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Trajectory Planning and Inverse Kinematics Solver for Real Biped Robot with 10 DOF-s

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Abstract: This work deals with the stability analysis of two legged (humanoid) robots during walking. This research area is characterized by the fact that there are a lots of publications, a method for synthesizing the gait of a planar biped walking on level ground is presented. Both the single support phase (SSP) and the double support phase (DSP) are considered. A complete step can be divided into a SSP and a DSP. The SSP is characterized by one limb (the swing limb) moving in the forward direction while another limb (the stance limb) is pivoted on the ground. This phase begins with the swing limb tip leaving the ground and terminates with the swing limb touching the ground. Its time period is denoted as TS. In the DSP, both lower limbs are in contact with the ground while the upper body can move forward slightly. The time period of this phase is denoted as TD. In the following step, the roles of the swing limb and the stance limb are exchanged. It has been noticed that the joint angle profiles can be determined if compatible trajectories for the hip and the tip of the swing limb can be prescribed. Also is presented in practical implementation in a real robot.

Keywords: Biped, algorithms, trajectory, inverse kinematics, DH.

1. INTRODUCTION

A rigid multi body system consists of a set of rigid objects, called links, joined together by joints such as introduced in humanoid robots and has been studied in biped locomotion articles [Hernndez-Santos et al. 2011]. Biped locomotion has been a topic of great attention in various researches performed on legged robots and is probably the most suitable method for robots to execute assigned maneuvers in a real environment with various obstacle conditions and geometry. Widespread studies have been conducted on biped walking, and now biped robots are capable of walking with a certain amount of stability. Trajectory control, motion planning and locomotion modeling is completely related to the kinematics analysis as it is fundamental in the study of linkage systems.

Forward and Inverse Kinematics are commonly implemented to determine main parameters affecting humanoid robot behavior and specify the reliable method to control motion and preserve stability [Azevedo et al. 2004]. The most frequently practiced parameters to be defined are joint parameters, including required drive torques, angles, and related twists [Guzmn et al. 2012, Hernndez-Santos et al. 2011].

1.1 The Denavit Hartenberg Convention

In this section, it is introduced the forward or configuration kinematics for a rigid robot. The forward kinematics problem represents the relationship between the individual joints of the robot humanoid and the position and orientation of the tool or end effector.

The Denavit Hartenberg parameters for the 10 DOF of the lower body (derived from Fig.4) are shown in the Table 1 and can be used further for forward and inverse kinematic. Transformations matrices of the end effector coordinate frame (x_e, y_e, z_e) to robot base frame (x_0, y_0, z_0) is:

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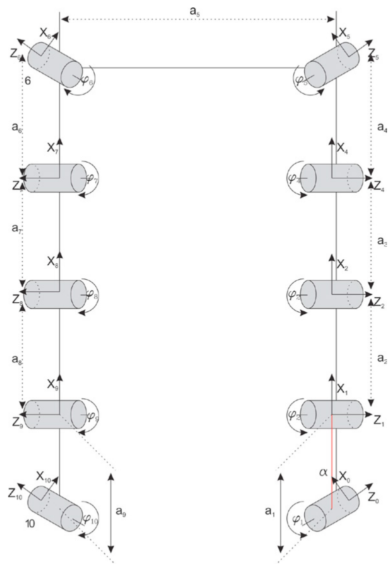

Fig. 1. Kinematic model of XA2S

$$T_{1-11} = A_1 + A_2 + ... + A_{11} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & d_x \\ c_{21} & c_{22} & c_{23} & d_y \\ c_{31} & c_{32} & c_{33} & d_z \\ c_{41} & c_{42} & c_{43} & d_{44} \end{bmatrix} \quad (1)$$

Using the D-H on above matrices (1) and after multiplying of these matrices it is calculated the c_{41}, c_{42}, c_{43} and c_{44} (2).

$$c_{41} = 0 \qquad c_{42} = 0 \qquad c_{43} = 0 \qquad c_{44} = 1 \quad (2)$$

The Initial position is defined as follows Fig. (2):

