

CSE4203: Computer Graphics

Chapter – 6 (part - C)

Transformation Matrices

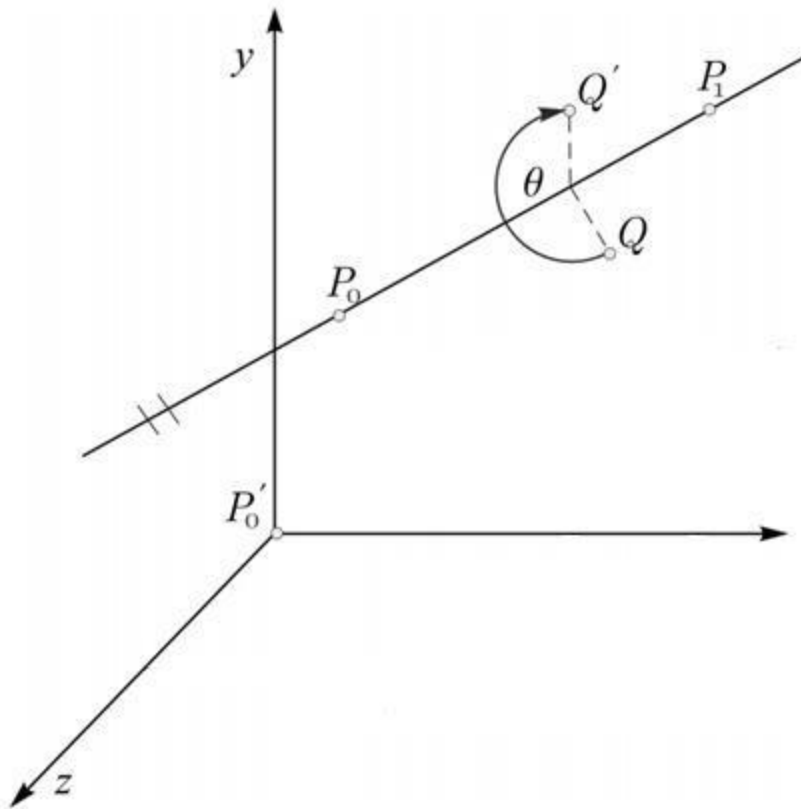
Outline

- 3D Transformation
- Rotation about an arbitrary line

References

- [http://ami.ektf.hu/uploads/papers/finalpdf/AMI 40 from175 to186.pdf](http://ami.ektf.hu/uploads/papers/finalpdf/AMI_40_from175_to186.pdf)
- [http://web.iitd.ac.in/~hegde/cad/lecture/L6 3dtrans.pdf](http://web.iitd.ac.in/~hegde/cad/lecture/L6_3dtrans.pdf)

