Macro Labor Reading Group
Entry, Exit, and Firm Dynamics

Samiran Dutta

18.03.24

The Backdrop

1. The labor markets are in a perpetual state of flux
   - Unemployed workers search for jobs
   - Employed workers either lose jobs or search for better alternatives
2. Productivity shocks drive firm growth and decline
   - Reallocation of resources across firms
   - Leading to job creation and destruction
3. Several papers combine frictional labor markets + firm dynamics
   - Elsby & Michaels (2013): Augments DMP with DRS production + shocks
   - Elsby & Gottfries (2021), Bilal *et al.* (2022): On-the-job search + firm dynamics
4. Today's session – workhorse model of firm dynamics (Hopenhayn, 1992)
   - Model and definition of stationary equlibrium
   - Quantitative application

Environment: Overview

1. Discrete time indexed $t = 0, 1, 2...$
2. Labor is the only factor (can generalize under homothetic production)
   - Aggregate labor stock N
   - Inelastic supply
3. No product or labor market power $\implies (p, w)$ given
4. Continuum of firms with no strategic interactions
   - Production: $y_i = x_i n_i^{\alpha}$; $\alpha < 1$
   - Entrants: draw initial productivity from $G(x)$
   - Incumbents: $x' \sim$ first-order Markov process with conditional cdf $F(x'|x)$
5. Fixed costs
   - Entry cost c_e
   - Per-period cost of operation c_f

Environment: Timing for Incumbents

Beginning of t

Productivity x_t observed

Taking state (x, p, w) given, decide on optimal labor demand

Leads to static profits $\pi(x; p, w) - c_f$

Expectation over shock transition $x_{t+1} \sim F(x_{t+1}|x_t)$, decide to stay or exit

Period t concludes

Optimization: Incumbents

1. Each period, incumbents maximize their static profits

$$\Pi(x;p,w) = \max_n \left\{ pxn^\alpha - wn - wc_f \right\}$$

where labor demand

$$n(x;p,w) = \left(\frac{\alpha p x}{w}\right)^{\frac{1}{1-\alpha}}$$

2. Value of an incumbent firm at period t

$$V_t(x;p,w) = \Pi(x;p,w) + \beta \max \left\{ \underbrace{\int V_{t+1}(x';p,w)dF(x'|x)}_{\text{stay continuation value}}, \underbrace{0}_{\text{exit scrap value}} \right\}$$

3. To define an exit threshold $\implies$ need V to be monotone in x
   - Π is clearly increasing in x
   - Assume that $F(x'|x)$ is decreasing in x

Then $\exists\ R$ s.t. $\forall x < R$, firms exit where R satisfies $\int V_{t+1}(x';p,w)dF(x'|x) = 0$

Environment: Timing for Entrants

Beginning of t

Pay c_e to set-up firm, allowing a draw $x_{t+1} \sim G(x)$

Expectation over $x_{t+1} \sim G(x)$

Evaluate whether value of entry $V_e > 0$, decide to enter or not

Period t concludes, production begins from $t+1$

Beginning of period $t+1$, becomes an incumbent with observed x_{t+1}

Optimization: Entrants

1. Define M as the mass of entrants
   - Pay c_e to draw $x \sim G(x)$
   - Start producing next period
2. Value of an entrant writes

$$V_{e,t}(x; p, w) = -c_e + \beta \int V_{t+1}(x'; p, w) dG(x')$$

3. Firms enter when $V_e > 0$, driving down profits until $V_e \leq 0$ in equilibrium
   - $V_e < 0$ would imply no entry *i.e.*, $M = 0$
   - $V_e = 0$ implies a positive mass of entrants $M > 0$
4. Then the free entry condition with $M > 0$

$$\beta \int V_{t+1}(x'; p, w) dG(x') = c_e$$

Distribution: Operational Firms

1. Define $\Phi_t([0,x'])$ as the p.d.f of firms over the productivity space $[0,x'] \subseteq \text{supp}(x)$

2. The distribution evolves according to a law of motion

$$\Phi_{t+1}([0,x']) = \underbrace{\int_{x \geq R} F(x'|x)\Phi_t(dx)}_{\text{Surviving Incumbents}} + \underbrace{M_{t+1}G(x')}_{\text{Entrants}}$$

