Ph22 Lab3

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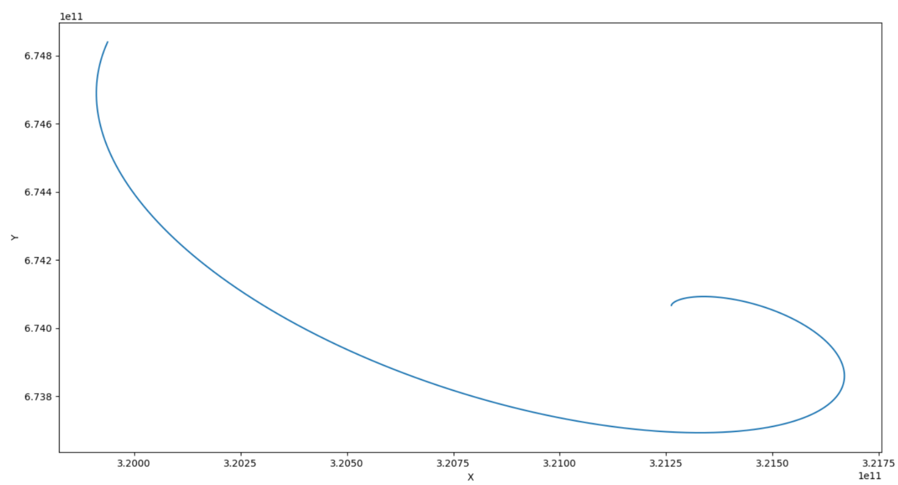
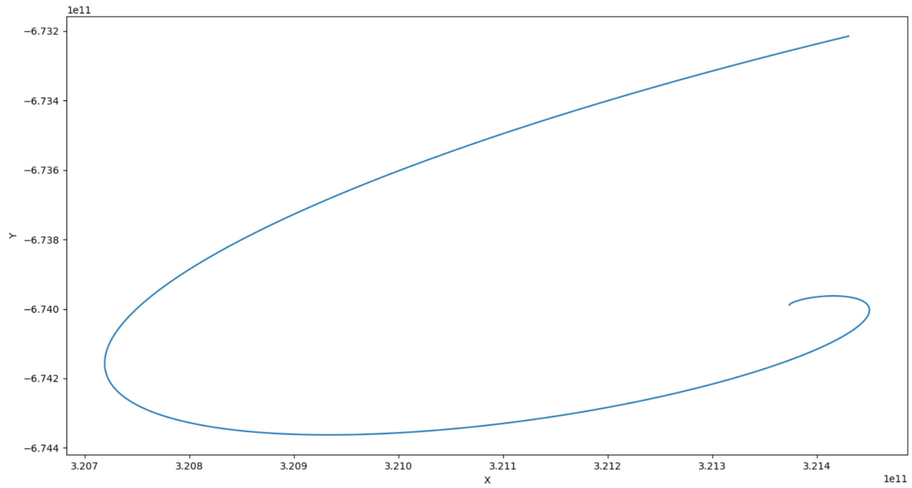
1 Jupiter-Sun Asteroid Orbits and Lagrangian Points

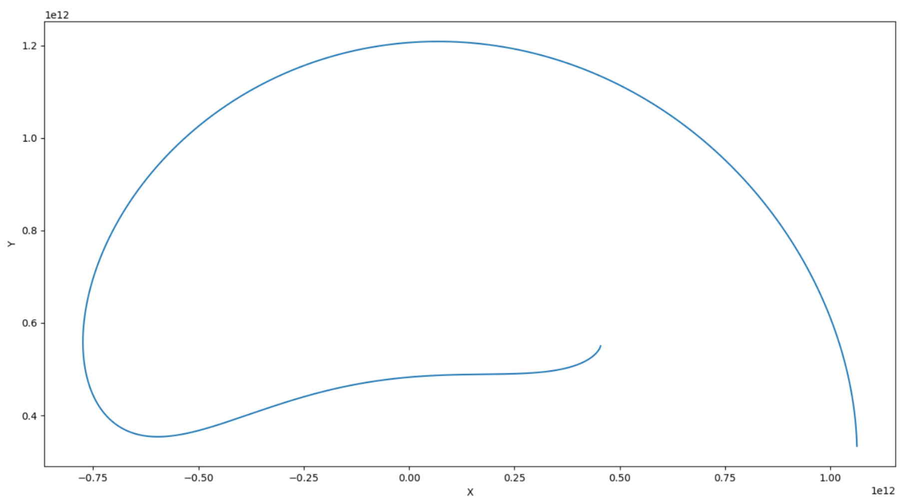
Below are plots for the evaluated orbits of an asteroid in the gravitational potential created by the Jupiter-Sun system around Lagrangian points L4 and L5. The plots were evaluated with $N = 2T$, $h = T/1000$, where T is the period of orbit given in the notes.



