Simplex Solver

October 20, 2023

Problem

Given the following linear system and objective function, find the optimal solution.

$$min(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_10 + x_11 + x_12 + x_13 + x_14 + x_15 + x_16 + x_17 + x_18 + x_1$$

$$\begin{cases}
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 - \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 + \\
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 - \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 - \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16 \\
y_1 + 2y_2 - y_3 - y_4 + 10y_5 - 7y_6 - 10y_7 - 2y_8 - 3y_9 - 3y_10 - 9y_11 - 10y_12 - 8y_13 - 4y_14 - 3y_15 + 6y_16 - \\
-y_1 - 5y_2 + 2y_3 + 3y_4 - 6y_5 - 7y_6 + 7y_7 - 8y_8 + 8y_9 + 3y_10 + 9y_11 - 10y_12 + 7y_13 - 4y_14 + 10y_15 + 7y_16
\end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases}
y_1 - y_2 + y_3 - y_4 + y_5 - y_6 + y_7 - y_8 + y_9 - y_10 + y_11 - y_12 + y_13 - y_14 + s_1 = 1 \\
2y_1 - 5y_2 + 2y_3 - 5y_4 + 2y_5 - 5y_6 + 2y_7 - 5y_8 + 2y_9 - 5y_10 + 2y_11 - 5y_12 + 2y_13 - 5y_14 + s_2 = 1 \\
-y_1 + 2y_2 - y_3 + 2y_4 - y_5 + 2y_6 - y_7 + 2y_8 - y_9 + 2y_10 - y_11 + 2y_12 - y_13 + 2y_14 + s_3 = 1 \\
-y_1 + 3y_2 - y_3 + 3y_4 - y_5 + 3y_6 - y_7 + 3y_8 - y_9 + 3y_10 - y_11 + 3y_12 - y_13 + 3y_14 + s_4 = 1 \\
10y_1 - 6y_2 + 10y_3 - 6y_4 + 10y_5 - 6y_6 + 10y_7 - 6y_8 + 10y_9 - 6y_10 + 10y_11 - 6y_12 + 10y_13 - 6y_14 + s_5 \\
-7y_1 - 7y_2 - 7y_3 - 7y_4 - 7y_5 - 7y_6 - 7y_7 - 7y_8 - 7y_9 - 7y_10 - 7y_11 - 7y_12 - 7y_13 - 7y_14 + s_6 = \\
-10y_1 + 7y_2 - 10y_3 + 7y_4 - 10y_5 + 7y_6 - 10y_7 + 7y_8 - 10y_9 + 7y_10 - 10y_11 + 7y_12 - 10y_13 + 7y_14 + s \\
-2y_1 - 8y_2 - 2y_3 - 8y_4 - 2y_5 - 8y_6 - 2y_7 - 8y_8 - 2y_9 - 8y_10 - 2y_11 - 8y_12 - 2y_13 - 8y_14 + s_8 = \\
-3y_1 + 8y_2 - 3y_3 + 8y_4 - 3y_5 + 8y_6 - 3y_7 + 8y_8 - 3y_9 + 8y_10 - 3y_11 + 8y_12 - 3y_13 + 8y_14 + s_9 = \\
-3y_1 + 3y_2 - 3y_3 + 3y_4 - 3y_5 + 3y_6 - 3y_7 + 3y_8 - 3y_9 + 3y_10 - 3y_11 + 3y_12 - 3y_13 + 3y_14 + s_10 = \\
-9y_1 + 9y_2 - 9y_3 + 9y_4 - 9y_5 + 9y_6 - 9y_7 + 9y_8 - 9y_9 + 9y_10 - 9y_11 + 9y_12 - 9y_13 + 9y_14 + s_11 = \\
-10y_1 - 10y_2 - 10y_3 - 10y_4 - 10y_5 - 10y_6 - 10y_7 - 10y_8 - 10y_9 - 10y_10 - 10y_11 - 10y_12 - 10y_13 - 10y_14 \\
-8y_1 + 7y_2 - 8y_3 + 7y_4 - 8y_5 + 7y_6 - 8y_7 + 7y_8 - 8y_9 + 7y_10 - 8y_11 + 7y_12 - 8y_13 + 7y_14 + s_13 = \\
-4y_1 - 4y_2 - 4y_3 - 4y_4 - 4y_5 - 4y_6 - 4y_7 - 4y_8 - 4y_9 - 4y_10 - 4y_11 - 4y_12 - 4y_13 - 4y_14 + s_14 = \\
