

# Simplex Solver

October 20, 2023

## Problem

Given the following linear system and objective function, find the optimal solution.

$$\min(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19})$$

$$\begin{cases} y_1 + 2y_2 - y_3 - y_4 \geq 1 \\ -y_1 - 5y_2 + 2y_3 + 3y_4 \geq 1 \end{cases}$$

## Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} y_1 - y_2 + s_1 = 1 \\ 2y_1 - 5y_2 + s_2 = 1 \\ -y_1 + 2y_2 + s_3 = 1 \\ -y_1 + 3y_2 + s_4 = 1 \end{cases}$$

Create the initial tableau of the new linear system.

$$\left[ \begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ 1 & -1 & 1 & 0 & 0 & 0 & 1 \\ 2 & -5 & 0 & 1 & 0 & 0 & 1 \\ -1 & 2 & 0 & 0 & 1 & 0 & 1 \\ -1 & 3 & 0 & 0 & 0 & 1 & 1 \\ \hline -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is  $y_1$  and the departing variable is  $s_2$ .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[ \begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ 0 & 3/2 & 1 & -1/2 & 0 & 0 & 1/2 \\ 1 & -5/2 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & -1/2 & 0 & 1/2 & 1 & 0 & 3/2 \\ 0 & 1/2 & 0 & 1/2 & 0 & 1 & 3/2 \\ \hline 0 & -7/2 & 0 & 1/2 & 0 & 0 & 1/2 \end{array} \right] \begin{matrix} s_1 \\ y_1 \\ s_3 \\ s_4 \end{matrix}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is  $y_2$  and the departing variable is  $s_1$ .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[ \begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 & 1/3 \\ 1 & 0 & 5/3 & -1/3 & 0 & 0 & 4/3 \\ 0 & 0 & 1/3 & 1/3 & 1 & 0 & 5/3 \\ 0 & 0 & -1/3 & 2/3 & 0 & 1 & 4/3 \\ 0 & 0 & 7/3 & -2/3 & 0 & 0 & 5/3 \end{array} \right] \begin{array}{l} y_2 \\ y_1 \\ s_3 \\ s_4 \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is  $s_2$  and the departing variable is  $s_4$ .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[ \begin{array}{cccccc|c} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & b \\ 0 & 1 & 1/2 & 0 & 0 & 1/2 & 1 \\ 1 & 0 & 3/2 & 0 & 0 & 1/2 & 2 \\ 0 & 0 & 1/2 & 0 & 1 & -1/2 & 1 \\ 0 & 0 & -1/2 & 1 & 0 & 3/2 & 2 \\ 0 & 0 & 2 & 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} y_2 \\ y_1 \\ s_3 \\ s_2 \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 0, s_2 = 2, s_3 = 1, s_4 = 0, x_1 = 2, x_2 = 0, x_3 = 0, x_4 = 1, y_1 = 2, y_2 = 1, z = 3$$