Magnetic Force

If a charge particle moves through a region in which both an electric field and a magnetic field are present the resulted force is the sum of electric force and magnetic force.

If both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ act on a charge q moving with a velocity $\vec{v}$, the total force on it can be represented as

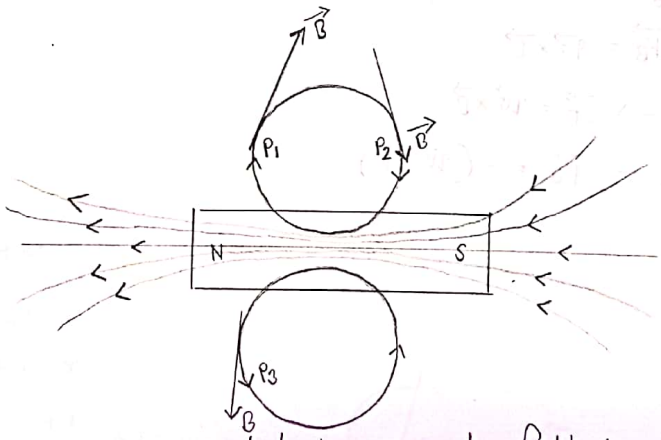
$$\vec{F} = qE + (q\vec{v} \times \vec{B})$$

↳ Lorents Force

$F_B = qV_B$

$B = \frac{F_B}{qv}$

Magnetic's field lines / Lines of induction :→



Magnetic field may be represented by magnetic field lines in the following ways.

1. The tangant to a field line at any point gives the direction of the magnetic field $\vec{B}$ at that point.
2. The field lines drawn so that the number of lines in per unit cross sectional area is proportional to the magnetic field B where the lines are also together B is large and where they are far a part B is small.

Scanned by CamScanner

Electromagnetism

When the charge is stracid magnetic field don't work upon that.

q_0 $\quad \vec{F_e} = q_0\vec{E}$

$$E = \frac{\vec{F_e}}{q_0}$$

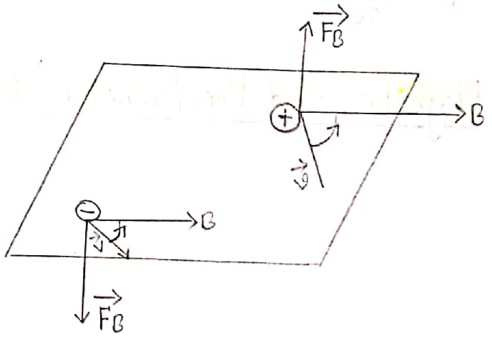
Magnetic Field :→ The field set up in the space around a current carying wire which produces a force on a moving charge is called magnetic field $\vec{B}$.

Magnetic Force :→ The force exerted by magnetic field on a moving charge is known as magnetic force.

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

$$\vec{F_B} = qv_B \sin\theta$$

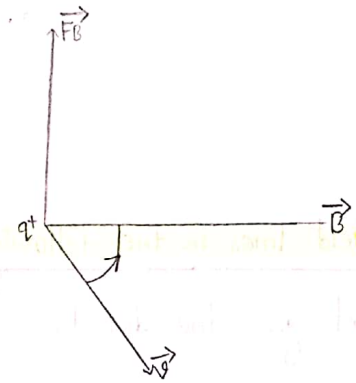
$$B = \frac{F_B}{qv\sin\theta}$$

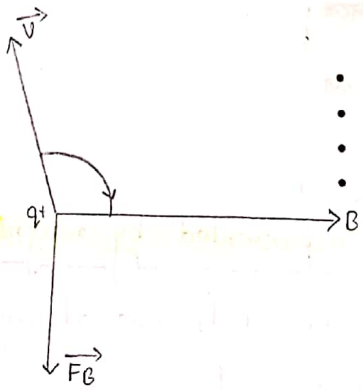


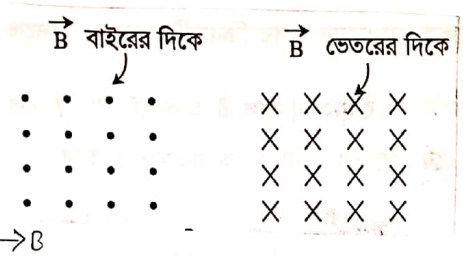
Magnetic Force → $\vec{F_B} = q\vec{v} \times \vec{B}$

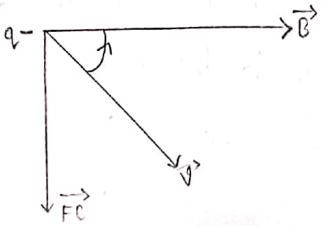
for positive charge → $\vec{F_B} = q\vec{v} \times \vec{B}$

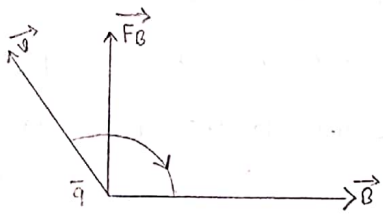
for negetive charge → $\vec{F_B} = -(q\vec{v} \times \vec{B})$

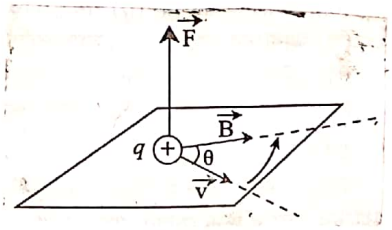












Scanned by CamScanner

Review and Summary

The Biot-Savart Law :→

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^2} \quad \text{(Biot-Savart law)}$$

Magnetic Field of a Long Straight Wire :→

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire)}$$

Magnetic Field of a Circular Arc :→

$$B = \frac{\mu_0 i \phi}{4\pi R} \quad \text{(at center of circular arc)}$$

Force Between Parallel Currents :→

$$F = \frac{\mu_0 L i_a i_b}{2\pi d} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Ampere's Law :→

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i \quad \text{(Ampere's law)}$$

Fields of a Solenoid and a Toroid :→

$$B = \mu_0 n i \quad \text{(ideal solenoid)}$$

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \quad \text{(toroid)}$$

Field of a Magnetic Dipole :→

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

Others

Force → $F = qvB\sin\theta$

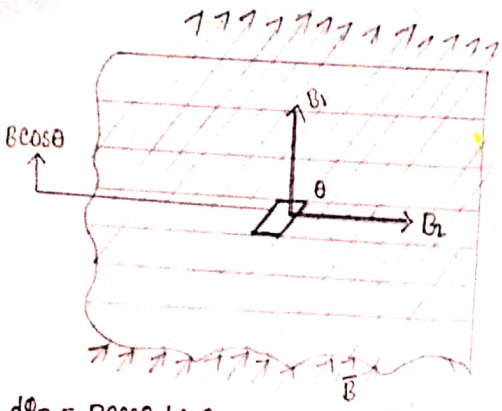
Electric Current → $B = \frac{\mu_0 N I}{2r}$

Force → $F = IlB\sin\theta$

Scanned by CamScanner

Magnetic Flux, φ_B

$$\varphi_B = \int \vec{B} \cdot d\vec{A}$$

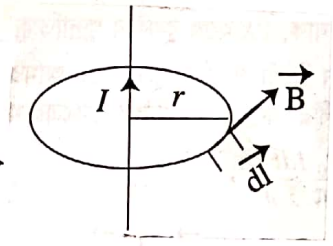


$$d\varphi_B = B\cos\theta \, dA \quad \varphi_B = \int d\varphi$$

Scanned by CamScanner

Ampere's Law

Ampere's Law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$



Experimentally found that

$B \propto \frac{i}{r}$ } $i \rightarrow$ Current in a wire
$r \rightarrow$ distance at point at which we want to know $\vec{B}$

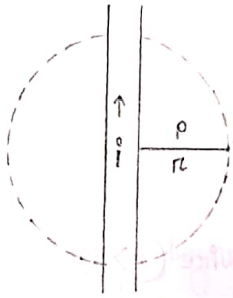
$B = \boxed{\frac{\mu_0}{2\pi}} \cdot \frac{i}{r}$

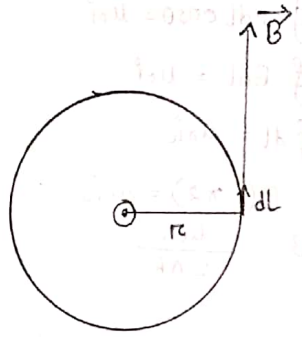
$\frac{\mu_0}{2\pi} \rightarrow$ Value of proportional constant

$\mu_0 \rightarrow$ Permeability constant

$B(2\pi r) = \mu_0 i$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$



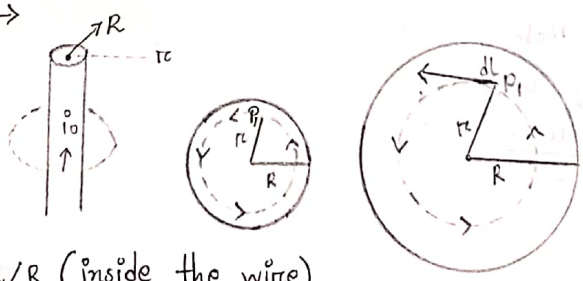


$\oint \vec{B} \cdot d\vec{l} = \oint B dl \cos\theta$
$= \oint B dl$
$= B \oint dl$
$= B(2\pi r)$

Applications of Ampere's Law :→

Question :→ B at a distance r from the centre of a long cylindrical wire of radius R carrying current i_0.

Answer :→



$B \propto i$
$= B \propto \frac{1}{r}$
$= B \propto \frac{1}{r}$
$\Rightarrow B = k \frac{i}{r}$
$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$

(i) $r < R$ (inside the wire)

$\oint \bar{B} \cdot \bar{dl} = \mu_0 i$ { $\bar{B}$ is not constant, B is constant }

$\oint B \cdot dl \cos\theta = \mu_0 I$

$\theta \rightarrow$ Angle between $\bar{B}$ and $\bar{dl}$

$\oint B dl = \mu_0 i$ $[\theta = 0°]$

$B(2\pi r) = \mu_0 i$

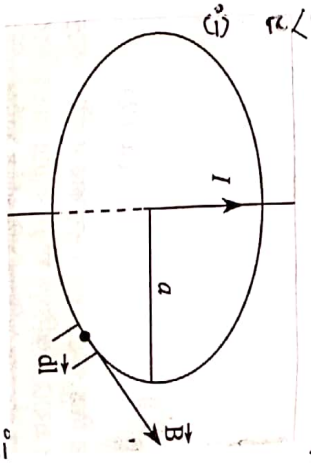
$B(2\pi r) = \mu_0 I_0 \frac{\pi r^2}{\pi R^2}$

For πR^2 the current is I_0
" 1 " " is $\frac{I_0}{\pi R^2}$
" πr^2 " " is $\frac{I_0 \pi r^2}{\pi R^2}$

$\therefore B = \frac{\mu_0 I_0 r}{2\pi R^2}$

$B(2\pi r) = \mu_0 I$
$= B(2\pi r)$
$= B \oint dl$
$= \oint B dl$
$= \oint B dl \cos\theta$
$= \oint \vec{B} \cdot d\vec{l}$

$\therefore \oint B \cdot dl = \mu_0 i$



Scanned by CamScanner

Ampere's Law: →

(ii) Surface of a wire

B at the surface of a wire $(r = R)$

$$B = \frac{\mu_0 I_0 R}{2\pi R^2}$$

$$B = \frac{\mu_0 I_0}{2\pi R}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

$$\Rightarrow \oint B dl \cos\theta = \mu_0 i$$

$$\Rightarrow \oint B dl = \mu_0 i$$

$$\Rightarrow B \oint dl = \mu_0 i_0$$

$$\therefore B(2\pi R) = \mu_0 i_0$$

$$B = \frac{\mu_0 i_0}{2\pi R}$$

(iii) B at the outside of the wire $(r > R)$

$$\oint \vec{B} \cdot dl = \mu_0 I_0$$

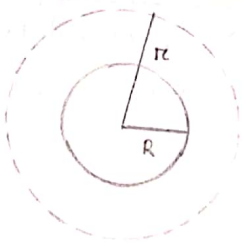
$$\Rightarrow \oint B dl \cos\theta = \mu_0 I_0$$

$$\Rightarrow \oint B dl \cos\theta = \mu_0 I_0$$

$$\Rightarrow B \oint dl = \mu_0 I_0$$

$$\therefore B(2\pi r) = \mu_0 I_0$$

$$\therefore B = \frac{\mu_0 I_0}{2\pi r}$$

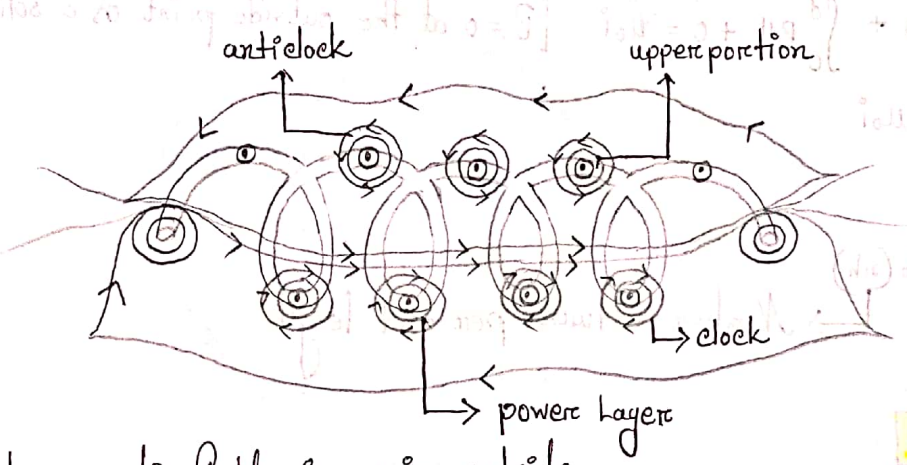


Scanned by CamScanner

Amperce's Law

B for Solinoid

Solinoid :→ A solinoid is a long wire wound in a closed packed helix and carrying a current i.

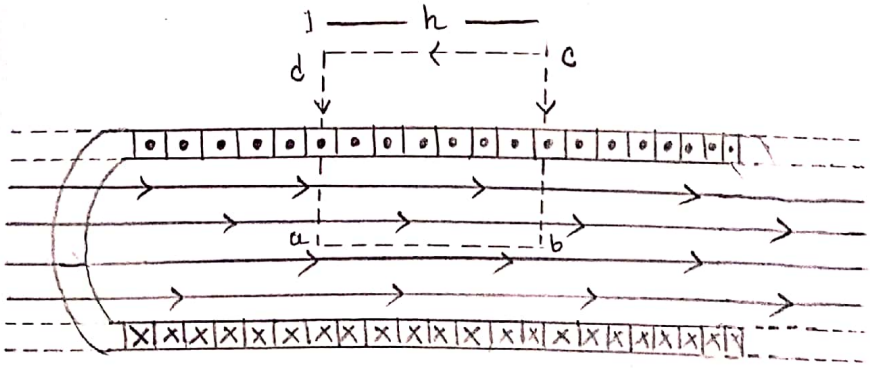


Solinoid's magnetic field is 0 in outside

Inside a magnetic field is parallel

B for Solinoid

Consider a section of an ideal solinoid maid of adjacent square turns equivalent to an infinitely long cylindrical current sheet