-3y_1 + 10y_2 - 3y_3 + 10y_4 - 3y_5 + 10y_6 - 3y_7 + 10y_8 - 3y_9 + 10y_10 - 3y_11 + 10y_12 - 3y_13 + 10y_14 + s_1 \\
6y_1 + 7y_2 + 6y_3 + 7y_4 + 6y_5 + 7y_6 + 6y_7 + 7y_8 + 6y_9 + 7y_10 + 6y_11 + 7y_12 + 6y_13 + 7y_14 + s_16 = \\
s_17 = 1 \\
-2y_1 - 7y_2 - 2y_3 - 7y_4 - 2y_5 - 7y_6 - 2y_7 - 7y_8 - 2y_9 - 7y_10 - 2y_11 - 7y_12 - 2y_13 - 7y_14 + s_18 = \\
-6y_1 - 8y_2 - 6y_3 - 8y_4 - 6y_5 - 8y_6 - 6y_7 - 8y_8 - 6y_9 - 8y_10 - 6y_11 - 8y_12 - 6y_13 - 8y_14 + s_19 = \\
-4y_1 + 2y_2 - 4y_3 + 2y_4 - 4y_5 + 2y_6 - 4y_7 + 2y_8 - 4y_9 + 2y_10 - 4y_11 + 2y_12 - 4y_13 + 2y_14 + s_20 = \\
-7y_1 - 2y_2 - 7y_3 - 2y_4 - 7y_5 - 2y_6 - 7y_7 - 2y_8 - 7y_9 - 2y_10 - 7y_11 - 2y_12 - 7y_13 - 2y_14 + s_21 = \\
y_1 - 9y_2 + y_3 - 9y_4 + y_5 - 9y_6 + y_7 - 9y_8 + y_9 - 9y_10 + y_11 - 9y_12 + y_13 - 9y_14 + s_22 = 1 \\
y_1 + 4y_2 + y_3 + 4y_4 + y_5 + 4y_6 + y_7 + 4y_8 + y_9 + 4y_10 + y_11 + 4y_12 + y_13 + 4y_14 + s_23 = 1 \\
9y_1 + 7y_2 + 9y_3 + 7y_4 + 9y_5 + 7y_6 + 9y_7 + 7y_8 + 9y_9 + 7y_10 + 9y_11 + 7y_12 + 9y_13 + 7y_14 + s_24 = \\
-y_1 - 8y_2 - y_3 - 8y_4 - y_5 - 8y_6 - y_7 - 8y_8 - y_9 - 8y_10 - y_11 - 8y_12 - y_13 - 8y_14 + s_25 = 1 \\
10y_1 - 9y_2 + 10y_3 - 9y_4 + 10y_5 - 9y_6 + 10y_7 - 9y_8 + 10y_9 - 9y_10 + 10y_11 - 9y_12 + 10y_13 - 9y_14 + s_26 \\
9y_1 - 3y_2 + 9y_3 - 3y_4 + 9y_5 - 3y_6 + 9y_7 - 3y_8 + 9y_9 - 3y_10 + 9y_11 - 3y_12 + 9y_13 - 3y_14 + s_27 = \\
y_1 - y_2 + y_3 - y_4 + y_5 - y_6 + y_7 - y_8 + y_9 - y_10 + y_11 - y_12 + y_13 - y_14 + s_28 = 1 \\
8y_1 - 8y_2 + 8y_3 - 8y_4 + 8y_5 - 8y_6 + 8y_7 - 8y_8 + 8y_9 - 8y_10 + 8y_11 - 8y_12 + 8y_13 - 8y_14 + s_29 = \\
-4y_1 - 3y_2 - 4y_3 - 3y_4 - 4y_5 - 3y_6 - 4y_7 - 3y_8 - 4y_9 - 3y_10 - 4y_11 - 3y_12 - 4y_13 - 3y_14 + s_30 = \\
-7y_1 - 5y_2 - 7y_3 - 5y_4 - 7y_5 - 5y_6 - 7y_7 - 5y_8 - 7y_9 - 5y_10 - 7y_11 - 5y_12 - 7y_13 - 5y_14 + s_31 = \\
2y_1 + y_2 + 2y_3 + y_4 + 2y_5 + y_6 + 2y_7 + y_8 + 2y_9 + y_10 + 2y_11 + y_12 + 2y_13 + y_14 + s_32 = 1 \\
-2y_1 + y_2 - 2y_3 + y_4 - 2y_5 + y_6 - 2y_7 + y_8 - 2y_9 + y_10 - 2y_11 + y_12 - 2y_13 + y_14 + s_33 = 1 \\
4y_1 - 4y_2 + 4y_3 - 4y_4 + 4y_5 - 4y_6 + 4y_7 - 4y_8 + 4y_9 - 4y_10 + 4y_11 - 4y_12 + 4y_13 - 4y_14 + s_34 = \\
10y_1 - 2y_2 + 10y_3 - 2y_4 + 10y_5 - 2y_6 + 10y_7 - 2y_8 + 10y_9 - 2y_10 + 10y_11 - 2y_12 + 10y_13 - 2y_14 + s_35 \\
y_1 + 10y_2 + y_3 + 10y_4 + y_5 + 10y_6 + y_7 + 10y_8 + y_9 + 10y_10 + y_11 + 10y_12 + y_13 + 10y_14 + s_36 =
\end{cases}$$

Create the initial tableau of the new linear system.