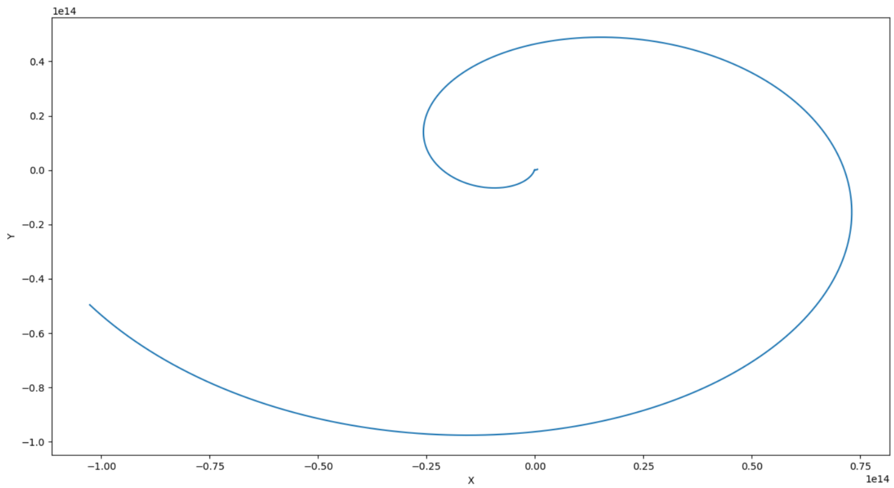
$\alpha = \frac{\pi}{3} + 0.0001$



$\alpha = -\frac{\pi}{3} + 0.0001$



$\alpha = \frac{\pi}{4}$



$\alpha = \frac{\pi}{8}$

2 Generic Three Body Problem

2.1 Analytic Confirmation of Equilateral Triangle Solution

We define our coordinate system with origin such that

$$\vec{r_1} + \vec{r_2} + \vec{r_3} = 0$$

We now assume that the distances between each respective object of mass M is given by d. Thus, substituting this into the equation of motion for the first object gives

$$\vec{a_1} = -\frac{GM}{d^3}\left(\vec{r_1} - \vec{r_2}\right) - -\frac{GM}{d^3}\left(\vec{r_1} - \vec{r_3}\right)$$

We can now simplify this using our choice of origin to decouple $\vec{r_1}$'s equation of motion from $\vec{r_2}$ and $\vec{r_3}$:

$$\vec{a_1} = \frac{d^2\vec{r_1}}{dt^2} = -\frac{GM}{d^3}\vec{r_1}$$

which corresponds to a circular equation of motion

$$\vec{r_1} = \left(\cos\sqrt{\frac{GM}{d^3}}t, \sin\sqrt{\frac{GM}{d^3}}t, 0\right)^T$$

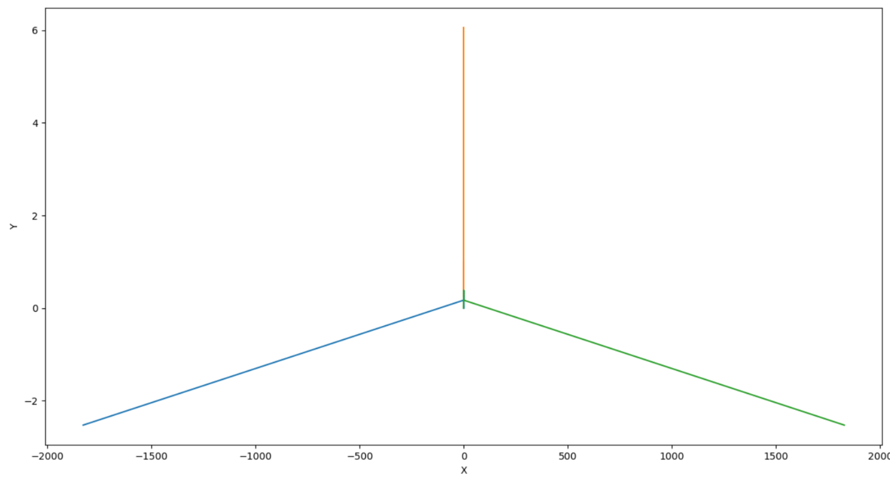
if we assume we are working in the x-y plane, WLOG. By symmetry, this holds for $\vec{r_2}$ and $\vec{r_3}$. Thus, this gives velocity

$$|\vec{v}| = |\sqrt{\frac{GM}{d^3}}\left(-\sin\sqrt{\frac{GM}{d^3}}t, \cos\sqrt{\frac{GM}{d^3}}t, 0\right)^T| = \sqrt{\frac{GM}{d^3}}$$

2.2 Plot

Below is a plot of the simple three body problem with $M1 = M2 = M3 = 1$ and $G = 1$. The initial conditions are

$$x_1 = 1, v_{x_1} = 0, y_1 = 0, v_{y_1} = 0, x_2 = 0, v_{x_2} = 0, y_2 = 1, v_{y_2} = 0, x_3 = -1, v_{x_3} = 0, y_3 = 0, v_{y_3} = 0$$



3 Figure 8 Plot

Below is a plot of the figure 8 orbit created from the initial conditions specified in the notes.