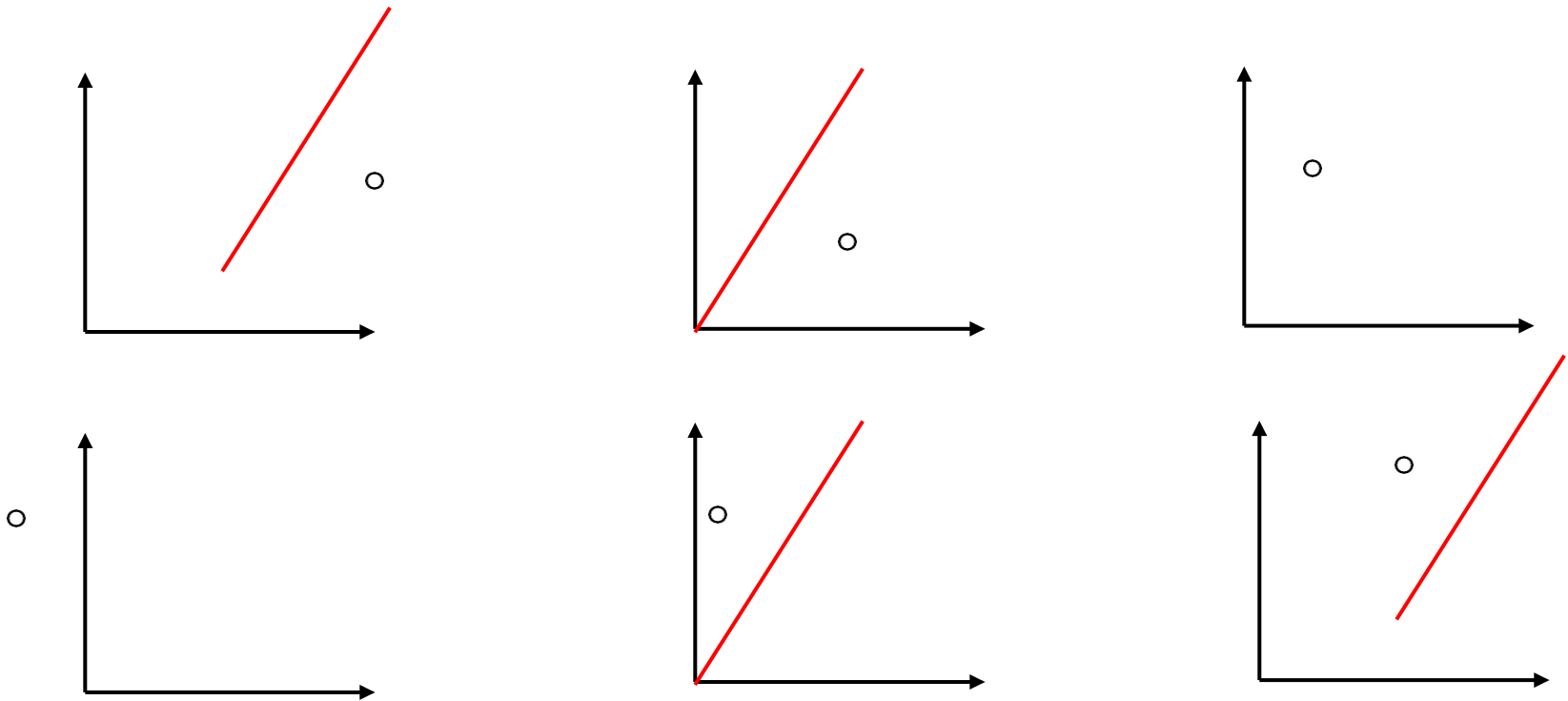
Rotation about an arbitrary line (1/1)



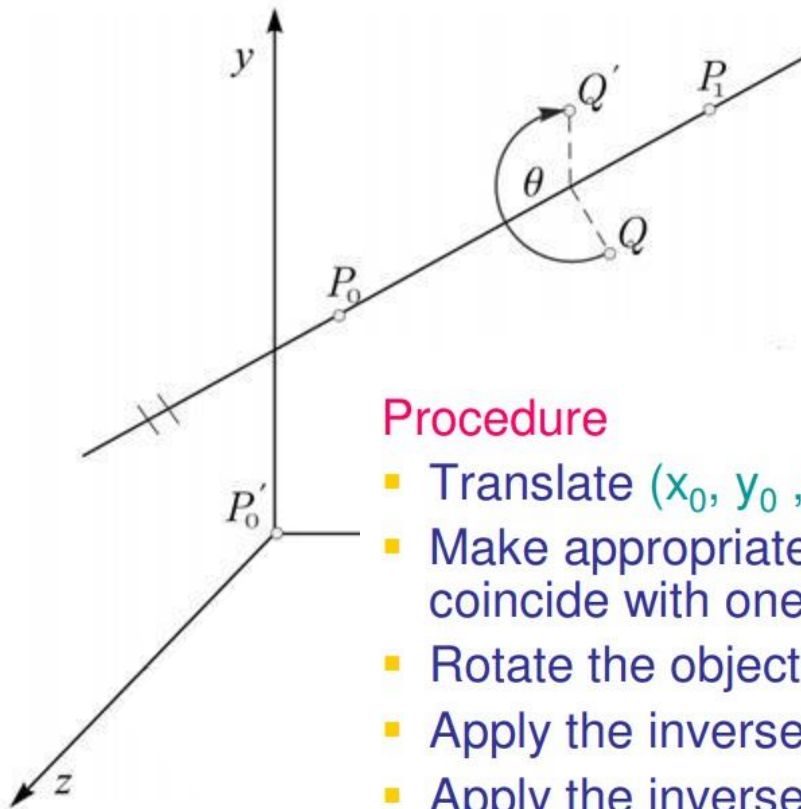
The basic idea is to make the arbitrary rotation axis coincide with one of the principle axis. Assume an arbitrary axis in space passing through the point **P_0 (x_0, y_0, z_0)** and **P_1 (x_1, y_1, z_1)** .

