Simplex Solver

October 20, 2023

Problem

Given the following linear system and objective function, find the optimal solution.

$$min(x_1 + x_2 + x_3 + x_4)$$

$$\begin{cases} y_1 + 2y_2 - y_3 - y_4 \geq 1 \\ -y_1 - 5y_2 + 2y_3 + 3y_4 \geq 1 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} y_1 - y_2 + s_1 = 1 \\ 2y_1 - 5y_2 + s_2 = 1 \\ -y_1 + 2y_2 + s_3 = 1 \\ -y_1 + 3y_2 + s_4 = 1 \end{cases}$$

Create the initial tableau of the new linear system.

$$\begin{array}{c} \left[\begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ \hline 1 & -1 & 1 & 0 & 0 & 0 & 1 \\ 2 & -5 & 0 & 1 & 0 & 0 & 1 \\ -1 & 2 & 0 & 0 & 1 & 0 & 1 \\ -1 & 3 & 0 & 0 & 0 & 1 & 1 \\ \hline -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ \\ \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_1 and the departing variable is s_2.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{c} \left[\begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ \hline 0 & 3/2 & 1 & -1/2 & 0 & 0 & 1/2 \\ 1 & -5/2 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & -1/2 & 0 & 1/2 & 1 & 0 & 3/2 \\ 0 & 1/2 & 0 & 1/2 & 0 & 1 & 3/2 \\ \hline 0 & -7/2 & 0 & 1/2 & 0 & 0 & 1/2 \end{array}\right] \end{array} \begin{array}{c} \\ s_1 \\ y_1 \\ s_3 \\ s_4 \\ \\ \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_2 and the departing variable is s_1.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{c} \left[\begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ \hline 0 & 1 & 2/3 & -1/3 & 0 & 0 & 1/3 \\ 1 & 0 & 5/3 & -1/3 & 0 & 0 & 4/3 \\ 0 & 0 & 1/3 & 1/3 & 1 & 0 & 5/3 \\ 0 & 0 & -1/3 & 2/3 & 0 & 1 & 4/3 \\ \hline 0 & 0 & 7/3 & -2/3 & 0 & 0 & 5/3 \end{array}\right] \begin{array}{c} \\ y_2 \\ y_1 \\ s_3 \\ s_4 \\ \\ \end{array} \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is s_2 and the departing variable is s_4.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\begin{array}{c} \left[\begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ \hline 0 & 1 & 1/2 & 0 & 0 & 1/2 & 1 \\ 1 & 0 & 3/2 & 0 & 0 & 1/2 & 2 \\ 0 & 0 & 1/2 & 0 & 1 & -1/2 & 1 \\ 0 & 0 & -1/2 & 1 & 0 & 3/2 & 2 \\ \hline 0 & 0 & 2 & 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{c} \\ y_2 \\ y_1 \\ s_3 \\ s_2 \\ \\ \end{array} \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$s_1 = 0, s_2 = 2, s_3 = 1, s_4 = 0, x_1 = 2, x_2 = 0, x_3 = 0, x_4 = 1, y_1 = 2, y_2 = 1, z = 3$