Current in each turn $i_s = i_0$

Consider a closed path of integration abcd

Scanned by CamScanner

$$\oint \vec{B}\cdot dl = \mu_0 i$$

$$\int_a^b \bar{B}\cdot dl + \int_b^c \bar{B}\cdot dl + \int_c^d \bar{B}\cdot dl + \int_d^a \bar{B}\cdot dl = \mu_0 i$$

$$\Rightarrow \int_a^b B dl \cos\theta + \int_b^c B dl \cos\theta + \int_c^d B dl \cos\theta + \int_d^a B dl \cos\theta = \mu_0 i$$

$$\Rightarrow \int_a^b B dl + 0 + \int_c^d B dl + 0 = \mu_0 i$$ [B = 0 at the outside point of a solinoid]

$$\Rightarrow Bh + 0 = \mu_0 i$$

$$\Rightarrow Bh = \mu_0 i$$

Here, $i = i_0 (nh)$

$\hookrightarrow$ Number of turns per unit length

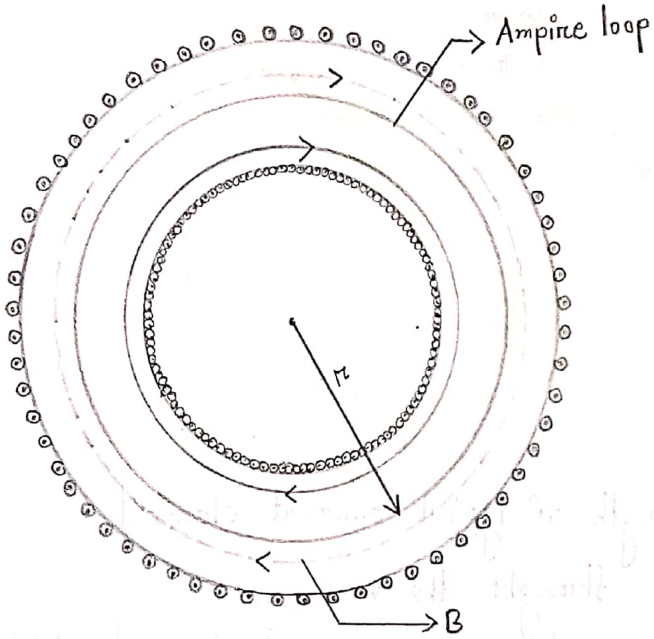
$$Bh = \mu_0 i_0 nh$$

$$\Rightarrow \boxed{B = \mu_0 i_0 n}$$

$$\Rightarrow B = \mu_0 i_0 \frac{N}{L}$$

Scanned by CamScanner

B at interior points of a toroid interior field

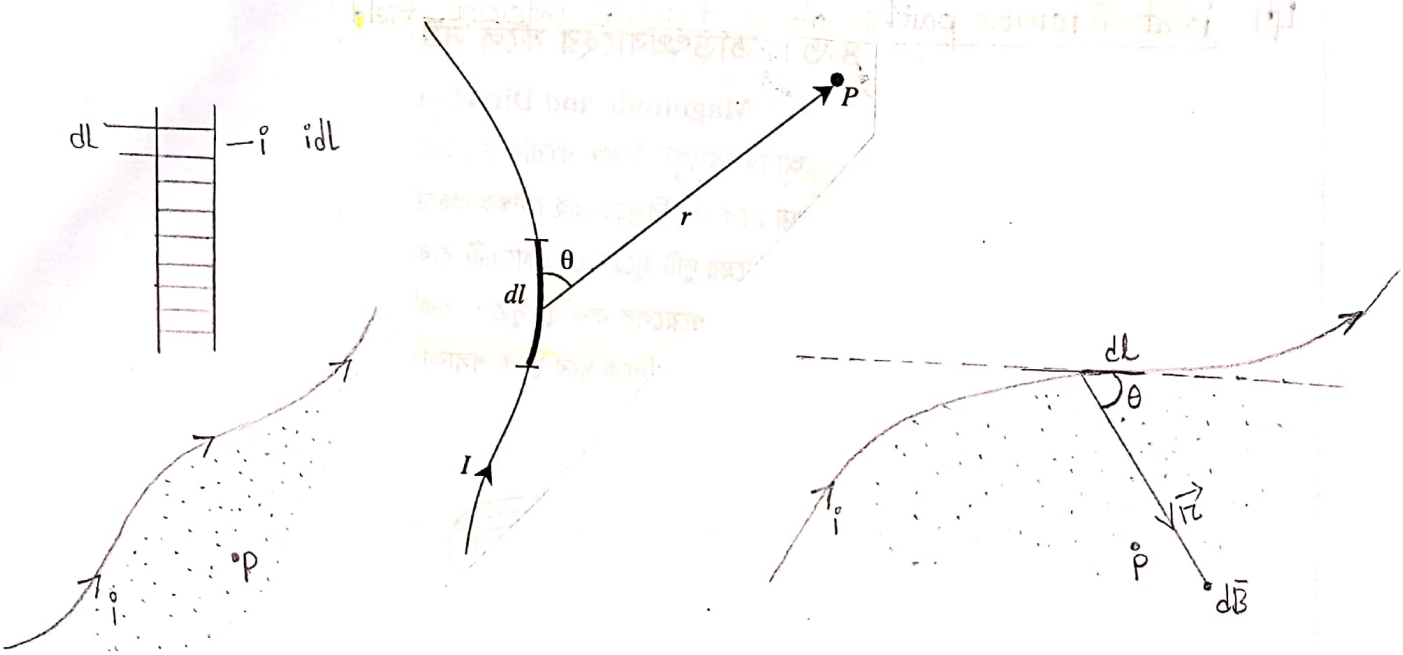


$B(2\pi r) = \mu_0 i N$ [i is the current in the toroid windings
[N is the number of turns]

$B = \frac{\mu_0 i N}{2\pi} \times \frac{1}{r}$ [B is not constant over the cross section]

Scanned by CamScanner

The Biot - Savart Law



Let $d\bar{l}$ be the length of typical current element

i be the current throught the wire

$\vec{r}$ is the displacement vector from $\bar{dl}$ to p at which the magnetic induction due to current i is $d\bar{B}$

According to the Biot savart Law

$$dB = \frac{\mu_0 i dl \sin\theta}{4\pi r^2}$$

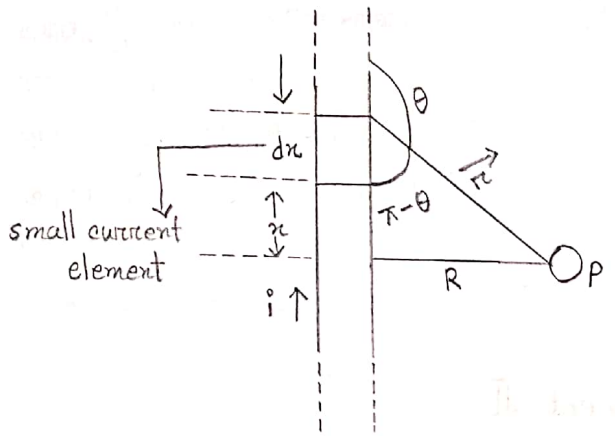
where θ is the angle between $d\bar{l}$ and $\vec{r}$

The direction of $d\bar{B}$ is that of the vector $(d\bar{l} \times \vec{r})$

$$d\bar{B} = \frac{\mu_0 i d\bar{l} \times \vec{r}}{r^3}$$

Scanned by CamScanner

A long state wire



According to the Biot severt's Law

The magnetic field / induction dB due to current element idx is $dB = \frac{\mu_0 i dx \sin\theta}{4\pi r^2}$

$$\therefore B = \int dB$$

$$= \int_{-\infty}^{+\infty} \frac{\mu_0 i dx \sin\theta}{4\pi r^2}$$

$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta \, dx}{r^2}$$

$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty} \frac{R dx}{(x^2+R^2)\sqrt{x^2+R^2}}$$

$$= \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty} \frac{R dx}{(x^2+R^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 i}{4\pi} \int_{-\infty}^{\infty} \frac{R dx}{(x^2+R^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 i}{4\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R^2 \sec^2\varphi \, d\varphi}{(R^2\tan^2\varphi + R^2)^{\frac{3}{2}}}$$

From figure

$r = \sqrt{R^2+x^2}$

from figure

$$\sin\theta = \sin(\pi-\theta)$$

$$= \frac{R}{r}$$

$$= \frac{R}{\sqrt{x^2+R^2}}$$

$x = R\tan\varphi$

$dx = R\sec^2\varphi \, d\varphi$

$-\frac{\pi}{2}$ to $+\frac{\pi}{2}$

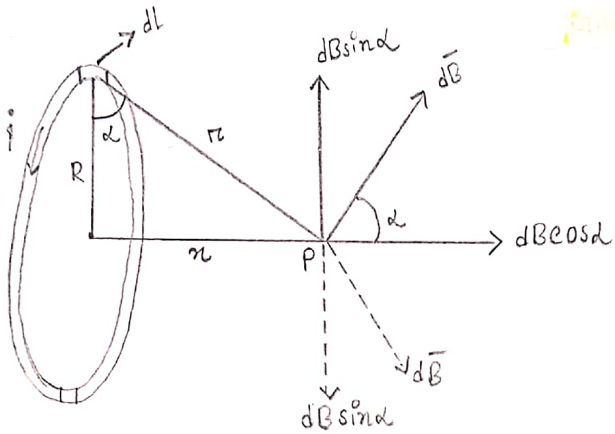
$$B = \frac{\mu_0 i}{4\pi R} \times 2$$

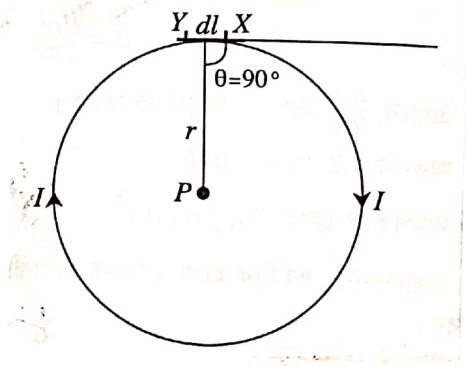
$$= \frac{\mu_0 i}{2\pi R}$$

$$\therefore \boxed{B = \frac{\mu_0 i}{2\pi R}}$$

Scanned by CamScanner

Magnetic field along the axis of a circular coil (Circular current loop)





$\overline{dB}$ at point p due to current element $d\bar{l}$

$$dB = \frac{\mu_0 i\, dl \sin\theta}{4\pi r^2}$$

$$\overline{dB} = \frac{\mu_0 i\, d\bar{l} \times \bar{r}}{4\pi r^3} \rightarrow \text{Vector form}$$

The $d\bar{l}$ and $\bar{r}$ are perpendicular to each other and the planed formed by $d\bar{l}$ and $\bar{r}$.

$$dB = \frac{\mu_0 i\, dl \sin 90^\circ}{4\pi r^2}$$

[always stay in 90° angle with r]

$$= \frac{\mu_0 i\, dl}{4\pi r^2} \quad \text{--- (i)}$$

$$B = \int dB \quad \text{--- (ii)}$$

dB resolves into two components

$dB_{\parallel}$ along the axis of the loop.

$dB_{\perp}$ normal to the axis of the loop.

From the semetry the vector sum of all perpendicular components $dB_{\perp}$ due to all the current elements $d\bar{l}$ is zero.

Only $dB_{\parallel}$ contributes to the total induction But point p

so, we can write

$$B = \int dB_{\parallel}$$

$$= \int dB\cos\alpha$$

$$= \int \frac{\mu_0 i\, dl \cos\alpha}{4\pi r^2} \quad \text{--- (3)}$$

Scanned by CamScanner

From figure $r = \sqrt{x^2+R^2}$

and $\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{x^2+R^2}}$

$$\therefore B = \int \frac{\mu_0}{4\pi} \frac{i\,dl\,R}{x^2+R^2\left(\sqrt{x^2+R^2}\right)}$$

$$= \int \frac{\mu_0}{4\pi} \frac{i\,dl\,R}{(x^2+R^2)^{\frac{3}{2}}}$$

I, R and x have the same value for all elements dl around the loop.

$$B = \frac{\mu_0 i R}{4\pi\,(x^2+R^2)^{\frac{3}{2}}} \int dl$$

$$= \frac{\mu_0 i R}{4\pi\,(x^2+R^2)^{\frac{3}{2}}} (2\pi R)$$

$$\boxed{\therefore B = \frac{\mu_0 i R^2}{2\,(x^2+R^2)^{\frac{3}{2}}}}$$

if $x \gg R$ then $B = \frac{\mu_0 i R^2}{2x^3}$

For loops with N no of turns

$$B = \frac{\mu_0 R^2 N}{2x^3}$$

$$= \frac{\mu_0 R^2 \pi N}{2\pi x^3} \quad (\pi R^2 = A)$$

$$= \frac{\mu_0 i A N}{2x^3\pi} \quad [\mu = NiA]$$

↳ Magnetic dipole moment

At centre of the loop, $x=0$

$$B = \frac{\mu_0 i R^2}{2\,(R^2+x^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 i R^2}{2R^3}$$

$$B = \frac{\mu_0 i}{2R}$$

for n turns $\boxed{B = \frac{\mu_0 N i}{2R}}$

Scanned by CamScanner

A uniform feild of magnetic induction $\overline{B}$ points horizontally from south to north; its magnetute $1.5\ w/m^2$. If a 5.0 Mev proton moves vertically downward through this field, what force will act on it?

Answer (solve) :→

Given Data

$B = 1.5\ w/m^2$

$\theta = 90°$

$q = 1.6 \times 10^{-19} C$

$KE = 5.0$ Mev (kinetic energy)

$= (5.0 \times 10^6 \times 1.6 \times 10^{-19})$ Joule

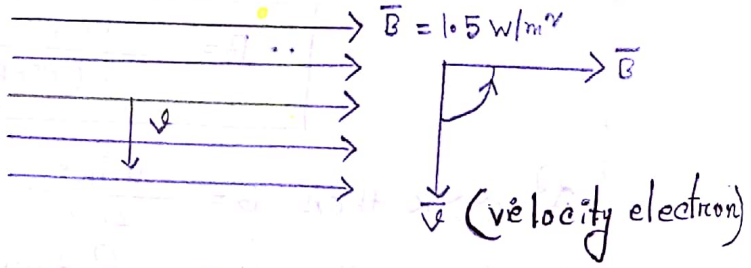
we have,

$KE = \frac{1}{2} m v^2$

$v = \sqrt{\frac{2KE}{m}}$

$= \sqrt{\frac{2 \times 5.0 \times 10^6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} kg}}$

$= 30.9529 \times 10^6$



we have the magnetic force

$F_B = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 30.9529 \times 10^6 \times 1.5 \times \sin 90°$

$= 7.7537 \times 10^{-12}$ N (Ans)

In the Bohr model of hydrozen atom the electrons circulates around the nucleus in a path of redius $5.1 \times 10^{-11} m$ at a frequency $\nu = 6.8 \times 10^{15}$ rev/sec

(a) What the value of B is set up at the centre of orbit.

(b) What is equivalent magnetic dipole moment?

(a) **Solve :→** we have B at the centre of the orbit

$B = \frac{\mu_0 i}{2R}$

$i = \frac{e}{t}$

$i = e\nu \quad (\nu = \frac{1}{t})$

$= 1.6 \times 10^{-19} \times 6.8 \times 10^{15}$ rev/sec

$= 1.1 \times 10^{-3} A$

Given that

$R = 5.1 \times 10^{-11} m$

$\mu_0 = 4\pi \times 10^{-7}\ W/Am^{-1}$ or TmA^{-1}

$\nu = 6.8 \times 10^{15}$ rev/sec

$B = \frac{4\pi \times 10^{-7} \times 1.1 \times 10^{-3}}{2 \times 5.1 \times 10^{-11}}$

$= 14\ w/m^2$ (Ans)

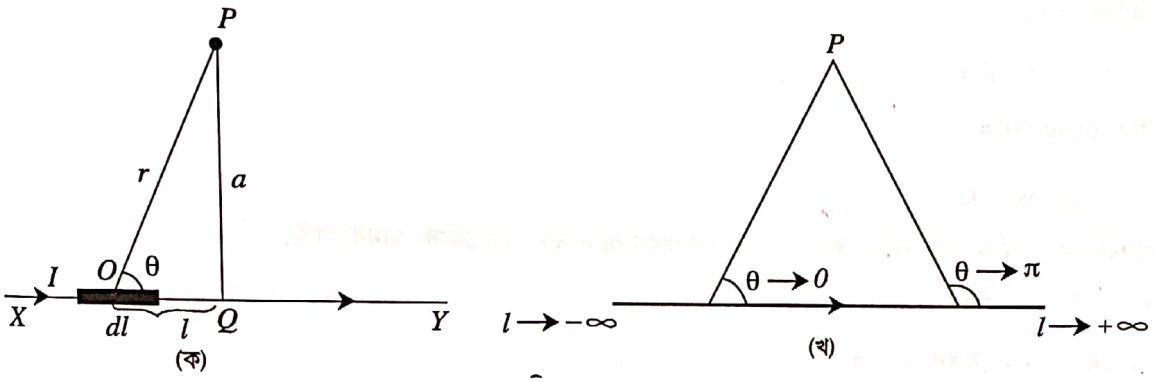
Scanned by CamScanner

(b) solve:→

৪.৫। বিয়োঁ-স্যাভার সূত্রের প্রয়োগ
Applications of Biot-Savart's Law

(ক) অসীম দৈর্ঘ্যের তড়িৎবাহী সরল তারের দরুন চৌম্বক ক্ষেত্র

বায়ু বা শূন্যস্থানে একটি দীর্ঘ ও সোজা পরিবাহী তার XY বিবেচনা করা যাক [চিত্র ৪·৮]। এর ভেতর দিয়ে X থেকে Y এর দিকে I প্রবাহ চলছে। এই তড়িৎ প্রবাহের ফলে P বিন্দুতে সৃষ্ট চৌম্বক ক্ষেত্র B হিসাব করতে হবে।



চিত্র : ৪.৮

ধরি,

$QP = a =$ পরিবাহীর মধ্যবিন্দু থেকে P বিন্দুর দূরত্ব।

$dl =$ পরিবাহীর মধ্যবিন্দু থেকে l দূরত্বে অবস্থিত পরিবাহীর ক্ষুদ্রাতিক্ষুদ্র দৈর্ঘ্য।

$r = dl$ এর মধ্যবিন্দু থেকে P বিন্দুর দূরত্ব।

$I =$ পরিবাহীতে তড়িৎ প্রবাহ।

$\theta =$ তড়িৎপ্রবাহ I বা dl এবং OP এর মধ্যবর্তী কোণ।

এখন বিয়োঁ-স্যাভাঁর সূত্র থেকে আমরা ক্ষুদ্র প্রবাহ উপাদানের জন্য P বিন্দুতে চৌম্বক ক্ষেত্রের মান পাই,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

এই সমীকরণকে যোগজীকরণ করে অসীম দৈর্ঘ্যের সরল পরিবাহীর জন্য P বিন্দুতে মোট চৌম্বক ক্ষেত্রের মান পাওয়া যাবে। যেহেতু পরিবাহীটি অসীম দৈর্ঘ্যের, সুতরাং যোগজীকরণের সীমা হবে $l = -\infty$ থেকে $l = \infty$ পর্যন্ত।

$$\therefore \quad B = \int dB = \int_{l=-\infty}^{l=\infty} \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$\text{বা, } B = \frac{\mu_0 I}{4\pi}\int_{l=-\infty}^{l=\infty}\frac{dl\sin\theta}{r^2} \quad \ldots \quad \ldots \quad (4.8)$$

