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EXTC-B

Tut 4. Calculus of variation

①
soln The trial soln is given by

$$y(x) = C_0 + C_1 x + C_2 x^2$$

$\therefore y(0) = 0$

$$0 = C_0 + C_1(0) + C_2(0)^2$$
$$\therefore C_0 = 0$$

$\therefore y(1) = 0$

$$0 = C_1(1) + C_2(1)$$
$$C_1 = -C_2$$

$\therefore$ Trial soln

$$\bar{y}(x) = C_0 + C_1 x + C_2 x^2$$
$$= C_1 x - C_1 x^2$$
$$\bar{y}(x) = C_1 x(1-x)$$

Now

$$I = \int_0^1 (y'^2 - 2y - 2xy)\,dx$$

$$= \int_0^1 [C_1(1-2x)]^2 - 2[C_1(x - x^2)] - 2x[C_1(x-x^2)]\,dx$$

$$= \int_0^1 c_1^2 x^2 - 2c_1^2 x^3 + c_1 x^4 - 2(c_1 x - c_1 x^2) - 2x(c_1 x - c_1 x^2)\, dx$$

$$= \int_0^1 (c_1^2 x^2 - 2c_1^2 x^3 + c_1^2 x^4 - 2c_1 x)\, dx$$

$$= \int_0^1 [c_1^2 x^2 + (2c_1 - 2c_1^2) x^3 + c_1^2 x^4 - 2c_1 x]\, dx$$

$$= \left[\frac{c_1^2 x^3}{3} + \frac{(2c_1 - 2c_1^2) x^4}{4} + \frac{c_1^2 x^5}{5} - c_1 x^2\right]_0^1$$

$$= \frac{c_1^3}{3} + \frac{2c_1}{4} - \frac{2c_2^2}{4} + \frac{c_1^2}{5} - c_1$$

$$= c_1^2\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) + c_1\left(\frac{2}{4} - 1\right)$$

$$I = \frac{c_1^2}{30} - \frac{c_1}{2}$$

$$\frac{dI}{dc_1} = 0$$

$$\frac{2c_1}{30} - \frac{1}{2} = 0$$

$$\frac{2c_1}{30} = \frac{1}{2}$$

$$c_1 = \frac{15}{2}$$

$$c_2 = -c_1 = -\frac{15}{2}$$

Now

$$y(x) = C_1 x(1-x)$$
$$= \frac{15}{2}(x - x^2)$$
$$= \frac{15}{2}x - \frac{15}{2}x^2$$

2) Sol^n

Trial sol^n is given as

$$y(x) = C_0 + C_1 x + C_2 x^2$$

$\therefore y(0) = 0$

$$0 = C_0 + C_1(0) + C_2(0)^2$$
$$\therefore C_0 = 0$$

$\therefore y(1) = 0$

$$0 = C_0 + C_1(1) + C_2(1)^2$$
$$C_1 = -C_2$$

$\therefore$ Trial sol^n is

$$y(x) = C_0 + C_1 x + C_2 x^2$$
$$= C_1 x - C_1 x^2$$

$$y(x) = C_1(x - x^2)$$

Now

$$I = \int_0^1 (2xy - y^2 - y'^2)\,dx$$

$$= \int_0^1 \{2xC_1(x-x^2) - [C_1(x-x^2)]^2 - (C_1(1-2x$$

$$= \int_0^1 \{2x^2 C_1 - 2x^3 C_1 - C_1^2 (x^2 - 2x^3 + x^4) - C_1^2 (1 - 4x + 4x^2)\} dx$$

$$= \int_0^1 C_1 (2x^2 - 2x^3) - C_1^2 (x^2 - 2x^3 + x^4) - C_1^2 (1 - 4x + 4x^2)$$

$$= \int_0^1 \{C_1 (2x^2 - 2x^3) - C_1^2 (x^4 - 2x^3 + 5x^2 - 4x + 1)\} dx$$

$$= C_1 \left[\frac{2x^3}{3} - \frac{2x^4}{4}\right]_0^1 - C_1^2 \left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{5x^3}{3} - \frac{4x^2}{2} + x\right]_0^1$$

$$= C_1 \left[2/3 - 1/2\right] - C_1^2 \left[1/5 - 1/2 + 5/3 - 2 + 1\right]$$

$$I = C_1/6 - C_1^2 \cdot 11/30$$

$$\frac{dI}{dC_1} = 0$$

$$\therefore 1/6 - \frac{22}{30} C_1 = 0$$

$$\therefore C_1 = 5/22$$

$\therefore$ The trial solⁿ is

$$\bar{y}(x) = C_1 (x - x^2)$$

$$= 5/22 (x - x^2)$$

$$= \frac{5}{22} x - 5/22\, x^2$$

(3)

$\Rightarrow$) $F = \dfrac{\sqrt{1+y'^2}}{y}$

$\because$ f does not contain x explicitly

$\therefore$ By euler lagrange's eqn reduce to

$$F - y'\frac{\partial f}{\partial y'} = C$$

$$\frac{\sqrt{1+y'^2}}{y} - y'\left(\frac{2y'}{2\sqrt{1+y'^2}\cdot y}\right) = C$$

$$\frac{\sqrt{1+y'^2}}{y} - \frac{y'^2}{y\sqrt{1+y'^2}} = C$$

$$\frac{1+y'^2 - y'^2}{y\sqrt{1+y'^2}} = C$$

$$\frac{1}{y\sqrt{1+y'^2}} = C$$

$$1 = Cy\sqrt{1+y'^2}$$

$$1 = C^2y^2(1+y'^2)$$

$$\frac{1}{C^2y^2} = 1+y'^2$$

$$\frac{1}{c^2y^2} - 1 = y'^2$$

$$\frac{1-c^2y^2}{c^2y^2} = y'^2$$

$$\therefore y' = \frac{\sqrt{1-c^2y^2}}{cy}$$

$$\frac{dy}{dx} = \frac{\sqrt{1-c^2y^2}}{cy}$$

put $1-(cy)^2 = t$

$$-2cy \cdot c\,dy = dt$$

$$\therefore cy\,dy = -\frac{dt}{2c}$$

$$\therefore -\frac{dt}{2c\,dx} = \sqrt{t}$$

$$\int \frac{dt}{\sqrt{t}} = \int -2c\,dx$$

$$\frac{t^{1/2}}{1/2} = -2cx$$

Resubstiting

$$2\left[1-(cy)^2\right]^{1/2} = -2cx.$$

$$1-(cy)^2 = c^2x^2$$

$$c^2y^2 = 1-c^2x^2$$

$$y^2 = \frac{1-c^2x^2}{c^2}$$

$$\therefore y = \frac{\sqrt{1-c^2x^2}}{c}$$

4) $\int_{x_1}^{x_2} (16y^2 - y''^2 + x^2)\,dx$

$$F = 16y^2 - y''^2 + x^2$$

$$\frac{\partial F}{\partial y} = 32y \quad \frac{\partial F}{\partial y'} = 0 \quad \frac{\partial F}{\partial y''} = -2y''$$

$\therefore$ By euler langrages of higher order

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y''}\right) = 0$$

$$32y - \frac{d}{dx}(0) + \frac{d^2}{dx^2}(-2y'') = 0$$

$$32y - 2y'' = 0$$

$$y^{iv} - 16y = 0$$

$$(D^4 - 16)y = 0$$

The auxilliary eqn is given as

$$D^4 - 16 = 0$$

$$(D^2-4)(D^2+4) = 0$$

$$D = \pm 2, \pm 2i$$

$CF = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x$

$PI = 0$

Now

$y = CF + PI$

$\therefore y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x$

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