1. In this exercise you are asked to parametrize the rational points on two varieties:
$$V_1 : x^4 - 1 = y^2 + 2y,$$
and
$$V_2 : 2x^6 - 1 = y^2 + 2y.$$
Hint: The first is reducible over $\mathbb{Q}$. The second is irreducible over $\mathbb{Q}$, but reducible over $\mathbb{Q}(\sqrt{2})$.

2. Let K be a field of characteristic $\neq 2$, with $A, B \in K$. Find a necessary and sufficient condition on A, B for the curve
$$C : y^2 = x^3 + Ax^2 + B$$
to be non-singular.

3. Find the primes of bad reduction for
$$C : y^2 = x^4 + 3^4 \times 5.$$
Show that C is birational (over $\mathbb{Q}$) to another curve which has good reduction at 3. (Don't be scared of the word birational. All you need is a change of variable of the form $y = \alpha y_1$, $x = \beta x_1$ such that when you simplify you end up with a curve of good reduction at 3.)

4. Let
$$E : y^2 = x^3 + 17.$$
Let
$$P = (-2, 3), \qquad Q = (2, 5).$$
Check that P and Q are on E. Calculate
$$2P, \qquad -Q, \qquad P + Q.$$