

Basic statistics

Mean

Definition of Mean in Statistics

Mean is the average of the given numbers and is calculated by dividing the sum of given numbers by the total number of numbers.

Mean = (Sum of all the observations/Total number of observations)

Example:

What is the mean of 2, 4, 6, 8 and 10?

Solution

First, add all the numbers.

$$2 + 4 + 6 + 8 + 10 = 30$$

Now divide by 5 (total number of observations).

$$\text{Mean} = 30/5 = 6$$

In the case of a discrete probability distribution of a random variable X , the mean is equal to the sum over every possible value weighted by the probability of that value; that is, it is computed by taking the product of each possible value x of X and its probability $P(x)$ and then adding all these products together.

Mean Symbol (X Bar)

The symbol of mean is usually given by the symbol ' \bar{x} '. The bar above the letter x, represents the mean of x number of values.

$$\bar{X} = (\text{Sum of values} \div \text{Number of values})$$

$$\bar{X} = (x_1 + x_2 + x_3 + \dots + x_n)/n$$

Example:

In a class there are 20 students and they have secured a percentage of 88, 82, 88, 85, 84, 80, 81, 82, 83, 85, 84, 74, 75, 76, 89, 90, 89, 80, 82, and 83.

Find the mean percentage obtained by the class.

Solution:

Mean = Total of percentage obtained by 20 students in class/Total number of students

$$= [88 + 82 + 88 + 85 + 84 + 80 + 81 + 82 + 83 + 85 + 84 + 74 + 75 + 76 + 89 + 90 + 89 + 80 + 82 + 83]/20$$

$$= 1660/20$$

$$= 83$$

Hence, the mean percentage of each student in the class is 83%.

Mean for Grouped Data

For grouped data, we can find the mean using either of the following formulas.

Direct method:

$$\text{Mean}, \bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

Assumed mean method:

$$\text{Mean}, (\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i}$$

Step-deviation method:

$$\text{Mean}, (\bar{x}) = a + h \frac{\sum f_i u_i}{\sum f_i}$$

Example:

Find the mean for the following distribution.

x_i	11	14	17	20
f_i	3	6	8	7

Solution:

For the given data, we can find the mean using the direct method

x_i	f_i	$f_i x_i$
11	3	33
14	6	84
17	8	136
20	7	140
	$\sum f_i = 24$	$\sum f_i x_i = 393$

$$\text{Mean} = \sum f_i x_i / \sum f_i = 393 / 24 = 16.4$$

Types of Mean

There are majorly three different types of mean value that you will be studying in statistics.

Arithmetic Mean

Geometric Mean

Harmonic Mean

Arithmetic Mean

When you add up all the values and divide by the number of values it is called Arithmetic Mean. To calculate, just add up all the given numbers then divide by how many numbers are given.

Example: What is the mean of 3, 5, 9, 5, 7, 2?

Now add up all the given numbers:

$$3 + 5 + 9 + 5 + 7 + 2 = 31$$

Now divide by how many numbers are provided in the sequence:

$$31 \div 6 = 5.16$$

5.16 is the answer.

Geometric Mean

The **geometric mean** of two numbers x and y is \sqrt{xy} . If you have three numbers x , y , and z , their geometric mean is $\sqrt[3]{xyz}$.

$$\text{Geometric Mean} = \sqrt[n]{x_1 x_2 x_3 \dots x_n}$$

Harmonic Mean

The harmonic mean is used to average ratios. For two numbers x and y , the harmonic mean is $\frac{2xy}{x+y}$. For, three numbers x , y , and z , the harmonic mean is $\frac{3xyz}{xy+xz+yz}$

$$= \frac{\frac{x_1}{f_1} + \frac{x_2}{f_2} + \frac{x_3}{f_3} + \frac{x_4}{f_4} + \dots + \frac{x_n}{f_n}}{N}$$

Harmonic mean:

Example

Suppose you have a dataset representing the daily temperatures for a week in degrees Celsius: [20, 22, 25, 24, 23, 21, 20].

To find the arithmetic mean (often just called “the mean”), you would sum up all the temperatures and then divide by the total number of days:

$$(20 + 22 + 25 + 24 + 23 + 21 + 20) / 7 = 155 / 7 \approx 22.14^\circ\text{C}.$$

So, the mean temperature for that week is approximately 22.14 degrees Celsius.

Example

Suppose you have a class where assignments are worth different percentages of the final grade. Here’s a breakdown:

- Assignment 1: 20% of the final grade, scored 85/100
- Assignment 2: 30% of the final grade, scored 90/100

- Assignment 3: 50% of the final grade, scored 95/100

To find the weighted mean, you would multiply each assignment's score by its weight, sum up these weighted scores, and then divide by the total weight:

$$\text{Weighted Mean} = (0.20 * 85) + (0.30 * 90) + (0.50 * 95)$$

$$= (17) + (27) + (47.5)$$

$$= 91.5$$

So, the weighted mean for this class's assignments is 91.5 out of 100.