UNIT 3 SYSTEMS FOR IMPRECISE/INCOMPLETE KNOWLEDGE

Structure **Page Nos.**

3.0 INTRODUCTION

In the earlier three units of the block, we discussed PL and FOPL systems for making inferences and solving problems requiring logical reasoning. However, these systems assume that the domain of the problems under consideration is complete, precise and consistent. But, in the real world, the knowledge of the problem domains is generally neither precise nor consistent and is hardly complete.

In this unit, we discuss a number of techniques and formal systems that attempt to handle some of these blemishes. To begin with, in Sections 4.2 to 4.5, we discuss **fuzzy systems** that attempt to handle **imprecision** in knowledge bases, specially, due to use of natural language words like hot, good, tall etc.

Then, we discuss **non-monotonic systems** which deal with **indefiniteness** of knowledge in the knowledge bases. The significance of these systems lies in the fact that most of the statements in the knowledge bases are actually based on **beliefs** of the concerned persons or actors. These beliefs get revised as better evidence for some other beliefs become available, where the later beliefs may be in conflict with the earlier beliefs. In such cases, the earlier beliefs my have to be temporarily suspended or permanently excluded from further considerations.

In Sections 4.7 and 4.8, we discuss two formal systems that attempt to handle **incompleteness** of the available information. These systems are called **Default Reasoning Systems** and **Closed World Assumption Systems**. Finally, we discuss some inference rules, viz, **abductive** inference rule and **inductive** inference rule that are, though not deductive, yet are quite useful in solving problems arising out of everyday experience.

3.1 OBJECTIVES

After going through this unit, you should be able to:

- enumerate various formal methods, which deal with different types of blemishes like incompleteness, imprecision and inconsistency in a knowledge base;
- discuss, why fuzzy systems are required;
- discuss, develop and use fuzzy arithmetic tools in solving problems, the descriptions of which involve imprecision;
- discuss default reasoning as a tool for handling incompleteness of knowledge;
- discuss Closed World Assumption System, as another tool for handling incompleteness of knowledge, and
- discuss and use non-deductive inference rules like abduction and induction, as tools for solving problems from everyday experience.

3.2 FUZZY SYSTEMS

In the symbolic Logic systems like, PL and FOPL, that we have studied so far, any (closed) formula has a truth-value which must be binary, viz., *True or False*. However, in our everyday experience, we encounter problems, the descriptions of which involve some words, because of which, to statements of situations, it is not possible to assign a truth value: *True or False*. For example, consider the statement: *If the water is too hot, add normal water to make it comfortable for taking a bath*.

In the above statement, for a number of words/phrases including 'too hot' 'add', 'comfortable' etc., it is not possible to tell when exactly water is too hot, when water is (at) normal (temperature), when exactly water is comfortable for taking a bath.

For example, we cannot tell the temperature T such that for water at temperature T or less, truth value False can be associated with the statement '*Water is too hot'* and at the same time truth-value True can also be associated to the same statement '*Water is too hot'* when the temperature of the water is, say, at degree T + 1, T + 2….etc.

Some other cases of Fuzziness in a Natural Language

***Healthy* Person:** we cannot even enumerate all the parameters that **determine** health. Further, it is even more difficult to tell for what value of a particular parameter, one is healthy or otherwise.

***Old/young person*:** It is not possible to tell exactly upto exactly what age, one is young and, by just addition of one day to the age, one becomes old. We age gradually. Aging is a **continuous** process.

***Sweet* Milk:** Add small sugar cube one at a time to glass of milk, and go on adding upto, say, 100 small cubes.

Initially, without sugar, we may take milk as not sweet. However, with addition of each one small sugar particle cube, the sweetness **gradually** increases. It is not possible to say that after addition of 100 small cubes of sugar, the milk becomes sweet, and, till addition of 99 small cubes, it was not sweet.

Pool, Pond, Lake,….., Sea, Ocean: for different sized water bodies, we can not say when exactly a pool becomes a pond, when exactly a pond becomes a lake and so on.

One of the reasons, for this type of problem of our inability to associate one of the two-truth values to statements describing everyday situations, is due to the use of natural language words like hot, good, beautiful etc. Each of these words does not denote something constant, but is a sort of linguistic variable. The context of a particular *usage* of such a word may delimit the scope of the word as a linguistic variable. The range of values, in some cases, for some phrases or words, may be very large as can be seen through the following three statements:

- Dinosaurs ruled the earth for a long period (*about millions of years*)
- It has not rained *for a long period (say about six months).*
- I had to wait for the doctor *for a long period (about six hours)*.

Fuzzy theory provides means to handle such situations. A **Fuzzy theory** may be thought as a technique of providing '**continuization**' to the otherwise binary disciplines like Set Theory, PL and FOPL.

Further, we explain how using fuzzy concepts and rules, in situation like the ones quoted below, we, the human beings solve problems, despite ambiguity in language.

Let us recall the case of crossing a road discussed in Unit 1 of Block 1. We Mentioned that a step by step method of crossing a road may consist of

(i) Knowing (exactly) the distances of various vehicles from the path to be followed to cross over.

(ii) Knowing the velocities and accelerations of the various vehicles moving on the road within a distance of, say, one kilometer.

(iii) Using Newton's Laws of motion and their derivatives like $s = ut + \frac{1}{2}at^2$, and calculating the time that would be taken by each of the various vehicles to reach the path intended to be followed to cross over.

(iv) Adjusting dynamically our speeds on the path so that no collision takes place with any of the vehicle moving on the road.

