STT 465 Exam 2 Formula Sheet

- An uncertain positive quantity λ follows a $Gamma(\alpha, \beta)$ then is:

$$p(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \quad ; for\ \lambda > 0$$

$$E(\lambda) = \frac{\alpha}{\beta} \qquad and \qquad Var(\lambda) = \frac{\alpha}{\beta^2}$$

- The Normal Model with known mean μ:

$$Y_1, Y_2, \ldots, Y_n | \mu, \lambda \ \sim iid\ N\left(\mu, \frac{1}{\lambda}\right)$$

$$\lambda \sim Gamma(\alpha, \beta) - Prior$$

$$\lambda | Y_1, Y_2, \ldots, Y_n \sim Gamma\left(\alpha + \frac{n}{2}, \beta + \frac{n}{2} S_{MLE}^2\right)$$

Where $S_{MLE}^2 = \frac{\sum_{i=1}^n (y_i - \mu)^2}{n}$ and the Precision $\lambda = \frac{1}{\sigma^2}$.

The posterior mean for λ: $E(\lambda | y_1, \ldots y_n) = \frac{\alpha + \frac{n}{2}}{\beta + \frac{n}{2} S_{MLE}^2}$

$(1-A)100\%$ HPD credible region (interval) for σ^2 is:

$$\left(\frac{2\beta + nS_{MLE}^2}{\chi_{\frac{A}{2}}^2(2\alpha+n)}, \frac{2\beta + nS_{MLE}^2}{\chi_{1-\frac{A}{2}}^2(2\alpha+n)}\right)$$

Where $\chi_{\frac{A}{2}}^2(2\alpha+n)$ is a chi square distribution with significant level A and degree of freedom $2\alpha + n$

- **Linear Regression Models**

  SLR model: $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$

  MLE estimates: $\widehat{\beta_2} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$ and $\widehat{\beta_1} = \bar{y} - \widehat{\beta_2}\bar{x}$

  MLR model: $\boldsymbol{Y} = \boldsymbol{X}\mathbf{B} + \boldsymbol{\varepsilon}$ where $\boldsymbol{X}$ is an $n \times p$ matrix and $\mathbf{B}$ is a p-parameter column vector.
  MLE Estimates:

$$\widehat{\boldsymbol{B}} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{Y} \qquad Var(\epsilon) = \widehat{\boldsymbol{\sigma}^2} = \frac{1}{n-p}\left(\boldsymbol{Y} - \boldsymbol{X}\widehat{\boldsymbol{B}}\right)^T\left(\boldsymbol{Y} - \boldsymbol{X}\widehat{\boldsymbol{B}}\right)$$

  Bayesian Estimates**:**

  Likelihood - $\{\boldsymbol{y}|\boldsymbol{X}, \boldsymbol{B}, \sigma_\varepsilon^2\} \sim multivariate\ Normal\ (\boldsymbol{XB}, \sigma_\varepsilon^2 \boldsymbol{I})$

Prior – $\beta_j \sim N\left(\beta_0, \sigma_b^2\right)$, $\sigma_\varepsilon^2 \sim Scaled\ inverse\ \chi^2(\ df_0, S_0)$

Full Posterior:

$$\beta_k |\ Else \sim N(\widetilde{\beta_k}\ , a^{-1})$$

Where $a = \frac{\sum_{i=1}^{n} x_{ik}^2}{\sigma_\varepsilon^2} + \frac{1}{\sigma_b^2}$ and $\widetilde{\beta_k} = \frac{\frac{\sum_{i=1}^{n} x_{ik}^2 \widetilde{y_i}}{\sigma_\varepsilon^2} + \frac{\beta_0}{\sigma_b^2}}{a}$

- **Nonlinear Regression Models**

Exponential model: $\mathrm{y_i} = \beta_2 \exp(\beta_3 \mathrm{x_i}) + \varepsilon_i$ where $\varepsilon_i \sim \mathrm{N}(\mathrm{o}, \sigma^2)$

Logistics Model: Response $Y_i = \begin{cases} 1 & , success \\ 0 & , failure \end{cases}$

$$y_i = E(Y_i) + \varepsilon_i$$

$$\eta_i = \log\left(\frac{\theta_i}{1-\theta_i}\right) = \tau + X_1\beta_1 + \cdots + X_p\beta_p$$

$$E(Y_i) = \theta_i = \frac{e^{\eta_i}}{1+e^{\eta_i}}$$

$$Odds_i = \frac{\theta_i}{1-\theta_i}$$

Odds Ratio (OR) for the Kth predictor variable:
$\widehat{OR} = \exp\,(\widehat{\beta_k})$