Algebraic Number Theory
Example Sheet 3

1. Show that the rings of integers of the following number fields are principal ideal domains (i.e. the have class number 1)
$$\mathbb{Q}(\sqrt{-3}), \qquad \mathbb{Q}(\sqrt{5}), \qquad \mathbb{Q}(\sqrt{-2}).$$

2. Show that the rings of integers of the following number fields have class number 2
$$\mathbb{Q}(\sqrt{-5}), \qquad \mathbb{Q}(\sqrt{-6}), \qquad \mathbb{Q}(\sqrt{-10}).$$

3. Solve the Diophantine equations
   (i) $x^2 + 2 = y^3$.
   (ii) $x^2 + 6 = y^3$.