Scanned by CamScanner

এই সমীকরণের r, θ এবং dl পরস্পর সম্পর্কযুক্ত হওয়ায় এই যোগজীকরণ সম্পন্ন করার জন্য এগুলোকে একটি মাত্র চলকের মাধ্যমে প্রকাশ করতে হবে। এখন ৪·৮ (ক) চিত্র থেকে-

$\cot\theta = \frac{-l}{a}$ [$\because$ l হচ্ছে Q বিন্দুর বাম দিকে]

$\therefore -l = a\cot\theta$ ... ... (4.9)

অন্তরীকরণ করে,

$dl = a\,\text{cosec}^2\theta\, d\theta$

আবার, $\text{cosec}\,\theta = \frac{r}{a}$

বা, $r = a\,\text{cosec}\,\theta$

যোগজীকরণের সীমা নির্ধারণের জন্য (4.9) সমীকরণ বা ৪.৮খ চিত্র থেকে আমরা পাই,

যখন $l = -\infty$, তখন $\theta = 0$

এবং যখন $l = \infty$, তখন $\theta = \pi$

সুতরাং (4.8) সমীকরণ দাঁড়ায়

$$B = \frac{\mu_0 I}{4\pi}\int_0^{\pi}\frac{(a\,\text{cosec}^2\theta\, d\theta)\sin\theta}{a^2\text{cosec}^2\theta}$$

$$= \frac{\mu_0 I}{4\pi a}\int_0^{\pi}\sin\theta\, d\theta$$

$$= -\frac{\mu_0 I}{4\pi a}\left[\cos\theta\right]_0^{\pi}$$

$$= -\frac{\mu_0 I}{4\pi a}\left[\cos\pi - \cos 0\right] = -\frac{\mu_0 I}{4\pi a}\left[-2\right]$$

$$\therefore B = \frac{\mu_0 I}{2\pi a}$$

সুতরাং অসীম দৈর্ঘ্যের সরল তড়িৎবাহী তারের জন্য চৌম্বক ক্ষেত্রের মান,

$$B = \frac{\mu_0 I}{2\pi a} \quad \ldots \quad \ldots \quad (4.10)$$

দিক : চৌম্বক ক্ষেত্রের দিক পাওয়া যায় ৪.৩ অনুচ্ছেদে বর্ণিত ম্যাক্সওয়েলের কর্ক-স্ক্রু সূত্র অথবা ফ্লেমিঙের ডান হস্ত সূত্র থেকে। এই দিক হয় বিবেচিত বিন্দুতে পরিবাহীর দৈর্ঘ্যের সাথে লম্ব বরাবর।

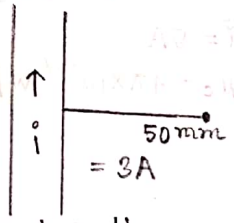
(খ) তড়িৎবাহী বৃত্তাকার কুণ্ডলীর কেন্দ্রে চৌম্বক ক্ষেত্র

একটি বৃত্তাকার কুণ্ডলী বিবেচনা করা যাক, যার ব্যাসার্ধ r। এই কুণ্ডলীর মধ্য দিয়ে I তড়িৎ প্রবাহ চলছে। কুণ্ডলীর কেন্দ্র P বিন্দুতে চৌম্বক ক্ষেত্র $\vec{B}$ এর মান নির্ণয় করতে হবে।

Scanned by CamScanner

(b) **solve:→**

$$\mu = NiA \qquad [A = \pi R^2]$$
$$= Ni\pi R^2$$
$$= 1.1\times10^{-3} \times 1 \times 3.1416 \times (5.1\times10^{-11})^2$$
$$= 90\times10^{-24}\ cm^{-}m^2 \text{ (Ans)}$$



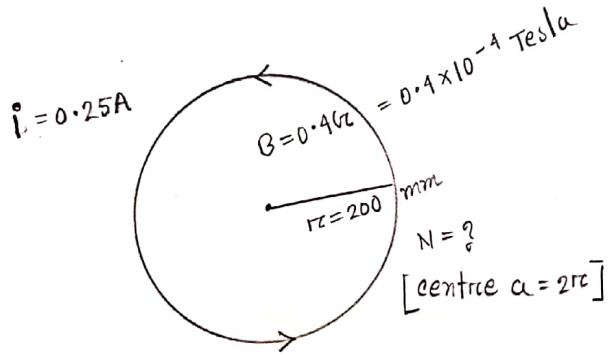
what is the magnetic field ?

solve:→

we know,

$$B = \frac{\mu_0 i}{2\pi r}$$
$$= \frac{4\pi\times10^{-7}\times3}{2\times3.1416\times 50\times10^{-3}}$$
$$= 12\times10^{-6}\ W/m^2 \text{ (Ans)}$$

Given that
$r = 50\times10^{-3}m$
$i = 3A$



solve:→

we know, $B = \frac{\mu_0 i N}{2r}$

$$\Rightarrow 0.4\times10^{-4} = \frac{4\pi\times10^{-7}\times0.25\times N}{2\times 200\times10^{-3}m}$$
$$\Rightarrow N = \frac{2\times200\times10^{-3}\times0.4\times10^{-4}}{4\pi\times10^{-7}\times0.25}$$
$$= 5.0929\times10^{-6} \text{ (Ans)}$$

For solenoid

$$B = \mu_0 i n$$
$$= \mu_0 i \frac{N}{L}$$

Scanned by CamScanner

Q A solenoid is 1m long and 3.0 cm in diameter. It has five layers of windings of 850 turns each and carries current of 5A.

a) solve :→ we know,

$$B = \mu_0 i n$$

$$B = \mu_0 i_0 \frac{N}{L}$$

$$= 4\pi \times 10^{-7} \times 5 \times 5 \times 850$$

$$= 26.70 \times 10^{-3} \ w/m^2 \ (Ans)$$

given Data

$$n = \frac{N}{L} = \frac{5 \times 850}{1m}$$

$$= 5 \times 850$$

$$i = 5A$$

$$\mu_0 = 4\pi \times 10^{-7} \ w/mA^{-1}$$

(b) solve :→

$$\varphi_B = \int \bar{B} \cdot d\bar{A}$$

$$\varphi_B = \int B dA \cos\theta$$

$$\varphi_B = B \int dA \cos 0^\circ$$

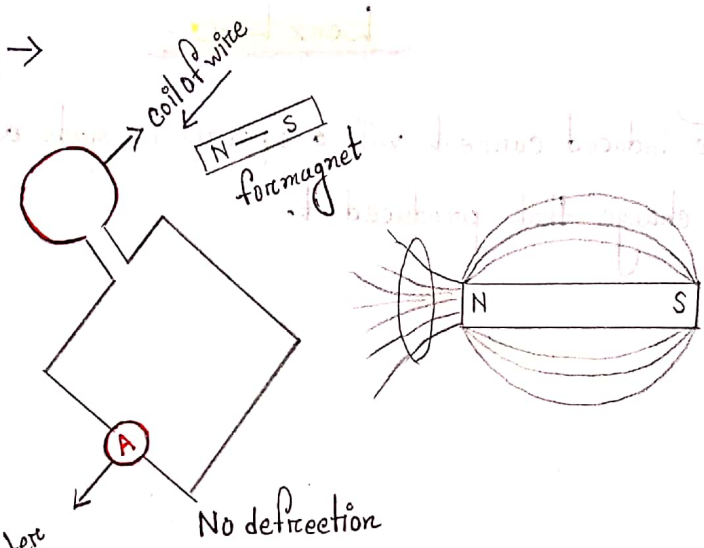
$$\varphi_B = B \int dA \quad [A = \pi R^2]$$

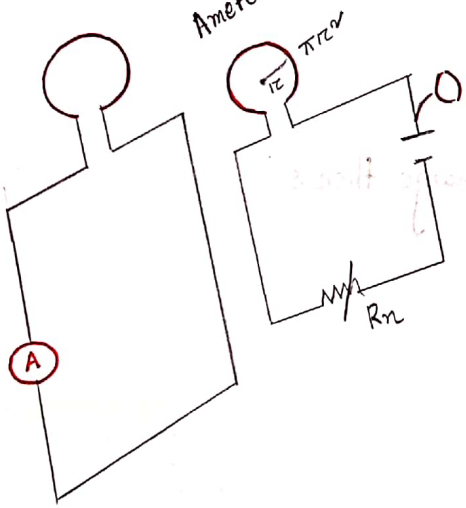
$$\varphi = BA \ (Ans) \quad [A = \pi \left(\frac{D}{2}\right)^2]$$

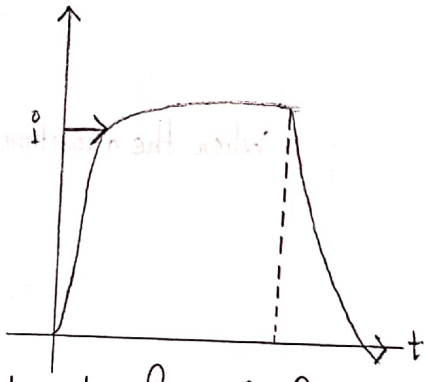
Scanned by CamScanner

Faraday's Law

Faraday's Experiment :→







Electromotive force (emf)

Faraday's Law

The induced emf in a current / coil is equal to the negetive rate at which the flux through the circit is changing i.e

$$\mathcal{E} = -\frac{d\varphi_B}{dt}$$

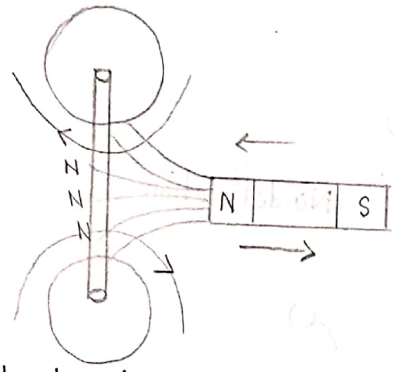
Where $\mathcal{E}$ is the induced emf and $\frac{d\varphi_B}{dt}$ if the rate of change of magnetic flux.

The minus sign is an indication of the direction of the induced emf.

Scanned by CamScanner

Lenz Law

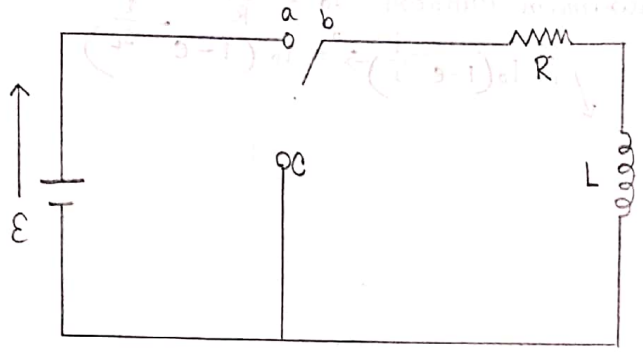
The induced current will a appear in such a direction that it opposed the change that produced it.

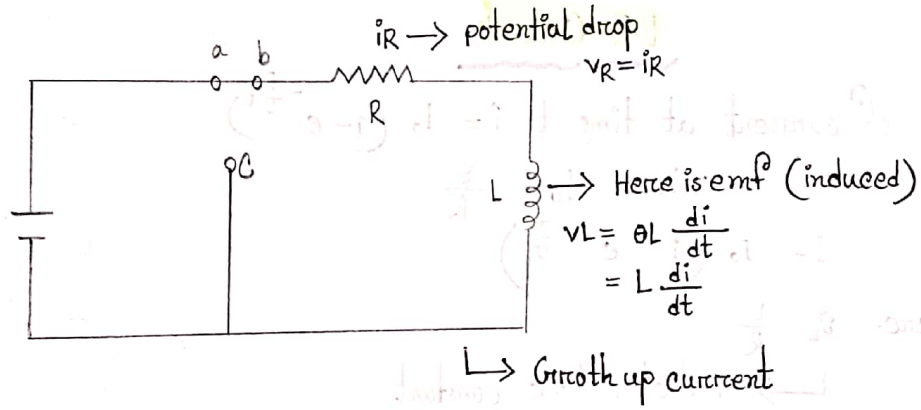


When the direction will change then s

Scanned by CamScanner

L.R Circuit





Growth up equation :→

$$-iR - L\frac{di}{dt} + \varepsilon = 0$$

$$\Rightarrow \varepsilon = iR + L\frac{di}{dt}$$

$$\Rightarrow \varepsilon - iR = L\frac{di}{dt}$$

$$v_L = L\frac{di}{dt}$$

$$\Rightarrow \frac{Rdi}{\varepsilon - iR} = \frac{R}{L}dt$$

$$\Rightarrow \int_0^i \frac{diR}{\varepsilon - iR} = \int_0^t \frac{R}{L}dt$$

$$\Rightarrow \int_0^i -\frac{dz}{z} = \int_0^t \frac{R}{L}dt$$

$$\Rightarrow \int_0^i -\ln z = \frac{R}{L}t$$

$$\Rightarrow \int_0^i \ln(\varepsilon - iR) = \frac{R}{L}t$$

$$\Rightarrow \ln(\varepsilon - iR) - \ln\varepsilon = -\frac{R}{L}t$$

$$\Rightarrow \ln\frac{\varepsilon - iR}{\varepsilon} = -\frac{R}{L}t$$

Let

$$\varepsilon - iR = z$$

$$-Rdi = dz$$

$$Rdi = -dz$$

$$\Rightarrow \frac{\varepsilon - iR}{\varepsilon} = e^{-\frac{R}{L}t}$$

$$\Rightarrow \varepsilon - iR = \varepsilon e^{-\frac{R}{L}t}$$

$$\Rightarrow iR = \varepsilon - \varepsilon e^{-\frac{R}{L}t}$$

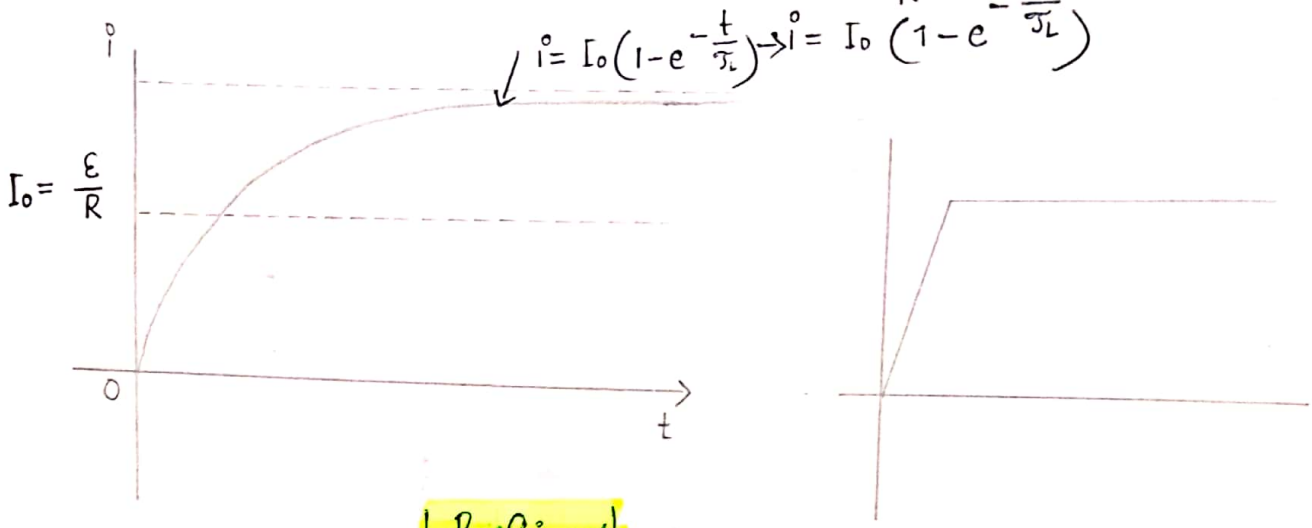
$$\Rightarrow iR = \varepsilon - \varepsilon e^{-\frac{R}{L}t}$$

$$\Rightarrow iR = \varepsilon\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{— (i)}$$