3. Define a policy function

$$\chi(x;p,w) = \begin{cases} 1 & ;\text{when incumbents decide to exit} \\ 0 & ;\text{when incumbents decide to stay} \end{cases}$$

4. Rewrite law of motion as

$$\Phi_{t+1}([0,x']) = \int F(x'|x)(1-\chi)\Phi_t(dx) + M_{t+1}G(x')$$

5. Steady state $\implies$ firms grow/shrink and enter/exit but distribution remains constant

$$\Phi_{t+1} = \Phi_t = \Phi$$

Equilibrium: Definition

A stationary recursive competitive equilibrium with positive entry consists of prices (p^*, w^*), aggregate production Y, aggregate labor demand N, mass of entrants M, value function $V(.)$ and a productivity distribution Φ such that

1. Goods market clears: $Y(p^*) = D(p^*) = \dfrac{\bar{D}}{p^*}$
2. Incumbents make optimal exit decisions such that $\int V(x'; p^*, w^*) dF(x'|x) = 0$
3. No further incentives to enter *i.e.* free-entry holds $V_e(x) = 0$
4. Entry and exit flows equalize, stationary dist. recursively defined by law of motion
$$\Phi([0, x']) = \int F(x'|x)(1-\chi)\Phi(dx) + MG(x')$$
5. Aggregates defined by
$$Y = \int y(x; p^*, w^*)\Phi dx$$
$$N = \int n(x; p^*, w^*)\Phi dx$$

Equilibrium: Algorithm

Walras's law $\implies$ set $w = 1$ and look for optimal p^* using the following steps

1. Discretize state space of x into n_x grid points, using Tauchen (1991) approx
   - Denote $i = 1, .., n_x$ as the grid point of state x
   - ..and f_{ij} as the transition probability from state i to j
2. Guess a price p_0
3. Solve for the Bellman equation using VFI, start with guess V^0 and compute V^1

$$V^1(x_i; p_0) = \Pi(x_i; p_0) + \beta \max\left\{ \sum_{j=1}^{n_x} f_{ij} V^0(x_j; p_0), 0 \right\} \qquad ; \forall i = 1, .., n_x$$

   - If $\max_i |V^1(x_i; p_0) - V^0(x_i; p_0)| < tol \implies$ converged V^*
   - Otherwise revise guess $V^0 = V^1$ and keep iterating
4. Use the converged V^* to collect exit decisions into a vector $n_x \times 1$

$$\chi(x_i; p_0) = 1 \text{ iff } \sum_{j=1}^{n_x} f_{ij} V^*(x_j; p_0) < 0 \qquad ; \chi(x_i; p_0) = 0 \text{ otherwise}$$

Equilibrium: Algorithm

5. Let g_i be the discretized PMF of $G(x)$ over grid x_i

6. Compute value of an entrant V_e

$$V_e(z_i; p_0) = -c_e + \beta \sum_{i=1}^{n_x} g_i V^*(x_i; p_0)$$

7. Free entry requires $V_e(x_i; p_0) = 0$, otherwise update price using bisection method
   - Guess interval $[\underline{p}, \bar{p}] \implies$ guess $p_0 = (\underline{p} + \bar{p})/2$
   - If $V_e(x_i; p_0) > 0 \implies$ reduce price to discourage entry, new interval $[\underline{p}, \bar{p} = p_0]$
   - If $V_e(x_i; p_0) < 0 \implies$ increase price to encourage entry, new interval $[\underline{p} = p_0, \bar{p}]$

