Simplex Solver

October 20, 2023

Problem

Given the following linear system and objective function, find the optimal solution.

$$min(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_10 + x_11 + x_12 + x_13 + x_14 + x_15 + x_16 + x_17 + x_18 + x_1$$

$$\begin{cases} y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 \\ -y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\ y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 \\ -y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\ y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 \\ -y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\ y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 + \\ -y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 + \\ y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 + \\ -y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$
\begin{cases}
y_1 - y_2 + y_3 - y_4 + y_5 - y_6 + y_7 - y_8 + y_9 - y_10 + s_1 = 1 \\
2y_1 - 5y_2 + 2y_3 - 5y_4 + 2y_5 - 5y_6 + 2y_7 - 5y_8 + 2y_9 - 5y_10 + s_2 = 1 \\
-y_1 + 2y_2 - y_3 + 2y_4 - y_5 + 2y_6 - y_7 + 2y_8 - y_9 + 2y_10 + s_3 = 1 \\
-y_1 + 3y_2 - y_3 + 3y_4 - y_5 + 3y_6 - y_7 + 3y_8 - y_9 + 3y_10 + s_4 = 1 \\
10y_1 - 6y_2 + 10y_3 - 6y_4 + 10y_5 - 6y_6 + 10y_7 - 6y_8 + 10y_9 - 6y_10 + s_5 = 1 \\
-7y_1 - 7y_2 - 7y_3 - 7y_4 - 7y_5 - 7y_6 - 7y_7 - 7y_8 - 7y_9 - 7y_10 + s_6 = 1 \\
-10y_1 + 7y_2 - 10y_3 + 7y_4 - 10y_5 + 7y_6 - 10y_7 + 7y_8 - 10y_9 + 7y_10 + s_7 = 1 \\
-2y_1 - 8y_2 - 2y_3 - 8y_4 - 2y_5 - 8y_6 - 2y_7 - 8y_8 - 2y_9 - 8y_10 + s_8 = 1 \\
-3y_1 + 8y_2 - 3y_3 + 8y_4 - 3y_5 + 8y_6 - 3y_7 + 8y_8 - 3y_9 + 8y_10 + s_9 = 1 \\
-3y_1 + 3y_2 - 3y_3 + 3y_4 - 3y_5 + 3y_6 - 3y_7 + 3y_8 - 3y_9 + 3y_10 + s_10 = 1 \\
-9y_1 + 9y_2 - 9y_3 + 9y_4 - 9y_5 + 9y_6 - 9y_7 + 9y_8 - 9y_9 + 9y_10 + s_11 = 1 \\
-10y_1 - 10y_2 - 10y_3 - 10y_4 - 10y_5 - 10y_6 - 10y_7 - 10y_8 - 10y_9 - 10y_10 + s_12 = 1 \\
-8y_1 + 7y_2 - 8y_3 + 7y_4 - 8y_5 + 7y_6 - 8y_7 + 7y_8 - 8y_9 + 7y_10 + s_13 = 1 \\
-4y_1 - 4y_2 - 4y_3 - 4y_4 - 4y_5 - 4y_6 - 4y_7 - 4y_8 - 4y_9 - 4y_10 + s_14 = 1 \\
-3y_1 + 10y_2 - 3y_3 + 10y_4 - 3y_5 + 10y_6 - 3y_7 + 10y_8 - 3y_9 + 10y_10 + s_15 = 1 \\
6y_1 + 7y_2 + 6y_3 + 7y_4 + 6y_5 + 7y_6 + 6y_7 + 7y_8 + 6y_9 + 7y_10 + s_16 = 1 \\
s_17 = 1 \\
-2y_1 - 7y_2 - 2y_3 - 7y_4 - 2y_5 - 7y_6 - 2y_7 - 7y_8 - 2y_9 - 7y_10 + s_18 = 1 \\
-6y_1 - 8y_2 - 6y_3 - 8y_4 - 6y_5 - 8y_6 - 6y_7 - 8y_8 - 6y_9 - 8y_10 + s_19 = 1 \\
-4y_1 + 2y_2 - 4y_3 + 2y_4 - 4y_5 + 2y_6 - 4y_7 + 2y_8 - 4y_9 + 2y_10 + s_20 = 1 \\
-7y_1 - 2y_2 - 7y_3 - 2y_4 - 7y_5 - 2y_6 - 7y_7 - 2y_8 - 7y_9 - 2y_10 + s_21 = 1 \\
y_1 - 9y_2 + y_3 - 9y_4 + y_5 - 9y_6 + y_7 - 9y_8 + y_9 - 9y_10 + s_22 = 1 \\
y_1 + 4y_2 + y_3 + 4y_4 + y_5 + 4y_6 + y_7 + 4y_8 + y_9 + 4y_10 + s_23 = 1 \\
9y_1 + 7y_2 + 9y_3 + 7y_4 + 9y_5 + 7y_6 + 9y_7 + 7y_8 + 9y_9 + 7y_10 + s_24 = 1
\end{cases}
$$

Create the initial tableau of the new linear system.