Scanned by CamScanner

If L is absent then the maximam current $I_0 = \frac{\varepsilon}{R}$



L.R Circuit

Growth of current at time t $i = I_0\left(1-e^{-\frac{R}{L}t}\right)$

where $I_0 = \frac{\varepsilon}{R}$

$$I = I_0\left(1-e^{-\frac{t}{\tau_L}}\right)$$

where, $\tau_L = \frac{L}{R}$

$\hookrightarrow$ inductive time constant

At time $I = I_0\left(1-e^{-\frac{t}{\tau_L}}\right)$

$\Rightarrow I = I_0\left(1-e^{-\frac{\frac{L}{R}}{\tau_L}}\right)$

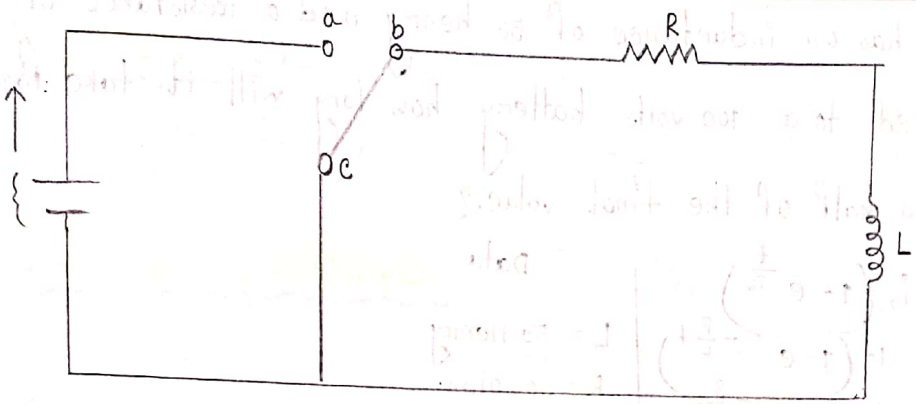
$\Rightarrow I = I_0\left(1-e\right)$

$\Rightarrow I = I_0\left(1-e^{-1}\right)$

$\Rightarrow i = I_0\left(1-0.37\right)$

$\Rightarrow i = i_0\ 0.63$

Scanned by CamScanner



$$\varepsilon = iR + L\frac{di}{dt}$$

For decay of current $\varepsilon = 0$

$$iR = -L\frac{di}{dt}$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides

$$\int_{i_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

$$\Rightarrow \ln[i]_{i_0}^{i} = -\frac{R}{L}t$$

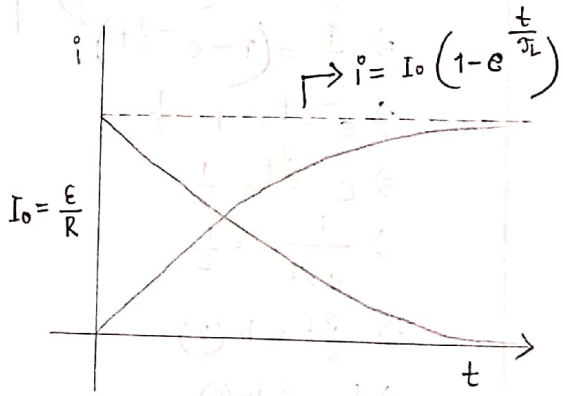
$$\Rightarrow \ln i - \ln i_0 = -\frac{R}{L}t$$

$$\Rightarrow \ln\frac{i}{i_0} = -\frac{R}{L}t$$

$$\Rightarrow \frac{i}{i_0} = e^{-\frac{R}{L}t}$$

$$\Rightarrow i = i_0 e^{-\frac{R}{L}t}$$

$$\Rightarrow i = i_0 e^{\frac{t}{\tau_L}}$$



Scanned by CamScanner

A solinoid has an inductance of 50 henry and a resistance of 30 ohms. It is connected to a 100 volts battery, how long will it take for the current to reach one half of the final value.?

Data

$L = 50$ Henry

$R = 300$ ohm

$i = \frac{i_0}{2}$

solve:→ $i = i_0\left(1 - e^{\frac{t}{\tau}}\right)$

$\Rightarrow i = i_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\Rightarrow \frac{i_0}{2} = i_0\left(1 - e^{-\frac{3}{5}t}\right)$

$\Rightarrow \frac{1}{2} = \left(1 - e^{-\frac{3}{5}t}\right)$

$\Rightarrow e^{-\frac{3}{5}t} = 1 - \frac{1}{2}$

$\Rightarrow e^{-\frac{3}{5}t} = \frac{1}{2}$

$\Rightarrow \frac{1}{e^{\frac{3}{5}t}} = \frac{1}{2}$

$\Rightarrow \frac{3}{5}t = \ln(2)$

$\Rightarrow t = \frac{5}{3}\ln(2)$

$= \frac{5}{3}(0.693)$

$\therefore t = 1.2$ sec

Electricity and Magnetism

$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\Rightarrow \frac{di}{dt} = I_0\, e^{-\frac{R}{L}t}$

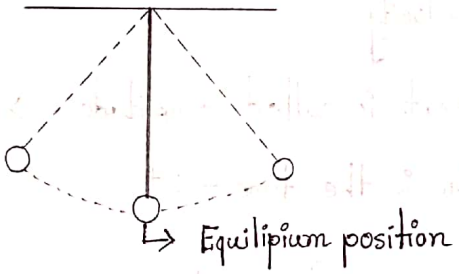
$\Rightarrow \frac{di}{dt} = \frac{\varepsilon}{R}\, e^{-\frac{R}{L}t}$

Scanned by CamScanner

Waves And Oscillation

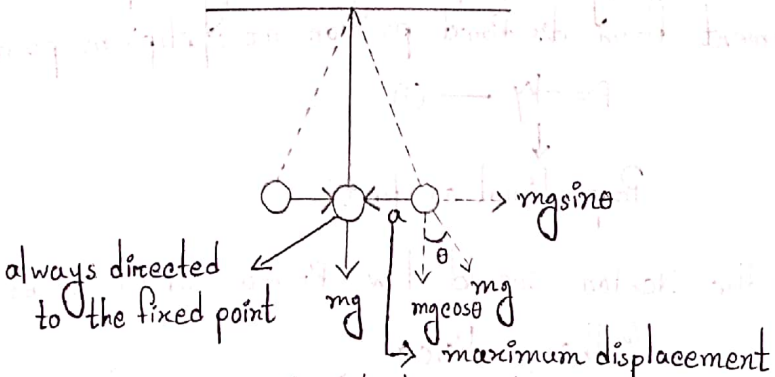
Periodic Motion :-> When a body moves in such a manner that in retraces in path after a regular time interval, it's motion is said to be preodic.

Oscillatory or Vibrating motion :-> If the motion of a body of a particel is reversed in direction after a regular interval of time it is called oscillatory or vibrating motion. Thus the oscillatory motion is a preodic to and fro (forward and backward) motion.



One Complete to and fro motion is called oscillation or vibration.

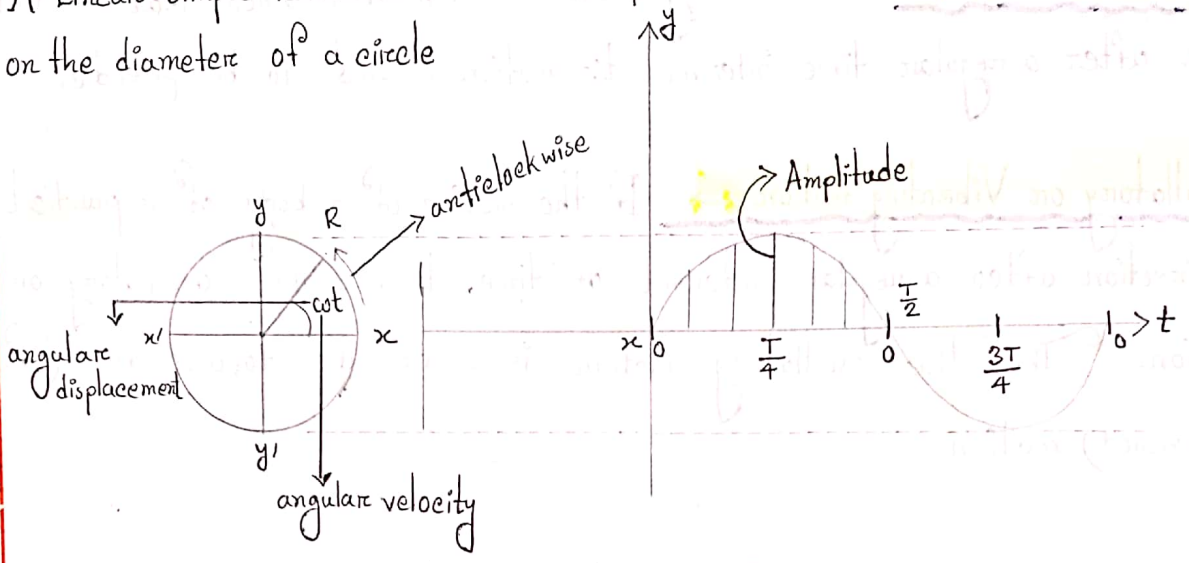
Simple Harmonic Motion :-> It is defined as the motion exicuted by a particle Oscillating to and fro along or in a straightline under the action of a force which is proportional to the displacement of the particle from a fixed point. and the direction of the force is always directed towards that point.



Force always opposite to the displacement $F = -kx$

Scanned by CamScanner

A Linear simple harmonic motion as a projection of a uniform circular motion on the diameter of a circle



The maximum displacement is called amplitude → a

One complete accillation is the time → t

Frequency :→ The number of complete oscillation in unit time.

Phase :→ The place of a particle in a particular time.

Differential Equation of Simple Harmonic Motion

If F be the force acting on a particle executing simple harmonic motion and y its displacement from it's fixed position or equilipium position and the

$$F = -ky \quad \text{— (1)}$$

k → Proportional constant

According to the Newtons second law $F = ma$, where a be the accilaration and m is the mass of the particle.

$$F = ma \quad \text{— (2)}$$

Scanned by CamScanner

From equation (1) and (2)

$$ma = -Ky$$

$$\Rightarrow m\frac{dv}{dt} = -Ky \qquad \Big| \; v = \frac{dy}{dt}$$

$$\Rightarrow m\frac{d}{dt}\left(\frac{dy}{dt}\right) = -ky$$

$$\Rightarrow m\frac{d^2y}{dt^2} = -Ky$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\frac{K}{m}y$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

where $\omega = \sqrt{\frac{K}{m}}$

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

Solⁿ:

Let $y = e^{mt}$

$$\Rightarrow \frac{dy}{dt} = me^{mt}$$

$$\Rightarrow \frac{d^2y}{dt^2} = m^2e^{mt}$$

$$\Rightarrow m^2e^{mt} + \omega^2 e^{mt} = 0$$

$$\Rightarrow (m^2 + \omega^2)e^{mt} = 0$$

Since $e^{mt} \neq 0$

so, $m^2 + \omega^2 = 0$

$$m^2 = -\omega^2$$

$$m = \sqrt{-\omega^2}$$

$$m = \pm i\omega \qquad \Big| \; i = \sqrt{-1}$$

$$\therefore y = e^{\pm i\omega t}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$y = A\sin\omega t + B\cos\omega t$$

$\cos\omega t + i\sin\omega t$ (For plus)

$\cos\omega t - i\sin\omega t$ (For minus)

$$y = Ae^{i\omega t} - Be^{-i\omega t}$$

$$\Rightarrow y = (A-B)\cos\omega t + \boxed{i(A\cdot B)\sin\omega t}$$

↳ displacement can't be immaginary

$$\Rightarrow y = (A+B)\cos\omega t$$

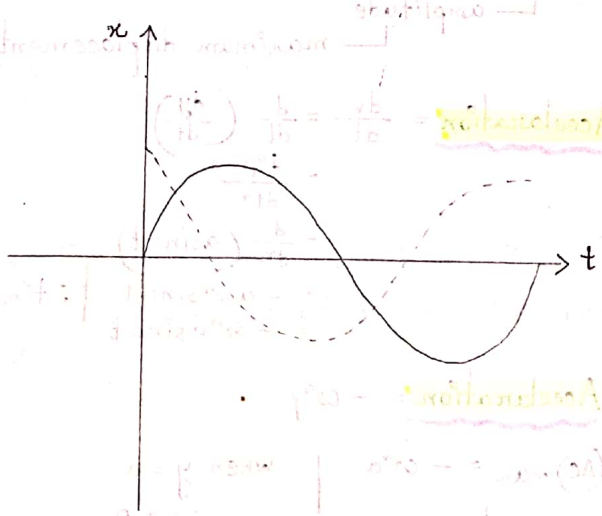
$$\Rightarrow y = A\cos\omega t$$

↳ phase

↳ Amplitude

$$\Rightarrow y = A\sin\omega t$$

$$\Rightarrow y = A\sin\omega t + B\cos\omega t$$



Velocity:→ $y = a\sin\omega t$

$$v = \frac{dy}{dt} = a\omega\cos\omega t$$

$$= a\omega\sqrt{1-\sin^2\omega t}$$

$$= a\omega\sqrt{1-\left(\frac{y}{a}\right)^2}$$

$$= a\omega\sqrt{\frac{a^2-y^2}{a^2}}$$

$$\therefore \boxed{v = \omega\sqrt{a^2-y^2}}$$

When $v = 0$, $v = \omega a$

Scanned by CamScanner

Simple Harmonic Motion

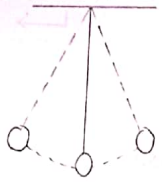
$y = a\sin\omega t$

$$\boxed{y = (a\sin\omega t + \varphi)}$$

velocity $v = a\omega\sqrt{1 - \frac{y^2}{a^2}}$

$= a\omega\sqrt{\frac{a^2 - y^2}{a^2}}$

$$\boxed{\therefore v = \omega\sqrt{a^2 - y^2}}$$



When, $y = 0$, $v = \omega a$

$V_{max} = \omega a$

$y = a$, $v = 0$

└ amplitude

└ maximum displacement

Accelaration $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right)$

$= \frac{d^2y}{dt^2}$

$= \frac{d}{d^2}(a\sin\omega t)$

$= -a\omega^2\sin\omega t$ | 2 times differentiation

$= -\omega^2 a\sin\omega t$

Accelaration $= -\omega^2 y$

$(A_c)_{max} = -\omega^2 a$ | when $y = a$

Accelaration $= 0$ | $\therefore y = 0$

Time period :→ $T = \frac{2\pi}{\omega}$ └ angular velocity

$\omega = \frac{2\pi}{T}$

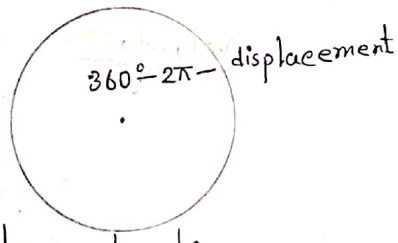
$\frac{d^2y}{dt^2} = -\omega^2 y$

$= -\omega^2 \times$ displacement

$\therefore \omega^2 = \frac{\text{accelaration}}{\text{displacement}}$

$\omega = \sqrt{\frac{\text{accelaration}}{\text{displacement}}}$

$\omega = \frac{2\pi}{T} = \sqrt{\frac{\text{accelaration}}{\text{displacement}}}$



complete accelaration → T

Scanned by CamScanner

Simple Harmonic Motion

1. Velocity $= \omega\sqrt{a^2 - y^2}$

2. Accelaration $= -\omega^2 y$

3. Time period $T = \frac{2\pi}{\omega}$

$$\omega = \sqrt{\frac{\text{accelaration}}{\text{displacement}}}$$

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{accelaration}}}$$

$$\text{or, } T = 2\pi\sqrt{\frac{m}{K}}$$

4. Frequency $f = \frac{1}{T}$

$$f = \frac{1}{2\pi\sqrt{\frac{m}{K}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$ $K \longrightarrow$ Proportional constant or Force Constant

Energy of Simple Harmonic Motion :→

Kinetic Energy :→

Let, the displacement of a particel exicuting S.H.M. at only instant be y. If the mass of the particel be m and its velocity at that instant be v. then the kinetic energy—

$$KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left(\frac{dy}{dt}\right)^2$$

$$y = a\sin(\underbrace{\omega t + \phi}_{\text{phase}})$$

$$\frac{dy}{dt} = a\omega\cos\omega t + \phi$$

So,
$$KE = \frac{1}{2}m\left[a\omega\cos(\omega t + \phi)\right]^2$$
$$= \frac{1}{2}ma^2\omega^2\cos^2(\omega t + \phi)$$

$$KE = \frac{1}{2}ma^2\frac{K}{m}\cos^2(\omega t + \phi)$$
$$= \frac{1}{2}a^2K\cos^2(\omega t + \phi)$$

Scanned by CamScanner

Potential Energy:→ The potential energy of a particle at the same instant is the amount of work that must be done in overcomming the force through a displacement y and is given by

$$U = \int_0^y F \cdot dy \longrightarrow \text{displacement}$$

Force. displacement = work done

$$PE = \int_0^y ma \cdot dy$$

$$= \int_0^y m\left(\frac{d^2y}{dx^2}\right) dy \longrightarrow \text{Rate of change of velocity}$$

$$\longrightarrow = \int_0^y m\omega^2 y \cdot dy = m\omega^2 \frac{y^2}{2} = \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} m \cdot \frac{k}{m} \cdot a^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} k a^2 \sin^2(\omega t + \phi)$$

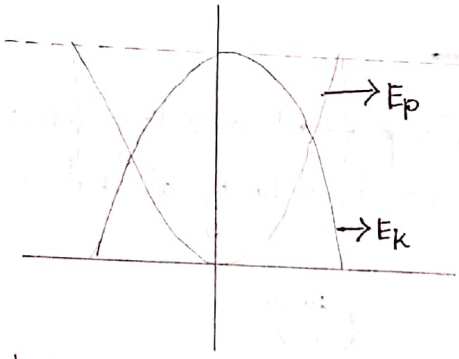
$$y = a\sin(\omega t + \phi)$$

$$\frac{d^2y}{dt^2} = -a\omega^2 \sin(\omega t + \phi) \longrightarrow \text{2 times}$$

$$= -\omega^2 - a\sin(\omega t + a) \quad [a\sin(\omega t + a) = y]$$

$$= -\omega^2 y$$

$$\boxed{\text{Total energy} = \frac{1}{2} k a^2}$$



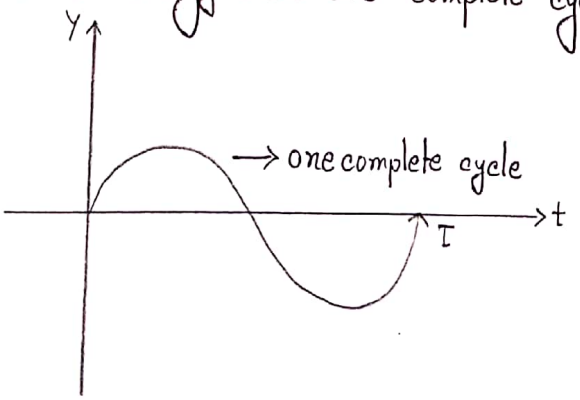
Average Value of Potential Energy:→

$$E_p = \frac{1}{2} k a^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t + \phi)$$

$$\left\{ \begin{array}{l} [\omega = \sqrt{\frac{k}{m}} \\ \omega^2 = \frac{k}{m} \\ k = \omega^2 m \end{array} \right.$$

Average potential energy over one complete cycle



Scanned by CamScanner

$$PE = \frac{\int_0^T \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t+\phi)\,dt}{\int_0^T dt}$$

$$= \frac{m\omega^2 a^2}{4T} \int_0^T 2\sin^2(\omega t+\phi)\,dt$$

$$= \frac{m\omega^2 a^2}{4T} \int_0^T [1-\cos 2(\omega t+\phi)]\,dt$$

$$= \frac{m\omega^2 a^2}{4T} \left[\int_0^T dt - \int_0^T \cos 2(\omega t+\phi)\,dt\right]$$

$$= \frac{m\omega^2 a^2}{4T} [T-0]$$

$$= \frac{1}{4} m\omega^2 a^2$$

$$= \frac{1}{4} ka^2$$ [This is the half of total energy]

Average kinetic Energy :→

$$KE = \frac{\int_0^T \frac{1}{2} m\omega^2 a^2 \cos^2(\omega t+\phi)\,dt}{\int_0^T dt}$$

$$= \frac{\int_0^T \frac{1}{4} m\omega^2 a^2\, 2\cos^2(\omega t+\phi)\,dt}{T}$$

$$= \frac{1}{4} ka^2$$ [half of total energy]

Ⅲ A body is vibrating with simple harmonic motion of amphilitude 15cm and frequency 4 Hz compute.