But, we know the human beings not only do not follow the above precise method but cannot follow the above precise method. **We, the human beings rather feel comfortable with fuzziness than precision.** We feel comfortable, if the instruction for crossing a road is given as follows:

Look on both your *left hand* and *right hand* sides, particularly in the beginning, to your right hand side. If there is no vehicle within *reasonable* distance, then attempt to cross the road. You may have to retreat back while crossing, from *somewhere* on the road. Then, try again.

The above instruction has a number of words like *left*, *right* (it may 45° to the right or 90° to the right) *reasonable*, each of which does not have a definite meaning. But we feel more comfortable than the earlier instruction involving precise terms.

Let us consider another example of *our being comfortable with imprecision than precision*. The statement: '*The sky is densely clouded*' is more comprehensible to human beings than the statement: '*The cloud cover of the sky is 93.5 %'*.

Thus is because of the fact that, we, the human beings are still better than computers in **qualitative** reasoning. Because of better qualitative reasoning capabilities

- just by looking at the eyes only and/or nose only, we may recognize a person.

- just by taking and feeling a small number of grains from cooking rice bowl, we can **tell** whether the rice is properly cooked or not.
- just by looking at few buildings, we can identify a locality or a city.

Achieving Human Capability

In order that computers achieve human capability in solving such problems, computers must be able to solve problems for which ***only incomplete and/or imprecise*** information/knowledge is available.

Modelling of Solutions and Data/Information/Knowledge

We know that for any problem, the plan of the proposed solution and the relevant information is fed in the computer in a form acceptable to the computer.

However, the problems to be solved with the help of computers are, in the first place, felt by the human beings. And then, the plan of the solution is also prepared by human beings.

It is conveyed to the computer mainly for execution, because computers have much better executional speed.

Summarizing the discussion, we conclude the following facts

(i) We, the human beings, sense problems, desire the problems to be solved and express the problems and the plan of a solution using imprecise words of a natural language.

(ii) We use computers to solve the problems, because of their executional power.

(iii) Computers function better, when the information is given to the computer in terms of ***mathematical entities*** like numbers, sets, relations, functions, vectors, matrices graphs, arrays, trees, records, etc., and when the ***steps of solution*** are generally precise, involving no ambiguity.

In order to meet the mutually conflicting requirements:

(i) *Imprecision* of natural language, with which the human beings are comfortable, where human beings feel a problem and plan its solution.

(ii) *Precision* of a formal system, with which computers operate efficiently, where computers execute the solution, generally planned by human beings
a new formal system viz. Fuzzy system based on the concept of 'Fuzzy' was suggested for the first time in 1965 by L. Zadeh.

In order to initiate the study of Fuzzy systems, we quote two statements to recall the difference between a precise statement and an imprecise statement.

A precise Statement is of the form: 'If income is more than 2.5 lakhs then tax is 10% of the taxable income'.

An ***imprecise*** statement may be of the form: 'If the ***forecast*** about the rain being **slightly less** than previous year ***is believed***, then there is around 30% **probability** that economy may suffer heavily'.

The **concept of 'Fuzzy',** which when applied as a **prefix/adjective to mathematical entities like set, relation**, **functions, tree,** etc., **helps us in modelling** the **imprecise** data, information or knowledge **through mathematical tools**.

Crisp Set/Relation vs. Fuzzy Set/Relation: In order to differentiate the sets, normally used so far, from the *fuzzy sets* to be introduced soon, we may call the normally called sets as *crisp sets*.

Next, we explain, how the fuzzy sets are defined, using mathematical entities, **to capture imprecise concepts**, through an example of the concept : tall.

In Indian context, we may say, a ***male adult***, is

(i) **definitely tall** if his height > 6 feet
(ii) **not at all** tall if height < 5 feet and
(iii) if his height = 5′ 2” a **little bit tall**
iv) if his height = 5′ 6” **slightly tall**
(v) if height = 5′ 9” **reasonably tall** etc.

Next step is **to model 'definitely tall' 'not at all tall', 'little bit tall', 'slightly tall' 'reasonably Tall'** etc. in terms of mathematical entities, e.g., numbers; sets etc.
In **modelling the vague concept like 'tall', through fuzzy sets,** the numbers in the **closed set [0, 1] of reals** may be used on the following lines:

(i) '***Definitely tall***' may be represented as '***tallness having value 1***'
(ii) '***Not at all tall***' may be represented as '***Tallness having value 0***'

other adjectives/adverbs may have values between 0 and 1 as follows:

(iii) '***A little bit tall***' may be represented as '***tallness having value say .2***'.
(iv) '***Slightly tall***' may be represented as '***tallness having value say .4***'.
(v) '***Reasonably tall***' may be represented as '***tallness having value say .7***'.
and so on.

Similarly, the values of other concepts or, rather, other **linguistic variables like sweet, good, beautiful**, etc. may be considered **in terms of real numbers between 0 and 1.**

Coming back to the ***imprecise concept of tall***, let us think of five male persons of an organisation, viz., Mohan, Sohan, John, Abdul, Abrahm, with heights 5' 2”, 6' 4”, 5' 9”, 4′ 8”, 5′ 6” respectively.
Then had we talked only of crisp set of tall persons, we would have denoted the

Set of tall persons in the organisation = {Sohan}

But, a fuzzy set, representing tall persons, include ***all the persons alongwith respective degrees of tallness***. Thus, ***in terms of fuzzy sets,*** we write:

Tall = {Mohan/.2; Sohan/1; John/.7; Abdul/0; Abrahm/.4}.

