Think about the last rail ticket you bought.

How much did it cost?

If it was 10% more expensive, would you still buy it?

would you change your ticket type?

would you change it for a bus, car or plane?

would you give up travelling?

Would you be willing to pay at least a bit more for the same ticket?

Modelling Conditional Fare Elasticities of Rail Demand

Samira Marx - MSc Transport Economics

Dr. Jeremy Toner - Supervisor

Modeling Conditional Fare Elasticities of Rail Demand

1. Statement of Research

1.1 Conditional Fare Elasticities

1.2 Problem of Research

1.3 Previous contributions

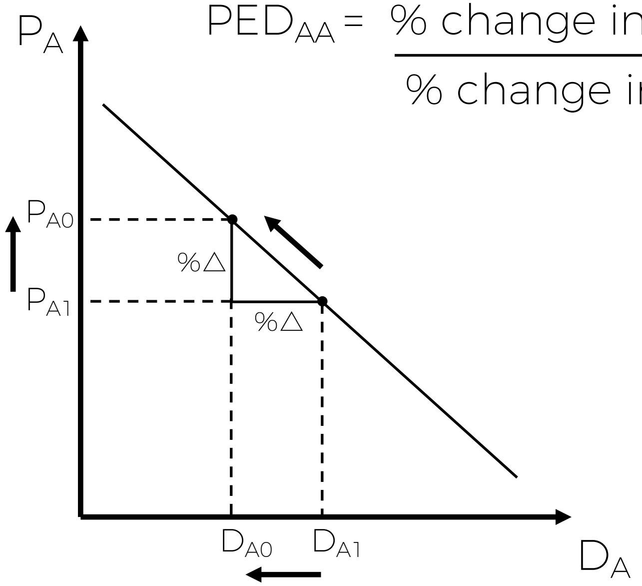
1.4 Objectives

CONDITIONAL ~~FARES~~ Price ELASTICITIES

OF RAIL DEMAND

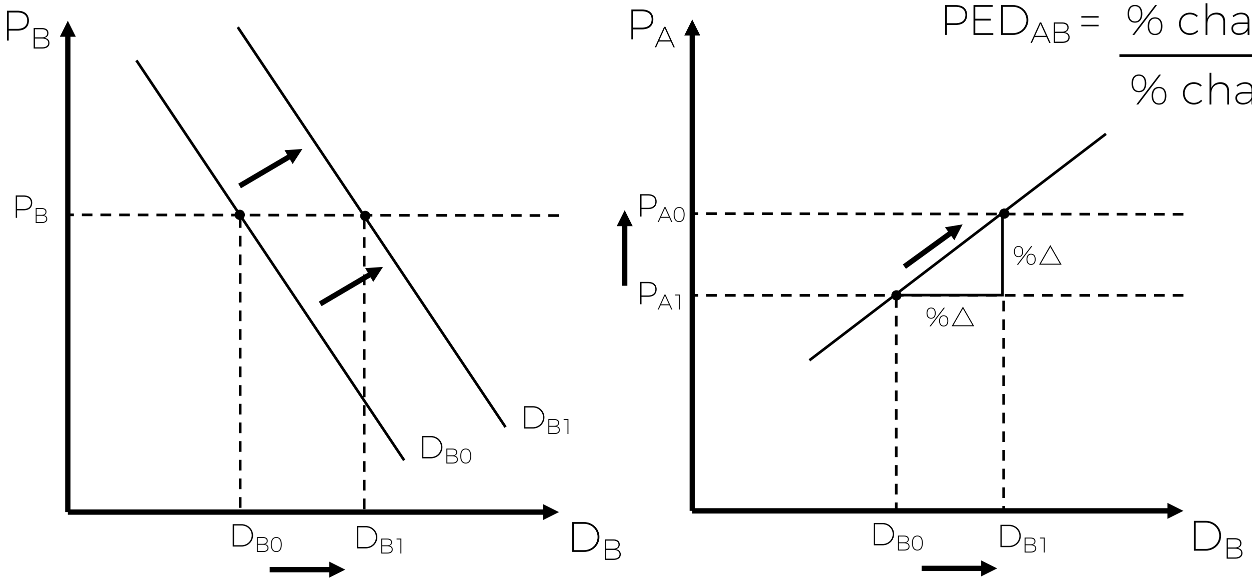
Own Elasticity

$$PED_{AA} = \frac{\%\text{ change in } P_A}{\%\text{ change in } D_A}$$



Cross Elasticity

$$PED_{AB} = \frac{\%\text{ change in } P_A}{\%\text{ change in } D_B}$$



Conditional fare elasticities are given by:

$$c_i = f_{ii} + \sum_k f_{ik}$$

, where:

c_i is the conditional elasticity of ticket i;

f_{ii} is the fare **own-elasticity** of ticket i;

f_{ik} are the fare **cross-elasticities** for other k tickets.

Statement of Research/ **Problem of Research**

WHY IS THAT A PROBLEM?

measurement of

price-effects

of rail tickets

DIFFERENT APPLICATIONS

RAIL DEMAND FORECAST

PRICE DISCRIMINATION

[PROFIT MAXIMIZATION]

1. Complexity increases geometrically.

For J type of tickets, there are J^2 possible effects.

