Return of the ODEs: Higher-order Methods

Samir Johnson

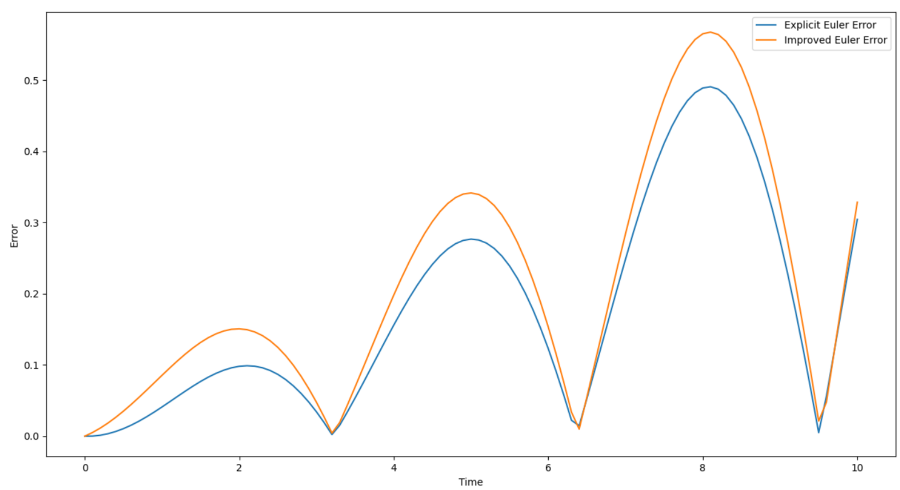
April 2020

1 Global Error of Explicit and Midpoint methods

Below we attach several plots of global error of both the Explicit Euler method and the Midpoint method for the simple harmonic oscillator ODE

$$\frac{dx}{dt} = v(t); \frac{dv}{dt} = -kx(t)$$

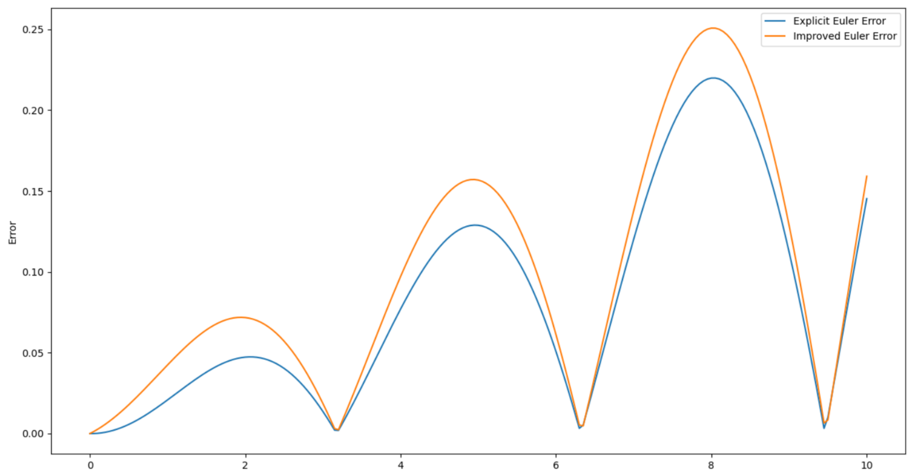
For simplicity, we set $k = 1$. With initial conditions $x(0) = 1, v(0) = 1$, this gives an analytic solution of $x_a(t) = sin(t)$. Thus, we can plot $\epsilon_{global}(t_n) = |x_a(t_n) - x_n|$, where x_n is the nth position generated by either explicit or midpoint methods.
We set $h = 0.1$ and $N = 10$ and plot the results.



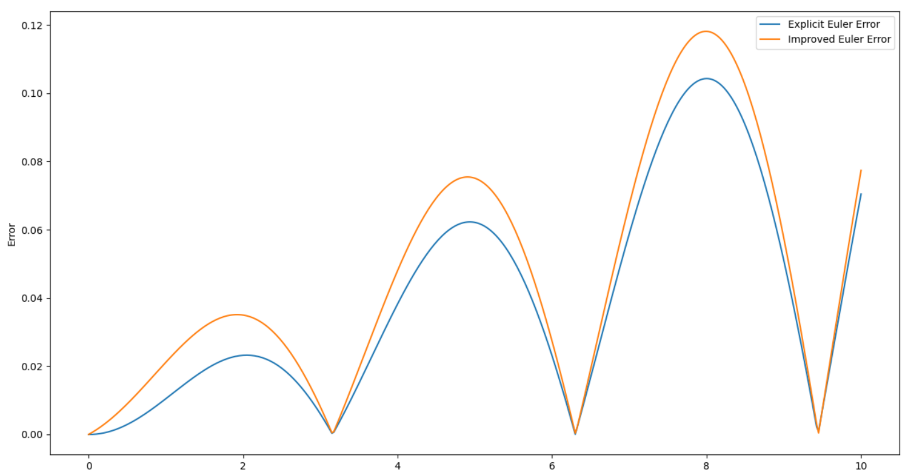
2 Convergence Plots of Explicit and Midpoint Methods