In 2D case (1/1)

Reflecting about an arbitrary line



Steps (1/1)

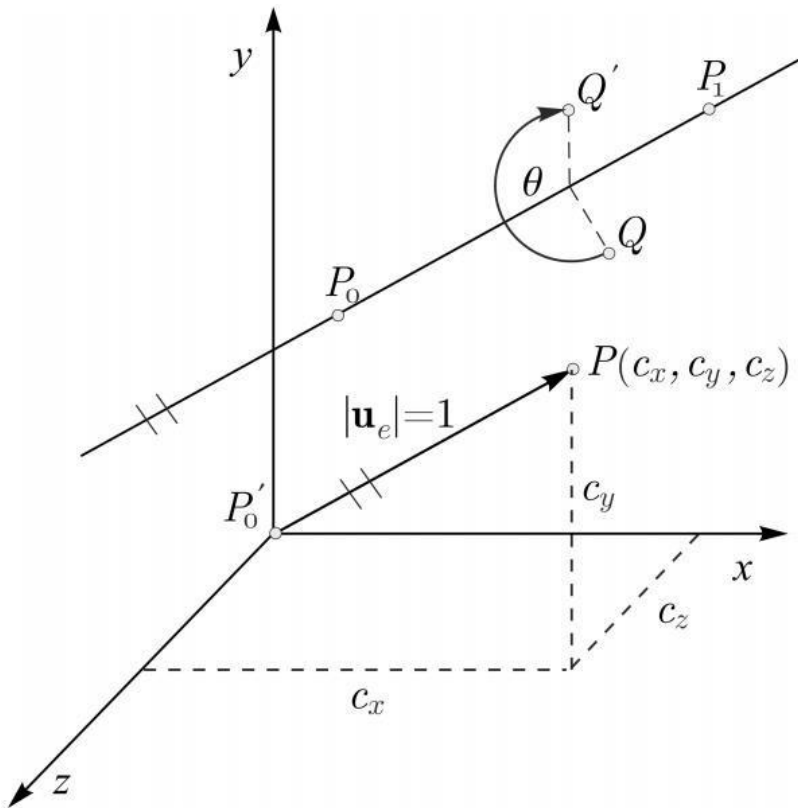


Procedure

- Translate (x_0, y_0, z_0) so that the point is at origin
- Make appropriate rotations to make the line coincide with one of the axes, say z -axis
- Rotate the object about z -axis by required angle
- Apply the inverse of step 2
- Apply the inverse of step 1