(a) The maximum values of accelaration and vilocity

(b) The accelaration and velocity when the displacement is 5cm.

solution :→ Vilocity$_{max}$ or $V_{max} = \omega A$

$= 25.1 \times 15$

$= 379.99 \text{ cm/s}$

Data

$a = 15\text{cm}$

$f = 4\text{Hz}$

$\omega = 2\pi f$

$= (2 \times 3.1416 \times 4) = 25.1 \text{ rad sec}^{-1}$

$A_{max} = -\omega^2 y$

$= -\omega^2 a$ [− no need cause its suggest the direction]

$= (25.1)^2 \times 15$

$= 9450.15 \text{ cm sec}^{-2}$

Scanned by CamScanner

田 The displacement of an occilating particle at an instant t is given by $y = a\cos\omega t + b\sin\omega t$. show that it is exicating a simple harmonic motion.

solve:→

$$\frac{dy}{dt} = -a\omega\sin\omega t + b\omega\cos\omega t$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\omega^2(a\cos\omega t + b\sin\omega t)$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\omega^2 y$$

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

Scanned by CamScanner

Composition of Simple Harmonic Motion

Composition of two simple harmonic rivocation of same frequency but different phase and amplitude

$$y_1 = a_1 \sin(\omega t + \alpha_1)$$
$$y_2 = a_2 \sin(\omega t + \alpha_2)$$

The result displacement will be $y = y_1 + y_2$

$$y = y_1 + y_2$$
$$\Rightarrow y = a_1 \sin(\omega t + \alpha_1) + (a_2 (\sin\omega t + \alpha_2))$$
$$= a_1 (\sin\omega t \cos\alpha_1 + \cos\omega t \sin\alpha_1) + a_2 (\sin\omega t \cos\alpha_2 + \cos\omega t \sin\alpha_2)$$
$$= \sin\omega t (a_1 \cos\alpha_1 + a_2 \cos\alpha_2) + \cos\omega t (a_1 \sin\alpha_1 + a_2 \sin\alpha_2)$$

Now Let,
$$a_1 \cos\alpha_1 + a_2 \cos\alpha_2 = A\cos\varphi \quad \text{—— (i)}$$
$$a_1 \sin\alpha_1 + a_2 \sin\alpha_2 = A\sin\varphi \quad \text{—— (ii)}$$

$$y = A\sin\omega t \cos\varphi + A\cos\omega t \sin\varphi$$
$$y = A(\sin\omega t \cos\varphi + \cos\omega t \sin\varphi)$$
$$y = A\sin(\omega t + \varphi)$$

$\hookrightarrow$ Amplitude of resulted vibration

$(i)^2 + (ii)^2 \Rightarrow$

$$A^2 \sin^2\varphi + A^2 \cos^2\varphi = (a_1 \sin\alpha_1 + a_2 \sin\alpha_2)^2 + (a_1 \cos\alpha_1 + a_2 \cos\alpha_2)^2$$
$$\Rightarrow A^2 (\sin^2\varphi + \cos^2\varphi) = a_1^2 \sin^2\alpha_1 + a_2^2 \sin^2\alpha_2 + 2a_1a_2 \sin\alpha_1 \sin\alpha_2 + a_1^2 \cos^2\alpha_1 + a_2 \cos^2\alpha_2 + 2a_1a_2 \cos\alpha_1 \cos\alpha_2$$
$$\Rightarrow A^2 = a_1^2 (\sin^2\alpha_1 + \cos^2\alpha_1) + a_2^2 (\sin^2\alpha_2 + \cos^2\alpha_2) + 2a_1a_2 (\sin\alpha_1 \sin\alpha_2 + \cos\alpha_1 \cos\alpha_2)$$
$$\Rightarrow A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos(\alpha_1 - \alpha_2)$$

$$\boxed{\therefore A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos(\alpha_1 - \alpha_2)}}$$

$$\frac{A\sin\varphi}{A\cos\varphi} = \frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2}$$
$$\tan\varphi = \frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2}$$

$$\boxed{\varphi = \tan^{-1}\left(\frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2}\right)}$$

Scanned by CamScanner

Special Case:

1. Same phase

$\alpha_1 = \alpha_2 = \alpha$

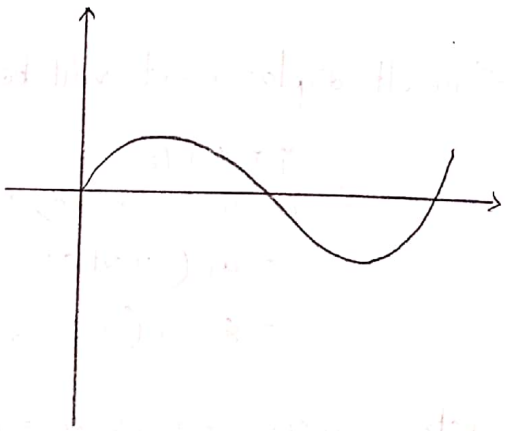
and $\alpha_1 - \alpha_2 = 0, 2\pi, 4\pi, \ldots 2n\pi : n = 0, 1, 2, 3, \ldots$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$$
$$= \sqrt{(a_1+a_2)^2}$$
$$= a_1 + a_2$$

$$\tan\varphi = \frac{(a_1+a_2)\sin\alpha}{(a_1+a_2)\cos\alpha} = \tan\alpha$$

$$\tan\varphi = \tan\alpha$$
$$\therefore \varphi = \alpha$$

$$Y = (a_1+a_2)\sin(\omega t + \alpha)$$



2. opposite phase

$A = a_1 - a_2$

if $a_1 = a_2$ for same phase

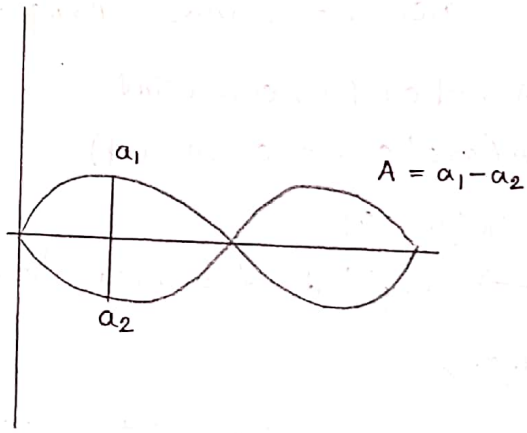
$A = a_1 + a_2 = a + a = 2a$

$A = 2a$

opposite $A = 0$

$Y = (2a\sin\omega t + \alpha)$

$Y = 0$



Example:→ $Y_1 = 2\sin\left(\omega t + \frac{\pi}{6}\right)$

$Y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$

$$Y = A\sin(\omega t + \varphi)$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_1 - \alpha_2)}$$
$$= \sqrt{(2)^2 + (3)^2 + 2\cdot2\cdot3\cos\left(\frac{\pi}{6} - \frac{\pi}{3}\right)}$$
$$= \sqrt{4 + 9 + 12\cos\left(-\frac{\pi}{6}\right)}$$
$$= 4.399$$

$$\varphi = \frac{4\pi}{15}$$

$$Y = 4.399\sin\left(\omega t + \frac{4\pi}{15}\right)$$

Scanned by CamScanner

Composition of two simple harmonic motion / vibrations at right angles to each other having equal frequencies but differing in phase and amplitude

Consider two simple harmonic vibrations, mutually perpendicular to each other.

$$x = a\sin(\omega t + \varphi) \quad \text{—— (1)}$$
$$y = b\sin\omega t \quad \text{—— (2)}$$

equation (1) can be written by

$$\frac{x}{a} = \sin(\omega t + \varphi)$$

$$\frac{x}{a} = \sin\omega t\cos\varphi + \cos\omega t\sin\varphi$$
$$= \sin\omega t\cos\varphi + \sqrt{1-\sin^2\omega t}\,\sin\varphi$$

from (2) $\frac{y}{b} = \sin\omega t$

$$\Rightarrow \frac{x}{a} = \frac{y}{b}\cos\varphi + \sqrt{1-\frac{y^2}{b^2}}\,\sin\varphi$$

$$\Rightarrow \frac{x}{a} - \frac{y}{b}\cos\varphi = \sqrt{1-\frac{y^2}{b^2}}\,\sin\varphi$$

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\cos\varphi\right)^2 = 1 - \frac{y^2}{b^2}\sin^2\varphi$$

$$\Rightarrow \frac{x^2}{a^2} - 2\frac{x}{a}\frac{y}{b}\cos\varphi + \frac{y^2}{b^2}\cos^2\varphi = \sin^2\varphi - \frac{y^2}{b^2}\sin^2\varphi$$

$$\Rightarrow \frac{x^2}{a^2} - 2\cdot\frac{xy}{ab}\cos\varphi + \frac{y^2}{b^2}\cos^2\varphi + \frac{y^2}{b^2}\sin^2\varphi = \sin^2\varphi$$

$$\Rightarrow \frac{x^2}{a^2} - 2\cdot\frac{xy}{ab}\cos\varphi + \frac{y^2}{b^2}(\sin^2\varphi + \cos^2\varphi) = \sin^2\varphi$$

$$\Rightarrow \frac{x^2}{a^2} - 2\cdot\frac{xy}{ab}\cos\varphi + \frac{y^2}{b^2} = \sin^2\varphi$$

$$\boxed{\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\varphi = \sin^2\varphi}$$

↳ Equation of ellipes.

Case-1:→ $\varphi = 0, 2\pi, 4\pi, \ldots 2n\pi$; $n = 0, 1, 2, 3$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\cdot\frac{xy}{ab} = 0$$

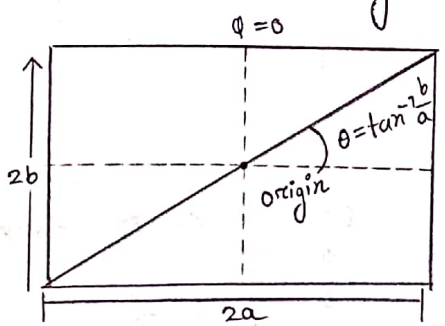
$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\Rightarrow \frac{x}{a} - \frac{y}{b} = 0$$

$$\Rightarrow \frac{y}{b} = \frac{x}{a}$$

$$\boxed{\Rightarrow y = \boxed{\frac{b}{a}}\cdot x}$$ → Similer to $y = mx$ ↳ straight line

slope



Scanned by CamScanner

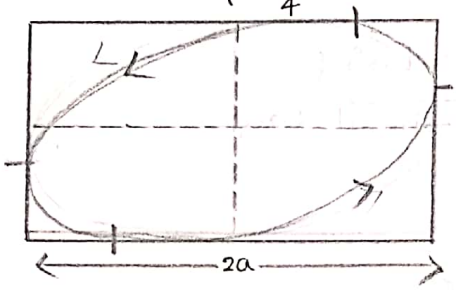
Case→2:→ if $\varphi = \frac{\pi}{4}$ radian

$$\cos = \frac{1}{\sqrt{2}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2 \cdot \frac{xy}{ab} \frac{1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\boxed{\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \sqrt{2}\,\frac{xy}{ab} = \frac{1}{2}}$$

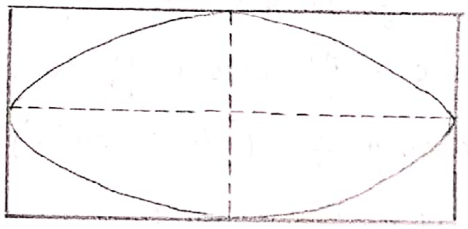
$\varphi = \frac{\pi}{4}$



Case→3:→ $\varphi = \frac{\pi}{2}$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \longrightarrow \text{ellipes}$$

$\varphi = \frac{\pi}{2}$ $a \neq b$

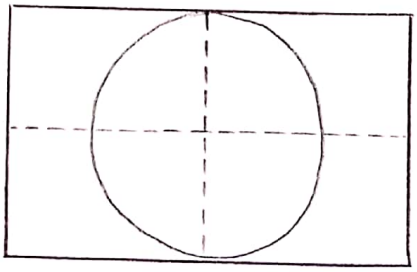


if $a = b$

$$\frac{x^2}{a^2} = \frac{y^2}{b^2} = 1$$

then it will be a circle

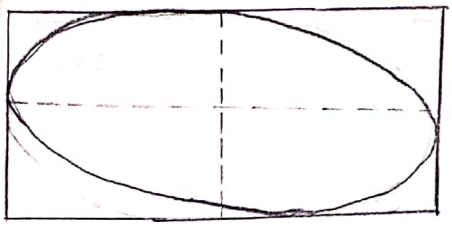
$\varphi = \frac{\pi}{2}$ and $a = b$



Case→4:→ $\varphi = \frac{3\pi}{4}$

$\cos\varphi = -\frac{1}{\sqrt{2}}$

$\sin\varphi = \frac{1}{\sqrt{2}}$



$\varphi = \pi$ — $\cos 180^\circ$ $\sin 180^\circ$

$\varphi = \frac{5\pi}{4}$

$\varphi = \frac{3\pi}{4}$, $\varphi = \frac{7\pi}{4}$

Scanned by CamScanner

The different equation of damped harmonic vibration

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = 0$$

where, $2\lambda = \frac{k^2}{m}$ and $\omega^2 = \frac{k}{m}$

solution:→

Let a trail solution

$$y = Ae^{nt}$$

$$\Rightarrow \frac{dy}{dt} = Ane^{nt}$$

$$\Rightarrow \frac{d^2y}{dt^2} = An^2e^{nt}$$

$$An^2e^{nt} + 2\lambda Ane^{nt} + \omega^2 Ae^{nt} = 0$$

$$\Rightarrow (An^2 + 2\lambda An + \omega^2 A)e^{nt} = 0$$

$$\Rightarrow Ae^{nt}(n^2 + 2\lambda n + \omega^2) = 0$$

since $Ae^{nt} \neq 0$ so,

$$n^2 + 2\lambda n + \omega^2 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow ax^2 + bx + c$$

$$\Rightarrow n = \frac{-2\lambda \pm \sqrt{4\lambda^2 - 4\omega^2}}{2}$$

$$= -\lambda \pm \sqrt{\lambda^2 - \omega^2}$$

$$A_1 = A_1 e^{\wedge}\left(-\lambda + \sqrt{\lambda^2 - \omega^2}\right)t + A_2 e^{\wedge}\left(-\lambda - \sqrt{\lambda^2 - \omega^2}\right)t$$

The value of A_1 and A_2 may be determined as follows:

$$\frac{dy}{dt} = A_1\left(-\lambda + \sqrt{\lambda^2 - \omega^2}\right)e^{\left(-\lambda + \sqrt{\lambda^2 - \omega^2}\right)t} + A_2\left(-\lambda - \sqrt{\lambda^2 - \omega^2}\right)e^{\left(-\lambda - \sqrt{\lambda^2 - \omega^2}\right)t}$$