The values .2, 1, .7, 0, .4 are called ***membership values or degrees***:

Note: ***Those elements which have value 0 may be dropped*** e.g.

Tall may also be written as Tall = {Mohan/.2; Sohan/1; John/.7;, Abrahm/.4}, neglecting Abdul, with associated degree zero.

If we ***define short/Diminutive*** as exactly ***opposite of Tall*** we may say
Short = {Mohan/.8; Sohan/0; John/.3; Abdul/1; Abrahm/.6}

3.3 RELATIONS ON FUZZY SETS

In the case of ***Crisp sets***, we have the concepts of *Equality of sets, Subset of a set, and Member of a set*, as illustrated by the following examples:

(i) **Equality of two sets**

Let A = {1, 4, 3, 5}
B = {4, 1, 3, 5}
C = {1, 4, 2, 5}

be three given sets.

Then, Set A is equal to set B *denoted by A = B*. But A is not equal to C, *denoted by* $A \neq C$.

(ii) **Subset**

Consider sets A = {1, 2, 3, 4, 5, 6, 7}
B = {4, 1, 3, 5}
C = {4, 8}

Then B is a subset of A, denoted by $B \subset A$. Also C is not a subset of A, denoted by $C \not\subset A$.

(iii) **Belongs to/is a member of**

If A = {1, 4, 3, 5}

Then each of 1, 4, 3 and 5 is called an *element or member* of A and the fact that *1 is a member of A* is denoted by $1 \in A$.

Corresponding Definitions/ concepts for Fuzzy Sets

In order to define for fuzzy sets, the concepts corresponding to the concepts of *Equality of Sets*, *Subset* and *Membership of a Set* considered so far only for crisp sets, first we illustrate the concepts through an example:

Let X be the set on which fuzzy sets are to be defined, e.g.,

X = {Mohan, Sohan, John, Abdul, Abrahm}.
Then X is called the ***Universal Set.***

Note: *In every fuzzy set, all the elements of X with their corresponding memberships values from 0 to 1, appear.*

(i) Degree of Membership: In respect of fuzzy sets, we do not speak of just '*membership*', but speak of '*degree of membership*'.
In the set
A = {Mohan/.2; Sohan/1; John/.7; Abrahm/.4},
Degree (Mohan) = .2, degree (John) =.4

For (ii) Equality of Fuzzy sets: Let A, B and C be fuzzy sets defined on X as follows:

Let A = {Mohan/.2; Sohan/1; John/.7; Abrahm/.4}
B = {Abrahm/.4, Mohan/.2; Sohan/1; John/.7}.

Then, as degrees of each element in the two sets, equal; we say fuzzy set A equals fuzzy set B, denoted as A = B
However, if C = {Abrahm/.2, Mohan/.4; Sohan/1; John/.7}, then
$A \neq C$.

(iii) Subset/Superset

Intuitively, we know

(i) The **Set of 'Very Tall' people** should be a **subset** of the set of **Tall people**.

(ii) If the **degree of 'tallness'** of a person **is say .5** then degree of **'Very Tallness'** for the person should be **lesser say .3**.

Combining the above two ideas we, may say that if
A = {Mohan/.2; Sohan/1; John/.7; Abrahm/.4} and
B = {Mohan/.2, Sohan/.9, John/.6, Abraham/.4}and further,
C = {Mohan/.3, Sohan/.9, John/.5, Abraham/.4},

then, in view of the fact that for each element, degree in A is greater than or equal to degree in B, **B is a subset of A** denoted as B $\subset$ A.
However, degree (Mohan) = .3 in C and degree (Mohan) =.2 in A,
,therefore, C is **not** a subset of A.
On the other hand degree (John) = .5 in C and degree (John) = .7 in A,
therefore, A is also not a subset of C.

We generalize the ideas illustrated through examples above

Let ***A and B be fuzzy sets*** on the universal set

$X = \{x_1, x_2, \ldots, x_n\}$
(X is called the Universe or Universal set)
s.t.
$A = \{x_1/v_1, x_2/v_2, \ldots, x_n/v_n\}$ and
$B = \{x_1/w_1, x_2/w_2, \ldots, x_n/w_n\}$

with that $0 \le v_i$, $w_i \le 1$.
Then fuzzy set A equals fuzzy set B, denoted as A = B, *if and only if*
$v_i = w_i$ for all i = 1,2,….,n.
Further if ***and*** $w \le v_i$ ***for all i.***

then B is a fuzzy subset of A.

Example: Let X = {Mohan, Sohan, John, Abdul, Abrahm}
A = {Mohan/.2; Sohan/1; John/.7; Abrahm/.4}
B = {Mohan/.2, Sohan/.9, John/.6, Abraham/.4}

Then B is a fuzzy subset of A.

In respect of fuzzy sets vis-à-vis (crisp) sets, we may note that:

- Corresponding to the concept of '**belongs to**' of **(Crisp) set**, we use the concept of '**degree of membership**' for fuzzy sets.
- It may be noted that every **crisp set** may be thought of as a **Fuzzy Set,** but **not conversely**. For example, if **Universal set is**
  X = {Mohan, Sohan, John, Abdul, Abrahm} and
  A = set of those members of X who are ***at least graduates,*** say,
  = {Mohan, John, Abdul}
  then we **can rewrite A as a fuzzy set** as follows:

A = {Mohan/1; **Sohan**/0; John/1; Abdul/1; **Abrahm**/0}, in which degree of each member of the crisp set, is taken as one and degree of each element of the universal set which does not appear in the set A, is taken as zero.

