Introduction:

This project involves the analysis of 20 assets from December 2010 to June 2011. Its main focus is on constructing and comparing portfolios that are created using three financial optimization models. Additionally, a market capitalization-weighted portfolio is used as a baseline, assuming that the 20 stocks represent the overall market.

Data Collection:

The historical data for the 20 assets, which are all constituents of the S&P 500, was obtained from Yahoo Finance. These assets are represented by symbols such as F (Ford Motor Co.), CAT (Caterpillar Inc.), DIS, MCD, KO, PEP, WMT, C, WFC, JPM, AAPL, IBM, PFE, JNJ, XOM, MRO, ED, T, VZ, and NEM. Additionally, the risk-free rate is estimated by collecting the treasury bill 30-year rate with the index ^TYX.

To estimate the parameters required for the optimization models, the project utilizes the monthly adjusted closing prices for each stock and the treasury bill from December 2007 to June 2011. After collecting the data, the mean and variance of returns for each stock are calculated. Furthermore, the covariance between the stocks has been calculated and is presented in Appendix-Table1 and Figure 1. It is shown that assets are not uncorrelated.

Market Portfolio

It is assumed that these 20 assets constitute the entire market, and therefore, the market weights are determined based on the proportional total market capitalization, which is collected from Yahoo Finance. Market capitalization refers to the total value of a company's outstanding shares of stock, calculated by multiplying the current market price per share by the total number of outstanding shares.

The capitalization weight of each asset is determined by the proportion of the asset's total capital value to the total market capital value. To calculate the market return and covariance, the following formulas are used:

$$\textit{Capitalization weights [i]} = \frac{market\ capitalization\ asset\ i}{total\ matket\ capital\ value}$$

$$\textit{Market Return} = \sum_{i=1}^{20} capitalization[i] * return[i]$$

Market Covariance = (Capitalization weights.T)@(Covariance@ Capitalization weights)

Using the risk-free rate, market return, market covariance, and the standard deviation of each stock, I calculate the expected rate of return (Table_2) and plot the capital market line (Figure-2), which represents the efficient assets. In Figure-2, the actual returns of assets are represented by blue dots. It is evident that while only a few assets have returns above the line, most assets do not provide the highest expected return for their level of risk.

Table 2_Return, Variance, Standard Deviation, Capitalization Weights and the Expected Return of each Asset

Asset	Return	Variance	Standard Deviation	Capitalization weights	Expected return
Risk Free	0.310	0.000	0.000	0.000	0.310
ED	0.756	20.625	4.541	0.005	0.879
MCD	1.236	21.397	4.626	0.034	0.889
JNJ	0.378	24.469	4.947	0.065	0.930
WMT	0.546	24.739	4.974	0.064	0.933
PEP	0.195	28.225	5.313	0.041	0.975
XOM	-0.031	29.823	5.461	0.069	0.994
KO	0.616	32.267	5.680	0.042	1.021
VZ	0.398	36.296	6.025	0.024	1.064
IBM	1.437	37.378	6.114	0.019	1.076
T	0.147	37.458	6.120	0.018	1.076
PFE	0.419	44.800	6.693	0.034	1.148
DIS	0.838	68.557	8.280	0.026	1.347
MRO	0.346	99.459	9.973	0.002	1.559
JPM	0.584	125.539	11.204	0.065	1.713
AAPL	1.945	130.214	11.411	0.423	1.739
NEM	0.990	140.249	11.843	0.005	1.793
CAT	1.995	175.589	13.251	0.018	1.969
WFC	1.094	215.108	14.667	0.025	2.146
C	-1.742	516.047	22.717	0.014	3.154
F	4.475	695.414	26.371	0.008	3.612

