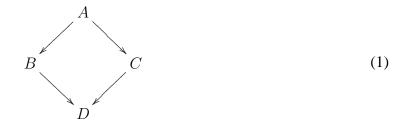
## Additional exercise 1.

Let X be the set  $\{-1, 0, 1\}$  with the following topology:

$$\mathrm{Open}(X) = \{\emptyset, \{0\}, \{-1, 0\}, \{0, 1\}, X\}.$$

a) Show that a presheaf on X naturally corresponds to a commutative diagram



of abelian groups.

- **b**) Determine the stalks of  $\mathcal{F}$  at all the points of X in terms of the diagram (1).
- c) Let  $\mathcal{F}$  be a presheaf on X. Which properties should the diagram (1) associated to  $\mathcal{F}$  satisfy in order that  $\mathcal{F}$  be a sheaf? Determine also the sheafification  $\mathcal{F}^+$  of  $\mathcal{F}$  in terms of (1).
- **d)** Consider the subpace  $Y = \{-1, 1\}$  of X, with the natural inclusion  $\iota : Y \hookrightarrow X$ . Give a surjective morphism

$$\phi: \mathbb{Z}_X \to \iota_* \mathbb{Z}_Y$$

such that there is an open  $U \subset X$  with  $\phi(U)$  not surjective. Here, for X a topological space,  $\mathbb{Z}_X$  denotes the constant sheaf with values in  $\mathbb{Z}$  on X.