The displacement is maximum at $t=0$ that is

$$y_{max} = a_0 \omega t = 0$$

Again, $y_{max} = A_1 + A_2 = a_0$

Again the velocity is zero at maximam displacement at time $t=0$, that is

$$\frac{dy}{dt} = 0 \text{ at } t = 0$$

Scanned by CamScanner

$$\Rightarrow A_1\left(-\lambda+\sqrt{\lambda^2-\omega^2}\right)+A_2\left(-\lambda-\sqrt{\lambda^2-\omega^2}\right)=0$$

$$\Rightarrow -A_1\lambda+A_1\sqrt{\lambda^2-\omega^2}-A_2\lambda-A_2\sqrt{\lambda^2-\omega^2}=0$$

$$\Rightarrow -\lambda\left(A_1+A_2\right)+\sqrt{\lambda^2-\omega^2}\left(A_1-A_2\right)=0$$

$$\Rightarrow -\lambda a_0+\sqrt{\lambda^2-\omega^2}\left(A_1-A_2\right)=0$$

$$\Rightarrow \sqrt{\lambda^2-\omega^2}\left(A_1-A_2\right)-\lambda a_0=0$$

$$\Rightarrow A_1-A_2=\frac{\lambda a_0}{\sqrt{\lambda^2-\omega^2}}$$

$$\Rightarrow 2A_1=a_0+\frac{\lambda a_0}{\sqrt{\lambda^2-\omega^2}}$$

$$\Rightarrow A_1=\frac{a_0}{2}\left(1+\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)$$

$$A_2=a_0-A_1$$

$$=a_0-\frac{a_0}{2}\left(1+\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)$$

$$=\frac{2a_0-a_0\left(-\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)}{}$$

$$=\frac{a_0-\frac{\lambda a_0^2}{\sqrt{\lambda^2-\omega^2}}}{2}$$

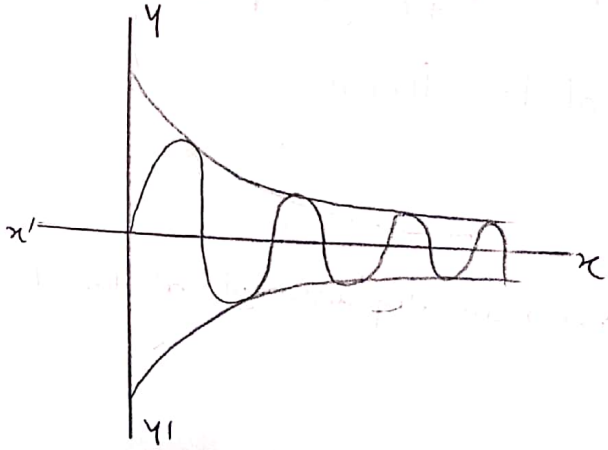
$$\therefore A_2=\frac{a_0}{2}\left(1-\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)$$

$$y=\frac{a_0}{2}\left(1+\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)e^{\left(-\lambda+\sqrt{\lambda^2-\omega^2}\right)t}+\frac{a_0}{2}\left(1-\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)e^{\left(-\lambda-\sqrt{\lambda^2-\omega^2}\right)t}$$

$$y=\boxed{\frac{a_0}{2}e^{-\lambda t}}\left[\left(1+\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)e^{\left(\sqrt{\lambda^2-\omega^2}\right)t}+\left(1-\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right)e^{\left(-\sqrt{\lambda^2-\omega^2}\right)t}\right]$$

amplitude of damped harmonic vibration

$y=a\sin\omega t$



Scanned by CamScanner

Damped Harmonic Vibration

$$y = A_1 e^{\left(-\lambda+\sqrt{\lambda^2-\omega^2}\right)t} + A_2 e^{\left(-\lambda-\sqrt{\lambda^2-\omega^2}\right)t}$$

↳ This term will be small than λ

$$y = A_1 e^{k_1 t} + A_2 e^{k_2 t}$$

$$y = \boxed{\frac{a_0}{2} e^{-\lambda t}} \left[\left(1+\frac{\lambda}{\sqrt{\lambda^2+\omega^2}}\right) e^{\left(\sqrt{\lambda^2-\omega^2}\right)t} + \left(1-\frac{\lambda}{\sqrt{\lambda^2-\omega^2}}\right) e^{\left(-\sqrt{\lambda^2-\omega^2}\right)t}\right]$$

↓ amplitude

(i) $\lambda^2 > \omega^2$

↳ If greater damping force will be greater

$\therefore \sqrt{\lambda^2+\omega^2}$ is positive and less than λ

$$y = A_1 e^{-k_1 t} + A_2 e^{-k_2 t}$$

(ii) $\lambda^2 = \omega^2$

$$\sqrt{\lambda^2-\omega^2} = 0$$

$$\lambda^2 \longrightarrow \omega^2$$

$$\sqrt{\lambda^2-\omega^2} = h$$

$$y = A_1 e^{(-\lambda+h)t} + A_2 e^{(-\lambda-h)t}$$

$$= e^{-\lambda t}\left(A_1 e^{ht} + A_2 e^{-ht}\right)$$

$$= e^{-\lambda t}\left[A_1\left(1+ht+\frac{(ht)^2}{2!}+\frac{(ht)^3}{3!}+\cdots\right) + A_2\left(1-ht+\frac{(ht)^2}{2!}+\cdots\right)\right]$$

$$y = e^{-\lambda t}\left[A_1(1+ht) + A_2(1-ht)\right]$$

$$y = e^{-\lambda t}\left[(A_1+A_2) + (A_1-A_2)ht\right]$$

$y = y_{max} = a_0$ and $\frac{dy}{dt} = 0$, at $t = 0$

$$A_1 + A_2 = a_0$$

$$A_1 - A_2 = \frac{\lambda a_0}{h}$$

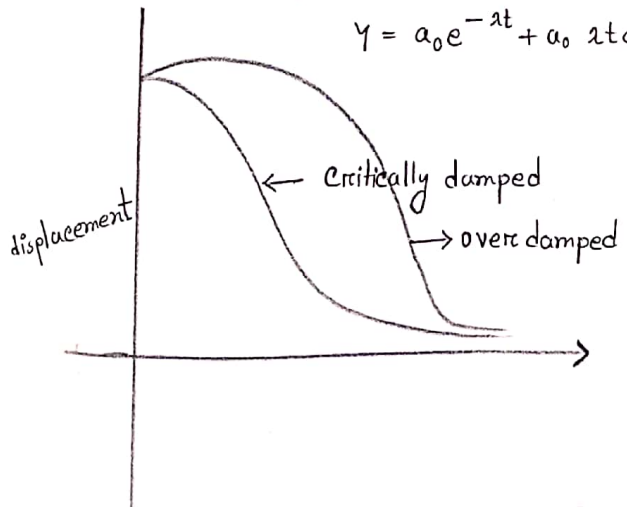
$$(A_1 - A_2)h = \lambda a_0$$

$$y = e^{-\lambda t}(a_0 + \lambda a_0 t)$$

$$y = a_0 e^{-\lambda t}(1+\lambda t)$$

↓ maximum amplitude

$$y = a_0 e^{-\lambda t} + a_0 \lambda t e^{-\lambda t}$$



Scanned by CamScanner

(iii) $\lambda^2 < \omega^2 = \sqrt{\lambda^2 - \omega^2}$.

$= \sqrt{-1}$

$= i$

$y^{-\lambda t} = \sqrt{-1}\ \sqrt{\omega^2 - \lambda^2}$

$= ig \quad [g = \sqrt{\omega^2 - \lambda^2}]$

$y = A_1 e^{(-\lambda + ig)t} + A_2 e^{(-\lambda - ig)t}$

$y = e^{-\lambda t}\left[A_1 e^{igt} + A_2 e^{-igt}\right]$

$e^{i\theta} = \cos\theta + i\sin\theta$

$= e^{-\lambda t}\left[A_1(\cos gt + i\sin gt) + A_2(\cos gt - i\sin gt)\right]$

$= e^{-\lambda t}\left[(A_1 + A_2)\cos gt + i(A_1 - A_2)\sin gt\right]$

$y = a_0 e^{-\lambda t}\sin(gt + \varphi)$

Scanned by CamScanner

Forced Harmonic Vibration

$$\boxed{\frac{d^2y}{dt^2} + 2\lambda \frac{dy}{dt} + \omega^2 y = \boxed{f_0 \sin pt}} \rightarrow f = \frac{p}{2\pi} \text{ [frequency of driving force]}$$

Differential equation of forced harmonic vibration

$$F = F_0 \sin pt$$

Let a particular solution of the above equation of vibration of the body after the steady states has been attained be

$$y = A\sin(pt - \theta)$$

$A \rightarrow$ Amplitude constant

$p \rightarrow$ Angular velocity

$$\frac{dy}{dt} = Ap\cos(pt-\theta)$$

$$\Rightarrow \frac{d^2y}{dt^2} = -Ap^2\sin(pt-\theta) \quad [y = A\sin(pt-\theta)]$$

$$= -p^2 y$$

$$\Rightarrow -Ap^2\sin(pt-\theta) + 2\lambda Ap\cos(pt-\theta) + \omega^2 A\sin(p-\theta) = f_0 \sin pt$$

$$\Rightarrow Ap^2\sin(pt-\theta) + 2\lambda Ap\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f_0\sin[(pt-\theta)+\theta]$$

$$\Rightarrow Ap^2\sin(pt-\theta) + 2\lambda Ap\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f_0\sin(pt-\theta)\cos\theta + f_0\cos(pt-\theta)\sin\theta$$

$$\Rightarrow A\sin(pt-\theta)(\omega^2 - p^2) + 2\lambda Ap\cos(pt-\theta) = f_0\sin(pt-\theta)\cos\theta + f_0\cos(pt-\theta)\sin\theta \quad \text{--- (1)}$$

$$A(\omega^2 - p^2) = f_0\cos\theta \quad \text{--- (2)}$$

$$2\lambda Ap = f_0\sin\theta \quad \text{--- (3)}$$

$$(2)^2 + (3)^2 \Rightarrow \quad A^2(\omega^2 - p^2)^2 + 4\lambda^2 A^2 p^2 = f_0^2 \cdot 1$$

$$\Rightarrow A = \sqrt{\frac{f_0^2}{(\omega^2 - p^2)^2 + 4\lambda^2 p^2}}$$

$$\Rightarrow \tan\theta = \frac{2\lambda Ap}{A(\omega^2 - p^2)}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2\lambda p}{\omega^2 - p^2}\right)$$

$F = F_0 \sin pt$

$\uparrow$

Periodic external force applied to compensate the damping force.

$$y = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4\lambda^2 p^2}} \sin\left[pt - \tan^{-1}\left(\frac{2\lambda p}{\omega^2 - p^2}\right)\right]$$

$$y = a_0 e^{-\lambda t}\sin(gt + \phi)$$

$$y = \boxed{a_0 e^{-\lambda t}}\sin(gt+\phi) + A\sin(pt - \theta)$$

$a_0 e^{-\lambda t} \rightarrow$ Amplitude

$g \rightarrow f = \frac{g}{2\pi}$

$A \rightarrow$ Amplitude

$p \rightarrow f = \frac{p}{2\pi}$

Scanned by CamScanner

When damping is finite the value of amplitude will be maximum when denominator in the equation has its minimum value.

when, $\frac{d}{dp}\left[(\omega^2-p^2)^2+4\lambda^2p^2\right]=0$

$$p=\sqrt{\omega^2-2\lambda^2}$$

$$f=\frac{p}{2\pi}=\frac{1}{2\pi}\sqrt{\omega^2-2\lambda^2}$$

↓

Resonance frequency

$$A_{max}=\frac{f_0}{2\lambda(\omega^2-\lambda^2)^{\frac{1}{2}}}$$

Stationary Wave

$$y=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

The amplitude $A=2a\cos\frac{2\pi x}{\lambda}$

The amplitude will be maximum if $\cos\frac{2\pi x}{\lambda}=\pm1$ and $\sin\frac{2\pi x}{\lambda}=0$

$$\frac{2\pi x}{\lambda}=0,\ \pi,\ 2\pi,\ \text{---}\ \text{etc}$$

$$x=\frac{n\pi}{2}$$

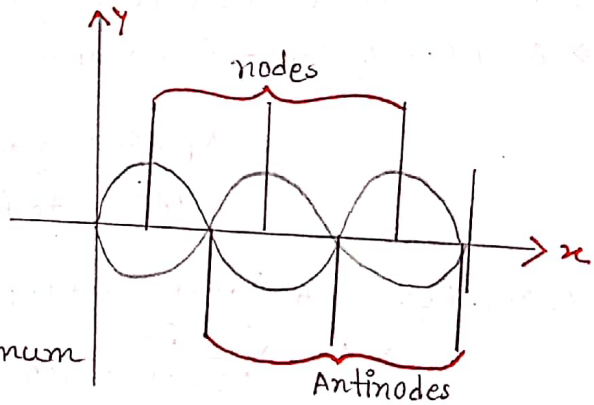
$$x=0,\ \underbrace{\frac{\lambda}{2},\ \frac{2\lambda}{2},\ \frac{3\lambda}{2}}$$

↓

Antinodes

↓

Position where the amplitudes are maximum



$$t=0,\ \frac{T}{2},\ \frac{2T}{2},\ \frac{3T}{2}$$

Amplitude minimum, when $\cos\frac{2\pi x}{\lambda}=0$ or $\sin\frac{2\pi x}{\lambda}=1$

$$\frac{2\pi x}{\lambda}=\frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2}\ \text{----}$$

$$=(2n+1)\frac{\pi}{2}$$

$$x=\frac{\lambda}{4},\ \frac{3\lambda}{4},\ \frac{5\lambda}{4}\ \text{----}$$

Velocity and Accelaration of standing wave

$$\frac{dy}{dt}=\frac{4\pi av}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$$

velocity is maximum when $\cos\frac{2\pi x}{\lambda}=\pm1$

$$x=0,\ \frac{\lambda}{2},\ \lambda\ \text{----}$$

Scanned by CamScanner

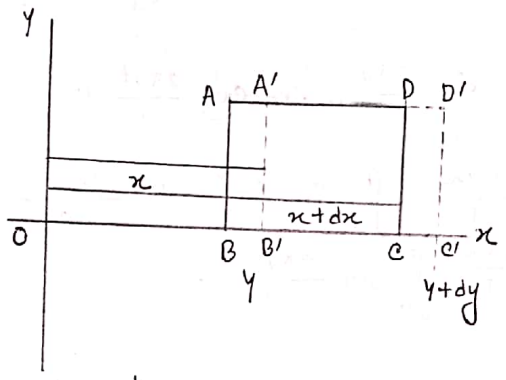
Similarly, minimum velocity $\rightarrow$ $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ----

Acceleration $\rightarrow$ $\frac{d^2y}{dt^2} = -\frac{8\pi^2 v^2}{\lambda^2} a\cos\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda}$

$= -\frac{4\pi^2 v^2}{\lambda^2} 2a\cos\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda}$

$= -\frac{4\pi v^2}{\lambda^2} y$

Energy



initial volume of ABCD $= \alpha dx$

$\hookrightarrow$ Crosssectional area

Final volume of A'B'C'D' $= \alpha[x+dx+y+dy-x-y]$

$= \alpha(dx+dy)$

changeing volume $= \alpha dy$

Hence strain produced in the medium $= \frac{\alpha dy}{\alpha dx} = \frac{dy}{dx}$

If the change in pressure is p

Bulk modulus $E = -\frac{p}{\frac{dy}{dx}}$

$p = E\left(-\frac{dy}{dx}\right)$

$\hookrightarrow$ Forced per unit area

From Newtons Law,

The velocity of logitudind

$v = \sqrt{\frac{E}{\rho}}$

$\Rightarrow v^2 = \frac{E}{\rho}$

$\Rightarrow E = v^2\rho$

$\therefore p = -v^2\rho\frac{dy}{dx}$

$p = v^2\rho\frac{4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$

$p = p_0 \sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$

Scanned by CamScanner

Where, $P_0 = v\rho \frac{4\pi a}{\lambda}$

$P_x = P_0 \sin \frac{2\pi x}{\lambda} \frac{2\pi vt}{\lambda}$

The velocity of particle $v = \frac{dy}{dx} = \frac{4\pi va}{\lambda} \cos \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}$

$= v_x \cos \frac{2\pi vt}{\lambda}$

$\therefore v_x = \frac{4\pi va}{\lambda} \cos \frac{2\pi x}{\lambda}$

work done per unit area $dw = p \cdot v \cdot dt$ $[w = Fx]$

$w = \int_0^T p \cdot v \cdot dt$

Energy transferred $= \int_0^T P_x \sin \frac{2\pi vt}{\lambda} \cdot v_x \cos \frac{2\pi vt}{\lambda} dt$

Rate of transferred $= \frac{P_x v_x}{T} \int_0^T \sin \frac{2\pi v}{\lambda} \cos \frac{2\pi vt}{\lambda dt}$

$= \frac{P_x v_x}{T} \int_0^T \sin \frac{4\pi vt}{\lambda} dt$

$= 0$

Wave velocity and Group Velocity

Progressive wave, $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$y = a \sin (\omega t - kx)$

where, $\omega = \frac{2\pi}{\lambda} = 2\pi f$ $\frac{\omega}{k} = \frac{\lambda}{t}$

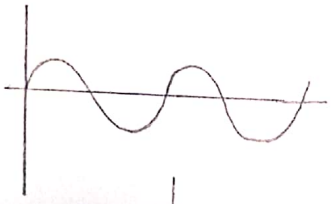
$\omega \rightarrow$ Angular velocity

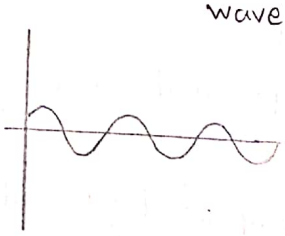
$k = \frac{2\pi}{\lambda}$ $\frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}}$, $\frac{\omega}{k} = f\lambda$