If A and B are substitutes then: $\triangle P_A \rightarrow D_A; D_B$

$\triangle P_B \rightarrow D_A; D_B$

… causing regression models to become complex:

$$V_A = P_A^{f_{AA}} * P_B^{f_{AB}} [\dots]$$

$$V_B = P_A^{f_{AB}} * P_B^{f_{BB}} [\dots]$$

2. Traditional regression models usually do not provide coherent estimates .

- own elasticities

+ cross elasticities

Previous Contributions

"Clearly the confidence intervals (…) mean these results are meaningless."

Author's conclusion on the first rail ticket elasticities

Joint RP-SP modelling

Estimation of elasticities based on SP data (market research) scaled by RP data (RUDD).

WEAKNESS: for some markets the results were not satisfactory.

OTHER TRIALS

- Free estimation
- Slutsky Symmetry Constraint
- Diversion Factors Constraint
- Own elasticities estimates constraining cross elasticities by "best evidence"
- Direct estimation of conditional elasticities

I have misplaced my phone somewhere in the home.
I can call my phone to locate it.

Where would I search for it?

Frequentist

I can hear the phone beeping.

I have a mental model to identify the area from which the sound is coming.

Therefore, upon the beeping, I infer the area I must search to locate it

Bayesian

I can hear the phone beeping.

Apart from my mental model which helps me to identify the area from which the sound is coming, I also know the places I have misplaced my phone in the past.

So I combine my inferences using the beeps and my prior information to identify the location I must search to locate the phone

1. Statement of Research/ **Objectives**

GENERAL OBJECTIVES:

Contribute to the studies of demand forecast in the rail industry;
Provide an alternative econometric model for estimation of conditional elasticities.

SPECIFIC OBJECTIVE

Review the conditional price elasticities of rail demand in the Great Britain applying bayesian inference.

STUDY'S BOUNDARIES

NLLD

Considered only Non-London Long Distance tickets in Great Britain.

1 F R A

Four ticket types:

- 1st Class;
- Full (anytime);
- Reduced (off-peak);
- Advance.

PDFH

Built upon the demand forecast methodology adopted in the Passenger Demand Forecast Handbook - PDFH.

2. Methodology

2.1 Data & Exploratory Analysis

2.2 Bayesian Statistics

Data Overview

This study will be based in the Rail Usage and Drivers Dataset (RUDD) for Non-London Long Distance (> 20miles).

PANEL DATA:

1996~2014	**6.184**	**852**
annual observations	OD pairs	rail stations

Industry Info

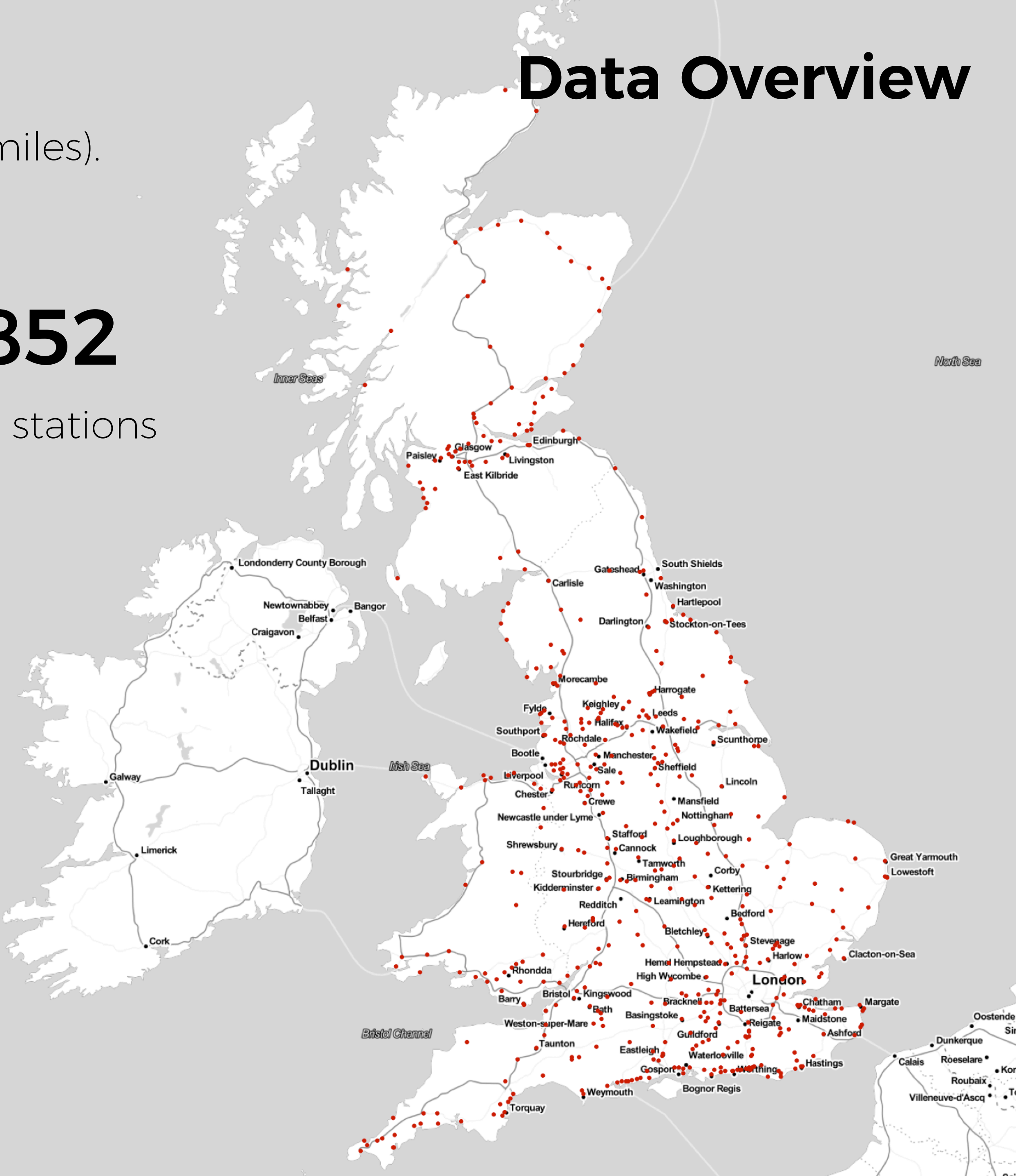
revenues and journeys by ticket type.