However, conversely, a fuzzy set **may not** be written as a **crisp set.** Let C be a fuzzy set denoting **Educated People,** where **degree of education is defined as** follows:

degree of education (Ph.D. holders) = 1
degree of education (Masters degree holders) = 0.85
degree of education (Bachelors degree holders) = .6
degree of education (10 + 2 level) = 0.4
degree of education (8^{th} Standard) = 0.1
degree of education (less than 8th) = 0.

Let us C = {Mohan/.85; Sohan/.4; John/.6; Abdul/1; Abrahm/0}.

Then, we cannot think of C as a crisp set.

Next, we define some more concepts in respect of fuzzy sets.

Definition: Support set of a Fuzzy Set, say C, is a crisp set, say D, containing all the elements of the universe X for which **degree** of membership in Fuzzy set **is positive. Let us consider again**

C = {Mohan/.85; Sohan/.4; John/.6; Abdul/1; Abrahm/0}.

Support of C = D = {Mohan, Sohan, John, Abdul}, where **the element Abrahm does not belong to D, because, it has degree 0 in C.**

Definition: Fuzzy Singleton is a fuzzy set in which there is exactly one element which has positive membership value.

Example:

Let us define a fuzzy set OLD on universal set X in which degree of OLD is zero if a person in X is below 20 years and Degree of Old is .2 if a person is between 20 and 25 years and further suppose that
Old = C = {Mohan/0; Sohan/0; John/.2; Abdul/0; Abrahm/0},
then support of old = **{John}** and hence old is a fuzzy singleton.

Ex. 1: Discuss equality and subset relationship for the following fuzzy sets defined on the Universal set X = { a, b , c, d, e}
A = { a/.3, b/.6, c/.4 d/0, e/.7}
B = {a/.4, b/.8, c/.9, d/.4, e/.7}
C = {a/.3, b/.7, c/.3, d/.2, e/.6}

3.4 OPERATIONS ON FUZZY SETS

For Crisp sets, we have the operations of **Union, intersection and complementation**, as illustrated by the example:

Let $X = \{x_1, x_2, \ldots, x_{10}\}$
$A = \{x_2, x_3, x_4, x_5\}$
$B = \{x_1, x_3, x_5, x_7, x_9\}$

Then $A \cup B = \{x_1, x_2, x_3, x_4, x_5, x_7, x_9\}$
$A \cap B = \{x_3, x_5\}$
A' or $X \sim A = \{x_1, x_6, x_7, x_8, x_9, x_{10}\}$

The concepts of Union, intersection and complementation for **crisp sets may be extended to FUZZY** sets after observing that for crisp sets A and B, we have

(i) $A \cup B$ is the **smallest** subset of X **containing** both A and B.
(ii) $A \cap B$ is the **largest** subset of X **contained in** both A and B.
(iii) **The complement A' is such that**

(a) A and A' **do not have any element in common** and
(b) Every element of the universal set **is in either A or A'.**

Fuzzy Union, Intersection, Complementation:
In order to motivate proper definitions of these operations, we may recall
(1) when a **crisp set** is treated as a **fuzzy set** then

(i) membership in a crisp set is indicated by degree/value of membership as 1 (one) in the corresponding Fuzzy set,

(ii) non-membership of a crisp set is indicated by degree/value of membership as **zero** in the corresponding Fuzzy Set.

Thus, **smaller the value** of degree of membership, a sort of **lesser it is a member** of the Fuzzy set.

(2) While taking union of Crisp sets, members of both sets are included, and none else. However, in each Fuzzy set, all members of the universal set occur but their degrees determine the level of membership in the fuzzy set.

The facts under (1) and (2) above, lead us to define:

The **Union of two fuzzy sets** A and B, is the set C with the same universe as that of A and B such that, the degree of an element of C is equal to the **MAXIMUM** of degrees of the element, in the two fuzzy sets.
(*if Universe A ≠ Universe B, then take Universe C as the union of the universe A and universe B)*
The **Intersection C of two fuzzy sets A and B is the fuzzy set in which**, the degree of an element of C is equal to the **MINIMUM** of degrees in the two fuzzy sets.

Example:

A = {Mohan/.85; Sohan/.4; John/.6; Abdul/1; Abrahm/0}
B = {Mohan/.75; Sohan/.6; John/0; Abdul/.8; Abrahm/.3}

Then

$A \cup B$ = {Mohan/.85; Sohan/.6; John/.6; Abdul/1; Abrahm /.3}
$A \cap B$ = {Mohan/.75; Sohan/.4; John/0; Abdul/.8; Abrahm/0}

and, the complement of A denoted by A′ is given by
C′ = {Mohan/.15; Sohan/.6; John/.4; Abdul/0; Abrahm /1}

Properties of Union, Intersection and Complement of Fuzzy Sets:
The following properties which hold for ordinary sets, also, hold for fuzzy sets

Commutativity

(i) $A \cup B = B \cup A$
(ii) $A \cap B = B \cap A$

We prove only (i) above just to explain, how the involved equalities, may be proved in general.

