Algebraic Geometry – Exercises 17 April 2007

1. *(3 points)* Let k be an algebraically closed field. In this exercise we will determine the automorphism group of $\mathbb{P}^1_k$ or, equivalently, of the function field $k(x)$.

a) Give a natural action of $\mathrm{GL}_2(k)$ on the graded algebra $k[x_0, x_1]$ and show that this action induces an action of $\mathrm{PGL}_2(k)$ on $\mathbb{P}^1_k$. The group $\mathrm{PGL}_2(k)$ is just $\mathrm{GL}_2(k)$ modulo its scalar matrices.

b) Deduce an action of $\mathrm{PGL}_2(k)$ on $k(x)$ and show that this gives an isomorphism $\mathrm{PGL}_2(k) \xrightarrow{\sim} \mathrm{Aut}_k(k(x))$.

2. *(3 points)* Hartshorne, IV.1.1.

3. *(3 points)* Hartshorne, IV.1.2.

4. *(3 points)* Let A be a unique factorisation domain and let K be its field of fractions. Prove that A is integrally closed in K.