$$\left[\begin{array}{cccccccccccccccccc}
y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_1 0 & y_1 1 & y_1 2 & y_1 3 & y_1 4 & s_1 & s_2 & s_3 & s_4 \\
\hline
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\
2 & -5 & 2 & -5 & 2 & -5 & 2 & -5 & 2 & -5 & 2 & -5 & 2 & -5 & 0 & 1 & 0 & 0 \\
-1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 & 0 & 0 & 1 & 0 \\
-1 & 3 & -1 & 3 & -1 & 3 & -1 & 3 & -1 & 3 & -1 & 3 & -1 & 3 & 0 & 0 & 0 & 1 \\
10 & -6 & 10 & -6 & 10 & -6 & 10 & -6 & 10 & -6 & 10 & -6 & 10 & -6 & 0 & 0 & 0 & 0 \\
-7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & 0 & 0 & 0 & 0 \\
-10 & 7 & -10 & 7 & -10 & 7 & -10 & 7 & -10 & 7 & -10 & 7 & -10 & 7 & 0 & 0 & 0 & 0 \\
-2 & -8 & -2 & -8 & -2 & -8 & -2 & -8 & -2 & -8 & -2 & -8 & -2 & -8 & 0 & 0 & 0 & 0 \\
-3 & 8 & -3 & 8 & -3 & 8 & -3 & 8 & -3 & 8 & -3 & 8 & -3 & 8 & 0 & 0 & 0 & 0 \\
-3 & 3 & -3 & 3 & -3 & 3 & -3 & 3 & -3 & 3 & -3 & 3 & -3 & 3 & 0 & 0 & 0 & 0 \\
-9 & 9 & -9 & 9 & -9 & 9 & -9 & 9 & -9 & 9 & -9 & 9 & -9 & 9 & 0 & 0 & 0 & 0 \\
-10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & 0 & 0 & 0 & 0 \\
-8 & 7 & -8 & 7 & -8 & 7 & -8 & 7 & -8 & 7 & -8 & 7 & -8 & 7 & 0 & 0 & 0 & 0 \\
-4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & 0 & 0 & 0 & 0 \\
-3 & 10 & -3 & 10 & -3 & 10 & -3 & 10 & -3 & 10 & -3 & 10 & -3 & 10 & 0 & 0 & 0 & 0 \\
6 & 7 & 6 & 7 & 6 & 7 & 6 & 7 & 6 & 7 & 6 & 7 & 6 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & 0 & 0 & 0 & 0 \\
-6 & -8 & -6 & -8 & -6 & -8 & -6 & -8 & -6 & -8 & -6 & -8 & -6 & -8 & 0 & 0 & 0 & 0 \\
-4 & 2 & -4 & 2 & -4 & 2 & -4 & 2 & -4 & 2 & -4 & 2 & -4 & 2 & 0 & 0 & 0 & 0 \\
-7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & -7 & -2 & 0 & 0 & 0 & 0 \\
1 & -9 & 1 & -9 & 1 & -9 & 1 & -9 & 1 & -9 & 1 & -9 & 1 & -9 & 0 & 0 & 0 & 0 \\
1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & 1 & 4 & 0 & 0 & 0 & 0 \\
9 & 7 & 9 & 7 & 9 & 7 & 9 & 7 & 9 & 7 & 9 & 7 & 9 & 7 & 0 & 0 & 0 & 0 \\
-1 & -8 & -1 & -8 & -1 & -8 & -1 & -8 & -1 & -8 & -1 & -8 & -1 & -8 & 0 & 0 & 0 & 0 \\
10 & -9 & 10 & -9 & 10 & -9 & 10 & -9 & 10 & -9 & 10 & -9 & 10 & -9 & 0 & 0 & 0 & 0 \\
9 & -3 & 9 & -3 & 9 & -3 & 9 & -3 & 9 & -3 & 9 & -3 & 9 & -3 & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
8 & -8 & 8 & -8 & 8 & -8 & 8 & -8 & 8 & -8 & 8 & -8 & 8 & -8 & 0 & 0 & 0 & 0 \\
-4 & -3 & -4 & -3 & -4 & -3 & -4 & -3 & -4 & -3 & -4 & -3 & -4 & -3 & 0 & 0 & 0 & 0 \\