Let $U = \{x_1, x_2\ldots..x_n\}$*. be universe for fuzzy sets A and B*
If $y \in A \cup B$*, then y is of the form {xi/di} for some i*
$y \in A \cup B \Rightarrow y = \{xi/ei\}$ *as member of A and*
$y = (xi/fi\}$ *as member of B and*
$di = \max\{ei, fi\} = \max\{fi, ei\}$
$\Rightarrow y \in B \cup A.$

Rest of the properties are stated without proof.
Associativity
(i) $(A \cup B) \cup C = A \cup (B \cup C)$
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

DeMorgan's Laws

$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

Involution or Double Complement

$(A')' = A$

Idempotence
$A \cap A = A$
$A \cup A = A$
Identity
$A \cup U = U \quad A \cup U = A$
$A \cap \phi = A \quad \phi \cap A = \phi$

where
ϕ: empty fuzzy set = $\{x/0 \text{ with } x \in U\}$
and
U: universe = $\{x/1 \text{ with } x \in U\}$

Ex. 2: For the following fuzzy sets
A = {a/.5, b/.6, c/.3, d/0, e/.9} and
B = { a/.3, b/.7, c/.6, d/.3, e/.6},
find the fuzzy sets $A \cap B$, $A \cup B$ and $(A \cap B)'$

3.5 OPERATIONS UNIQUE TO FUZZY SETS

Next, we discuss three operations, viz., *concentration, dilation and normalization*, that are relevant only to fuzzy sets and can not be discussed for (crisp) sets.

(1) Concentration of a set A is defined as
CON (A) = $\{x/m^2_A(x) | x \in U\}$
Example:
If A = {Mohan/.5; Sohan/.9; John/.7; Abdul/0; Abrahm/.2}
then
CON (A) = {Mohan/.25; Sohan/.81; John/.49; Abdul/0; Abrahm/.04}.

In respect of concentration, it may be noted that the associated values being between 0 and 1, on squaring, become smaller. In other words, the values concentrate towards zero. This fact may be used for giving increased emphasis on a concept. If *Brightness* of articles is being discussed, then *Very bright* may be obtained in terms of *CON. (Bright).*

(2) Dilation (Opposite of Concentration) of a fuzzy set A is defined as

$$\text{DIL (A)} = \{x/\sqrt{m_A(x)} | x \in U\}$$

Example:

If A = {Mohan/.5; Sohan/.9; John/.7; Abdul/0; Abrahm/.2}
then
DIL (A) = {Mohan/.7; Sohan/.95; John/.84; Abdul/0; Abrahm/.45}

The associated values, that are between 0 and 1, on taking square-root get increased, e.g., if the value associated with x was .01 before dilation, then the value associated with x after dilation becomes .1, i.e., ten times of the original value.
This fact may be used for *decreased emphasis*. For example, if colour say '*yellow*' has been considered already, then '*light yellow*' may be considered in terms of already discussed '*yellow*' through Dilation.

(3) Normalization of a fuzzy set, **is defined as**

$$NORM\,(A) = \left\{ x \,/ \left(\frac{m_A(x)}{Max} \right) | \; x \in U \right\}.$$

NORM (A) and is a fuzzy set in which membership values are obtained by dividing values of the membership function of A **by the maximum membership function**.

The resulting fuzzy set, called the **normal**, (or **normalized) fuzzy set**, has the maximum of membership function value of 1.

Example:

If A = {Mohan/.5; Sohan/.9; John/.7; Abdul/0; Abrahm/.2}

Norm (A) = {Mohan/(.5 ÷.9 = .55.); Sohan/1; John /(.7 ÷.9 = .77.); Abdul/0; Abrahm/(.2 ÷.9 = .22.)}

Note: If one of the members has value 1, then Norm (A) = A,

Relation & Fuzzy Relation

We know from our earlier background in Mathematics that a relation from a set A to a set B is a subset of A x B.
For example, The relation of father may be written as {{Dasrath, Ram), …}, *which is a subset of A* $\times$ *B, where A and B are sets of persons living or dead.*

The relation of *Age* may be written as
{(Mohan, 43.7), (Sohan, 25.6), …},
where A is set of living persons and B is set of numbers denoting years.

Fuzzy Relation

In fuzzy sets, every element of the universal set occurs with some degree of membership. **A fuzzy relation may be defined in different ways.** One way of defining fuzzy relation is to assume the underlying sets as crisp sets. We will discuss only this case.

Thus, a **relation from A to B, where we assume A and B as crisp sets, is a fuzzy set, in which with each** element of A × B is associated a degree of membership between zero and one.

For example:

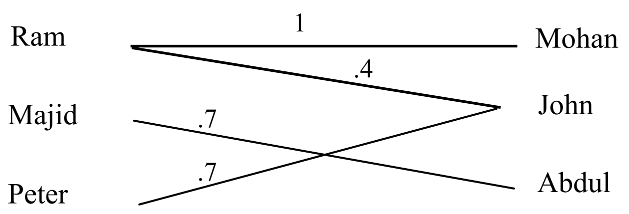
We may define the relation of UNCLE as follows:
(i) *x* is an UNCLE of *y* with degree **1** if *x* is brother of mother or father,
(ii) *x* is an UNCLE of *y* with degree **.7** if *x* is a brother of an UNCLE of *y*, and *x* is not covered above,
(iii) *x* is an UNCLE of *y* with degree **.6** if *x* is the son of an UNCLE of mother or father.

Now suppose

Ram is UNCLE of Mohan with degree **1,** Majid is UNCLE of Abdul with degree **.7** and Peter is UNCLE of John with degree **.7**. Ram is UNCLE of John with degree**.4** Then **the relation of UNCLE can be written** as a set of ordered-triples as follows:

{(Ram, Mohan, 1), (Majid, Abdul, .7), (Peter, John, .7), (Ram, John, .4)}.