8. Using optimal price p^*, use law of motion to find stationary distribution Φ

$$\underbrace{\Phi}_{n_x \times 1} = \underbrace{\hat{F}_{ij}}_{n_x \times n_x} \underbrace{\Phi}_{n_x \times 1} + \underbrace{Mg}_{n_x \times 1} \qquad \implies \Phi = M(I - \hat{F}_{ij})^{-1} g$$

$\hat{F}_{ij}$ is element-wise multiplication of (transition matrix)' and exit vector $F' \times (1 - \chi)$

Equilibrium: Algorithm

We have been assuming $M > 0$ so far, need to make sure this is indeed the case.

9. Stationary distribution Φ is linear in $M \implies$ can write $\Phi(p^*, M) = M \times \Phi(p, 1)$

10. The goods market must clear

$$Y(p^*, M) = D(p^*) \implies \sum_{i=1}^{n_x} y(x_i; p_0)\Phi(p^*, M) = \frac{\bar{D}}{p^*}$$

$$M \sum_{i=1}^{n_x} y(x_i; p_0)\Phi(p^*, 1) = \frac{\bar{D}}{p^*}$$

$$M = \frac{\bar{D}/p^*}{\sum_{i=1}^{n_x} y(x_i; p_0)\Phi(p^*, 1)}$$

11. If $M > 0$ then we have found an equilibrium with positive entry!

Quantitative Application: Parameters

1. Assume technology and preferences

$$y = xn^{\alpha}, \quad D(p) = \bar{D}/p$$

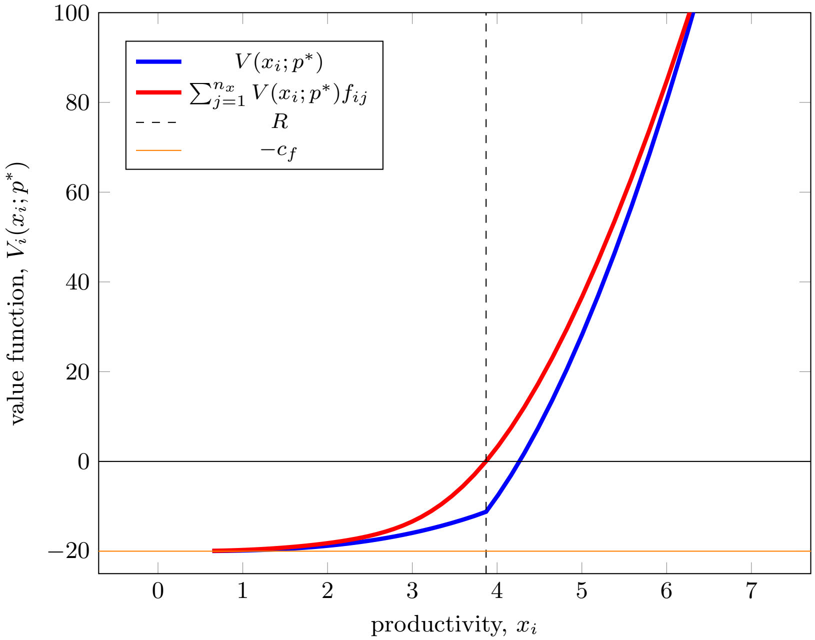
2. Firm productivity follows log-normal AR(1), approx. by Markov chain on 101 nodes

$$\log x_{t+1} = (1-\rho)\log \bar{x} + \rho \log(x_t) + \sigma\epsilon_{t+1} \quad ; \epsilon \sim N(0,1)$$