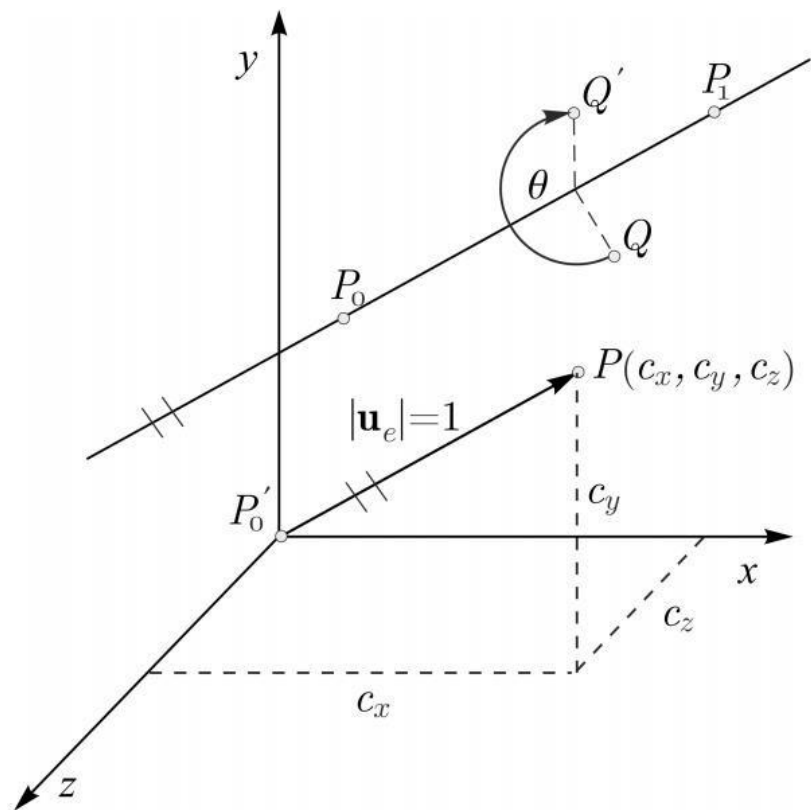
Direction Cosine (1/2)

- Direction cosines of a vector are the cosines of the angles between the vector and the three coordinate axes.



Direction Cosine (2/2)

Equivalently, they are the contributions of each component of the basis to a unit vector in that direction.