As in the case of ordinary relations, we can use matrices and graphs to represent FUZZY relations, e.g., the relation of UNCLE discussed above, may be graphically denoted as



Fuzzy Graph

Fuzzy Reasoning

In the rest of this section, we just have a fleeting glance on Fuzzy Reasoning.
Let us recall the well-known Crisp Reasoning Operators

(i) AND
(ii) OR
(iii) NOT
(iv) *IF* P *THEN* Q
(v) P *IF AND ONLY IF* Q

Corresponding to each of these operators, there is a fuzzy operator discussed and defined below. For this purpose, we assume that P and Q are fuzzy propositions with associated degrees, respectively, deg (P) and deg (Q) between 0 and 1.

The deg (P) = 0 denotes *P is False* and deg (P) =1 denotes *P is True.*

Then the operators are defined as follows:

(i) Fuzzy AND to be denoted by $\wedge$, is defined as follows:

For given fuzzy propositions P and Q, the expression P $\wedge$ Q denotes a fuzzy proposition with Deg (P $\wedge$ Q) = **min** (deg (P), deg (Q))

Example: Let P: *Mohan is tall* with deg (P) = .7
Q: *Mohan is educated* with deg (Q) = .4
Then P $\wedge$ Q denotes: *'Mohan is tall and educated'* with degree (**(min)** {.7, .4}) = **.4**

(ii) Fuzzy OR to be denoted by $\vee$, is defined as follows:
For given fuzzy propositions P and Q, P $\vee$ Q is a fuzzy proposition with
Deg (P $\vee$ Q) = **max** (deg (P), deg (Q))

Example: Let P: *Mohan is tall* with deg (P) = .7
Q*: Mohan is educated* with deg (Q) = .4
Then P $\vee$ Q denotes*: 'Mohan is tall or educated'* with degree (**(max)** {.7, .4}) = **.7**

3.6 NON-MONOTOMIC REASONING SYSTEMS

Monotonic Reasoning: The conclusion drawn in PL and FOPL are only through (valid) deductive methods. When some axiom is *added* to a PL or an FOPL system, then, through deduction, we can draw *more* conclusions. Hence, more additional facts become available in the knowledge base with the addition of each axiom. Adding of axioms to the knowledge base increases the amount of knowledge contained in the knowledge base. Therefore, the set of facts through inferences in such systems **can only grow larger** with addition of each axiomatic fact. Adding of new facts can not reduce the size of K.B. Thus, amount of knowledge **monotonically** increases with the number of independent premises due to new facts that become available.

However, in everyday life, many times in the light of new facts that become available, we may have to revise our earlier knowledge. For example, we consider a sort of deductive argument in FOPL:

(i) Every bird can fly long distances
(ii) Every pigeon is a bird. (iii) Tweety is a pigeon.
Therefore, Tweety can fly long distances.

However, later on, we come to know that Tweety is actually a hen and a hen cannot fly long distances. Therefore, we have to revise our belief that Tweety can fly over long distances.

This type of situation is not handled by any monotonic reasoning system including PL and FOPL .This is appropriately handled by Non-Monotomic Reasoning Systems, which are discussed next.

A **non-monotomic reasoning system** is one which allows *retracting* **of old knowledge due to discovery of new facts** which contradict or invalidate a part of the current knowledge base. Such systems also take care that retracting of a fact may necessitate a chain of retractions from the knowledge base or even reintroduction of earlier retracted ones from K.B. Thus, *chain-shrink* and *chain emphasis* of a K.B and reintroduction of earlier retracted ones are part of a non-monotomic reasoning system.

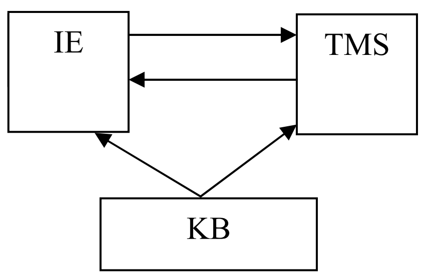
To meet the requirement for reasoning in the real-world, we need non-monotomic reasoning systems also, in addition to the monotomic ones. This is true specially, in view of the fact that it is not reasonable to expect that **all the knowledge needed** for a set of tasks could be acquired, validated, and loaded into the system just at the outset. In general, initial knowledge is an *incomplete* set of *partially true* facts. The set may also be redundant and may contain inconsistencies and other sources of uncertainty.

Major components of a Non-Monotomic reasoning system

Next, we discuss a typical non-monotomic reasoning system **(NMRS)** consists of the following three major components:

(1) Knowledge base (KB),
(2) Inference Engine (IE),
(3) Truth-Maintenance System (TMS).

The **KB** contains information, facts, rules, procedures etc. relevant to the type of problems that are expected to be solved by the system. The component **IE** of NMRS gets facts from KB to draw new inferences and sends the new facts discovered by it (i.e., IE) to KB. The component **TMS**, after addition of new facts to KB. either from the environment or through the user or through IE, checks for validity of the KB. It may happen that the new fact from the environment or inferred by the IE may conflict/contradict some of the facts already in the KB. In other words, an *inconsistency* may arise. In case of inconsistencies, TMS retracts some facts from KB. Also, it may *lead to a chain of retractions* which may require interactions between KB and TMS. Also, some new fact either from the environment or from IE, may invalidate some earlier retractions *requiring reintroduction* of earlier retracted facts. This may lead to a chain of reintroductions. These retrievals and introductions are taken care of by TMS. The IE is completely relieved of this responsibility. Main job of IE is *to conclude* new facts when it is supplied a set of facts.