Operational Info

distance traveled, generalized journey time, journey time, interchange penalty, average service interval.

Socio-demographic

GDP, population, household income, inflation, employment, competitor modal's cost, car ownership,



why a bayesian approach?

"Bayesian econometrics and how to get rid of those wrong signs."

William Griffiths, 1988

cited by 20 (Google Scholar)

"The Time Has Come: Bayesian Methods for Data Analysis in the Organizational Sciences. "

John K. Kruschke, Herman Aguinis, and Harry Joo, 2012

ADVANTAGES

- natural interpretation of the results of a statistical investigation then the frequentist approach.
- provides a formal framework for incorporating prior knowledge.

[ALLEGED] DISADVANTAGES

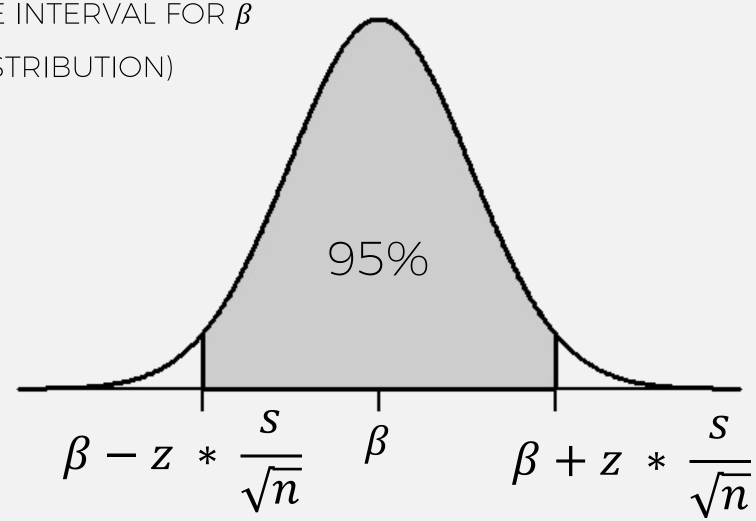
- It is subjective since you need to choose a prior distribution for your parameters;
- It is computationally intensive;
- It is redundant

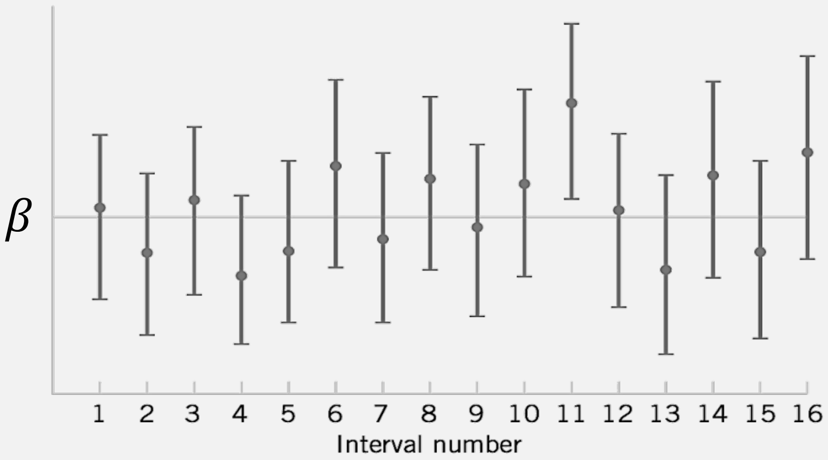
Interpreting reported information about β

Frequentist

CONFIDENCE INTERVAL FOR β

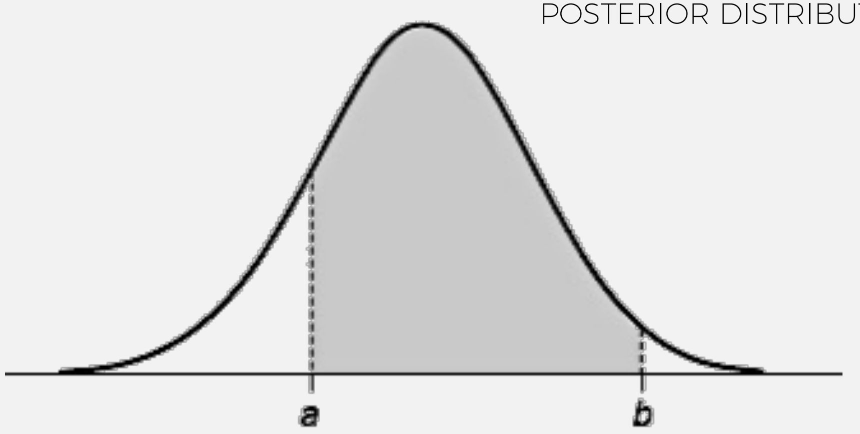
(NORMAL DISTRIBUTION)