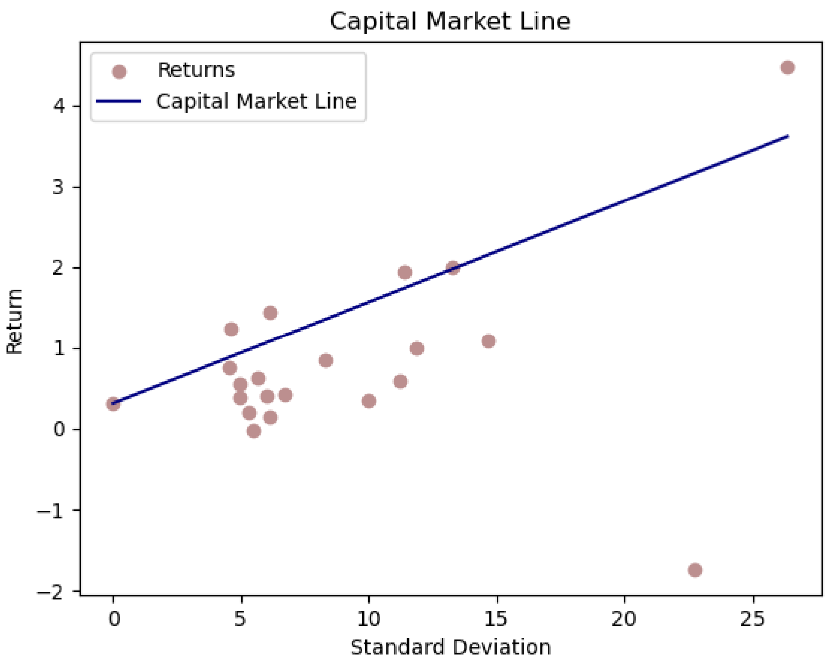

Figure 2-Capital Market Line

Portfolio Construction:

To construct the portfolios, I used three financial optimization models:

- Mean-variance optimization (MVO)
- Robust mean-variance optimization (RMVO)
- Risk Parity optimization (with no short selling)

The financial optimization model is Mean-variance optimization. The goal is to construct a portfolio with a maximize risk adjusted return.

$$\max(x)\ \mu^T x - \lambda x^T Qx$$

$$\text{s.t}\ \ 1^T x = 1$$

In this model, λ, is the risk-aversion coefficient which represents the expected trade-off between risk and return. It quantifies the rate at which an investor is willing to sacrifice expected return in exchange for reducing variance.

$$\lambda = \frac{E(r) - (risk\ free\ rate)}{variance\ \ of\ the\ market}$$

In this formula $E(r)$ is the market return. As a result the risk aversion coefficient is 0.01861 for time horizon between December 2007 to July 2011.

For the second model, I used Robust mean-variance optimization with a box uncertainty set.

$$\max(x, y)\ \mu^T x - \lambda x^T Qx - \theta^T y$$

$$\text{s.t}\ \ 1^T x = 1$$

$$y \geq x$$

$$y \geq -x$$

Mean-Variance Optimization (MVO) ignores estimation errors. To address this issue, I used Robust Mean-Variance Optimization which takes into account the uncertainty .

In this model:

$\theta = \varepsilon\ (\Theta^{1/2})ii$

ε = 1.64485 for confidence level 90%

ε = 1.96 for for confidence level 95%

$\Theta = \sigma\ (i) / \sqrt{T}$, T in this project is 42 month for time horizon December 2007 to July 2011, which changes in the part two based on the time horizon.

Also it is assumed that short selling is allowed.

The third model, Risk Parity optimization, is used to diversify risk. In this model, each asset have the same risk contribution risk.

$$\min(x, \theta) \ \sum_{i=1}^{n} (x(i)(Qx)i - \theta)^2$$

$$\text{s.t} \quad 1^T x = 1$$

$$x \geq 0$$

In this model , I assumed that short selling is not allowed and θ is an auxiliary unconstrained variable.

Part One:

In the first part, portfolios were constructed and analyzed to assess their performance in terms of returns in July 2011.

Portfolio Analysis:

Figure 3 illustrates the stock weights of the portfolios. It is evident that the Market portfolio exhibits a high degree of concentration , and a significant weight is allocated for asset one, surpassing 40%.Since short selling is allowed, in MVO and RMVO, certain assets have negative weights, indicating short positions. The weights of assets in the risk parity portfolio range between 2% and 10%.