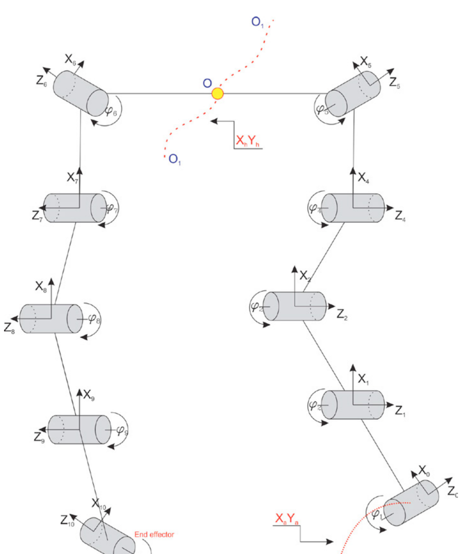

Fig. 2. Initial position of XA2S

Where x_{a1} and x_h are:

$$x_{a1} = \begin{cases} x_a(t,k) \quad if \quad 0 < t \leq s \\ x_a(s,k) \quad if \quad s < t \leq 2s + 2d \\ [x_a[t-(2s+2d),2k] + x_{a1}(2s+2d) - x_a(0,2k)] \Rightarrow \\ \Rightarrow if \quad 2s + 2d < t \leq 3d + 2d \end{cases} \quad (3)$$

$$x_h = \begin{cases} x_{hs}(t,k) \quad if \quad 0 < t \leq s \\ x_{hD}(t,k) \quad if \quad s < t \leq s + d \\ x_{hs}[t-(s+d),2k] + x_h(s+d) - \Rightarrow \\ \Rightarrow -x_{hs}(0,2k) \quad if \quad s + d < t \leq 2s + d \\ x_{hD}[t-(s+d),2k] + x_h(1.3) - \Rightarrow \\ \Rightarrow x_{hD}(s,2k) \quad if \quad 2s + d < t \leq 2s + 2d \\ x_{hs}[t-(2s+2d+0.01),3k] + x_h(2s+2d) \Rightarrow \\ \Rightarrow -x_{hD}(0,3k) \quad if \quad 2s + 2d < t \leq 3s + 2d \end{cases} \quad (4)$$

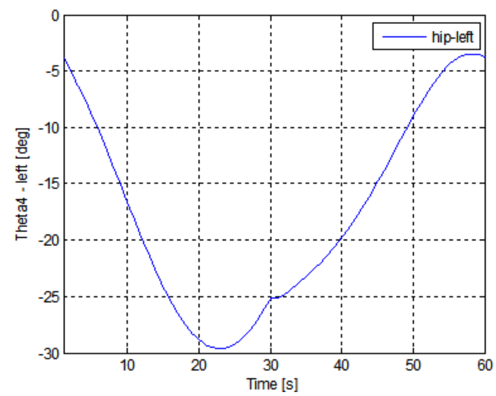

Fig. 3. Joint angle, hip left for two steps

From equations (1), (2), (3), and (4) follows for the angle θ_4 (5) where as a result we have generated trajectory Fig. (3).

$$\theta(t)_{4left-hip} = acos\left(\frac{(x_h - x_{a1}) - l_1 cos(\theta_1)}{l_2}\right) \quad (5)$$

From Fig. (2) follows for the angle $\theta_{3left-knee}$ where as a result we have generated trajectory Fig. (5).

$$\begin{aligned} O_1 &= x_h - x_{a1}, \\ O_2 &= x_h - x_{a2}, \\ F_1 &= y_h - y_{a1}, \\ F_2 &= y_h - y_{a2}, \\ E_1 &= \left(\frac{(x_h - x_{a2})^2 - l_1^2 + (y_h - y_{a1})^2 - l_2^2}{2l_1^2}\right), \\ \theta_{3left-knee} &= acos\left([(x_h - x_{a1})E_1 + (y_h - y_{a1})] \Rightarrow \right. \\ &\left. \frac{\Rightarrow \sqrt{(x_h - x_{a1})^2 + (y_h + y_{a1})^2 - E_1^2}}{(x_h - x_{a1})^2 + (y_h + y_{a1})^2}\right) \end{aligned} \quad (6)$$