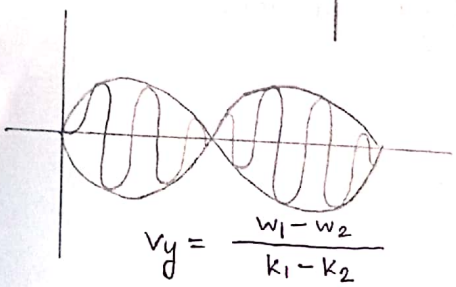
$k \rightarrow$ Wave number

$\therefore \frac{\omega}{k} = \boxed{v_p} \rightarrow$ phase velocity

wave velocity







$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2}$

$v_f = \frac{\Delta \omega}{\Delta k}$

Doplers Effect

$n' = \left(\frac{v}{v-a}\right) n$ $n' > n$

$n' \rightarrow$ listener's frequency

$v \rightarrow$ velocity of sound wave

$a \rightarrow$ listeners velocity

Scanned by CamScanner

Doplen's Effect

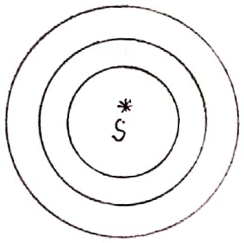
$$n' = \left(\frac{v}{v-a}\right)n$$

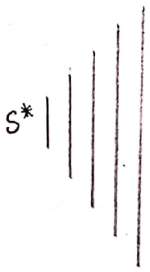
$n' > n$

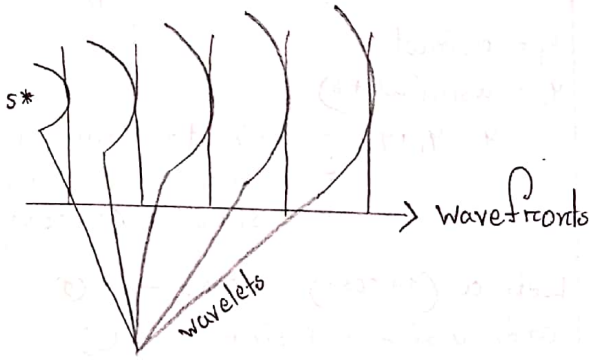
$$n' = \frac{v}{(v+a)n}$$

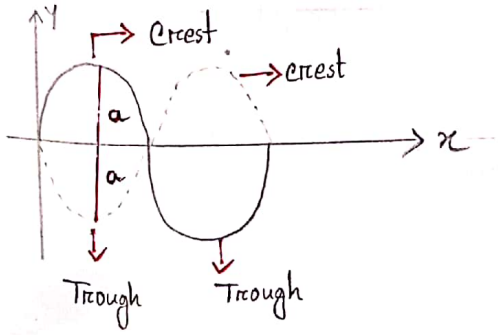
Optics (Physical)

Huygen's Principle :→ All points on a wavefronts serves as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.



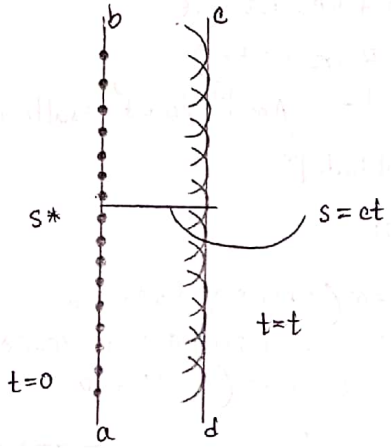






$A = a - a = 0$

$I = A^2 = 0$

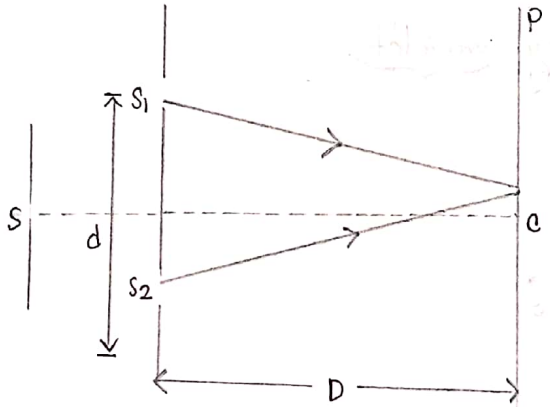


Scanned by CamScanner

Interference :→ When two light waves of same frequency and amplitude cross each other at a certain point in medium then the resultant intensity will be eighter great or less than that which would be given by one wave alone. Due to the super position of two waves. This phenominon is known as interference of light.

1. Constructive interference → Intensity is more
2. Destructive interference → Intensity is less.

Analytical Calculations of Interference



$y_1 = a\sin\omega t$

$y_2 = a\sin(\omega t + \delta)$

$$y = y_1 + y_2 = a\sin\omega t + a\sin(\omega t + \delta)$$
$$= a\sin\omega t + a\sin\omega t\cos\delta + a\cos\omega t\sin\delta$$
$$= a\sin\omega t + (1+\cos\delta) + a\cos\omega t\sin\delta \quad \text{—— (1)}$$

Let, $a(1+\cos\delta) = R\cos\theta$ —— (2)

and, $a\sin\delta = R\sin\theta$ —— (3)

$$y = R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$$
$$y = R\sin(\omega t + \theta)$$

↳ Amplitude of resultant wave

Intencity = $|\text{amplitude}|^2$

$I = R^2$ —— (4)

$$R^2(\sin^2\theta + \cos^2\theta) = a^2(1+\cos\delta)^2 + a^2\sin^2\delta$$
$$= a^2 + 2a^2\cos\delta + a^2\cos^2\delta + a^2\sin^2\delta$$
$$= a^2 + a^2(\sin^2\delta + \cos^2\delta) + 2a^2\cos\delta$$

$$R^2 = a^2 + a^2 + 2a^2\cos\delta$$
$$= 2a^2(1+\cos\delta)$$
$$= 2a^2\, 2\cos^2\frac{\delta}{2} = 4a^2\cos^2\frac{\delta}{2}$$

$$\boxed{\therefore I = 4a^2\cos^2\frac{\delta}{2}}$$

Scanned by CamScanner

Condition of Maxima and Minima

1. **Maxima:→** when $\delta = 0, 2\pi, 2(2\pi), \ldots n(2\pi)$ Then it will be bright phase

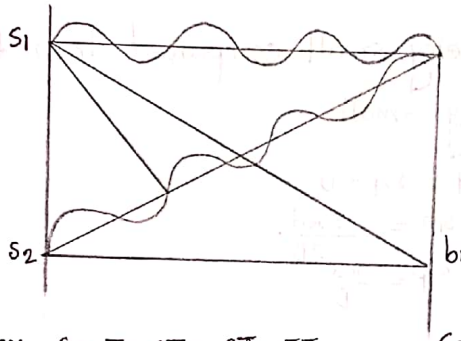
Intencity will be maximum that is →

$$\text{path difference} = \frac{\lambda}{2\pi} \times \text{phasediff}$$

$$I_{max} = 4a^2$$

path difference $= 0, \lambda, 2\lambda, 3\lambda, \ldots n\lambda$

if two phase are same



bright phase

2. **Minima:→** when $\delta = \pi, 2\pi, 3\pi, 5\pi, \ldots (2n+1)\pi$

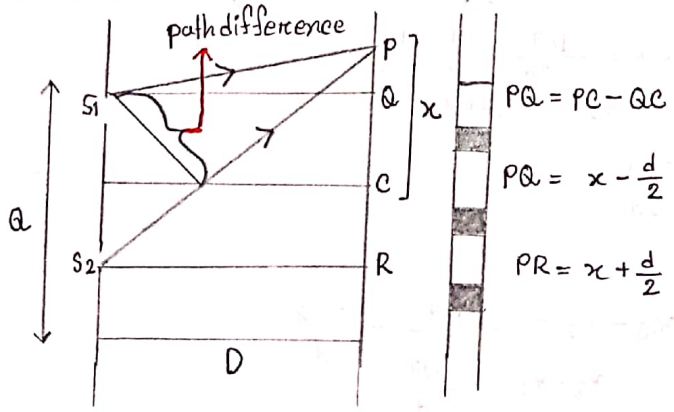
$$x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots (2n+1)\frac{\lambda}{2}$$

Intencity minimum

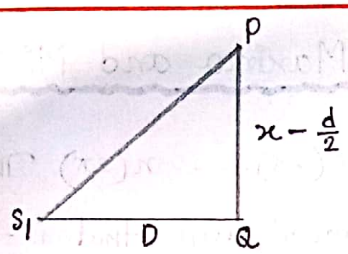
$I_{min} = 0$ Dark phase

Crest - Crest or Top - Top interfaire

Positions of Interference Fringes:→



Scanned by CamScanner



$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x+\frac{d}{2}\right)^2\right] - \left[D^2 + \left(x-\frac{d}{2}\right)^2\right]$$

$$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = D^2 + x^2 + xd + \frac{d^2}{4} - D^2 - x^2 + xd - \frac{d^2}{4}$$

$$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = 2xd$$

$$\Rightarrow (S_2P - S_1P) = \frac{2xd}{(S_2P - S_1P)}$$

In practice d and x are very small compared to D the separation distance between S_1 and S_2 is very small

$$S_1P \approx S_2P \approx D$$

$$S_2P - S_1P = \frac{2xd}{2D}$$

$$S_2P - S_1P = \frac{xd}{D}$$

Bright Fringes:→

$$\frac{xd}{D} = n\lambda \; ; \; n = 0, 1, 2, 3, \ldots$$

$$x_n = \frac{nD\lambda}{d}$$

$$n = 0, \; x_0 = 0$$

$$n = 1, \; x_1 = \frac{D\lambda}{d}$$

$$x_3 = \frac{3D\lambda}{d}$$

Dark Fringes:→

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2} \; ; \; n = 0, 1, 2, \ldots$$

$$x_n = \frac{(2n+1)D\lambda}{2d}, \; x_3 = \frac{5D\lambda}{2d}$$

$$n = 1, \; x = \frac{D\lambda}{2d} = \frac{1}{2}\frac{D\lambda}{d}$$

$$x_2 = \frac{3D\lambda}{2d}$$

$$x_3 - x_2 = \frac{5D\lambda}{2\lambda} - \frac{3D\lambda}{2d}$$

$$\beta = \frac{D\lambda}{d}$$

Fringe Width:→

$$x_3 - x_2 = \frac{3D\lambda}{2} - \frac{2D\lambda}{d}$$

$$x_3 - x_2 = \frac{D\lambda}{d}$$

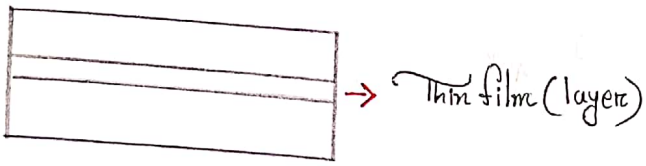
$$\beta = \frac{D\lambda}{d}$$

$$\beta \propto \lambda$$

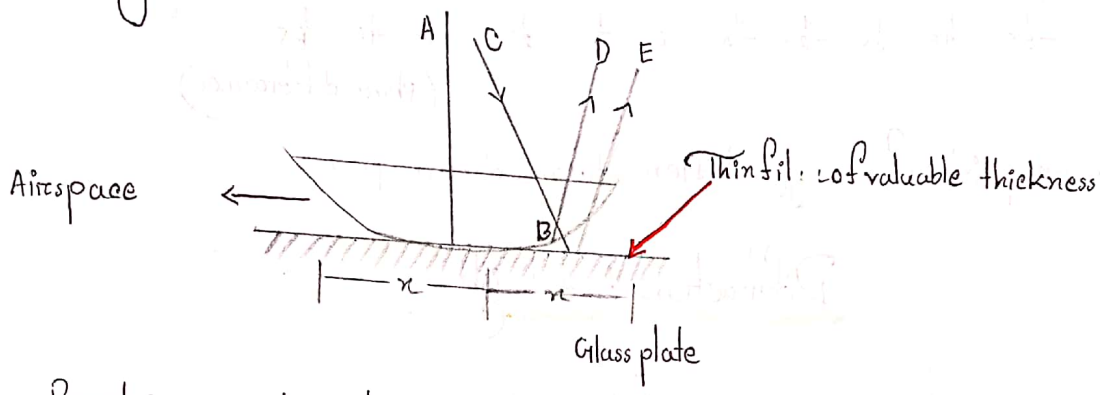
If colour will change then fringe width will be change

Scanned by CamScanner

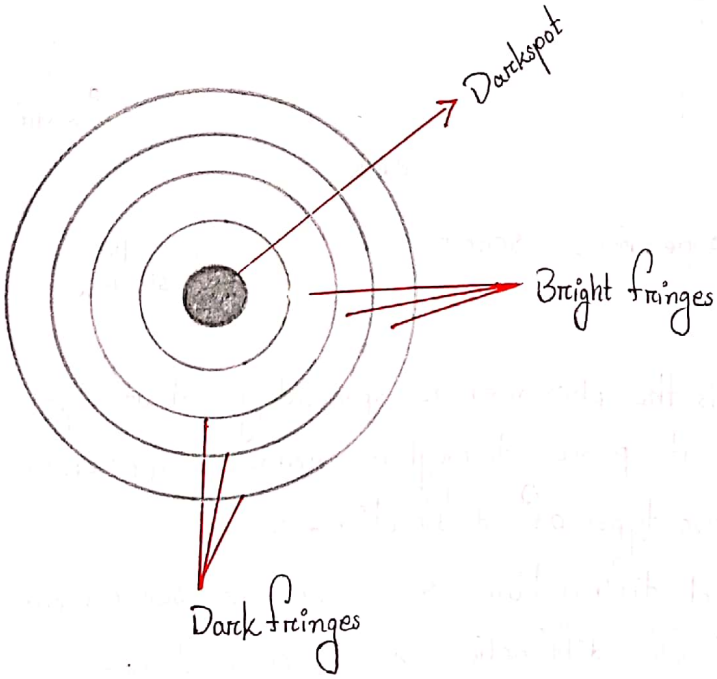
Thin Film Interference :->



Newton rings :->



B and C are coherent source



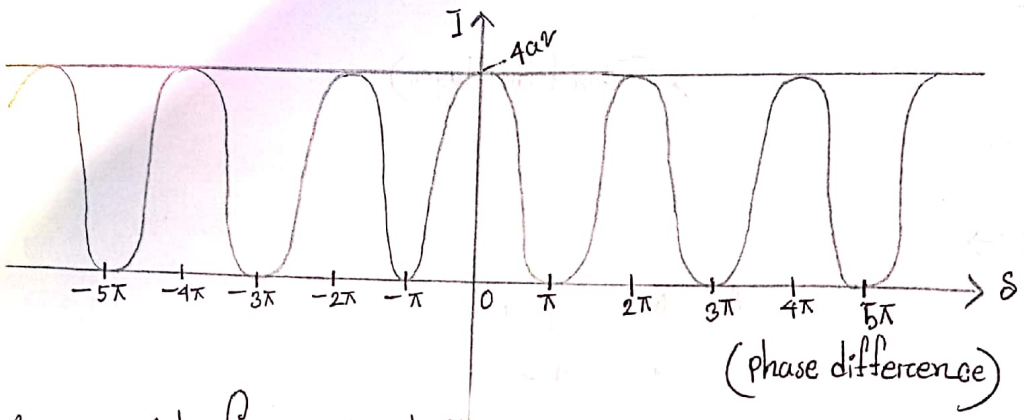
$$R = \frac{DM^2p - Dn^2}{4p^2}$$

↳ Difference between two ring

$$I = 4a^2\cos^2\frac{\delta}{2}$$

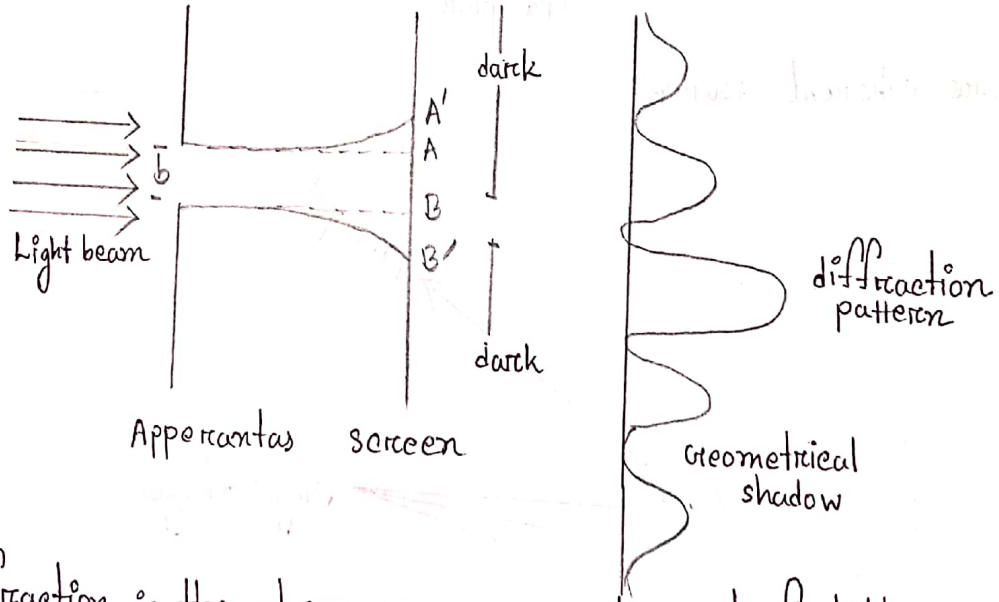
Scanned by CamScanner

Intencity or Energy Distribution:→



Minimum point of energy shiften to maximum point

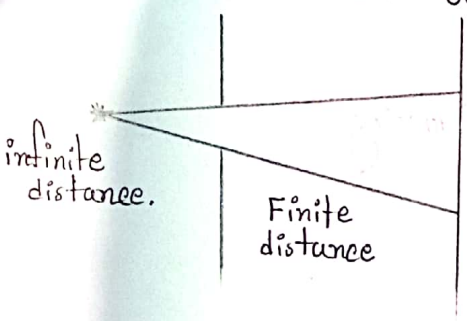
Diffraction And Light



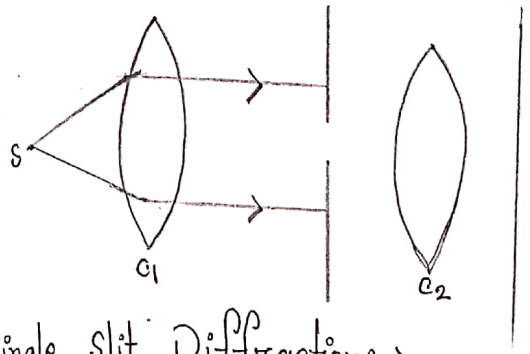
Diffraction is the phenomenon spreading out of light accross the geometrical shadow when it passes through a narrow apperture.

