The Successive Overrelaxation Method

The Successive Over relaxation Method, or SOR, is devised by applying extrapolation to the Gauss-Seidel method. This extrapolation takes the form of a weighted average between the previous iterate and the computed Gauss-Seidel iterate successively for each component:

$$x_i^{(k)} = \omega \bar{x}_i^{(k)} + (1-\omega)x_i^{(k-1)} \qquad (3.38)$$

(where $\bar{x}$ denotes a Gauss-Seidel iterate, and ω is the extrapolation factor). The idea is to choose a value for ω that will accelerate the rate of convergence of the iterates to the solution.

In matrix terms, the SOR algorithm can be written as follows:

$$x^{(k)} = (D-\omega L)^{-1}(\omega U + (1-\omega)D)x^{(k-1)} + \omega(D-\omega L)^{-1}b. \qquad (3.39)$$

Example 13 : Solve the 3 by 3 system of linear equations **Ax = b** where

$$A = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 6 & -5 \\ 0 & -5 & 11 \end{bmatrix} \qquad b = \begin{bmatrix} 8 \\ -29 \\ 43 \end{bmatrix}$$

by SOR method .

Solution : For SOR iterations , the system can be written as

$$x_1^{(new)} = (1-\omega)x_1^{(old)} + \omega(\frac{1}{2}x_2^{(old)} + 2)$$

$$x_2^{(new)} = (1-\omega)x_2^{(old)} + \omega(\frac{1}{3}x_1^{(new)} + \frac{5}{6}x_3^{(old)} - \frac{29}{6})$$

$$x_3^{(new)} = (1-\omega)x_3^{(old)} + \omega(\frac{5}{11}x_1^{(new)} + \frac{43}{11})$$

Start with $x^0 = (0, 0, 0)^T$, for $\omega = 1.2$ we get the following solution

```
>> SOR_f(A,b,x0,1.2,0.001,50)
   1.0000    2.4000   -4.8400    2.0509

   2.0000   -0.9840   -3.1747    2.5491

   3.0000    0.6920   -2.3392    2.9052

   4.0000    0.8581   -2.0838    2.9733

   5.0000    0.9781   -2.0187    2.9951

   6.0000    0.9931   -2.0039    2.9989

   7.0000    0.9991   -2.0007    2.9998

SOR method converged
   8.0000    0.9997   -2.0001    3.0000
```

Infact the required number of iterations for different values of relaxation parameter ω for tolerence value 0 .00001 is as follows

Ω	0.8	0.9	1.0	1.2	1.25	1.3	1.4
No. of iterations	44	36	29	18	15	13	16