Where x_{a2}, y_{a1} and y_{a2} are:

$$x_{a2} = \begin{cases} 0 \quad if \quad 0 < t \leq s + d \\ x_a[t-(s+d),2k] - x_a(0,2k) \quad if \quad s + d < t \leq 2 \\ x_{a2}(2s+d) \quad if \quad 2s + d < t \leq 3s + 3d \end{cases} \quad (7)$$

$$y_{a1} = \begin{cases} y_a(t,k) & if \quad 0 < t \leq s \\ 0 \quad if & s < t \leq 2s + 2d \\ y_a[t-(2s+2d), 2k] & if \;\; 2s+d < t \leq 3s+2d \end{cases} \tag{8}$$

$$y_{a2} = \begin{cases} 0 \quad if & 0 < t \leq s + d \\ y_a[t-(s+d), 2k] & if \;\; s+d < t \leq 2s+d \\ 0 \quad if & 2s+d < t \leq 3s+3d \end{cases} \tag{9}$$

According Fig. (2) and equations (5) and (6), the angle can be calculated (10) where as a result we have generated trajectory Fig. (4).

$$\theta_{2Left-ankle} = 90 - (\theta_{3Left-knee} + \theta_{4Left-hip}) \tag{10}$$

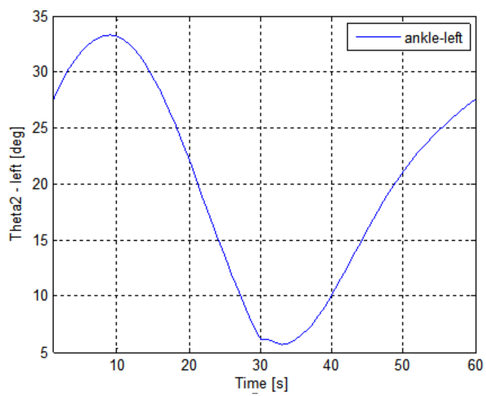

Fig. 4. Joint angle, ankle left for two steps

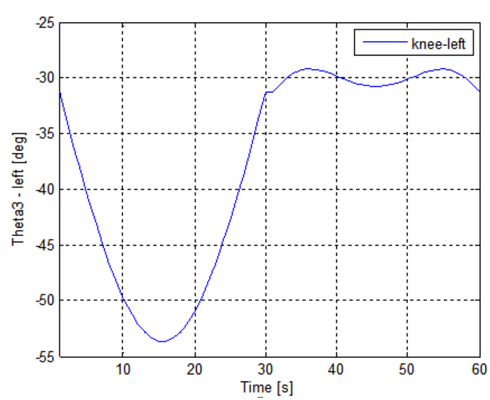

Fig. 5. Joint angle, knee left for two steps

From Fig. (2) and equations (4) and (5), can be obtained the angle as follows (11) where as a result we have generated trajectory Fig. (6).

$$\theta_{11Right-ankle} = 90 - (\theta_{Right-hip} + \theta_{10Right-knee}) \tag{11}$$

In Fig.(7) and Fig.(8) we can see generated trajectory for right knee and right hip.

Now all the angles for the joint are calculated and therefore it is possible to determine the position and orientation of the right/left hip. The kinematic model is derived such that the global positions, velocity and acceleration of each