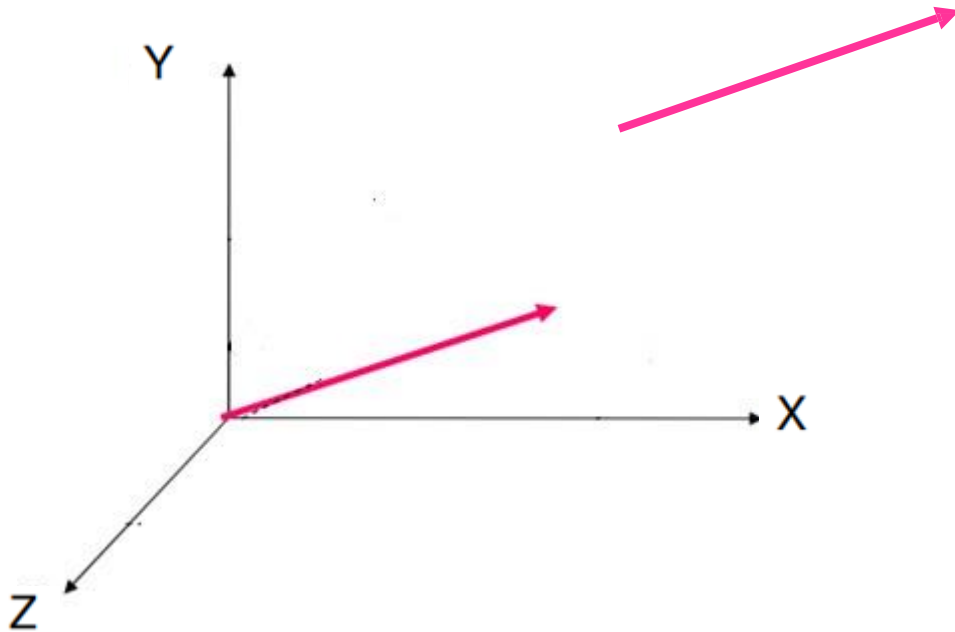
$$\mathbf{u} = P_1 - P_0$$

$$\mathbf{u}_e := \frac{\mathbf{u}}{|\mathbf{u}|} = (c_x, c_y, c_z)$$

$$c_x^2 + c_y^2 + c_z^2 = 1$$

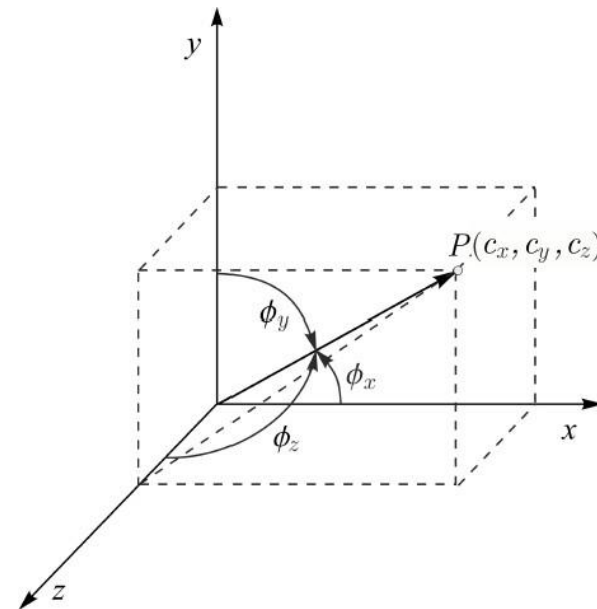
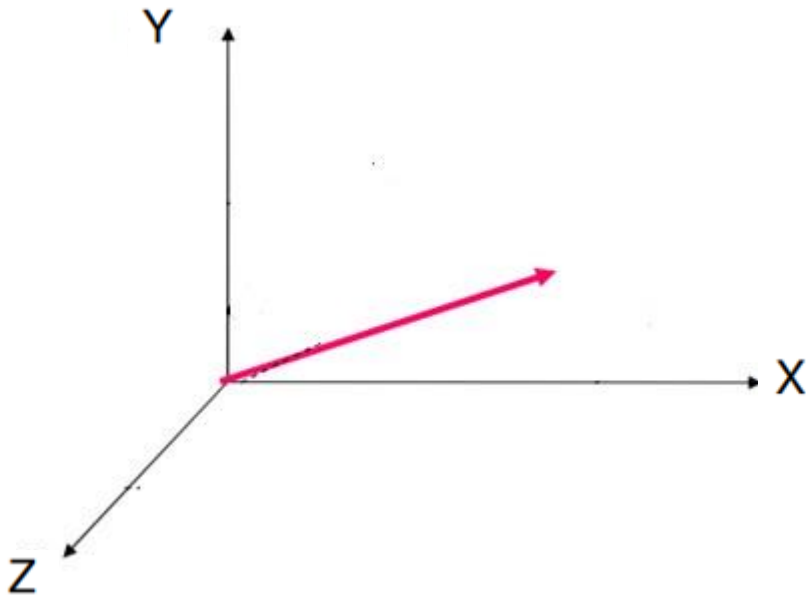
$$\cos \phi_x = c_x, \quad \cos \phi_y = c_y, \quad \cos \phi_z = c_z$$

Coinciding the line with Principal axis (1/5)