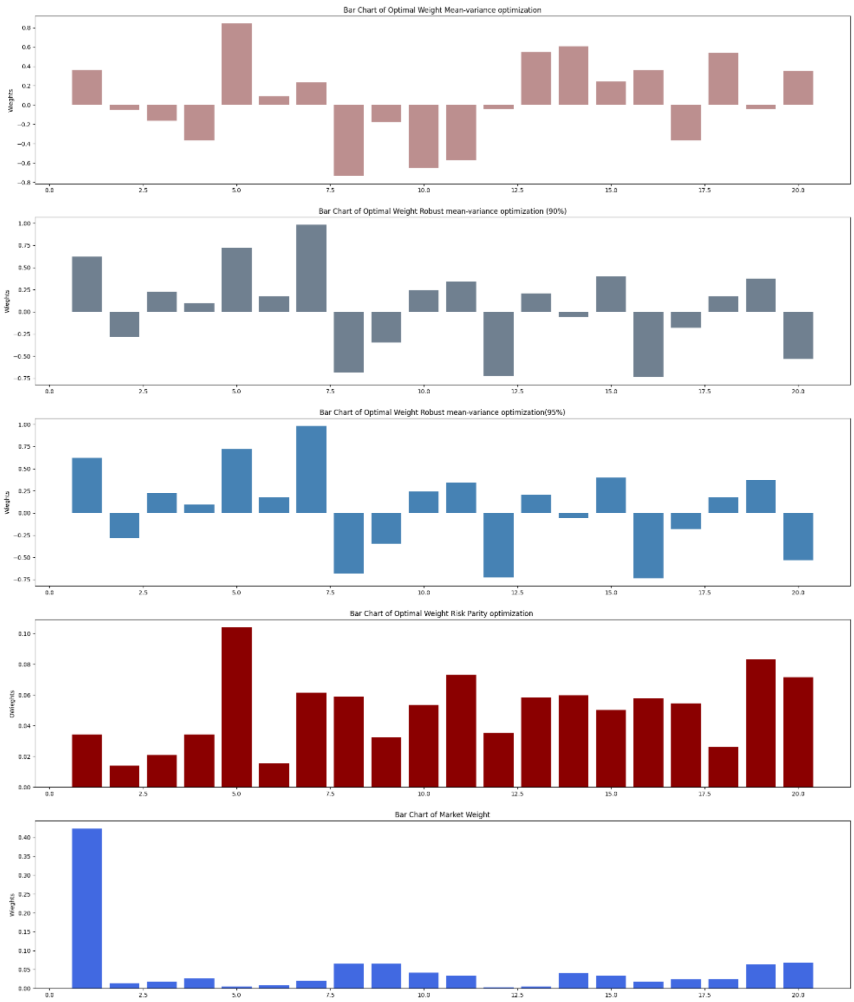

Figure 3_ Weights of different Portfolio

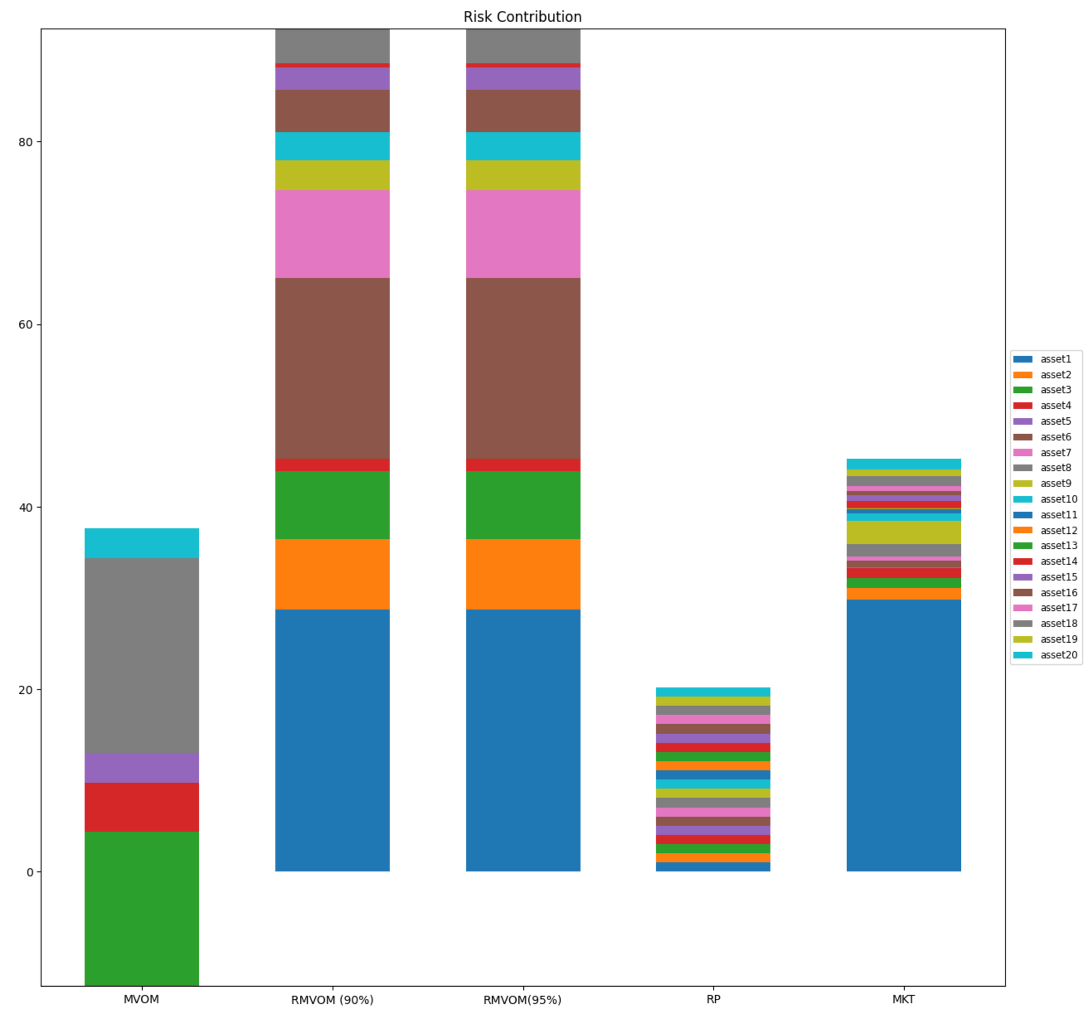

Figure 4_ Risk Contribution of each assets in their portfolio

In Figure 4, the risk contribution of each asset in their portfolios is depicted. It is evident that the RMVO portfolios exhibit higher overall risk compared to the other portfolios. In all portfolios, except for the Risk Parity model, assets with higher returns (weights * return) also tend to have higher risks. However, the Risk Parity model stands out as it ensures equal contribution of risk from all assets. In the MVO and RMVO models, certain assets exhibit negative risk contributions, indicating their opposite performance to the market. These assets act as hedgers, effectively reducing the overall risk of the portfolio.

Table 3_ Return, Variance, Standard Deviation and Sharpe Ratio for July 2011

Portfolio	Return	Variance	Standard Deviation	Sharpe Ratio
Mean Variance	-5.348	37.636	6.135	-0.872
Robust Mean Varian(90%)	1.133	81.068	9.004	0.126

Robust Mean Varian(95%)	1.133	81.068	9.004	0.126
Risk Parity	-1.265	20.212	4.496	-0.281
Market Cap	-2.487	45.264	6.728	-0.370

In analysing the portfolios, it is evident that the RMVO models exhibit higher returns as well as higher risks compared to the other portfolios. On the other hand, the risk parity models demonstrate the lowest levels of risk while still delivering returns higher than that of the market. The Sharpe ratio, which measures risk-adjusted returns, indicates the efficiency of each portfolio.

Notably, the Mean Variance Optimization (MVO) model shows negative returns, suggesting that it may not be an optimal choice for investors. Meanwhile, the Robust Mean Variance models (90% and 95%) provide positive returns, but they also entail higher levels of risk and volatility.

Investors' preferences and risk tolerance play a crucial role in selecting between the Robust Mean Variance and risk parity models. Hedgers, who aim to reduce risk, may favour the risk parity model due to its equal risk contribution from all assets. On the other hand, investors with a higher risk appetite might opt for the Robust Mean Variance model, which offers potentially higher returns but also exposes them to increased risk.

Part two:

In the second part, the portfolio was analysed across various time horizons, and the results are presented as follows.

Portfolio Analysis

In this part, portfolios are analysed in different time horizons and as a result a a cumulative wealth plot is acured from Julyy 2011 to Novomber 2011.