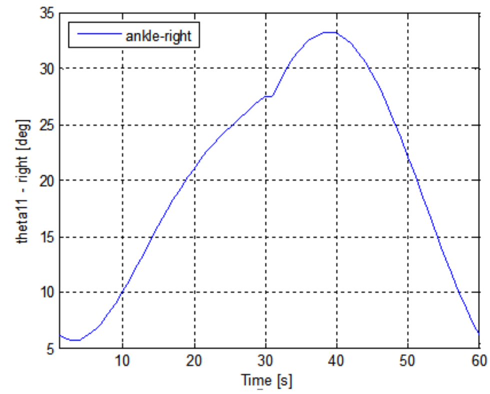

Fig. 6. Joint angle, ankle right for two steps

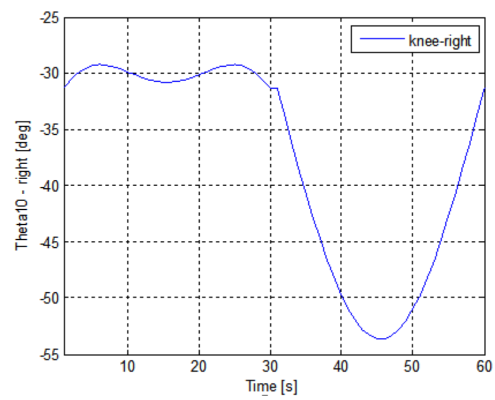

Fig. 7. Joint angle, knee right for two steps

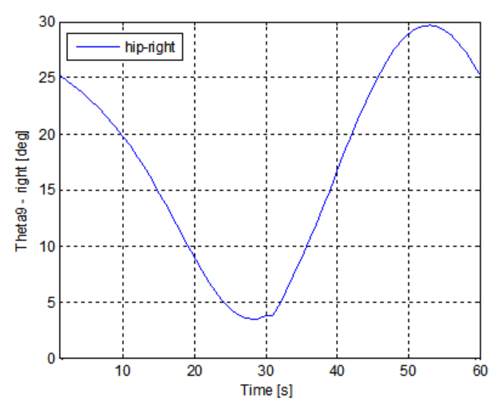

Fig. 8. Joint angle, hip right for two steps

link and their Center of Mass (CoM) are calculated by given the joint angles [Kajita and Tani, 1991].

2. ALGORITHMS FOR MOTION PLANNING IN THE SAGITTAL PLANE

In this section planning motion for the sagittal plane (axes x, y) is accomplished, considering only the phase of double support, and considering that the transition between the foot support is instantaneous [Mu X., and Wu Q., 2004] and [Omer et al. 2009]. To accomplish this task are required some parameters, they determining the configuration of walk, as shown in Fig. (9).

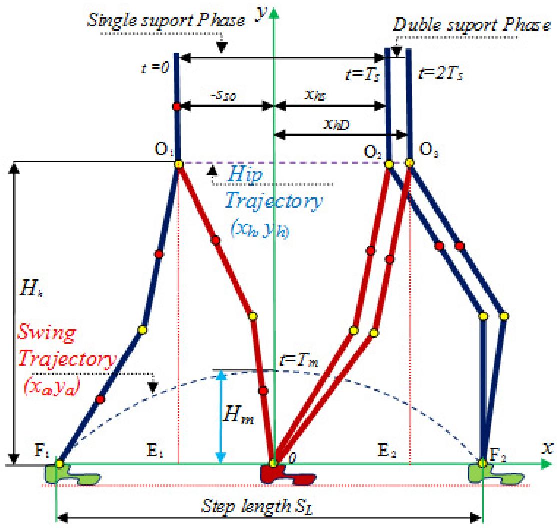

Fig. 9. Coordinates for planning motion in the sagittal plane [Bajrami X., 2013, Bajrami et al. 2013]

The explanations below for abbreviations that are used in Fig (9).

- T_s - Time for each step,
- S_l length of step
- H_{hip} twisting of the hip
- H_m height of the floating leg
- H_h height of the floating hip
- $x_{a1}, x_{a2}, y_{a1} and y_{a2}$, are trajectories of foots on x respectively y-axes
- x_{hs}, y_{hs}, are trajectories of hip on x respectively y-axes during single support Phase "SSP"
- x_{hD}, y_{hD}, are trajectories of hip on x respectively y-axes during double support Phase "DSP"

2.1 Trajectories of the swing limb

The trajectory of the tip of the swing limb during the SSP is an important factor in biped walking [Mu X., and Wu Q., 2004].

$$X_a : x_a = \begin{cases} x_a = a_0 + a_1 * t + a_2(k) * \Rightarrow \\ \Rightarrow *t^2 + a_3(k) * t^3 \\ y_a = b_0 + b_1 * t + b_2(k) * \Rightarrow & 0 < t \leq T_s \\ \Rightarrow *t^2 + b_3(k) * t^3 + \Rightarrow \\ \Rightarrow +b_4(k) * t^4 + b_5(k) * t^5 \end{cases} \tag{12}$$

for the x_a and y_a separately [9].

Next the constraint equations that can be used for solving the coefficients, a_i and $b_j (i = 0...3)$ and $(j = 0...5)$ are derived. Casting the gait patterns in terms of four basic quantities: step length S_l, step period for the SSP T_s, maximum clearance of the swing limb H_m and its location S_m. Other constraints used for designing the swing limb motion are repeatable gait and minimizing the effect of impact [Mu X., and Wu Q., 2004].