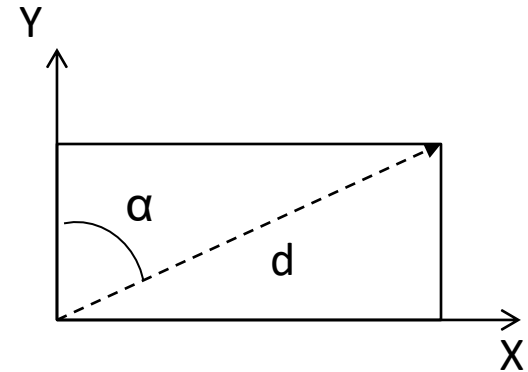
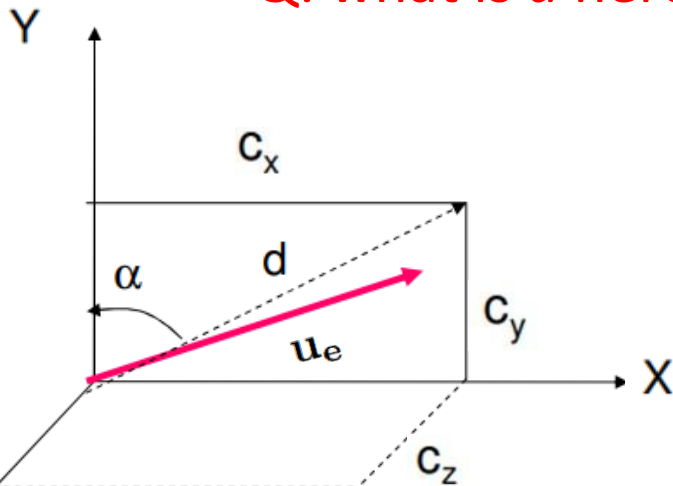
Coinciding the line with Principal axis (2/5)

Coinciding the arbitrary axis with any axis
the rotations are needed about other two axes



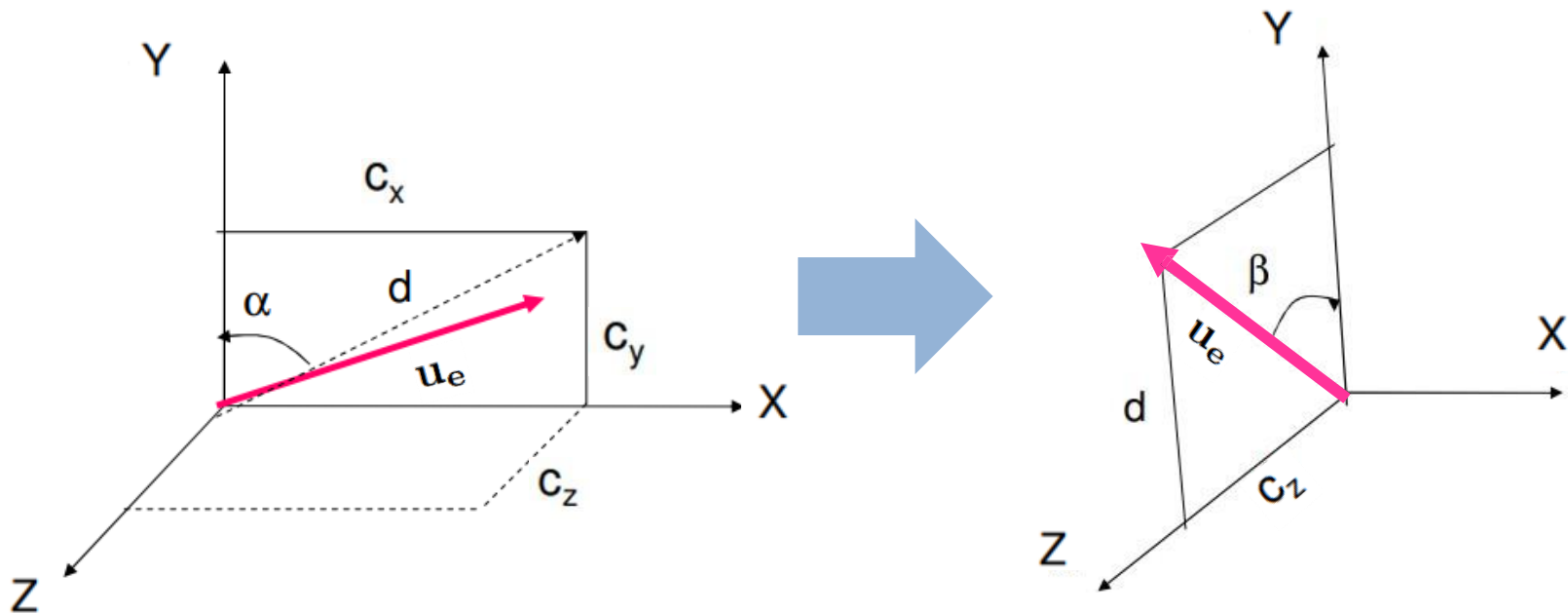
Coinciding the line with Principal axis (3/5)

Q: what is d here?



