PCA Math Solution

Example:

Data

x	y
2.5	2.4
0.5	0.7
2.2	2.9
1.9	2.2
3.1	3.0
2.3	2.7
2	1.6
1	1.1
1.5	1.6
1.1	0.9

Sol^n: **Step 1:** Get some data

Total X = 18.1   Total Y = 19.1

Avg X, $\bar{X}$ = 1.81   Avg $\bar{Y}$ = 1.91

Step 2: Subtract the mean

$x_i - \bar{x}$	$y_i - \bar{y}$
0.69	0.49
−1.31	−1.21
0.39	0.99
0.09	0.29
1.29	1.09
0.49	0.79
0.19	−0.31
−0.81	−0.81
−0.31	−0.31
−0.71	−1.01

Mean of X
$$= \frac{\sum_{i=1}^{n} x_i}{n}$$
Mean of Y
$$= \frac{\sum_{i=1}^{n} y_i}{n}$$

$\times\ a^2+b^2+c^2 = (a+b+c)^2$

Step 3: Calculate the covariance matrix :-

$$\text{Cov} = \begin{pmatrix} \text{Cov}(x,x) & \text{Cov}(x,y) \\ \text{Cov}(y,x) & \text{Cov}(y,y) \end{pmatrix}$$

We know,

$$\text{Cov}(x,y) = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

$$\text{Cov}(x,x) = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

$\text{Cov}(x,y) = 0.6154444444$

$\text{Cov}.(x,x) = 0.6165555556$

$\text{Cov}(y,y) = 0.7165555556$

$$\therefore \text{Cov} = \begin{pmatrix} \text{Cov}(x,x) & \text{Cov}(x,y) \\ \text{Cov}(y,x) & \text{Cov}(y,y) \end{pmatrix}$$

$$= \begin{pmatrix} 0.6165555556 & 0.6154444444 \\ 0.6154444444 & 0.7165555556 \end{pmatrix}$$

Step 4: Calculate Eigenvectors and Eigenvalues of covariance matrix

Suppose, $(A - \lambda I)X = 0$

$$A = \begin{pmatrix} 0.6165555556 & 0.6154444444 \\ 0.6154444444 & 0.7165555556 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\therefore A - \lambda I = \begin{pmatrix} 0.6165555556 & 0.6154444444 \\ 0.6154444444 & 0.7165555556 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\det \begin{pmatrix} 0.6165555556 - \lambda & 0.6154444444 \\ 0.6154444444 & 0.7165555556 - \lambda \end{pmatrix} = 0$$

$$\Rightarrow (0.6165555556 - \lambda)(0.7165555556 - \lambda) - 0.3787718641 = 0$$

$$\Rightarrow 0.4417963087 - 0.6165555556\lambda - 0.7165555556\lambda + \lambda^2 - 0.3787718641 = 0$$

Formula:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \lambda^2 - 1.333111111\lambda + 0.06302444446 = 0$$

$$\therefore \lambda = \frac{-(-1.33311111) \pm \sqrt{(-1.33311111)^2 - 4 * 1 * 0.0630244446}}{2 \cdot 1}$$

$$\therefore \lambda_2 = 0.04908339907$$

$$\lambda_1 = 1.284027712$$

$$\therefore \text{Eigenvalues} = \begin{pmatrix} 0.04908339907 \\ 1.284027712 \end{pmatrix}$$

Step 5: Choosing components and forming a feature vector.

For, $\lambda_1 = 1.284027712$

$$\begin{pmatrix} 0.6165555556 - 1.284027712 & 0.6154444444 \\ 0.6154444444 & 0.7165555556 - 1.284027712 \end{pmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -0.66747 & 0.61544 \\ 0.61544 & -0.56747 \end{pmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

Now,

$$-0.66747x_1 + 0.61544y_1 = 0 \quad \text{--- (i)}$$

$$0.61544x_1 - 0.56747y_1 = 0 \quad \text{--- (ii)}$$

(i) $\Rightarrow$

$$x_1 = \frac{0.61544}{0.66747} y_1$$

$$= 0.922\, y_1$$

(ii) $\Rightarrow$ $x_1 = \frac{0.56747}{0.61544} y_1$

$$= 0.922\, y_1$$

$$\therefore\ e_1 \sim \begin{bmatrix} \frac{0.922}{A} \\ \frac{1}{A} \end{bmatrix}$$

Now, $A = \sqrt{(0.922)^2 + (1)^2} = 1.3602$

$$\therefore\ e_1 \sim \begin{bmatrix} 0.6778 \\ 0.7352 \end{bmatrix}$$

Again,

for. $\lambda_2 = 0.04908339907$

$$\begin{pmatrix} 0.6165555556 - 0.04908339907 & 0.6154444444 \\ 0.6154444444 & 0.7165555556 - 0.04908339907 \end{pmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$= \begin{pmatrix} 0.56747 & 0.61544 \\ 0.61544 & 0.66747 \end{pmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

Now, $0.56747 x_1 + 0.61544 y_1 = 0$ .... (iii)

$0.61544 x_1 + 0.66747 y_1 = 0$ .... (iv)

(iii) $\Rightarrow$ $0.56747 x_1 + 0.61544 y_1 = 0$

$\Rightarrow x_1 = -\dfrac{0.61544 \, y_1}{0.56747}$

$= -1.0845 \, y_1$

(iv) $\Rightarrow$ $x_1 = -\dfrac{0.66747}{0.61544} \, y_1$

$= -1.0845 \, y_1$

$$e_2 \sim \begin{bmatrix} \frac{-1.0845}{A} \\ \frac{1}{A} \end{bmatrix}$$

Now, $A = \sqrt{(-1.0845)^2 + 1^2} = 1.47517$

$$\therefore e_2 = \begin{bmatrix} -0.735 \\ 0.677 \end{bmatrix}$$

So, Eigen vector $= \begin{pmatrix} 0.6778 & -0.735 \\ 0.7352 & -0.677 \end{pmatrix}$

$\therefore$ We get PCA $= \begin{pmatrix} 0.6778 \\ 0.7352 \end{pmatrix}$ for max^m

eigenvalue $\lambda = 1.284037712$