Topic 2.4 Unification and Resolution in FOL

a) Definition

- Unification is a process of making sentences equal using substitution.
- A unifier is a substitution that makes sentences equal.
- Most general unifier (mgu) is one, which may have some instances, but which is not an instance of any other.

b) Basic idea of the process

Matching Rules that are involved during Unification

- Different predicates or constants don't match.
- A variable matches a term if the term does not contain the variable itself.

c) Simplified UNIFY algorithm

We assume that

- Two same-predicate atomic sentences are given in a set, S;

- The algorithm returns the mgu, θ;

- The algorithm can be used repeatedly for a set with more than 2 such sentences in a chain.

Major steps:

1)Set $\theta = \{\}$.

2)If SUBST(θ, S) contains only one element, then stop; θ is an mgu of S. Otherwise, find the first disagreeing pair of SUBST(θ, S).

3)If the disagreeing pair contains a variable v and a term t such that v does not occur in t (that is, t is another variable / a constant / a function without v), then obtain

$\theta = \theta \cup \{v / t\}$, and go to step 2. Otherwise stop; S is not unifiable.

Example:

S = {P1(x, y, z), P1(F1("Km"), "Bn", u)}

1. $\theta = \{\}$, SUBST(θ, S) = S.

2. θ = { x / F1("Km") },
   SUBST(θ, S) =
   {P1(F1("Km"), y, z), P1(F1("Km"), "Bn", u)}.

3. θ = { x / F1("Km"), y / "Bn" },
   SUBST(θ, S) =
   {P1(F1("Km"), "Bn", z), P1(F1("Km"), "Bn", u)}.

4. θ = { x / F1("Km"), y / "Bn", z / u },
   SUBST(θ, S) = {P1(F1("Km"), "Bn", u)}.
   Terminate.

d) The Resolution rule of FOL

Example:

$P1(x) \vee P5(\text{“Km”}, x) \vee P3(y)$

$\neg P4(\text{“Sm”}, z) \vee \neg P5(\text{“Km”}, \text{“Rm”}) \vee P6(\text{“Hm”})$

$P1(\text{“Rm”}) \vee P3(y) \vee \neg P4(\text{“Sm”}, z) \vee P6(\text{“Hm”})$

The **rule**:

From two clauses,

$p_1 \vee p_2 \vee \ldots \vee p_i$ and

$q_1 \vee q_2 \vee \ldots \vee q_j$

we resolve

$SUBST(\theta, p_1 \vee p_2 \vee \ldots \vee p_{k-1} \vee p_{k+1} \vee \ldots \vee p_i \vee q_1 \vee q_2 \vee \ldots \vee q_{l-1} \vee q_{l+1} \vee \ldots \vee q_j)$,

where

p_x, q_y are FOL literals (positive or negative),

θ is a substitution and θ = UNIFY (p_k, $\neg q_l$).