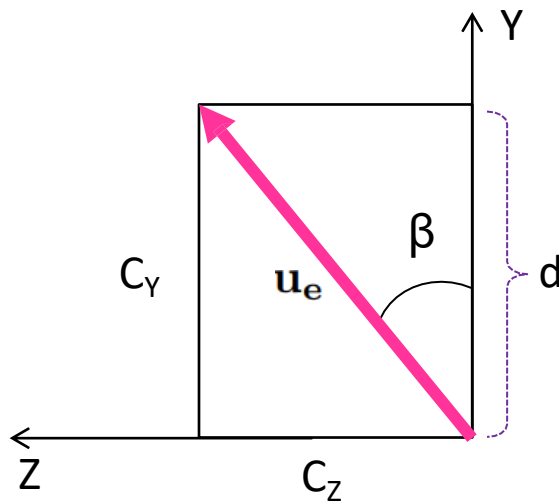
$$d = \sqrt{c_x^2 + c_y^2} \quad \cos \alpha = \frac{c_y}{d} \quad \sin \alpha = \frac{?}{d}$$

Coinciding the line with Principal axis (4/5)

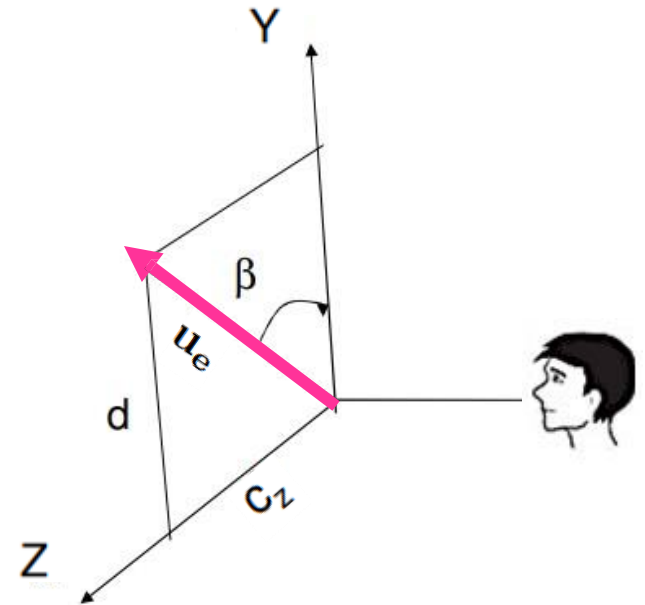


What is the rotation matrix?