-7 & -5 & -7 & -5 & -7 & -5 & -7 & -5 & -7 & -5 & -7 & -5 & -7 & -5 & 0 & 0 & 0 & 0 \\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\
-2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\
4 & -4 & 4 & -4 & 4 & -4 & 4 & -4 & 4 & -4 & 4 & -4 & 4 & -4 & 0 & 0 & 0 & 0 \\
10 & -2 & 10 & -2 & 10 & -2 & 10 & -2 & 10 & -2 & 10 & -2 & 10 & -2 & 0 & 0 & 0 & 0 \\
1 & 10 & 1 & 10 & 1 & 10 & 1 & 10 & 1 & 10 & 1 & 10 & 1 & 10 & 0 & 0 & 0 & 0 \\
\hline
-1 & -1 & -3 & 3 & -4 & -8 & 2 & 10 & 5 & -3 & 7 & -2 & 8 & -7 & 0 & 0 & 0 & 0
\end{array}\right.$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_6 and the departing variable is $s_1 5$.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{ccccccccccccccc}
y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_10 & y_11 & y_12 & y_13 & y_14 & s_1 \\
\hline
7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 1 \\
1/2 & 0 & 1/2 & 0 & 1/2 & 0 & 1/2 & 0 & 1/2 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\
-2/5 & 0 & -2/5 & 0 & -2/5 & 0 & -2/5 & 0 & -2/5 & 0 & -2/5 & 0 & -2/5 & 0 & 0 \\
-1/10 & 0 & -1/10 & 0 & -1/10 & 0 & -1/10 & 0 & -1/10 & 0 & -1/10 & 0 & -1/10 & 0 & 0 \\
41/5 & 0 & 41/5 & 0 & 41/5 & 0 & 41/5 & 0 & 41/5 & 0 & 41/5 & 0 & 41/5 & 0 & 0 \\
-91/10 & 0 & -91/10 & 0 & -91/10 & 0 & -91/10 & 0 & -91/10 & 0 & -91/10 & 0 & -91/10 & 0 & 0 \\
-79/10 & 0 & -79/10 & 0 & -79/10 & 0 & -79/10 & 0 & -79/10 & 0 & -79/10 & 0 & -79/10 & 0 & 0 \\
-22/5 & 0 & -22/5 & 0 & -22/5 & 0 & -22/5 & 0 & -22/5 & 0 & -22/5 & 0 & -22/5 & 0 & 0 \\
-3/5 & 0 & -3/5 & 0 & -3/5 & 0 & -3/5 & 0 & -3/5 & 0 & -3/5 & 0 & -3/5 & 0 & 0 \\
-21/10 & 0 & -21/10 & 0 & -21/10 & 0 & -21/10 & 0 & -21/10 & 0 & -21/10 & 0 & -21/10 & 0 & 0 \\
-63/10 & 0 & -63/10 & 0 & -63/10 & 0 & -63/10 & 0 & -63/10 & 0 & -63/10 & 0 & -63/10 & 0 & 0 \\
-13 & 0 & -13 & 0 & -13 & 0 & -13 & 0 & -13 & 0 & -13 & 0 & -13 & 0 & 0 \\
-59/10 & 0 & -59/10 & 0 & -59/10 & 0 & -59/10 & 0 & -59/10 & 0 & -59/10 & 0 & -59/10 & 0 & 0 \\
-26/5 & 0 & -26/5 & 0 & -26/5 & 0 & -26/5 & 0 & -26/5 & 0 & -26/5 & 0 & -26/5 & 0 & 0 \\
-3/10 & 1 & -3/10 & 1 & -3/10 & 1 & -3/10 & 1 & -3/10 & 1 & -3/10 & 1 & -3/10 & 1 & 0 \\
81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-41/10 & 0 & -41/10 & 0 & -41/10 & 0 & -41/10 & 0 & -41/10 & 0 & -41/10 & 0 & -41/10 & 0 & 0 \\
-42/5 & 0 & -42/5 & 0 & -42/5 & 0 & -42/5 & 0 & -42/5 & 0 & -42/5 & 0 & -42/5 & 0 & 0 \\
-17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & 0 \\