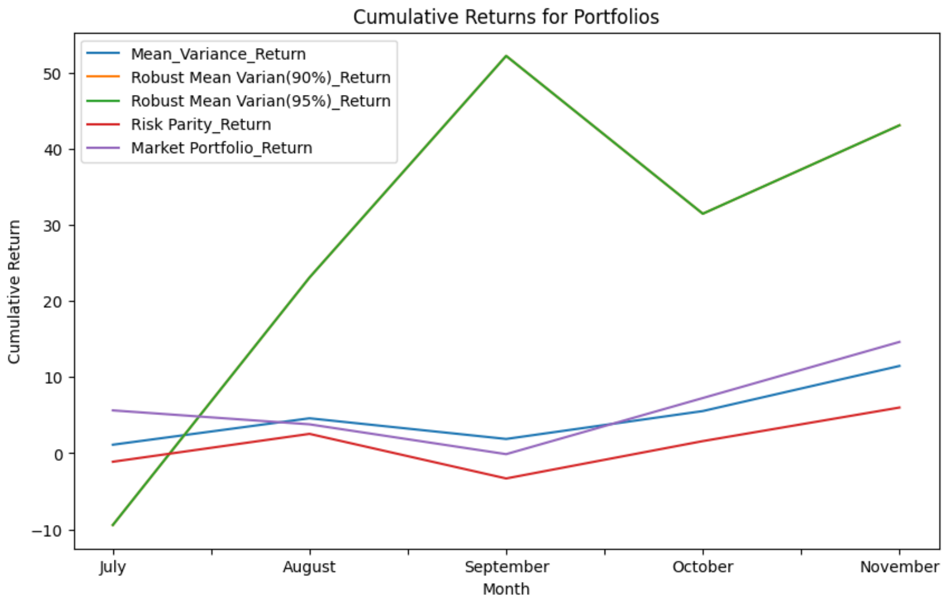

Figure 5_Cumulative Return of portfolios

In this part, portfolios are analysed over different time horizons, and the results are presented through a cumulative wealth plot from July 2011 to November 2011. Figure 5 displays the cumulative wealth plot, showing that the portfolio created using the robust mean-variance model has a higher return compared to the other portfolios. Additionally, it is observed that the returns of the other portfolios are lower than that of the market.

Figure 6 depicts the graph of the Sharpe ratio during this time frame. The market Sharpe ratio is represented by the blue line as a baseline. It is evident that the sharp ration of all portfolios lie below the market line (except on October), indicating that none of them are efficient. This implies that there is no trade-off between their risk and return.

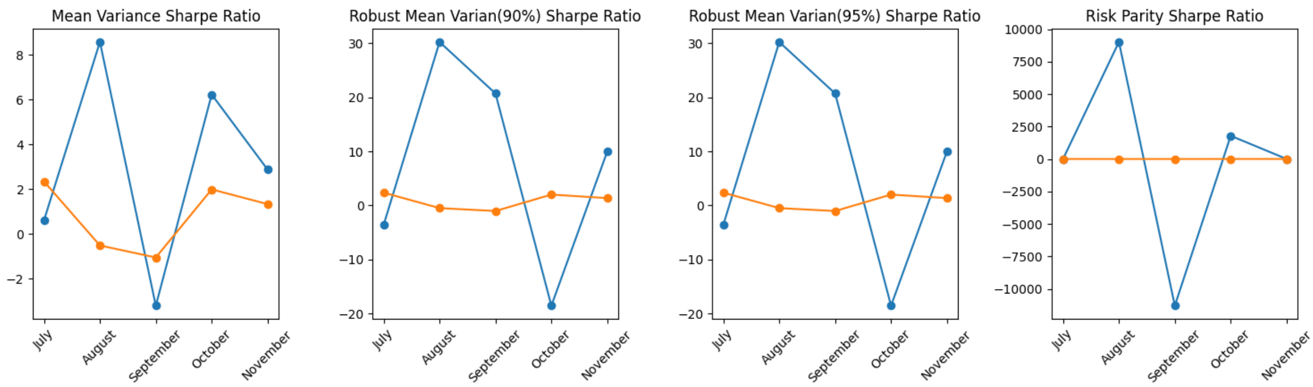

Figure 6_Comparison of Sharpe Ratio of different portfolios with Market(blue line)

Conclusion and Recommendations

In this project, three financial optimization models were utilized, assuming that all available assets represent the overall market. The portfolios constructed by the Robust Mean Variance model exhibited higher returns and risks compared to the other portfolios. It is recommended that hedgers choose the risk parity model, while investors willing to accept higher risk should opt for the Robust Variance Model. However, the Mean Variance portfolio is not advisable. Upon comparing the performance of these portfolios with the market, it can be concluded that there is no trade-off between risk and return for these portfolios.