Bayesian

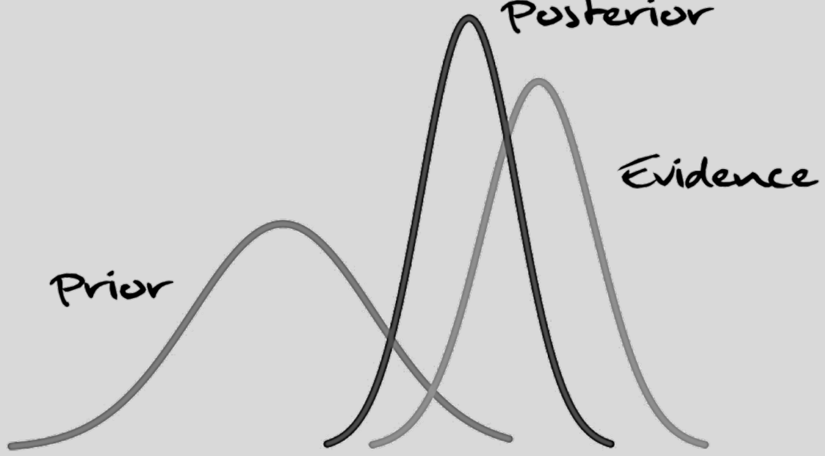
POSTERIOR DISTRIBUTION OF β



The output of a bayesian analysis is a full probability density function for β.

Therefore one can speak a probability of β lying between a and b (as the area below the curve).

what's that about?



"Probability is orderly opinion, and that inference from data is nothing other than the ***revision of such opinion in the light of relevant new information.****"*

BAYES' THEOREM

POSTERIOR DENSITY

Given the data, what do we know about $\boldsymbol{\theta}$?

PRIOR DENSITY

Belief, theory, accumulated knowledge.

LIKELIHOOD

Function of the parameters given a data, assuming a probability distribution.

$$p(\boldsymbol{\theta} \mid \mathrm{Data}) = \frac{p(\boldsymbol{\theta}) * p(\mathrm{Data} \mid \boldsymbol{\theta})}{p(\mathrm{Data})}$$

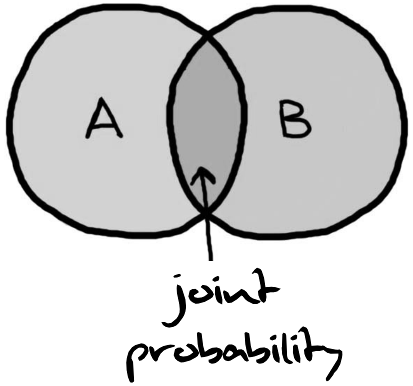
EVIDENCE

Bayes' rule is based in simple rules of probability:

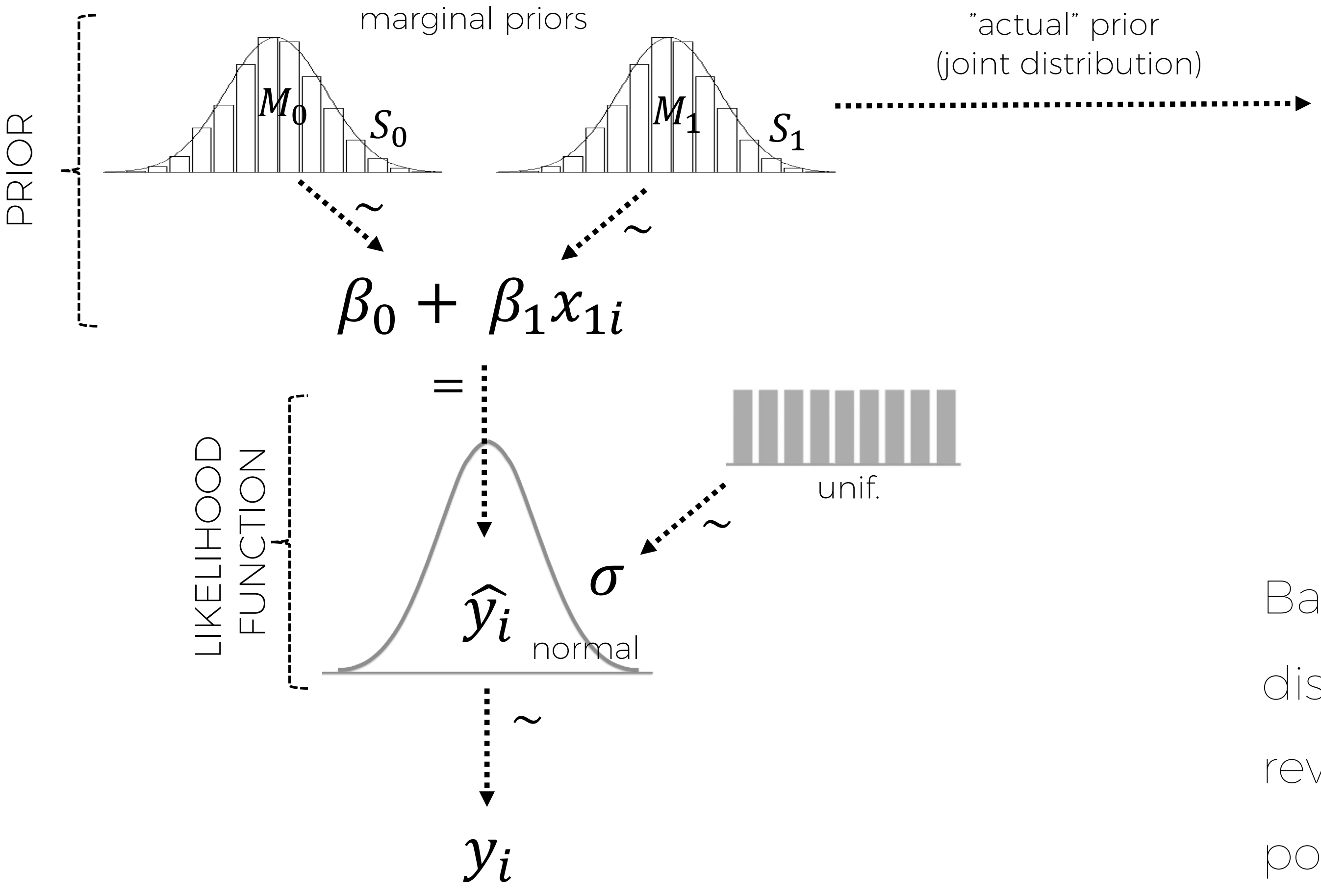
$$p(A,B) = p(B|A)*p(A) \quad \text{or} \quad p(A,B) = p(A|B)*p(B)$$