-38/5 & 0 & -38/5 & 0 & -38/5 & 0 & -38/5 & 0 & -38/5 & 0 & -38/5 & 0 & -38/5 & 0 & 0 \\
-17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & 0 \\
11/5 & 0 & 11/5 & 0 & 11/5 & 0 & 11/5 & 0 & 11/5 & 0 & 11/5 & 0 & 11/5 & 0 & 0 \\
111/10 & 0 & 111/10 & 0 & 111/10 & 0 & 111/10 & 0 & 111/10 & 0 & 111/10 & 0 & 111/10 & 0 & 0 \\
-17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & -17/5 & 0 & 0 \\
73/10 & 0 & 73/10 & 0 & 73/10 & 0 & 73/10 & 0 & 73/10 & 0 & 73/10 & 0 & 73/10 & 0 & 0 \\
81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 81/10 & 0 & 0 \\
7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 7/10 & 0 & 0 \\
28/5 & 0 & 28/5 & 0 & 28/5 & 0 & 28/5 & 0 & 28/5 & 0 & 28/5 & 0 & 28/5 & 0 & 0 \\
-49/10 & 0 & -49/10 & 0 & -49/10 & 0 & -49/10 & 0 & -49/10 & 0 & -49/10 & 0 & -49/10 & 0 & 0 \\
-17/2 & 0 & -17/2 & 0 & -17/2 & 0 & -17/2 & 0 & -17/2 & 0 & -17/2 & 0 & -17/2 & 0 & 0 \\
23/10 & 0 & 23/10 & 0 & 23/10 & 0 & 23/10 & 0 & 23/10 & 0 & 23/10 & 0 & 23/10 & 0 & 0 \\
-17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & -17/10 & 0 & 0 \\
14/5 & 0 & 14/5 & 0 & 14/5 & 0 & 14/5 & 0 & 14/5 & 0 & 14/5 & 0 & 14/5 & 0 & 0 \\
47/5 & 0 & 47/5 & 0 & 47/5 & 0 & 47/5 & 0 & 47/5 & 0 & 47/5 & 0 & 47/5 & 0 & 0 \\
4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 0 \\
\hline
-17/5 & 7 & -27/5 & 11 & -32/5 & 0 & -2/5 & 18 & 13/5 & 5 & 23/5 & 6 & 28/5 & 1 & 0
\end{array}\right.$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_5 and the departing variable is s_36.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{ccccccccccccccccccccccc}
y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_1 0 & y_1 1 & y_1 2 & y_1 3 & y_1 4 & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
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\end{array}\right.$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is $s_1 5$ and the departing variable is $s_2 4$.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{ccccccccccccccccccccccc}
y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_1 0 & y_1 1 & y_1 2 & y_1 3 & y_1 4 & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\
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\end{array}\right.$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = \frac{88}{83}, s_1 0 = \frac{68}{83}, s_1 1 = \frac{38}{83}, s_1 2 = \frac{193}{83}, s_1 3 = \frac{51}{83}, s_1 4 = \frac{127}{83}, s_1 5 = \frac{12}{83}, s_1 6 = \frac{9}{83}, s_1 7 = 1, s_1 8 = \frac{145}{83}, s_1 9 =$$