Below we attach several of the same plots as in the previous section, with each successive plot scaling the timestep h by a factor of one half. That is, the first plot uses a timestep of $h_2 = \frac{h}{2} = 0.05$, the second plot uses a timestep of $h_4 = \frac{h}{4} = 0.025$, and so on.

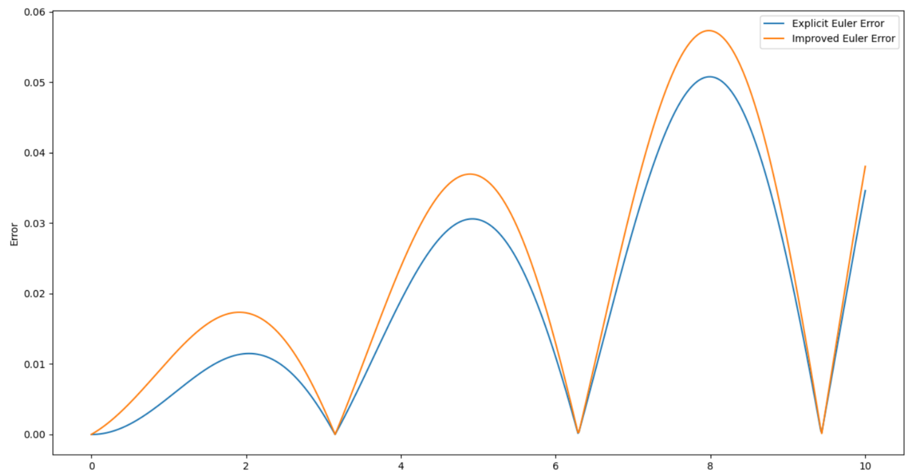
$\frac{h}{2}$



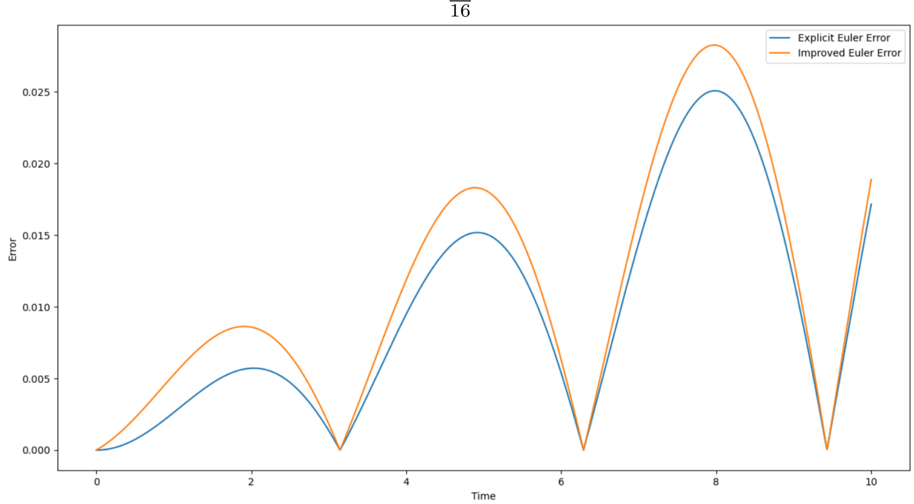
$\frac{h}{4}$



$\frac{h}{8}$



$$\frac{h}{16}$$



Note how each time the timestep scales by a factor of one half the global error also scales by a factor of one half.

3 Orbital Plot Computed with 4th order Runge-Kutta

Below is the plot of the computed orbit of an object in a gravitational potential. The initial conditions are $x(0) = 1, x'(0) = 0, y(0) = 0, y'(0) = 1$ with $R = 1$. The plot was computed with $N = 10$ and a stepsize $h = 0.0001$.