L- 4: Descriptive Statistics

Today…..

- Recall the past for a while_ Conditional probability and Baye’s theorem & some examples
- Random variables
- Probability distribution
- Examples

Conditional Probability and Baye's theorem –

'Recap'

conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

or

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

ie

$$P(A \cap B) = P(A|B)\,P(B)$$

$$= P(B|A)\,P(A)$$

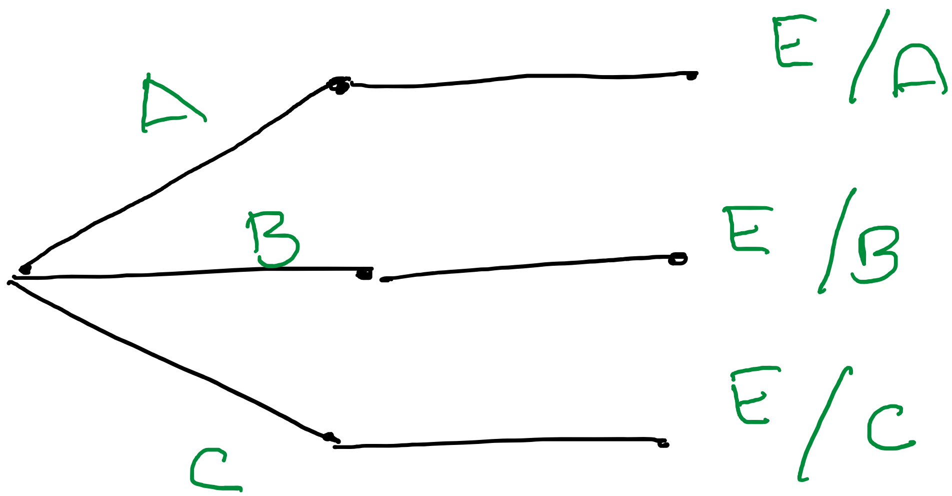
$$P(A/E) = \frac{P(A \cap E)}{P(E)}$$

$$= \frac{P(E/A)\,P(A)}{\sum_i P(E|A_i)\,P(A_i)}$$

$\downarrow$

'Baye's theorem'

Baye's theorem



Total prob

$$= \sum_{A,B,C} P(E|A)\,P(A)$$

ie
$$P(A/E) = \frac{P(E|A)\;P(A)}{\sum P(E|A)\;P(A)} \checkmark$$

or
$$P(B|E) = \frac{P(E/B)\;P(B)}{\sum P(E|A)\;P(A)} \checkmark$$

Expecting some examples
or case studies
or applications from
Participants

Example :-

In a factory, three machines A, B and C manufactures 40%, 35% & 25% of the total output. From the past record, it is observed that of their out-put 2, 4, 5 percents are defective. A product is drawn at random and is found to be defective. what is the prob. that it was manufactured by A | B | C ? what is the observation?

Highest

A 0.40 — 0.02 — E/A

B 0.35 — 0.04 — E/B

C 0.25 — 0.05 — E/C

$$E/A = \frac{0.4 \times 0.02}{0.0345} = 0.232$$

$$E/B = \frac{0.35 \times 0.04}{''} = 0.406$$

$$E/C = \frac{0.25 \times 0.05}{''} = 0.362$$

Total probability

$$= P(E/A)\,P(E) + P(E|B)\,P(B) + P(E/c)\,P(C)$$

$$= (0.02)(0.4) + (0.04)(0.35) + (0.05)(0.25)$$

$$= 0.0345$$

Conclusion

?

Random Variable

Random Variables

We now introduce a new term

Instead of saying that the possible outcomes are 1,2,3,4,5 or 6, we say that ***random variable*** X can take values {1,2,3,4,5,6}. **A random variable is an expression whose value is the outcome of a particular experiment.**

The random variables can be either *discrete* or *continuous*.

It's a convention to use the upper case letters (X, Y) for the names of the random variables and the lower case letters (x,y) for their possible particular values.

Random Variables

Definition

A discrete random variable is a random variable with a finite (or countably infinite) range.
A continuous random variable is a random variable with an interval (either finite or infinite) of real numbers for its range.