Next, We explain the ideas discussed above through an example:

Let us assume KB has two facts P and $\sim Q \rightarrow \sim P$ and a rule called *Modus Tollens*. When **IE** is supplied these knowledge items, it concludes Q and sends Q to KB. However, through interaction with the environment, **KB** is later supplied with the information that ~ P is more appropriate than P. Then **TMS,** on the addition of ~ P to KB, finds that KB is no more consistent, at least, with P. The knowledge that ~ P is more appropriate, suggests that P be retracted. Further Q was concluded *assuming P as True*. But, in the new situation in which P is assumed to be not appropriate, Q also becomes inappropriate. **P and Q are not deleted from KB, but are just marked as dormant or ineffective.** This is done in view of the fact that later on, if again, it is found appropriate to include P or Q or both, then, instead of requiring some mechanism for adding P and Q, we just remove marks that made these dormant.

Non-monotomic Reasoning Systems deal with

1) Revisable belief systems

2) incomplete K.B. —— Default Reasoning

—— Closed World assumption

3.7 DEFAULT REASONING SYSTEMS

In the previous section, we discussed *uncertainty due to beliefs* (which are not necessarily *facts*) where beliefs are changeable. Here, we discuss *another form of uncertainty* that occur as a result of *incompleteness* of the available knowledge at a particular point of time.

One method of handling uncertainty due to **incomplete** KB is through **default reasoning** which is also a form of non-monotomic reasoning and is based on the following mechanism:

*Whenever, for any entity relevant to the application, information is not in the KB, then a **default value** for that type of entity, is assumed and is assigned to the entity. The default assignment is not arbitrary but is based on experiments, observations or some other rational grounds. However, the typical value for the entity is removed if some information contradictory to the assumed or default value becomes available.*

The advantage of this type of a reasoning system is that we need not store all facts regarding a situation. **Reiter has given one theory of default reasoning, which is expressed as**

$$\frac{a(x) : M\, b_1\,(x),\ldots\ldots, Mb_k\,(x)}{C(x)} \qquad (A)$$

where M is a *consistency operator.*

The inference rule (A) states that if a(x) is true and none of the conditions b_k (x) is in conflict or contradiction with the K.B, then you can deduce the statement C(x)

The idea of default reasoning is explained through the following example:

Suppose we have

(i) $$\frac{Bird(x) : Mfly(x)}{Fly(x)}$$

(ii) Bird (twitty)

M fly (x) stands for a statement of the form *'KB does not have any statement of the form that says x does not have wings etc, because of which x may not be able to fly'.* In other words, *Bird (x) : M fly (x)* may be taken to stand for the statement *'if x is a normal bird and if the normality of x is not contradicted by other facts and rules in the KB.'* then we can assume that *x can fly*. Combining with *Bird (Twitty),* we conclude that if KB does not have any facts and rules from which, it can be inferred that *Twitty can not fly*, then, we can conclude that *twitty can fly.*

Further, suppose, KB also contains

(i) Ostrich (twitty)

(ii) Ostrich (x) $\rightarrow$ ~ FLY (x).

From these two facts in the K.B., it is concluded that Twitty being an ostrich, can not fly. In the light of this knowledge the fact that Twitty can fly has to be withdrawn. Thus, Fly (twitty) would be locked. Because, default Mfly (Twitty) is now inconsistent.

Let us consider another example:

$$\frac{\text{Adult}(x) : \text{M drive}(x)}{\text{Drive}(x)}$$

The above can be interpreted in the default theory as:

If a person x is an adult and in the knowledge base there is no fact (*e.g., x is blind, or x has both of his/her hands cut in an accident etc*) which tells us something making x incapable of driving, **then *x*** can drive, is assumed.

3.8 CLOSED WORLD ASSUMPTION SYSTEMS

Another mechanism of handling incompleteness of a KB is called 'Closed World Assumption' (CWA).

This mechanism is useful in applications where *most of the facts are known* and therefore it is reasonable *to assume that if a proposition cannot be proved, then it is FALSE. This is called CWA with failure as negation.*

This means if a ground atom P(a) is not provable, then assume ~ P(a). *A predicate like LESS (x, y) becomes a **ground atom** when the variables x and y are replaced by constants say x by 2 and y by 3, so that we get the ground atom LESS (2, 3).*

Example of an application where CWA is reasonable is that of *Airline reservation* where city-to-city flight not explicitly entered in the flight schedule or time table, *are assumed not to exist*.

AKB is **complete** if for each ground atom P(a); either P(a) or ~ P(a) can be proved.

By the use of CWA any incomplete KB becomes complete **by the addition of the meta rule**:

If P(a) can not be proved then assume ~ P (a).

Example of an incomplete K.B.: Let our KB contain only

(i) P(a).
(ii) P(b).
(iii) P(a) $\rightarrow$ Q(a).
(iv) Rule of Modus Ponens: From P and P $\rightarrow$ Q, conclude Q.

The above KB is *incomplete* as we can not say anything about Q(b) (or ~ Q(b)) from the given KB.

Remarks: In general, KB argumented by CWA need *not be* consistent i.e., it may contain two mutually conflicting *wffs*. For example, if our KB contains only P(a) $\vee$ Q(b).

(Note: from P (a) $\vee$ Q (b), we can not conclude either of P (a) and Q (b) with definiteness)

As neither P(a) nor Q(b) is provable, therefore, we add ~ P(a) and ~ Q(b) by using CWA.
But, then, the set of P(a) $\vee$ Q(b), ~ P(a) and ~Q(b) is inconsistent.