$$p(B|A)*p(A) = p(A|B)*p(B)$$

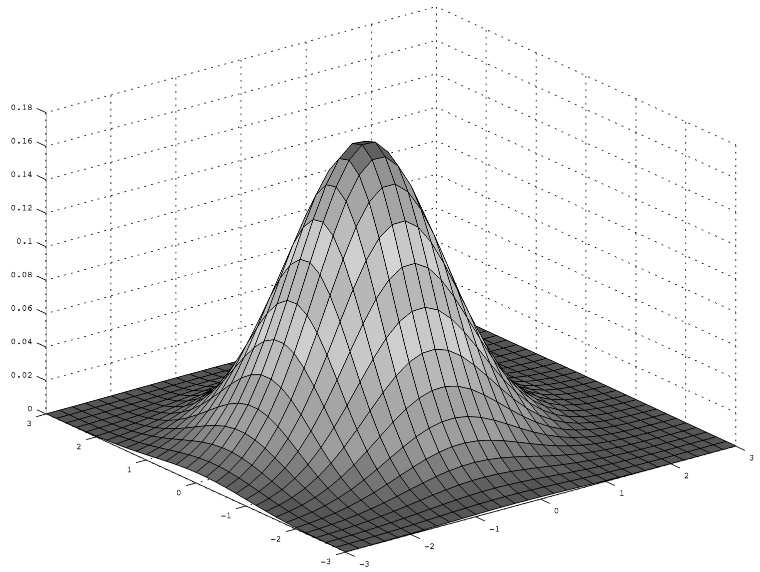
$$p(B|A) = \frac{p(A|B)*p(B)}{p(A)}$$



Bayesian regression/ **INPUTS: Prior and Likelihood**

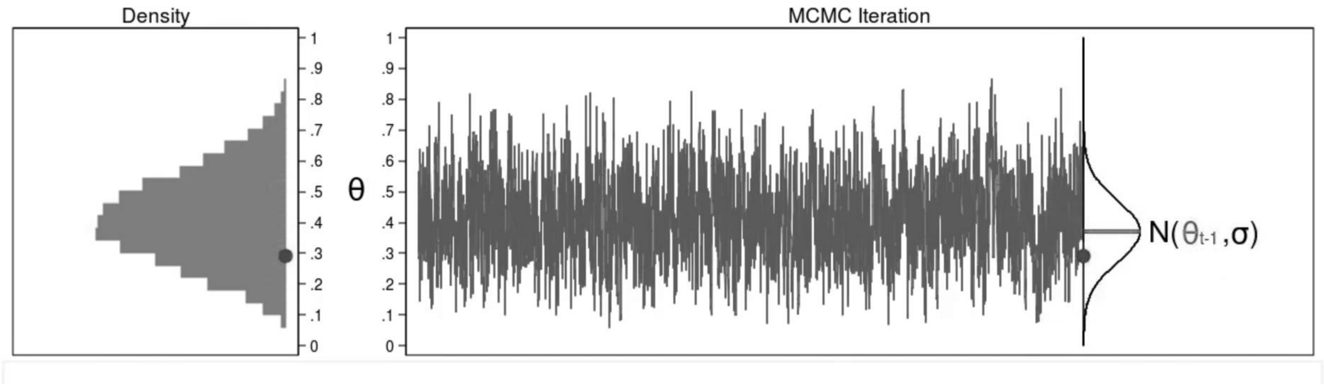


PROBABILITY SPACE FOR A BIVARIATE NORMAL

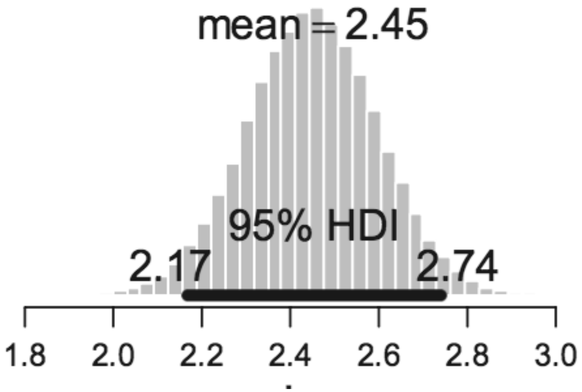


Bayesian analysis yields a complete distribution over the joint parameter space, revealing the relative credibility of all possible combinations of parameter values.