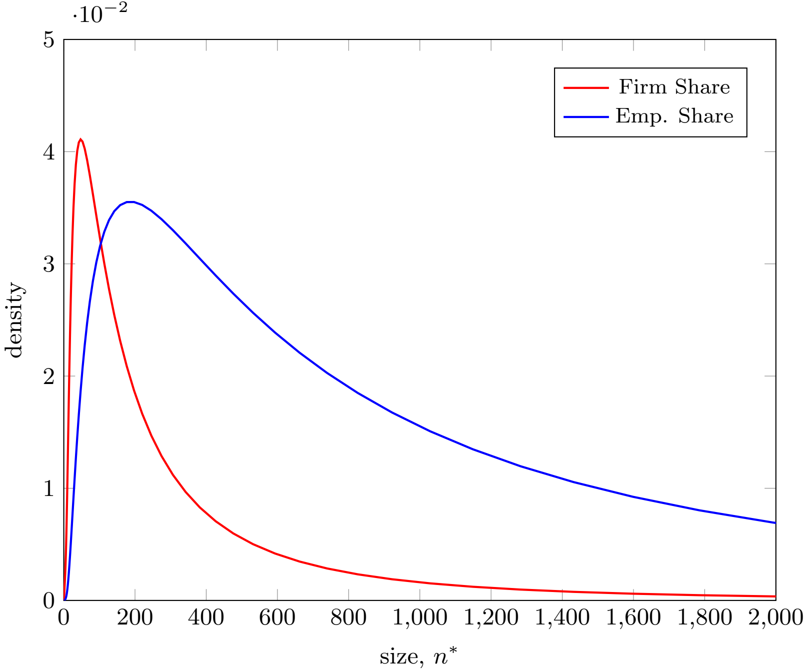
3. Benchmark parameter values

Description	Parameter	Value
Labor Share	α	2/3
Discount Factor	β	0.80
Fixed operating cost	c_f	20
Fixed entry cost	c_e	40
Mean of LN AR(1)	$\log \bar{x}$	1.39
Standard Deviation	σ	0.20
Persistence	ρ	0.9
Demand curve constant	$\bar{D}$	100

Results: Value Function



Results: Stationary Distributions



Empirics: Implications

1. Labor demand is proportional to productivity, $\uparrow$ productivity $\iff$ $\uparrow$ firm size
2. Lifecycle predictions
   - Firms enter small, with a productivity draw enough to compensate for entry cost
   - Survive in the industry if they get favourable productivity draws, becoming larger
   - Larger firms are thus older and more efficient
3. Rationalizing lifecycles in the developing world $\implies$ add frictions

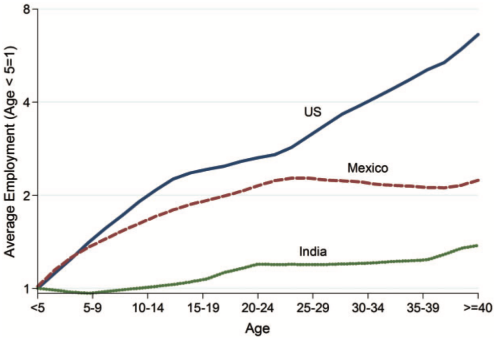


Figure: Plant Employment by Age in Cross-Section, *Source:* Hseih and Klenow (2014)

Way Forward

1. The baseline model is efficient $\implies$ compet. eq is the solution to planner's problem
   - Implies no misallocation of resources
   - Hopenhayn and Rogerson (1993) adds labor adjustment costs to induce misallocation
   - Leads to inaction region $\implies$ firms do not hire more even if productivity is high
2. The model is silent about wage dispersion
   - Absent firm dynamics, models of on-the-job search leads to wage dispersion
   - Burdett and Mortensen (1998) is a classic example
   - Postel-Vinay and Robin (2002) talks about wage posting with offer-matching
3. Can combine labor market frictions + firm dynamics to say a lot more..
   - Elsby and Gottfries (2021) adds labor market frictions through a matching function
   - Firm dynamics from evolving productivity
   - Job-to-job transitions from on-the-job search $\implies$ Wage dispersion
   - Provides a novel theory of misallocation, with firms managing turnover and output