References :

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Table 1_Covariance Matrix

	AAPL	C	CAT	DIS	ED	F	IBM	JNJ	JPM	KO	MCD	MRO	NEM	PEP	PFE	T	VZ	WFC	WMT	XOM
AAPL	130.21	87.28	74.75	53.69	19.85	105.24	33.93	24.88	29.50	17.72	15.84	57.12	-3.13	17.44	9.28	39.38	33.10	20.35	6.54	20.67
C	87.28	516.05	153.75	112.22	41.95	228.15	43.92	56.04	188.64	51.79	49.51	86.75	6.90	52.98	89.87	36.17	32.22	251.90	45.42	32.36
CAT	74.75	153.75	175.59	76.31	7.26	209.24	41.51	34.72	73.46	42.99	21.48	74.01	46.69	38.13	41.43	51.33	49.24	94.79	28.26	33.33
DIS	53.69	112.22	76.31	68.56	13.42	114.99	28.56	24.48	50.03	22.23	19.00	56.56	11.20	19.81	27.91	20.37	20.53	69.91	7.70	21.01
ED	19.85	41.95	7.26	13.42	20.63	4.72	6.60	11.36	12.24	7.98	8.47	11.42	-9.81	9.79	9.97	10.64	12.16	8.30	2.23	6.93
F	105.24	228.15	209.24	114.99	4.72	695.41	63.41	32.55	159.18	48.56	16.01	86.72	10.36	38.56	21.60	50.37	57.38	194.66	40.61	32.30
IBM	33.93	43.92	41.51	28.56	6.60	63.41	37.38	15.63	30.47	11.94	4.65	40.97	10.32	14.46	10.19	6.86	9.24	18.63	8.16	6.53
JNJ	24.88	56.04	34.72	24.48	11.36	32.55	15.63	24.47	23.12	15.21	14.27	19.52	14.03	17.24	18.52	12.82	13.12	28.99	9.39	9.94
JPM	29.50	188.64	73.46	50.03	12.24	159.18	30.47	23.12	125.54	21.82	12.38	26.05	-16.51	21.74	37.99	15.37	15.81	137.67	26.54	7.16
KO	17.72	51.79	42.99	22.23	7.98	48.56	11.94	15.21	21.82	32.27	14.93	20.92	26.94	22.17	18.46	18.53	17.71	30.29	9.91	12.83
MCD	15.84	49.51	21.48	19.00	8.47	16.01	4.65	14.27	12.38	14.93	21.40	12.74	16.66	13.69	16.82	10.77	10.29	26.04	7.48	11.84
MRO	57.12	86.75	74.01	56.56	11.42	86.72	40.97	19.52	26.05	20.92	12.74	99.46	24.15	23.36	23.60	12.74	23.56	37.34	2.68	26.52
NEM	-3.13	6.90	46.69	11.20	-9.81	10.36	10.32	14.03	-16.51	26.94	16.66	24.15	140.25	12.67	13.68	10.27	12.01	-14.29	9.46	15.08
PEP	17.44	52.98	38.13	19.81	9.79	38.56	14.46	17.24	21.74	22.17	13.69	23.36	12.67	28.22	15.10	12.18	13.68	25.92	10.97	8.40
PFE	9.28	89.87	41.43	27.91	9.97	21.60	10.19	18.52	37.99	18.46	16.82	23.60	13.68	15.10	44.80	16.79	16.55	58.16	14.26	11.79
T	39.38	36.17	51.33	20.37	10.64	50.37	6.86	12.82	15.37	18.53	10.77	12.74	10.27	12.18	16.79	37.46	29.39	14.54	11.28	10.76
VZ	33.10	32.22	49.24	20.53	12.16	57.38	9.24	13.12	15.81	17.71	10.29	23.56	12.01	13.68	16.55	29.39	36.30	15.44	11.92	16.88
WFC	20.35	251.90	94.79	69.91	8.30	194.66	18.63	28.99	137.67	30.29	26.04	37.34	-14.29	25.92	58.16	14.54	15.44	215.11	25.56	13.41
WMT	6.54	45.42	28.26	7.70	2.23	40.61	8.16	9.39	26.54	9.91	7.48	2.68	9.46	10.97	14.26	11.28	11.92	25.56	24.74	7.26
XOM	20.67	32.36	33.33	21.01	6.93	32.30	6.53	9.94	7.16	12.83	11.84	26.52	15.08	8.40	11.79	10.76	16.88	13.41	7.26	29.82