2.2 Repeatability of the gait

The requirement for repeatable gait imposes the initial posture and angular velocities to be identical to those at the end of the step [Mu, X. and Wu, Q., 2004]. Furthermore, since during the DSP both tips are in contact with the ground and remain stationary, the initial velocities in both horizontal and the vertical direction must remain zero [Kajita et al. 2003, Kajita and Tani, 2004]. Subsequently, the following relations must hold:

$$x_a(k) = -\frac{S_l}{2}; x_a(T_s) = \frac{S_l}{2}; \dot{x}_a(k) = 0; \dot{x}_a(T_s) = 0. \tag{13}$$

Where:

$$K(k) = \begin{bmatrix} T_m^2 & T_m^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & T_s^2 & T_s^3 & T_s^4 & T_s^5 \\ 0 & 0 & T_m^2 & T_m^3 & T_m^4 & T_m^5 \\ 0 & 0 & 2T_m & 3T_m^2 & 4T_m^3 & 5T_m^4 \\ T_s^2 & T_s^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2T_s & 3T_s^2 & 4T_s^3 & 5T_s^4 \end{bmatrix}^{-1} * \Rightarrow$$
$$\Rightarrow * \begin{bmatrix} S_m - a_0(k) - a_1 * T_m \\ -b_0 - b_1 * T_s \\ H_m - b_0 - b_1 * T_s \\ -b_1 \\ \frac{S_L(k)}{2} - a_0(k) - a_1 * T_s \\ -a_1 \end{bmatrix} \tag{14}$$

$$SL(k) = \begin{cases} 0.7 & if \quad 0 \leq k \leq 0.7 \\ ... \\ 0.7 & if \quad 9.7 \leq k \leq 10.5 \end{cases} \tag{15}$$

From equation (12) variables are derived as follows:

Table 1. Optimization parameters for foot trajectory

Parameters					
a_0	a_1	a_2	a_3		
-0.35	0	5.833	-6.481		
b_0	b_1	b_2	b_3	b_4	b_5
0	0	8.889	-29.63	24.691	$1.645 * 10^{-13}$

2.3 Trajectories of hip

Hip motion has significant effect on the stability of the biped system [Mu X., and Wu Q., 2004]. Here, the trajectory of the hip is designed for the SSP and the DSP, separately, which are denoted by the coordinate of the hip position as $X_{hS} : (x_{hS}(t), y_{hS}(t))$, in the SSP and X_{hD} :

$(x_{hD}(t), y_{hD}(t))$ in the DSP. A third order polynomial function is used to describe X_{hS} and X_{hD}, respectively. With a general function of vertical hip motion, they are shown below:

$$x_h S = c_0(k) + c_1 * t + c_2(k) * t^2 + c_3(k) * t^3 \Rightarrow \\ \Rightarrow T_s < t \leq T_D \\ x_h D = d_0(k) + d_1(k) * t_D + d_2(k) * t_D^2 + \Rightarrow \\ \Rightarrow +d_3(k) * t_D^3, \quad T_s < t_D \leq T_D \tag{16}$$

$$y_{hS} = h_h, \quad T_s < t \leq T_D; y_{hD} = h_h, \quad T_s < t_D \leq T_D \tag{17}$$

The constraint relations are described as follows:

Vertical hip motion: One desired feature of biped gait is to keep the minimum vertical motion of the gravity center, which requires minimum vertical motion of the hip [Mu X., and Wu Q., 2004]. For the sake of simplicity, we assume Y_{hS} and Y_{hD} a constant at any time during the whole gait cycle, i.e.

$$y_{hS}(t) = H_h; y_{DS}(t) = H_h \tag{18}$$

H_h should be given such that the robot does not go through the singular configurations.

2.4 Repeatability of the gait

To keep the gait repeatable, the posture and angular velocity at the beginning of the SSP must be identical to that at the end of the DSP [Mu X., and Wu Q., 2004] thus, the following relations must hold:

$$x_{hS}(0) = -S_S O; x_{hD}(k) = \frac{S_L(k)}{2} - S_{S0} \tag{19}$$

$$\dot{x}_{hS}(0) = V_{h1}; \dot{x}_{hD}(k) = V_{h1} \tag{20}$$

Where:

$$xh_0(k) = \frac{-S_L(k)}{4}, S_{s0}(k) = |x_a(0,k) - xh_0(k)| \tag{21}$$

Variables that change, if:

