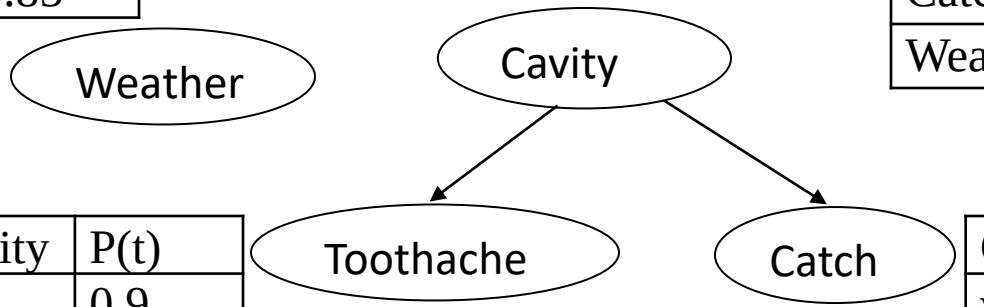
6.2. Probabilistic Reasoning using Bayesian Networks

A. An example of Bayesian Networks

P(warm)	P(cold)
0.08	0.85

P(v)
0.002

Cavity	v	¬v
Toothache	t	¬t
Catch	h	¬h
Weather	warm	cold



Cavity	P(t)
v	0.9
¬v	0.03

Cavity	P(h)
v	0.95
¬v	0.02

- $P(t \wedge v) = P(t \wedge v \wedge h) + P(t \wedge v \wedge \neg h) = P(t \mid v) * P(v) * P(h \mid v) + \ldots =$
  $P(t \mid v) * P(v) = 0.9 * 0.002 = \ldots$ [No Catch! No Weather, obviously!]
- $P(t \wedge \neg v) = P(t \mid \neg v) * P(\neg v) = 0.03 * 0.998 = \ldots$
- $P(\neg h \wedge v) = P(\neg h \mid v) * P(v) = 0.05 * 0.002 = \ldots$
- $P(t) = P(t \wedge v \wedge h) + P(t \wedge v \wedge \neg h) + P(t \wedge \neg v \wedge h) + P(t \wedge \neg v \wedge \neg h) =$
  $\ldots = P(t \mid v) * P(v) + P(t \mid \neg v) * P(\neg v) = \ldots$

- Formula: $P((X_1 = x_1) \wedge (X_2 = x_2) \wedge \ldots \wedge (X_n = x_n)) =$
  $P(x_1, x_2, \ldots, x_n) = \prod_{i=1:n} P(x_i \mid \text{parents}(X_i))$, where parents($X_i$) means specific values of Parents(X_i)

  - $P(t \vee v) = P(t) + P(v)$ = ---------- **[Avoid duplicate atomic events.]**

  ✓ $P(t \mid (v \vee h)) = ??$

  ✓ $P(h) = P(t \wedge v \wedge h) + P(\neg t \wedge v \wedge h) + P(t \wedge \neg v \wedge h) + P(\neg t \wedge \neg v \wedge h) = ??$

B. Specification of a Bayesian Network

1. A BN is a data structure represented by a directed acyclic graph.

2. A node represents a random variable.

3. An arrow (arc) represents a 'parent-child' relationship.

4. A node X_i is assigned a conditional probability table (CPT) that quantifies the effect of the parents on the node, that is, the distribution,

$$P(X_i \mid \text{Parents}(X_i)).$$