ML.exer.chap6

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1 Exercise 6.2:

Give some finite domain set, χ, and a umber $\kappa \leq |\chi|$, figure out the VC-dimansion of each of the following classes (and prove your claims):

1. $H^{\chi}_{=\kappa} = \{h \in \{0,1\} : |\{x : h(x) = 1\}| = \kappa\}$. that is, the set of all functions that assign the value 1 to exactly elements of χ.

proof:
we claim $VC - dimansion = min\{\kappa, |\chi| - \kappa\}$ then we show that $VCdim(H_{=\kappa}) \leq \kappa$ for that we define set $C \subseteq \chi$ with size $\kappa + 1$ and we show thet it does not exist $h \in H_{=\kappa}$ with labeling 1 to the all $\kappa + 1$ elements $x \in C$ because $|\{x : h(x) = 1\}| = k$.
also we show $VC - dimansion(H_{=k}) =\leq |x| - k$ therefore we define set $C \subseteq \chi$ with size $|\chi| - \kappa + 1$ we should show that there is not $h \in H$ thet labeling 0 to the all elements $(x \in C)$ because we have $|\{x : h(x) = 0\}| = |\chi| - \kappa$.

2. $H_{at-most-k} = \{h \in \{0,1\}^{\chi} : |\{x : h(x) = 1\}| \leq \kappa \quad or \quad |\{x : h(x) = 0\}| \leq \kappa\}$

proof: we claims $VCdim(H) = \kappa$. then first, we show $VCdim(H) \leq \kappa$ we define the set $C = \{x_1, ..., x_{k+1}\} \subseteq \chi$ we know that there is not $h \in H$ s.t. labeling all the elements with 1. because we have k or less elements with 1 labelin ih H. on the other side we show that $VCdim(H) \geq \kappa$
let $(y_1, ..., y_m) \in \{0,1\}^{\chi}$ be a vector of labels with the size $m \leq \kappa$ this labeling makes by the $h \in H$ s.t $h(x_i) = y_i$ for every $x_i \in C$ and $h(x) = 0$ for every other $x \in \chi \setminus C$ and we know C shattered by H.
Therefore $VCdim(H) = \kappa$.

2 Exercise 6.4:

we prove sauer's lemma by proving that for every class H of finite VCdimension d, and every subset A of the domain,
$|H_A| \leq |\{B \subseteq A : HshattersB\}| \leq \sum_{i=0}^{d} \binom{|A|}{i}$.
solution: Let $X = \mathbb{R}^2$ We will demonstrate all the 4 combinations using hypothesis classes defined over $X \times \{0,1\}$ and named $\delta = \sum_0^2 \binom{|A|}{i}$
$(=,=) : C = \{x_1 = (0,1), x_2 = (1,1)\}$
$|H_A| = |\{(0,0), (1,1), (1,0), (0,1)\}| = 4$

$|\{\varnothing, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_3, x_2\}\}| = 7$
$\delta = 4$

$(<, =) : C = \{(0, 1), (1, 0), (1, 1)\}$
$|H_A| = |\{(0, 0, 0), (1, 1, 1), (1, 0, 0), (0, 0, 1), (0, 1, 1), (1, 1, 0)\}| = 6$
$|\{\varnothing, \{x_2\}\}| = 2$
$\delta = 7$

$(=, <) : C = \{x_1 = (0, 0), x_2 = (0, 1)\}$
$|H_A| = |\{(1, 0), (1, 1)\}| = 2$ $|\{\varnothing, \{x_2\}\}| = 2$
$\delta = 4$

$(<, <) : C = \{x_1 = (1, 1), x_2 = (2, 2)\}$
$|H_A| = |\{(0, 0), (1, 1)\}| = 2$
$|\{\varnothing, \{x_1\}, \{x_2\}\}| = 3$
$\delta = 4$

3 Exercise 6.9:

Let H be class of signed intervals,that is, $H = \{h_{a,b,s} : a \leq b, s \in \{0, 1\}\}$ where

$$h_{a,b,s}(x) = \begin{cases} s & if x \in [a, b] \\ -s & if x \notin [a, b] \end{cases}$$

Calculate VCdim(H).
proof: first Let $C = \{c_1 = 1, c_2 = 2, c_3 = 3\}$ then we see that this C shattered by H.

a	b	c_1	c_2	c_3
0	4	+	+	+
0	2.5	+	+	-
1.5	2.5	+	-	+
1.5	4	-	+	+
0	1.5	+	-	-
1.5	2.5	-	+	-
2.5	4	-	-	+
4	5	-	-	-

therefore $VCdim(H) \geq 3$ now we define C by 4 points $C = \{c_1, c_2, c_3, c_4\}$ then we see the labeling $(+, -, +, -)$ does not obtained. Therefore $VCdim(H) \leq 3$ we showed that $VCdim(H) = 3$.