Algebraic Geometry — Exercises 20 February 2007

1. *(3 points)* Let A be a ring (as usual commutative with unit).

a) Show that A has a maximal ideal if A is not the zero ring.

b) Show that an ideal $I \subset A$ is maximal if and only if A/I is a field.

c) Show that an ideal $I \subset A$ is prime if and only if A/I is an integral domain.

d) Let $A^\times$ be the multiplicative group of A. Show that $A^\times$ and $\bigcup_{\substack{\mathfrak{m} \subset A \\ \text{max. id.}}} \mathfrak{m}$ are each other's complements in A.

e) The ring A is called *local* if it has exactly one maximal ideal. Show that A is local if and only if $A - A^\times$ is an ideal of A.

2. *(3 points)* Let k be an algebraically closed field, and let $a_1, \ldots, a_r$ be distinct elements of $\mathbf{A}^1(k) = k$.

a) Give a basis for the k-vector space of regular functions on $\mathbf{A}^1(k) - \{a_1, \ldots, a_r\}$.

b) Give a basis for the k-vector space of $\mathcal{O}_{\mathbf{A}^1(k),0}$, the stalk at 0 of the sheaf of regular functions on $\mathbf{A}^1(k)$.

3. *(3 points)* Show that $k[x, y]$ is the k-algebra of regular functions on $\mathbf{A}^2(k) - \{(0, 0)\}$.

4. *(2 points)* Hartshorne, Exercise II.1.14.

5. *(2 points)* Hartshorne, Exercise II.1.15.