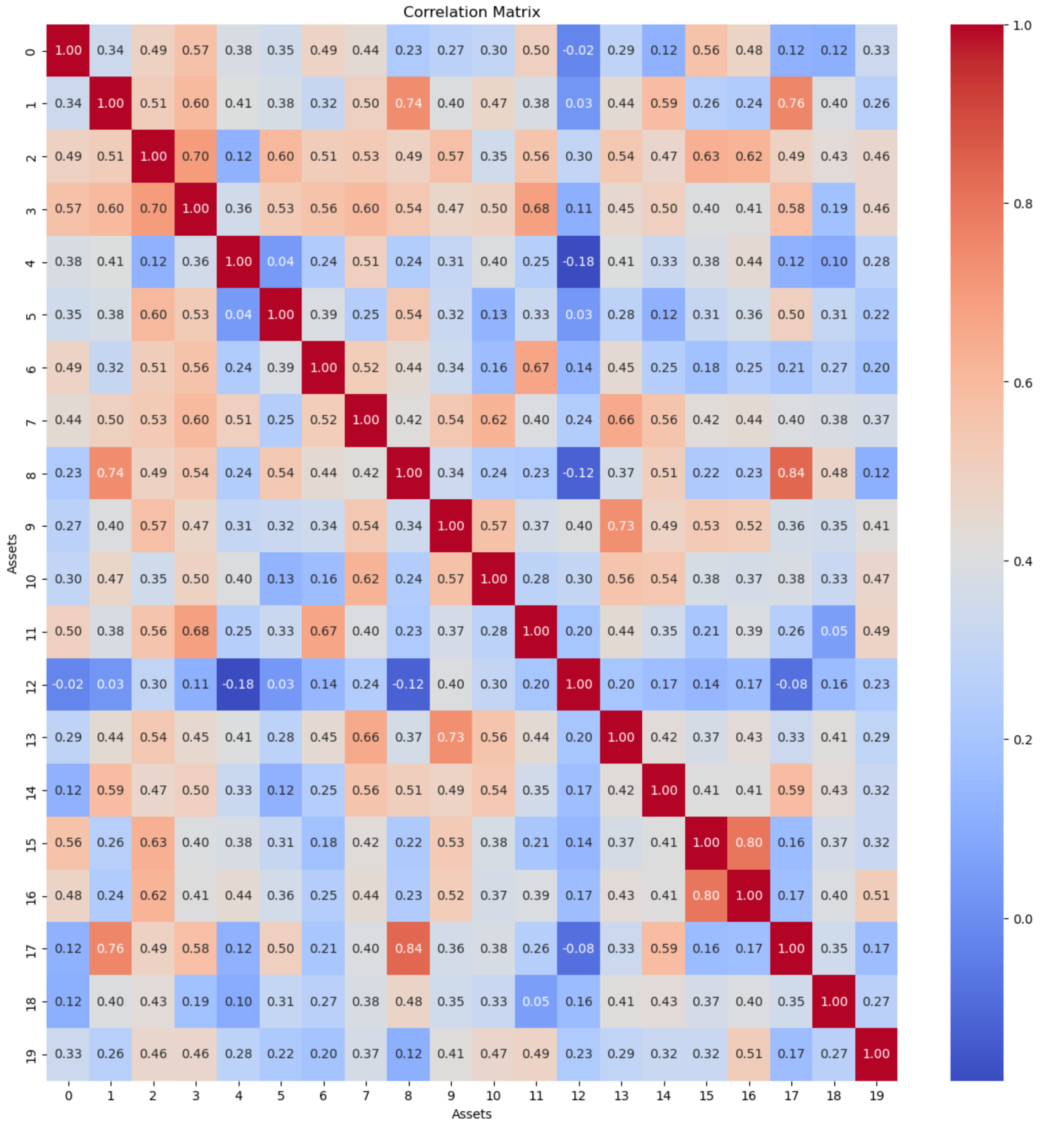

Figure 1_Correlation Heat Map