Introduction to Econometrics
PS #1 - Probability review

1 Preliminaries of Random Variables

Consider an experiment of flipping a fair coin three times.

1. Write down the sample space denoted by S. Is each outcome equally likely?
2. What is the set identified by the event that the number of heads is 2? Compute the event probability.
3. Denote by X a RV that represents the number of heads after three flips. Graph the PMF and CDF of X.
4. Suppose you repeat the experiment two times. The outcome is (H, T, T) and (H, H, T) on the first and second trial, respectively. What is the average based on these two trials?
5. Compute $E(X)$ and $V(X)$.

2 Moments of a discrete RV

After some years of experience, the instructor of the Statistics course asks to meet the coordinator of the master to present him the following PDF of X, the number of students who miss his evening class on Fridays:

$X = x$	0	1	2	3	4	5	6	7	8	≥ 9
$f(x)$	0.35	0.15	0.10	0.00	0.00	0.05	0.05	0.10	0.20	0.00

1. Compute the mean, median, mode, and the standard deviation of the distribution.
2. Is the distribution symmetric?

The instructor then shows the PDF of Y, the number of students who miss his evening class on Wednesdays

$Y = y$	0	1	2	3	4	5	6	≥ 7
$f(y)$	0.05	0.10	0.20	0.30	0.20	0.10	0.05	0.00

1. Compare the first two moments of this distribution with the previous one.
2. Based on this comparison, what might be the argument of the instructor?

3 Linear function of a discrete RV

You are the data scientist for a popular fitness app that tracks users' daily exercise activity. The app categorizes users into different levels based on their daily exercise time. Let X be the RV that codes exercise activity into 3 categories - low, moderate and high, with the following PDF

$X = x$	0	1	2
$f(x)$	0.30	0.50	0.20

1. The developers are interested in the average (expected) activity and the variability (variance) in the users' activity. Compute $E(X)$ and σ_X.
2. Suppose that the developers want to create a reward system, giving points according to $Y = 5X + 2$. Report the expected value and variance of Y.

4 Joint distribution of two RVs

		Y		
		2	4	6
	1	1/8	1/4	1/8
X	3	1/24	1/4	1/24
	9	1/12	0	1/12

1. What is the marginal distribution of X?
2. Find the conditional probability of X given $Y = 2$ i.e., $P(X = x|Y = 2)$
3. Report the expected value of X and Y, denoted by μ_x and μ_y
4. Find the covariance of X and Y, denoted by $cov(X, Y)$.
5. Check whether X and Y are independent.

5 Function of two random variables

Suppose that a firm holds an investment portfolio consisting of two stocks A and B, both bought in equal amounts. Denote the random variables that are the returns on A and B by X and Y (respectively). The firm knows that $E(X) = 10\%$, $\sigma_X = 15$, $E(Y) = 15\%$ and $\sigma_Y = 25$.

Let Z be the RV that represents the combined portfolio i.e., $Z = X + Y$

1. Compute the expected return on the portfolio.
2. Assume that $\text{Corr}(X, Y) = 1$, compute the variability of the company's combined portfolio.
3. Suppose that the company purchases twice the amount of stock B, such that $Z = X + 2Y$. Compute the new expected value and variance.

6 Discrete to continuous RV

Suppose that a group of fishermen are interested in measuring the depth of a nearby lake. Due to limited technological prowess, the fishermen are only able to make measurements to the nearest meter. Let $M =$ maximum depth (in meters). Define X as the discrete RV of depth measurement, with the following PDF

$X = x$	1	2	≥ 3
$f(x)$	0.30	0.50	0.20

1. Graph the PDF of X and the associated probability histogram.
2. Assume that the meteorological department steps in with improved depth measurement technology, leading to the new PDF of X

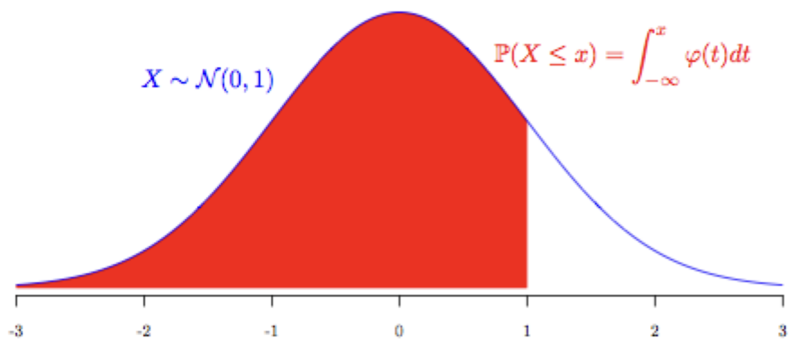
$X = x$	1	1.5	2	2.5	≥ 3
$f(x)$	0.25	0.05	0.40	0.10	0.20

   Graph the new probability histogram.
3. If we continue to measure the depth as finely as possible, how does the PDF change?

7 The standard normal distribution

The life length (in years) of a well-known mobile phone follows a normal distribution with mean 3.1 and variance 2.25. Let the RV be defined by $X \sim \mathcal{N}(3.1, 2.25)$

1. What is the probability that the phone works for less than a year?
2. What is the probability that the phone works for 4 years or more?
3. What is the probability that the phone will work for 1 to 3.5 years?



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441