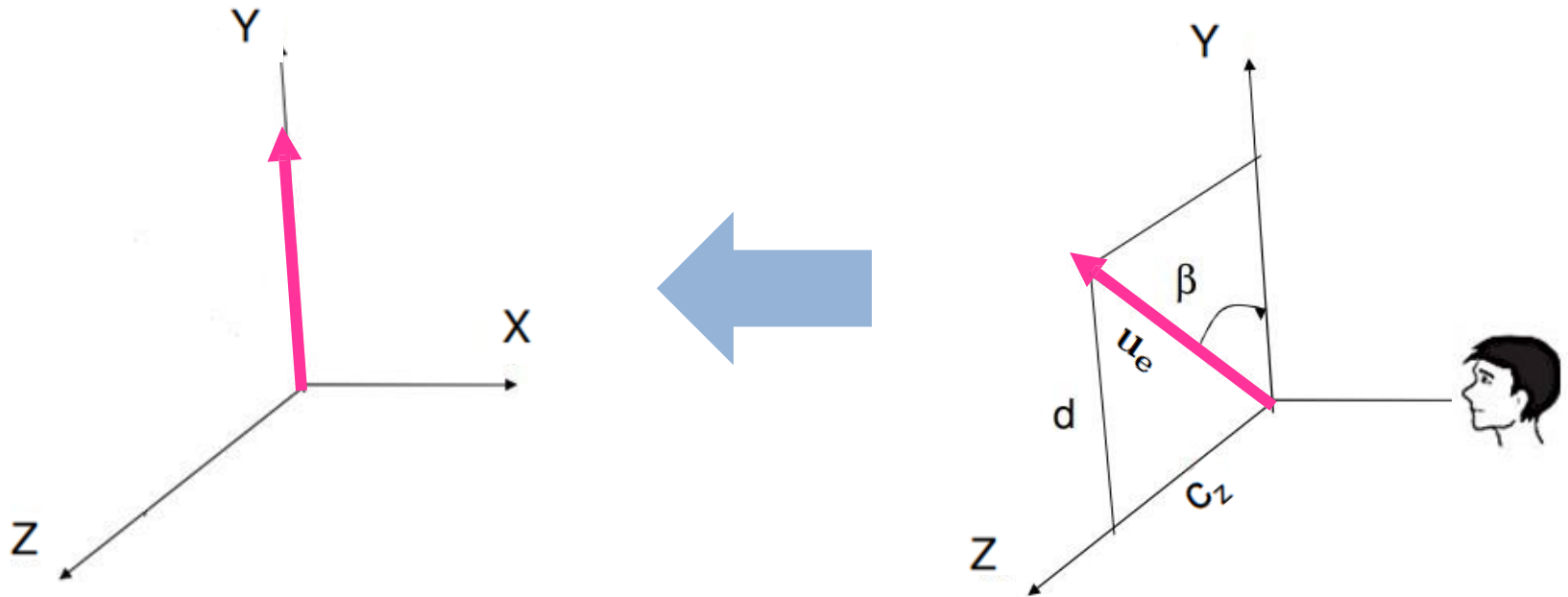
Coinciding the line with Principal axis (5/5)



$$\cos \beta = d \quad \sin \beta = ?$$

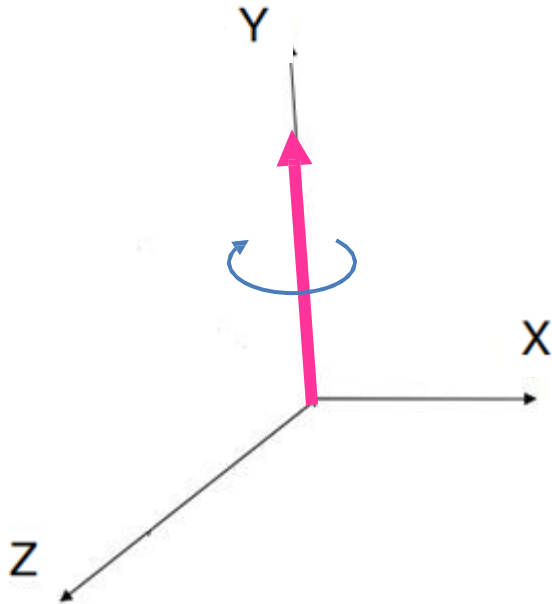


Rotating about the principal axis (1/2)



What is the rotation matrix?

Rotating about the principal axis (2/2)



What is the rotation matrix?

Undoing the steps (1/1)

- Q: What are the undoing steps?

Composite Transformation (1/1)

- $M = T^{-1} * R_x^{-1}(-\beta) * \dots ?$

- $Q' = M * Q$

Practice Problem

- AB is a line and P is a point in 3D space; where the points A,B and P are (1,1,1), (3,3,4) and (2,2,4) respectively. We want to rotate P along AB by +90 degree. Determine the composite transformation matrix to do the task and calculate the rotated point P'.