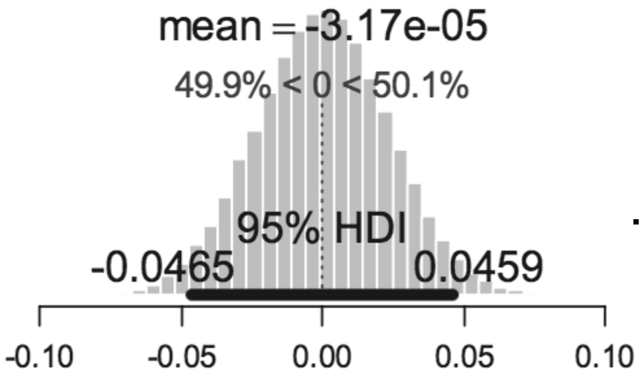
Bayesian regression/ **OUTPUT: Posterior Distribution**



The posterior distribution is approximated by a very large sample of combinations of the parameters (β_0, β_1 and σ) through a Markov Chain of Monte Carlo, which selects "credible" draws based in the priors and the likelihood.



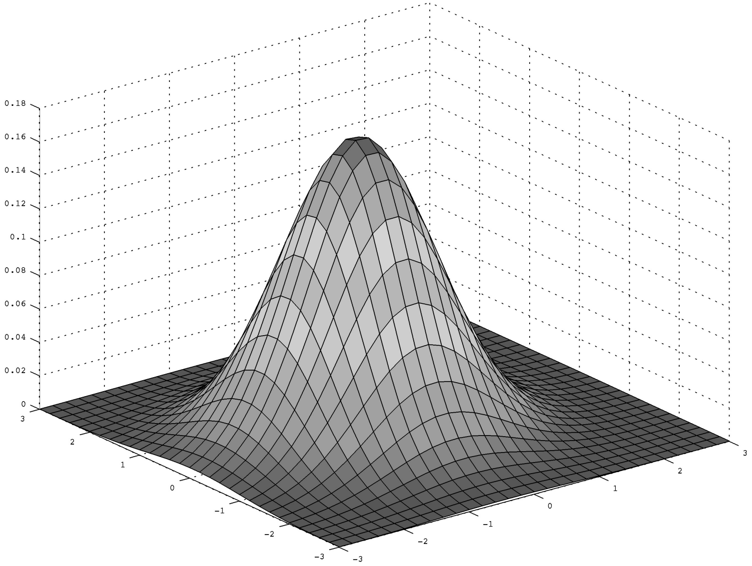
β_0



β_1

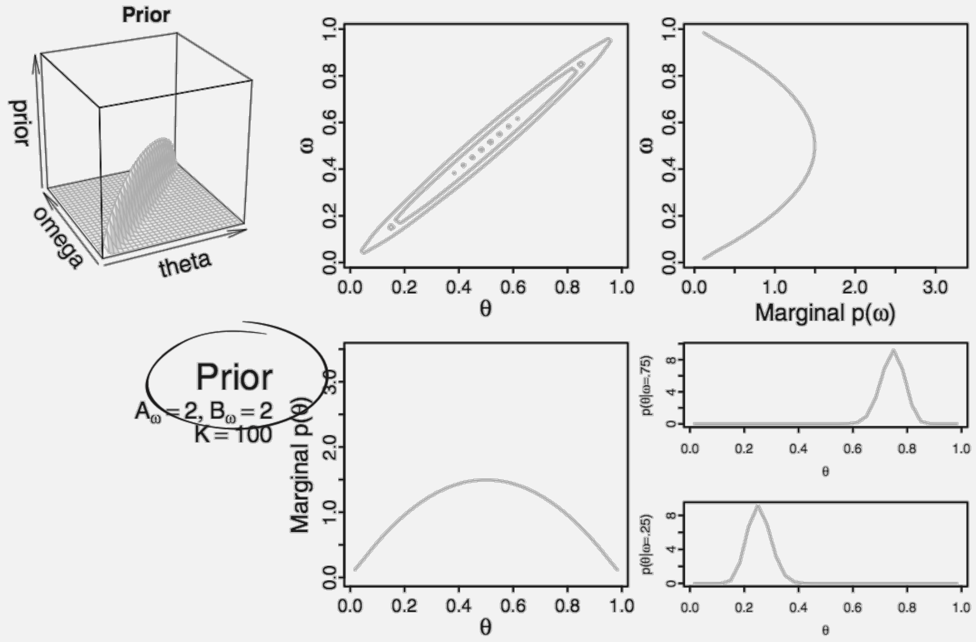
marginal posterior

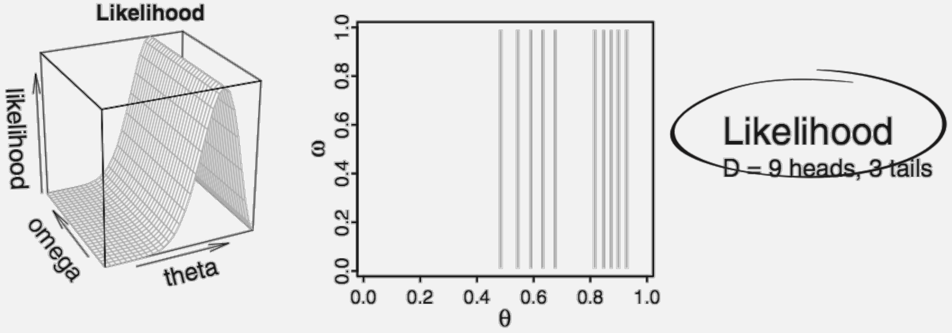
"actual" posterior
(joint distribution)