There are two types of diffraction →

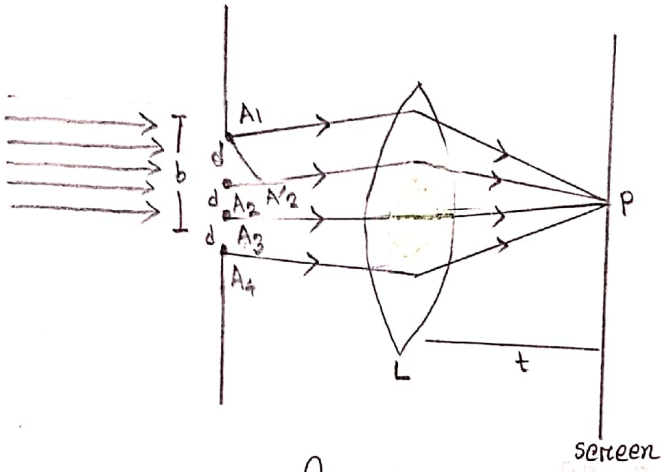
1. Frennel diffraction:→ Source and screen are at finite distance.
2. Fraunhoffer diffraction:→ Source and screen are at infinite distance.



Scanned by CamScanner



Single slit Diffraction:→



There are n number of point sources separation distances between consecutive two point source is d

then $b = (n-1)d$

path difference $A_2A'_2 = d\sin\theta$

phase difference $\varphi = \frac{2\pi}{\lambda} d\sin\theta$

$$E_1 = a\cos\omega t$$
$$E_2 = a\cos(\omega t - \varphi)$$
$$E_3 = a\cos(\omega t - 2\varphi)$$
$$E = a\cos[\omega t - (n-1)\varphi]$$

$$E = E_1 + E_2 + E_3 + \cdots + E_n$$
$$= a\cos\omega t + a\cos(\omega t - \varphi) + a\cos(\omega t - 2\varphi) + \cdots a\cos[\omega t - (n-1)\varphi]$$
$$= a \frac{\sin\frac{n\varphi}{2}}{\sin\frac{\varphi}{2}} \cos\left[\omega t - \tfrac{1}{2}(n-1)\varphi\right]$$
$$= E_\theta \cos\left[\omega t - \tfrac{1}{2}(n-1)\varphi\right]$$

where, $E_\theta = a \frac{\sin n\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}$

↓

Amplitude of resultant wave.

Scanned by CamScanner

In the limit $n \to \infty$ and

$d \to 0$

$b = nd$

If φ is very small then

$\sin\varphi = \varphi$

$$E_\theta = \frac{a \sin n\frac{\varphi}{2}}{\frac{\varphi}{2}}$$

$$= \frac{a \sin \frac{n}{2} \frac{2\pi}{\lambda} d\sin\theta}{\frac{\pi}{\lambda} d\sin\theta}$$

$$= a \frac{\sin \frac{nd\sin\theta}{\lambda}}{\frac{\pi}{\lambda} \frac{nd\sin\theta}{n}}$$

$$E_\theta = na \frac{\sin \frac{\pi}{\lambda} b\sin\theta}{\frac{\pi}{\lambda} \frac{b\sin\theta}{n}}$$

$$E_\theta = A \frac{\sin\beta}{\beta}$$

where, $A = na$ and $\beta = \frac{\pi}{\lambda} b\sin\theta$

$$\text{Intensity} = (\text{Amplitude})^2$$

$$= \left(A \frac{\sin\beta}{\beta}\right)^2$$

$$= A^2 \frac{\sin^2\beta}{}$$

$$= I_0 \frac{\sin^2\beta}{\beta^2}$$

Intensity of central position

$$I = 4a^2\cos^2\frac{\delta}{2}$$

$$I = I_0 \cos^2\frac{\delta}{2}$$

Scanned by CamScanner

Single Slit Diffraction

The intensity distribution

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

$$E = A \frac{\sin\beta}{\beta} \cos(\omega t - \beta)$$

Position of maximum and minimum

The intensity is zero

$I = 0$ when

$\beta = m\pi$; $m = \pm 1, \pm 2, \pm 3$

$$\frac{\pi b \sin\theta}{\lambda} = m\pi$$

$$b\sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{b}\right)$$

First minima $\theta = \sin^{-1}\left(\frac{\pi}{b}\right)$

Second minima $\theta = \sin^{-1}\left(\frac{2\lambda}{b}\right)$

Third minima $\theta = \sin^{-1}\left(\frac{3\lambda}{b}\right)$

$I = I_0$ when

$\beta = 0$, since $\lim_{\beta \to 0} \frac{\sin\beta}{\beta} = 1$

$$\frac{dI}{d\beta} = I_0 \left[\frac{\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3} \right] = 0$$

$$= \sin\beta\,(\beta - \tan\beta) = 0$$

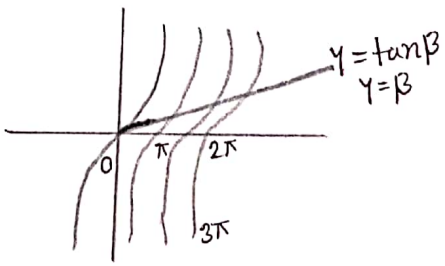
$\tan\beta = 0$

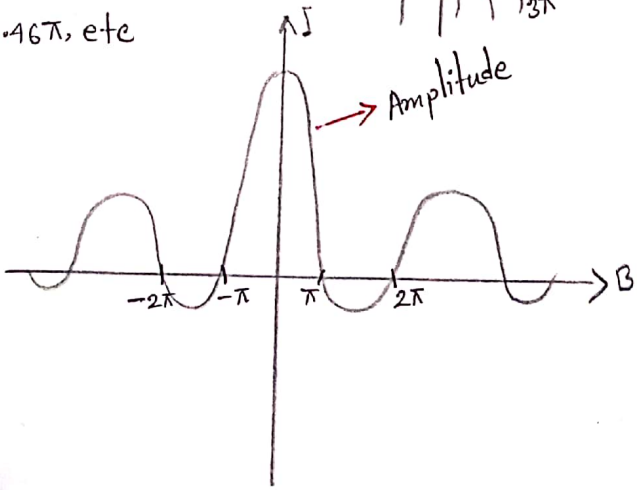
$\sin\beta = 0$

$y = \beta$

$y = \tan\beta$

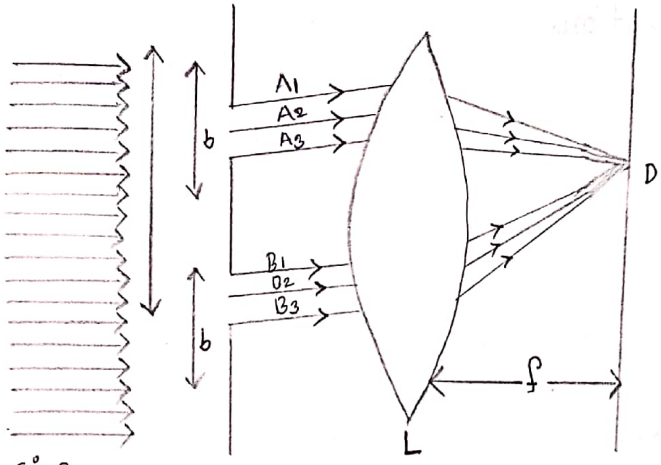
$\beta = 0$; 1.43π, 2.46π, etc





Scanned by CamScanner

Double-Slit Diffraction



$$E_1 = A \frac{\sin\beta}{\beta} \cos(\omega t - \beta)$$

$$E_2 = A \frac{\sin\beta}{\beta} \cos(\omega t - \beta - \varphi_I)$$

$\varphi_I \rightarrow$ Phase difference between E_1 and E_2

$$\varphi_I = \frac{2\pi}{\lambda} d\sin\theta$$

$$E = E_1 + E_2$$

$$= A \frac{\sin\beta}{\beta} \left[\cos(\omega t - \beta) + \cos(\omega t - \beta - \varphi_I)\right]$$

$$= 2A \frac{\sin\beta}{\beta} \cos\left(\frac{2\omega t - 2\beta - \varphi_I}{2}\right) \cos$$

$\varphi_{I/2}$

$$\boxed{E = 2A \frac{\sin\beta}{\beta} \cos(\omega t - \beta - \varphi_{I/2}) \cos\gamma}$$

where, $\gamma = \varphi_{I/2} = \frac{\pi}{\lambda} d\sin\theta$

$$E = \boxed{2A \frac{\sin\beta}{\beta} \cos\gamma} \cdot \cos(\omega t - \beta - \varphi_{I/2})$$

$\rightarrow$ Amplitude

$$I = 4A^2 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma$$

$$I = 4I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma$$

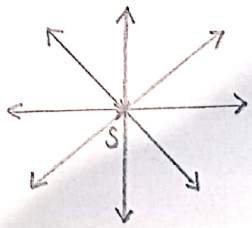
$$I = 4I_0 \boxed{\frac{\sin^2\beta}{\beta^2}} \cos^2\gamma$$

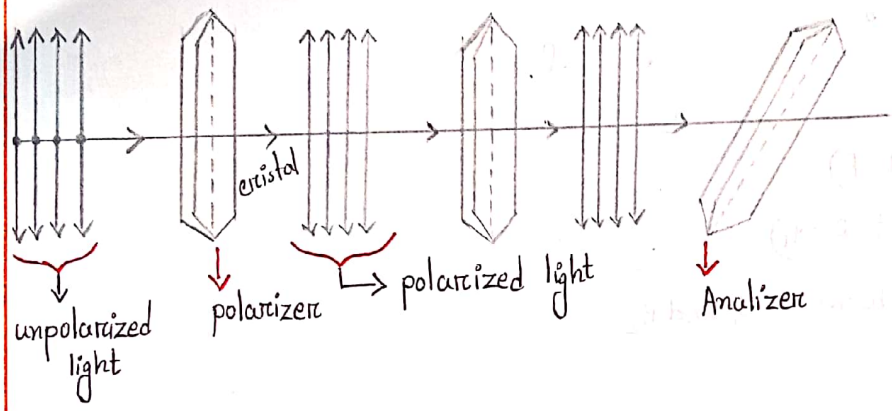
diffraction pattern produced by single plet

Scanned by CamScanner

Polarization of Light

The process of confining the vibrations of electric vector of light waves to one direction is called polarization.

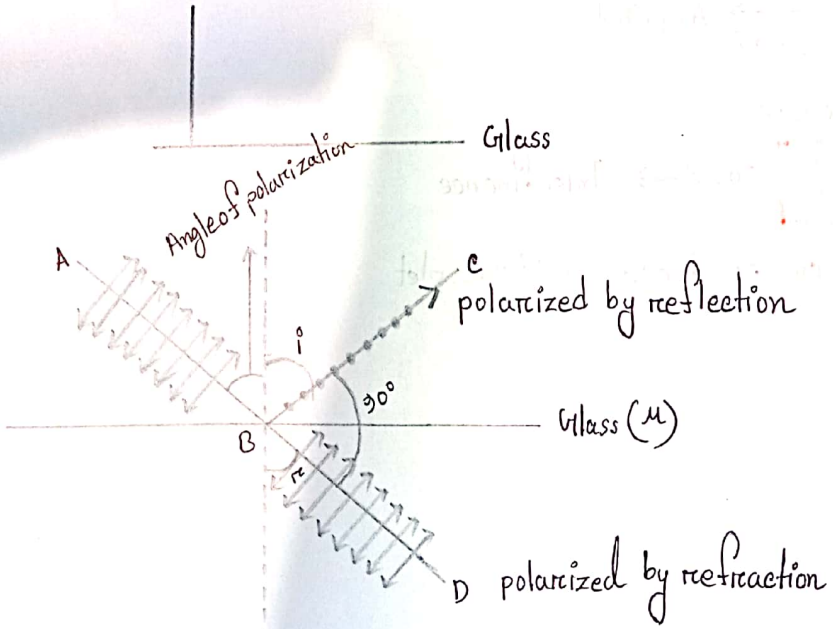




Process of Polarization:→

1. By reflection
2. By refraction
3. Double refraction

Polarization by reflection:→



Scanned by CamScanner

Brewster's Law:→ The tangent of angle of polarization

Snails Law:→ $\mu = \frac{\sin i}{\sin r}$

Brewster's Law:→ $\mu = \tan i$

$$\frac{\sin i}{\sin r} = \tan i$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{\sin i}{\cos i}$$

$$\sin r = \cos i$$

$$\Rightarrow \cos i = \cos\left(\frac{\pi}{2} - r\right)$$

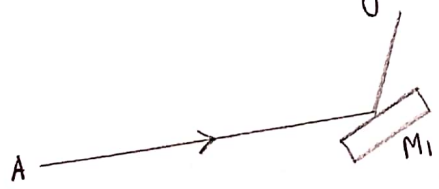
$$\Rightarrow i = \left(\frac{\pi}{2} - r\right)$$

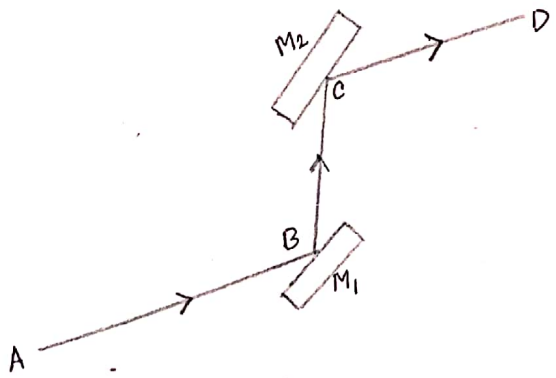
$$i + r = \frac{\pi}{2}$$

$$\angle DBC = 180 - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

Malus Law:→ The intensity of the polarized light transmitted the analyzer varies as the square of the cosine of angle between plane of transmission of the analyzer and plane of polariser



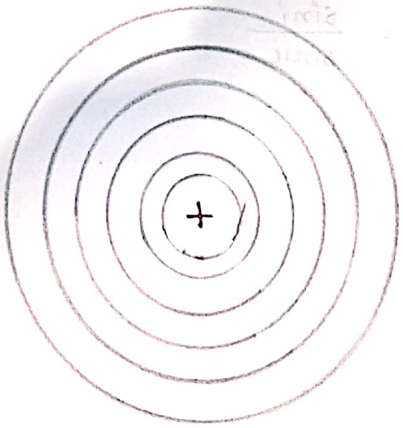


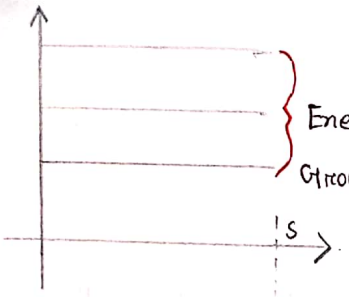
$E_1 = E\cos^2\theta$

Scanned by CamScanner

Semi- conducter

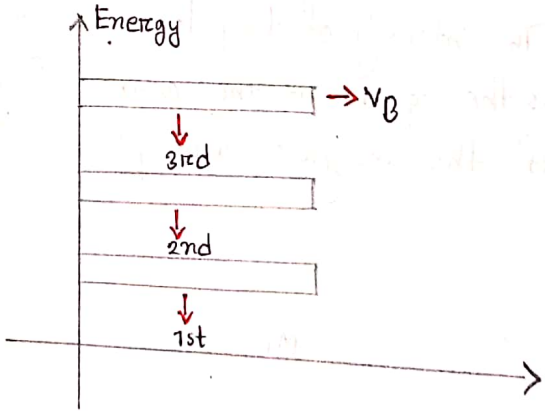
Energy Band :→





Hydrozen atom(-1·36 ev)

1. Valance bond
2. Conduction bond
3. Forbidden energy bond



Scanned by CamScanner