STT 465 – Fall 2019

Homework 1 – Due 09/13/2019 (In Class)

1. Consider a system of three Bernoulli random variables (X, Y, Z). Suppose that $p(Z = 1) = 0.6$ The conditional distribution $p(X, Y|Z)$ is given on the table below:

	p(X,Y|Z=0)			p(X,Y|Z=1)	
	Y=0	Y=1		Y=0	Y=1
X=0	0.06	0.24	X=0	0.12	0.28
X=1	0.14	0.56	X=1	0.18	0.42

Show complete work when answering the following questions:

i. Are (X, Y) conditionally independent?
ii. Are (X, Y) independent?
iii. Are (X, Y) exchangeable?

2. Suppose $\theta \sim p(\theta)$ and $X_1, X_2, \dots, X_n$ are conditionally independent and identically distributed (i.i.d) (discrete) random variables given θ $(assumed\ to\ be\ discrete)$. Show that the marginal (Unconditional on θ) distributions of $X_1, X_2, \dots, X_n$ are exchangeable.

3. Logan (1983) reported the following joint density on $(Y_1, Y_2) = (\ Father's\ Occupation\ , Son's\ Occupation)$

Father's Occup	farm	operatives	craftsmen	sales	professional
farm	0.018	0.035	0.031	0.008	0.018
operatives	0.002	0.112	0.064	0.032	0.069
craftsmen	0.001	0.066	0.094	0.032	0.084
sales	0.001	0.018	0.019	0.010	0.051
professional	0.001	0.029	0.032	0.043	0.130

Use the joint probability distribution to calculate the following distributions:

i. The marginal distribution of a father's occupation
ii. The marginal distribution of a son's occupation
iii. The conditional distribution of a son's occupation given that the father's occupation is a craftsman.

4. A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a “false positive” result for 1% of healthy persons tested. If 0.5% of the population actually have the disease,
   i. What is the probability that the test result is positive?
   ii. What is the probability that a randomly chosen person has the disease given that the test result is positive?