3.9 OTHER NON-DEDUCTIVE SYSTEMS

PL and FOPL are *deductive* inferencing systems: i.e., the conclusions drawn are *invariably true* whenever the premises are *true*. However, due to limitations of these systems for making inferences, as discussed earlier, we must have other systems inferences. In addition to *Default Reasoning systems* and *Closed World Assumption systems*, we have the following useful reasoning systems:

1) **Abductive inference** System, which is based on the use of causal knowledge to explain and justify a (*possibly invalid*) conclusion.

Abduction Rule

$$\frac{\begin{array}{c} P \rightarrow Q \\ Q \end{array}}{P}$$

Note that *abductive inference rule is different form Modus Ponens inference rule* in that in abductive inference rule, the *consequent* of P $\rightarrow$ Q, i.e., Q is assumed to be given as True and the *antecedent* of P $\rightarrow$ Q, i.e., P *is inferred.*

The abductive inference is useful in *diagnostic applications*. For example while diagnosing a disease (*say P*), the doctor asks for the symptoms (*say Q*). Also, the doctor knows that for given the disease, say, Malaria (P); the symptoms include high fever starting with feeling of cold etc. (Q)

i.e., doctor knows P$\rightarrow$Q

The doctor then attempts to diagnose the disease (i.e., P) from symptoms. However, it should be noted that the conclusion of the disease from the symptoms may not always be correct. In general, abductive reasoning leads to correct conclusions, but the conclusions may be *incorrect* also. In other words, Abductive reasoning is not a **valid form** of reasoning.

Inductive Reasoning is a method of generalisation from a finite number of instances.

The rule, generally, denoted as

$$\frac{P(a_1),\ P(a_2)......,P(a_n)}{(x)\ P(x)},$$

states that from n instances P(a_i) of a predicate/property *P(x),* we infer that *P(x) is True for all x.*

Thus, from a finite number of observations about some property of objects, we generalize, i.e., make a *general* statement *about all* the elements of the domain in respect of the property.

For example, we may, conclude that: *all cows are white,* after observing a large number of white cows. However, this conclusion may have some exception in the sense that we may come across a black cow also. Inductive Reasoning like Abductive Reasoning, Closed World Assumption Reasoning and Default Reasoning is not *irrefutable*. In other words, these reasoning rules lead to conclusions, which may be True, but not necessarily always.

However, all the rules discussed under Propositional Logic (PL) and FOPL, including Modus Ponens etc are deductive i.e., lead to irrefutable conclusions.

3.10 SUMMARY

In this unit, we briefly discussed some formal systems which take care of at least one of the blemishes in the knowledge base, namely, of inconsistency, imprecision and incompleteness of the knowledge base. In Sections 4.2 to 4.5, we discuss Fuzzy systems, which attempt to handle imprecision due to use of words, having multiple meanings, of a natural language. The words appear in the description of the problems to be solved by man-machine systems.

In Section 4.6, we briefly discuss non-monotonic (formal) systems, which mainly deal with problems involving *beliefs,* in stead of *facts*. A belief may be revised by the believer, when strong evidence becomes available for the revision of the belief.

In Sections 4.7 and 4.8, we discuss two formal systems which attempt to deal with *incompleteness* of the available knowledge of the problem domain. *Default reasoning* systems discussed in Section 4.7, attempt to handle the problem of incompleteness of knowledge, through assumption of default values for the missing values. The default values may be withdrawn, in case some knowledge contrary to the default values, becomes available.

On the other hand, another formal system, viz., *closed world assumption system* discussed in Section 4.8, assume that, if the truth of a statement is not available in the knowledge base, then assume the statement false. Finally, in Section 4.9, we discuss some inference rules, namely, *abductive* and *inductive* rules, which though are not deductive, yet prove quite useful in everyday problems, particularly, in diagnostic problems.

3.11 SOLUTIONS/ANSWERS

Ex. 1: Both A and C are subsets of the fuzzy set B, because deg (x min A) ≤ deg (x in B) for all $x \in X$

Similarly degree (x in C) ≤ degree (x in B) for all $x \in X$
Further, A is not a subset of C, because,
deg (c in A) = .4 > .3 = degree (c in C)
Also, C is not a subset of A, because,
degree (b in C) = .7 > .6 = degree (b in A)

Ex. 2: $A \cap B$ = {a/.3, b/.6, c/.3, d/0, e/.6},
where degree (x in $A \cap B$) = min { degree (x in A), degree (x in B)}.
$A \cup B$ = {a/.5, b/.7, c/.6, d/.3, e/.9},
where degree (x in $A \cup B$) = max {degree (x in A), degree (x in B)}.

The fuzzy set $(A \cap B)'$ is obtained from $A \cap B$, by the rule:

degree (x in $(A \cap B)'$) = 1 − degree (x in $A \cap B$).
Hence
$(A \cap B)' = \{ a/.7, b/.4, c/.7, d/1, e/.4\}$

3.12 FURTHER READINGS

1. Munikata, Toshinori *Chapter 5 of Fundamentals of the New Artificial Intelligence: Beyond Traditional Paradigm* (springer, 1998).
2. Nguyen, H.T. Walker E.A. *A First Course in Fuzzy Logic* (CRC Press, 1997)
3. Patterson, D.W. *Introduction to Artificial Intelligence and Expert Systems*(Prentice-Hall of India, 2001)