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_10	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_10
1	−1	1	−1	1	−1	1	−1	1	−1	1	0	0	0	0	0	0	0	0	0
2	−5	2	−5	2	−5	2	−5	2	−5	0	1	0	0	0	0	0	0	0	0
−1	2	−1	2	−1	2	−1	2	−1	2	0	0	1	0	0	0	0	0	0	0
−1	3	−1	3	−1	3	−1	3	−1	3	0	0	0	1	0	0	0	0	0	0
10	−6	10	−6	10	−6	10	−6	10	−6	0	0	0	0	1	0	0	0	0	0
−7	−7	−7	−7	−7	−7	−7	−7	−7	−7	0	0	0	0	0	1	0	0	0	0
−10	7	−10	7	−10	7	−10	7	−10	7	0	0	0	0	0	0	1	0	0	0
−2	−8	−2	−8	−2	−8	−2	−8	−2	−8	0	0	0	0	0	0	0	1	0	0
−3	8	−3	8	−3	8	−3	8	−3	8	0	0	0	0	0	0	0	0	1	0
−3	3	−3	3	−3	3	−3	3	−3	3	0	0	0	0	0	0	0	0	0	1
−9	9	−9	9	−9	9	−9	9	−9	9	0	0	0	0	0	0	0	0	0	0
−10	−10	−10	−10	−10	−10	−10	−10	−10	−10	0	0	0	0	0	0	0	0	0	0
−8	7	−8	7	−8	7	−8	7	−8	7	0	0	0	0	0	0	0	0	0	0
−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	0	0	0	0	0	0	0	0	0	0
−3	10	−3	10	−3	10	−3	10	−3	10	0	0	0	0	0	0	0	0	0	0
6	7	6	7	6	7	6	7	6	7	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
−2	−7	−2	−7	−2	−7	−2	−7	−2	−7	0	0	0	0	0	0	0	0	0	0
−6	−8	−6	−8	−6	−8	−6	−8	−6	−8	0	0	0	0	0	0	0	0	0	0
−4	2	−4	2	−4	2	−4	2	−4	2	0	0	0	0	0	0	0	0	0	0
−7	−2	−7	−2	−7	−2	−7	−2	−7	−2	0	0	0	0	0	0	0	0	0	0
1	−9	1	−9	1	−9	1	−9	1	−9	0	0	0	0	0	0	0	0	0	0
1	4	1	4	1	4	1	4	1	4	0	0	0	0	0	0	0	0	0	0
9	7	9	7	9	7	9	7	9	7	0	0	0	0	0	0	0	0	0	0
−1	−1	−3	3	−4	−8	2	10	5	−3	0	0	0	0	0	0	0	0	0	0

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_6 and the departing variable is s_15.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_10	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
7/10	0	7/10	0	7/10	0	7/10	0	7/10	0	1	0	0	0	0	0	0	0
1/2	0	1/2	0	1/2	0	1/2	0	1/2	0	0	1	0	0	0	0	0	0
−2/5	0	−2/5	0	−2/5	0	−2/5	0	−2/5	0	0	0	1	0	0	0	0	0
−1/10	0	−1/10	0	−1/10	0	−1/10	0	−1/10	0	0	0	0	1	0	0	0	0
41/5	0	41/5	0	41/5	0	41/5	0	41/5	0	0	0	0	0	1	0	0	0
−91/10	0	−91/10	0	−91/10	0	−91/10	0	−91/10	0	0	0	0	0	0	1	0	0
−79/10	0	−79/10	0	−79/10	0	−79/10	0	−79/10	0	0	0	0	0	0	0	1	0
−22/5	0	−22/5	0	−22/5	0	−22/5	0	−22/5	0	0	0	0	0	0	0	0	1
−3/5	0	−3/5	0	−3/5	0	−3/5	0	−3/5	0	0	0	0	0	0	0	0	0
−21/10	0	−21/10	0	−21/10	0	−21/10	0	−21/10	0	0	0	0	0	0	0	0	0
−63/10	0	−63/10	0	−63/10	0	−63/10	0	−63/10	0	0	0	0	0	0	0	0	0
−13	0	−13	0	−13	0	−13	0	−13	0	0	0	0	0	0	0	0	0
−59/10	0	−59/10	0	−59/10	0	−59/10	0	−59/10	0	0	0	0	0	0	0	0	0
−26/5	0	−26/5	0	−26/5	0	−26/5	0	−26/5	0	0	0	0	0	0	0	0	0
−3/10	1	−3/10	1	−3/10	1	−3/10	1	−3/10	1	0	0	0	0	0	0	0	0
81/10	0	81/10	0	81/10	0	81/10	0	81/10	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
−41/10	0	−41/10	0	−41/10	0	−41/10	0	−41/10	0	0	0	0	0	0	0	0	0
−42/5	0	−42/5	0	−42/5	0	−42/5	0	−42/5	0	0	0	0	0	0	0	0	0
−17/5	0	−17/5	0	−17/5	0	−17/5	0	−17/5	0	0	0	0	0	0	0	0	0
−38/5	0	−38/5	0	−38/5	0	−38/5	0	−38/5	0	0	0	0	0	0	0	0	0
−17/10	0	−17/10	0	−17/10	0	−17/10	0	−17/10	0	0	0	0	0	0	0	0	0
11/5	0	11/5	0	11/5	0	11/5	0	11/5	0	0	0	0	0	0	0	0	0
111/10	0	111/10	0	111/10	0	111/10	0	111/10	0	0	0	0	0	0	0	0	0
−17/5	7	−27/5	11	−32/5	0	−2/5	18	13/5	5	0	0	0	0	0	0	0	0

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_5 and the departing variable is s_24.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_10	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_10	s_11	s_12	s_13
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	7	1	11	0	0	6	18	9	5	0	0	0	0	0	0	0	0	0	0	0	0	0

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = \frac{40}{37}, s_10 = \frac{28}{37}, s_11 = \frac{10}{37}, s_12 = \frac{87}{37}, s_13 = \frac{17}{37}, s_14 = \frac{57}{37}, s_15 = 0, s_16 = \frac{3}{37}, s_17 = 1, s_18 = \frac{67}{37}, s_19 = \frac{75}{37}, s_2$$