Algebraic Geometry – Exercises 13 March 2007

1. *(3 points)* Let A be a ring. Consider the map

$$V : \{\text{ideals of } A\} \to \{\text{closed subsets of } \operatorname{Spec}(A)\}$$

defined by $\mathfrak{a} \mapsto \{\mathfrak{p} \in \operatorname{Spec}(A) : \mathfrak{p} \supset \mathfrak{a}\}$ and the map

$$I : \{\text{closed subsets of } \operatorname{Spec}(A)\} \to \{\text{ideals of } A\}$$

defined by $Z \mapsto \{a \in A : a \in \mathfrak{p} \text{ for all } \mathfrak{p} \in Z\}$. Show that V and I give a bijection between the set of radical ideals of A and the set of closed subsets of $\operatorname{Spec}(A)$.

2. *(3 points)* Hartshorne, Exercise II.2.1.

3. *(3 points)* Hartshorne, Exercise II.2.2.

4. *(3 points)* Hartshorne, Exercise II.2.4.

5. *(2 points)* Hartshorne, Exercise II.2.5.