Random Variables

Examples of Random Variables

Examples of **continuous** random variables:

electrical current, length, pressure, temperature, time, voltage, weight

Examples of **discrete** random variables:

number of scratches on a surface, proportion of defective parts among 1000 tested, number of transmitted bits received in error

$S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$

X = no of heads

3 2 2 2 1 0 1 1

$= 0, 1, 2, 3$

X	0	1	2	3
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$P(x) = ?$

Random variable

Discrete

$P(x)$, $x = 1, 2, 3 \ldots$

(i) $0 \le P(x_i) \le 1$

(ii) $\sum_i P(x_i) = 1$

Prob. distribution function

continuous

$f(x)$

, $x \in (\ )$

(i) $0 \le f(x) \le 1$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

density f

Validation

i) $P(x) = \dfrac{x-3}{2}$

$x = 1, 2, 3, 4$

another

$$P(x) = \frac{x^2}{5}, \quad x = 0, 1, 2, 3, 4$$

innovate achieve lead

Example:-

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) K value: ?

(ii) $P(x < 6)$

(iii) $P(x \geq 6)$

(iiii) $P(3 < x \leq 6)$

Example:-

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) K value:? $\quad \sum P(x) = 1$

$\Rightarrow 0 + K + 2K + \cdots = 1$

$10K^2 + 9K - 1 = 0$

$K = -1, 1/10$

(ii) $P(x < 6)$

(iii) $P(x \geq 6)$

(iiii) $P(3 < x \leq 6)$

innovate achieve lead

Example:-

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) K value : ? $\longrightarrow K = 1/10$

(ii) $P(x < 6) \longrightarrow P(0) + \cdots P(5)$
$= K^2 + 8K = 81/100$

(iii) $P(x \geq 6) \longrightarrow P(6) + P(7) = \frac{19}{100}$

(iv) $P(3 < x \leq 6) \longrightarrow P(4) + \cdots$
$= 33/100$

Example:

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

$K = ?$

$P(1 < x < 2)$

Example:

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

$$\int f(x)\,dx = 1 \Rightarrow \int_0^3 Kx^2\,dx = 1$$

$$\Rightarrow K\left[\frac{x^3}{3}\right]_{x=0}^{3} = 1$$

$$\Rightarrow \frac{K}{3}\left[27-0\right] = 1$$

ie $K = 1/9$

Example:

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

$K = ? \longrightarrow 1/9$

$P(1 < x < 2) \longrightarrow = \int_1^2 Kx^2\,dx$

$= 1/9 \left[\frac{x^3}{3}\right]_{x=1}^{2}$

$= 1/27\,[8 - 1] = 7/27$

Expectation of a random Variable

$$E(x) = \sum_i x_i \, p(x_i)$$

$$= \int x \, f(x) \, dx$$

μ = Mean of a random variable

Variance of a r.v

$$Var(x) = \sigma^2 = E(x-\mu)^2$$

$$= E(x^2 + \mu^2 - 2\mu x)$$

$$= E(x^2) + \mu^2 - 2\mu E(x)$$

$$= E(x^2) + \mu^2 - 2\mu \cdot \mu$$

$$= E(x^2) - \mu^2$$

$$= E(x^2) - [E(x)]^2$$

ie Mean $\mu = E(x) = \sum x\, p(x)$

$= \int x\, f(x)\, dx$

Variance $\sigma^2 = E\left((x-\mu)^2\right)$

$= E(x^2) - \left[E(x)\right]^2$

$E(x^2) = \sum x^2\, p(x) \longrightarrow$ discrete

$\int x^2 f(x)\, dx \longrightarrow$ continu.

Probability Distributions

Discrete

Binomial

Poisson

Continuous

Normal
or
Gaussi

Discrete

Binomial

$$P(x) = {}^{n}C_x \, p^{x} \, q^{n-x}$$

no. of trials

no of succ

prob of succ

$q = 1-p$

poisson

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$x = 0, 1, 2 \ldots$

Thanks