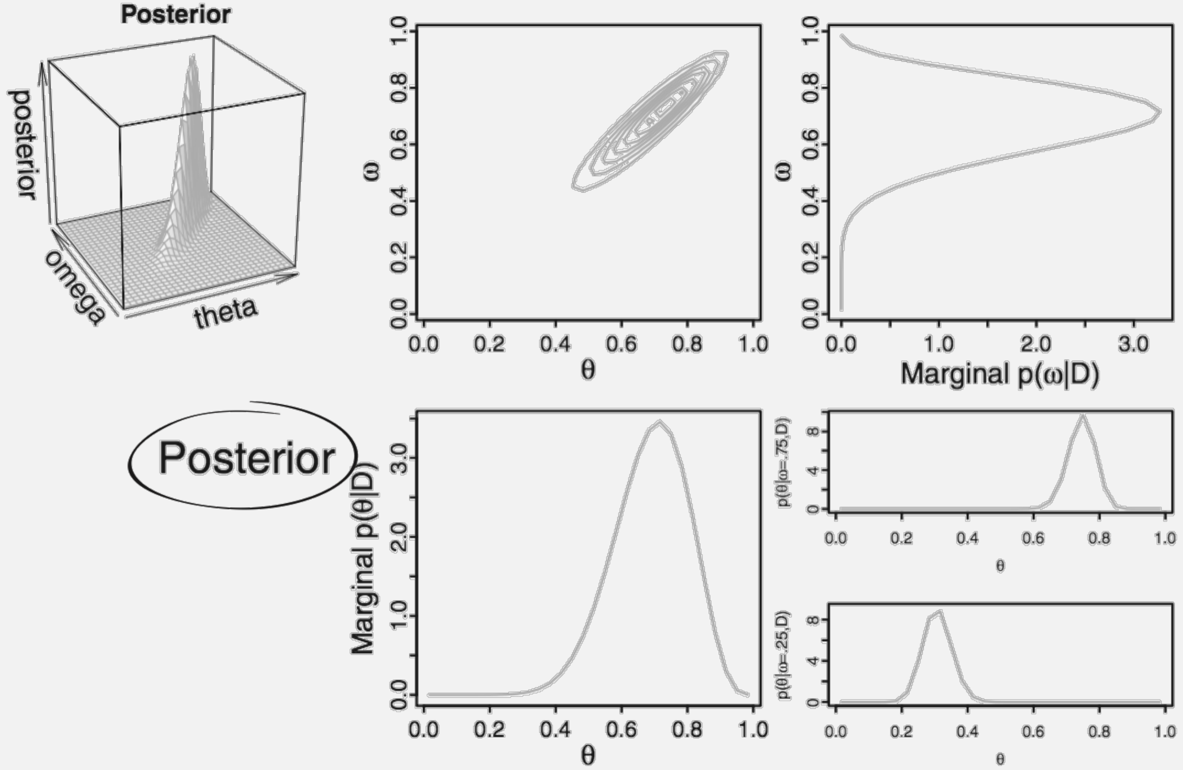


PROBABILITY SPACE FOR A BIVARIATE NORMAL

visualizing probability densities







NEXT STEPS

1.

PREPARE THE DATA

Check consistency across the data set;

Clean it;

Interpret variables and its values.

2.

SET UP THE MODEL

Specify the model,

$$y_i = \beta_0 + \ \beta_1 x_{1i} + \cdots + \beta_{ni}$$

and define the priors – which must be justified for a skeptical and scientific audience.

3.

CONDITION IT ON THE OBSERVED DATA

Calculate and interpret the posterior distribution

4.

EVALUATE THE FIT AND SENSITIVENESS

How well does the model fit the data?

Are the conclusions reasonable?

How sensitive is the model to the priors?

5.

ITERATE

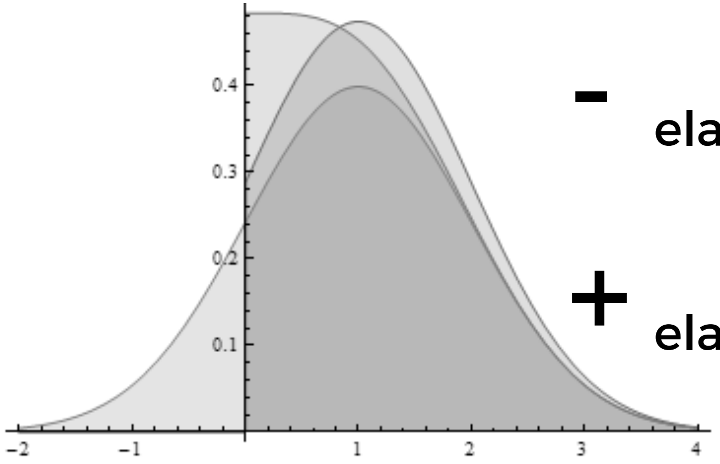
Add complexity in layers

Having a first model as simple as possible and add richness and complexity in layers decrease the scope of error at each iteration.

