a. Client's Portfolio

| | |
|---|---|
| Risky | $r_p = 20\%$<br>$\sigma_p = 30\%$ |
| T-bill | $r_f = 5\%$ |

Expected return

$r_c = 20\% * 70\% + 5\% * 30\%$

$r_c = 15.5\%$

Standard Deviation

$\sigma_c = 30\% * 70\% + 0\% * 30\%$

$\sigma_c = 21\%$

B. Portfolio Proportions

| | Calculation | Position Proportion |
|---|---|---|
| Stock A | $30\% * 70\%$ | 21% |
| Stock B | $40\% * 70\%$ | 28% |
| Stock C | $30\% * 70\%$ | 21% |
| Risk-free | | 30% |

c.

$Sharpe = \frac{Risk\ premium}{SD\ of\ excess\ return} = \frac{E(r)-r_f}{\sigma}$

Sharpe ratio of Risky Portfolio

$\frac{20\%-5\%}{30\%} = \frac{15\%}{30\%} = 50\%$

Sharpe ratio of Client's Portfolio

$\frac{20\%-5\%}{21\%} = \frac{15\%}{21\%} = 71.4\%$

d.

Proportion $y$

$15\% = 20\% * y + 5\% * (1 - y)$

$15\% = 20\% * y + 5\% - 5\% * y)$

$15\% = 15\% * y + 5\%$

$10\% = 15\% * y$

$y = 67\%$

Portfolio Proportions for an expected return of 15%

| | Calculation | Position Proportion |
|---|---|---|
| Stock A | 30% * 67% | 20.1% |
| Stock B | 40% * 67% | 26.8% |
| Stock C | 30% * 67% | 20.1% |
| Risk-free | | 33% |

e.

Investment proportion to limit standard deviation to 10%

$\sigma_p = y * \sigma_c$

$0.1 = 0.3y$, $y = 0.33 = 33\%$

Expected rate of return

$r_p = 33\% * 20\% + 67\% * 5\% = 9.95\%$

f.

Optimized y

$y^* = \frac{E(r_p) - r_f}{A\sigma_p^2} = \frac{0.2 - 0.05}{3*(0.3)^2} = 56\%$

Optimized Portfolio

Expected Return

$E[r_c] = 0.56 * 20\% + 0.44 * 5\% = 13.4\%$

Standard Deviation

$\sigma_c = 0.56 * 30\% = 16.8\%$

Utility Score

$U = E[r_c] - \frac{1}{2}A\sigma_c^2 = 13.4\% - \frac{1}{2}(3)(16.8\%)^2 = 0.09166$

Indifference curve

(see below – Samir will add this)