Algebraic Number Theory (MA3A6) - Problem Sheet 8

This sheet is not for submission, but you may ask for help or feedback if you wish. You could also review questions from previous sheets.

1. Compute the class number of $\mathbb{Q}(\sqrt{-d})$ for
   - $d = 23$
   - $d = 163$
   - $d = 29$
   - $d = 15$
2. Let $\mathcal{O}_K$ be the ring of algebraic integers of a number field. Prove that $\mathcal{O}_K$ has infinitely many prime ideals.
3. Let I, J be non-zero integral ideals of a Dedekind domain R. Prove that there exists an integral ideal K such that IK is principal and $K + J = R$.
4. Give an example of a Dedekind domain that is not a field that has finitely many prime ideals.
5. Show that the absolute value of the discriminant d_K of a number field K tends to ∞ with the degree n of the field K.
6. Let L/K be an extension of number fields. If I is an ideal of $\mathcal{O}_K$, then $I\mathcal{O}_L$ is the ideal of $\mathcal{O}_L$ generated by the elements of I.

   Let I be an integral ideal of K and $I^m = (a)$. Show that I becomes a principal ideal in the field $L = K(\sqrt[m]{a})$, in the sense that $I\mathcal{O}_L = (\alpha)$ for some $\alpha \in L$.