$$SD_0(k) = \frac{S_L(k)}{2} - 0.2, then\ with \\ C_0(k) = -S_{s0}(k), c_1 = v_{h1} \tag{22}$$

follows

$$K(k) = \begin{bmatrix} T_s^2 & T_s^3 & 0 & 0 & 0 & 0 \\ 2T_s & 3T_s^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T_s & T_s^2 & T_s^3 \\ 0 & 0 & 1 & T_D & T_D^2 & T_D^3 \\ 0 & 0 & 0 & 1 & 2T_s & 3T_s^2 \\ 0 & 0 & 0 & 1 & 2T_D & 3T_D^2 \end{bmatrix}^{-1} * \Rightarrow \\ \Rightarrow * \begin{bmatrix} S_{DO}(k) - (c_0(k) - a_1 * T_s) \\ V_{h2} - c_1 \\ S_{DO}(k) \\ \frac{S_L(k)}{2} - S_{SO}(k) \\ V_{h2} \\ V_{h1} \end{bmatrix} \tag{23}$$

2.5 Continuity of the gait

The hip trajectory must be continuous during the whole gait cycle, i.e., the horizontal displacements and velocities of the hip at the end of the SSP and the beginning of the DSP must be identical respectively, which leads to [Mu X., and Wu Q., 2004]:

$$x_{hD}(0) = S_{DO}; x_{hS}(k) = S_{DO} \tag{24}$$

$$\dot{x}_{hD}(0) = V_{h2}; \dot{x}_{hS}(k) = V_{h2} \tag{25}$$

From equations (16) and (17) follows for the variables $c_0(k)$, $c_1(k)$, $c_2(k)$, $c_3(k)$, $d_0(k)$, $d_1(k)$, $d_2(k)$, $d_3(k)$ and are presented below with the respective values:

Table 2. Optimization parameters for hip trajectory

Parameters			
c_0	c_1	c_2	c_3
-0.175	0.5	0.958	-1.481
d_0	d_1	d_2	d_3
-0.42	3.65	-7.5	5

Where: x_{a1}, x_{a2}, y_{a1}, y_{a12} are trajectories of foots on x respectively y-axes [Bajrami Xhevahir et al. 2013, Bajrami Xh. et al. 2013].

3. CONCLUSION

The main goal was the derivation of a walking model for the dynamic behavior of a Humanoid

Table 3. Optimal parameters for the foots

Parameters			
x_{a1}	x_{a2}	y_{a1}	y_{a2}
-0.35	0	0	0
...	...	...	...
0	0.6	0	0

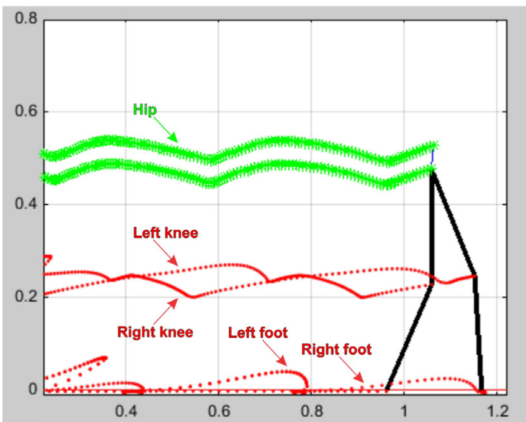

Fig. 10. Hip, knee and foot trajectory of the biped in sagittal view (X, Z)

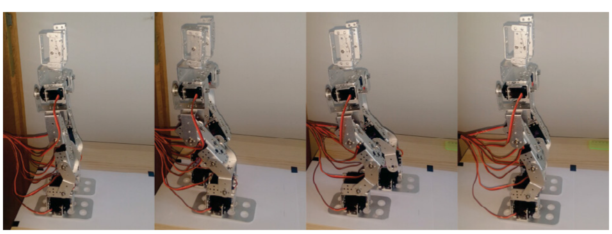

Fig. 11. Real biped robot walking in sagittal view

robot. For the two different locomotion phases a new approach was created. In this work are given trajectories of movements for legs and hip. After these trajectories for biped walking are generated Fig.(10), they also have been tested on real biped Fig.(11), and with full convince we can say that real biped showed a very good performance while walking. Knowing that walking synchronization and practical implementation it's a hard work to implement, we remain hopeful that this work will serve to young researchers in this field.

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