Sample problem list for numerical methods.

1. Use Fixed Point Iteration Method to evaluate: $f(x) = x^3 - 0.75$, with $x_0 = 0.8$ correct to four significant figures.
2. Use Newton Raphson Method and Secant Method to evaluate following function , correct to 3 decimal places and also draw the graphs of their convergence.
$$f(x) = cosx - 2x$$
For Newton Raphson method x0 = 0.5 and for Secant method x0=1, x1=0.5.

3. Consider we have one species of fish population and they are feeding on three species of insects. The problem is to determine the average caloric value a fish is getting from a particular species of insect.
Let say in a lake there are a lot of trout and they are feeding on moths, midges and worms. We want to determine how much caloric value a trout is getting from moths, midges and worms. This is a difficult task. To make things less complicated, we will assume that all the trout in the lake only feeding on these three types of insects. To start the process, we can catch some trout and examine the content in the stomach. We can count the head capsule of each insects (moths, midges and worms) to determine how many moths, midges and worms the trout had eaten. Then we will use a calorimeter to measure the total caloric value of the stomach's content. Suppose we do this for three fish and find the following data:

Fish	Number of Moths	Number of Midges	Number of Worms	Total Calorie Value at the Stomach
A	12	18	15	660
B	8	14	13	580
C	5	9	10	295

Let x_1 represent the average caloric content of midges eaten, x_2 represent the average caloric content of moths eaten and x_3 represent the average caloric content of worms eaten.

Now answer the following questions:

Question 1: Write down the system of linear equations for fish A, B and C.

Question 2: Solve x_1, x_2 and x_3 using Gauss elimination methods.

Question 3: Perform Gauss Jordan methods and find out row echelon form of the matrix.

Question 4: Complete the Gauss-Seidel iteration of our example. Write your result in a table starting from the first iteration. You should do as many as you need before you convince that you get your solution. Does the solution converge to the same solution we got before?

4. Derive a normal equations for evaluating the parameters a and b to fit data to
   a. Power function model of the form $y = ax^b$

b. Population growth model of the form $y = ae^{bx}$ using the principle of least square regression.

5. Construct a divide difference formula for four points.
6. The velocity of a particle is presented by:

$$v(t) = \frac{\sin(t)}{(t+1)^2 \exp(t)}$$

If the initial position of the particle is $x(0) = 0$ then estimate the position $x(2)$ using the integral:

$$\int_0^t v(t)dt$$

By applying a suitable Newton-Cotes formula
7. Use Romberg Integration to evaluate: $\int_0^2 (e^{x^2} - 1)dx$
8. The following table gives the velocity of an object at various points in time:

Time (Seconds)	1	1.2	1.6	1.8	2.2	2.4	2.8	3
Velocity (m/sec)	9.0	9.5	10.2	11.0	13.2	14.7	18.7	22.0

Find the acceleration of the object at T = 2.0 seconds. Assume a suitable value for h.
9. Evaluate the following integral:

$$\int_0^2 (3x^2 + 2x - 5)\ dx$$

i. Using Trapezoidal rule
ii. Using Simpson's 1/3 rule
iii. Using Simpson's 3/8 rule
10. Estimate the first derivative of $f(x) = \ln x\ at\ x = 1$ using the first order.
i. First order forward difference formula
ii. First order backward difference formula
iii. Second order central difference formula
Compare the result with true value 1.

11. Evaluated the first derivative at $x = -3\ and\ x = 0$ of the following table function:

x	− 3	− 2	− 1	0	1	2	3
y	− 33	− 12	− 3	0	3	12	33

12. Use Euler's method to solve the differential equation $y' = \frac{-y}{2y+1}$ with the initial condition y(0) = 1 with step size 0.25 and find the value of y(1).

13. Solve the differential equation $10y'' + 2(y') + 6x = 0$ with y(0) = 1 and y'(0) = 0 by Heun's method to estimate *y*(0.2) using h = 0.1

14. You have to measure the flow rate of water through a small pipe. In order to do it, you place a bucket at the pipe's outlet and measure the volume in the bucket as a function of time as tabulated below. Estimate the flow rate at $t = 7\,s$ using any best numerical differentiation method.

t, s	0	1	5	8
$Volume, cm^3$	0	1	8	16.4