3. Expected Results

Expected Results

Compute coherent own and cross elasticities for rail tickets.



- own elasticities

+ cross elasticities

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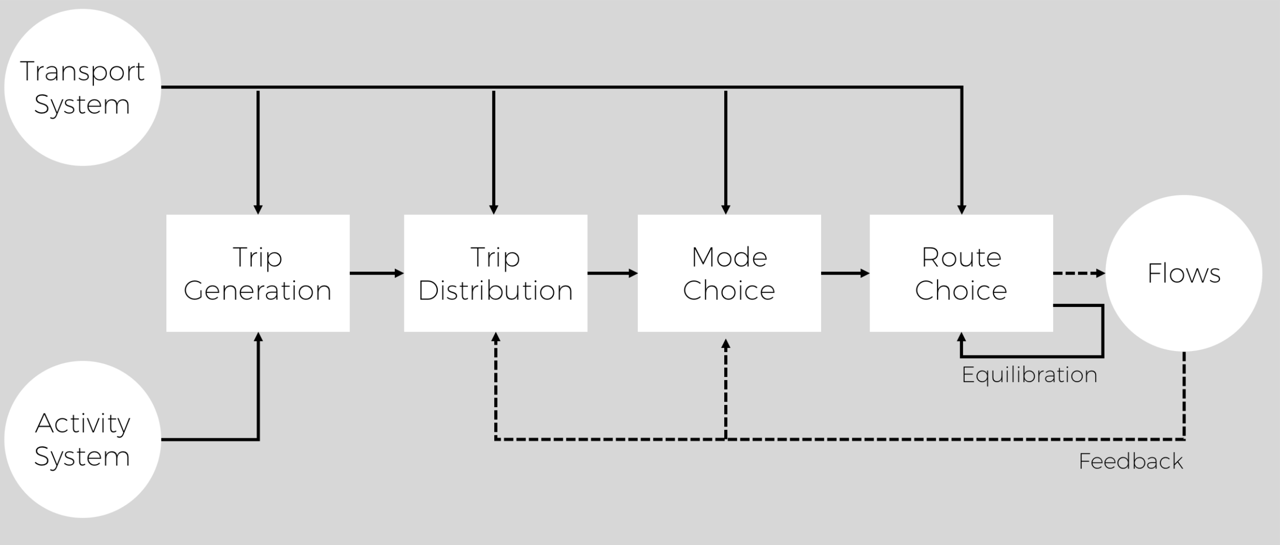
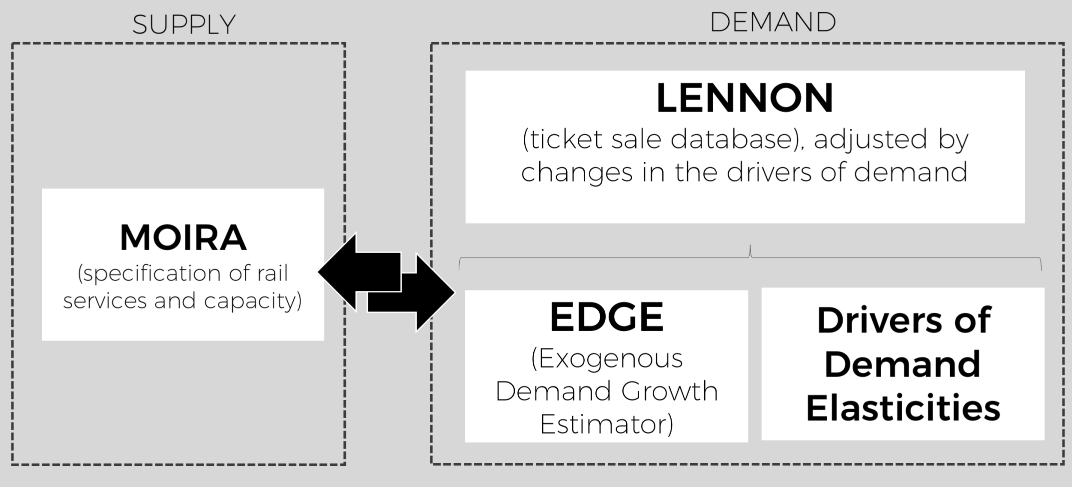
PASSENGER DEMAND FORECAST IN THE RAIL INDUSTRY

In place of a traditional forecast method, i.e. the four step model, rail demand models are based on the relationship between changes in the volume of passengers and travelling by rail and changes in the factors known as **drivers of demand** (Worsley, 2012).



The Four Step Model (Traditional Forecast)

x



Rail Forecast Framework

Drivers of

DEMAND

are factors that affect the rail patronage. The PDFH (Passenger Demand Forecast Handbook - PDFH) identifies all of the known drivers, broadly classified as:

External Factors
GDP; employment; population etc.

Quality of services

Fares
including interaction between different types of tickets.

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SUPPLY

MOIRA
(specification of rail services and capacity)

DEMAND

LENNON
(ticket sale database), adjusted by changes in the drivers of demand

EDGE
(Exogenous Demand Growth Estimator)

Drivers of Demand Elasticities

Rail Forecast Framework

WHAT ARE THE DRIVERS OF DEMAND?

They are factors that affect the rail patronage. The PDFH (Passenger Demand Forecast Handbook - PDFH) identifies all of the known drivers, broadly classified as:

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