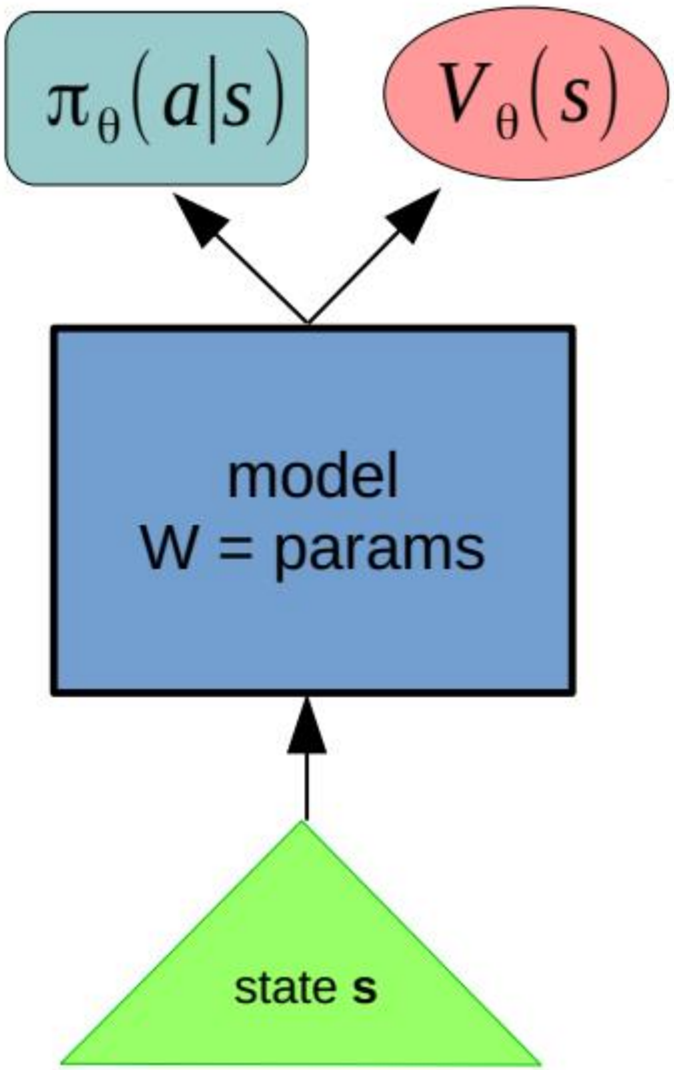
Bias in Actor-Critic Algorithms

- Approximating the policy gradient introduces bias
- A biased policy gradient may not find the right solution
  - e.g. if $Q_w(s, a)$ uses aliased features, can we solve gridwold example?
- Luckily, if we choose value function approximation carefully

Then we can avoid introducing any bias

- i.e. We can still follow the *exact* policy gradient

Advantage Actor-Critic (aka A2C)



A(s,a) = Q(s,a) - V(s)= $r+\gamma \cdot V(s') - V(s)$

Improve policy:

$$\nabla J_{actor} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot A(s,a)$$

Improve value:

$$L_{critic} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} (V_\theta(s) - [r + \gamma \cdot V(s')])^2$$

Tricks & tips

- V(s) errors less important than in Q-learning
  - actor still learns even if critic is random, just slower
- Regularize with entropy (hello exploration)
  - to prevent premature convergence
- learn on parallel sessions
  - or super-small experience replay
- Use F.log_softmax() for numerical stability

Summary of Policy Gradient methods

- The policy gradient has many equivalent forms:

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\, \mathrm{G}_t\right] && \text{REINFORCE} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\; Q^w(s,a)\right] && \text{Q Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\; A^w(s,a)\right] && \text{Advantage Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\; \delta\right] && \text{TD Actor-Critic}
\end{aligned}$$

- Each leads a stochastic gradient ascent algorithm
- Critic uses policy evaluation (e.g. MC or TD learning) to estimate $Q_\pi(s, a)$, $A_\pi(s, a)$ or $V_\pi(s)$

Questions?