

Ahsanullah University of Science & Technology

Department of
Computer Science and Engineering

ASSIGNMENT



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2.2

a) $\underline{x}y + \underline{x}y'$

$$= x(y + y')$$

$$= x \cdot 1 \quad [\text{Complement Law}]$$

$$= x$$

b) $(x + \underline{y})(x + \underline{y}')$

$$= x \cdot x + x \cdot \underline{y}' + x \cdot \underline{y} + \underline{y} \cdot \underline{y}'$$

$$= x + x\underline{y}' + x\underline{y} + 0$$

$$= x(1 + \underline{y}' + \underline{y})$$

$$= x \cdot 1 \quad [\text{Complement Law}]$$

$$= x$$

c) $\underline{x}yz + x'y + \underline{xyz}'$

$$= \underline{x}y(z + z') + x'y$$

$$= \underline{x}y \cdot 1 + x'y$$

$$= \underline{y}(x + x')$$

$$= \underline{y} \cdot 1 \quad [\text{Complement Law}]$$

$$= \underline{y}$$

$$\underline{d)} \quad (A+B)' (A'+B')'$$

$$= A'B' \cdot AB$$

$$= 0$$

$$\underline{e)} \quad (a+b+c') (a'b'+c)$$

$$= ((a+b)+c') ((a+b)'+c)$$

$$= (a+b)(a+b)' + (a+b)c + c'(a+b)' + cc'$$

$$= 0 + c(a+b) + a'b'c' + 0 \quad [\text{Complement Law}]$$

$$= a'b'c' + c(a+b)$$

$$\underline{f)} \quad a'b'c + abc' + abc + a'b'c'$$

$$= ab(c+c') + a'b(c+c')$$

$$= ab \cdot 1 + a'b \cdot 1$$

$$= b(a+a')$$

$$= b \cdot 1 \quad [\text{Complement Law}]$$

2.3

$$\text{a) } ABC + A'B + ABC'$$

$$= AB(C + C') + A'B$$

$$= AB \cdot 1 + AB$$

$$= B(A + A')$$

$$= B \cdot 1 \quad [\text{Complement Law}]$$

$$= B$$

b) $x'y'z + xz$

$$= z(x + x'y)$$

$$= z(x + y) \quad [\because A + A'B = A+B]$$

c) $(x+y)'(x'+y')$

$$= x'y' \cdot (x'+y')$$

$$= x'x'y' + x'y'y'$$

$$= x'y' + x'y'$$

$$= x'y' \quad [\text{Idempotent Law}]$$

d) $xy + xwz + xwz'$

$$= xy + xw(z + z')$$

$$= xy + xw \cdot 1 \quad [\text{Complement Law}]$$

$$= x(y + w)$$

$$\underline{\text{Q1}} \quad (BC' + A'D)(AB' + CD')$$

$$= ADB'C' + BCC'D' + AA'D'D + A'CDD'$$

$$= 0 + 0 + 0 + 0 \quad [\text{Complement Law}]$$

$$= 0$$

$$\underline{\text{Q2}} \quad (a' + c')(a + b' + c')$$

$$= aa' + a'b' + a'c' + ac' + b'c' + c'c$$

$$= 0 + a'b' + a'c' + ac' + b'c' + c'$$

$$= c'(1+a+a') + a'b' + b'c'$$

$$= c' \cdot 1 + a'b' + b'c'$$

$$= c' (1+b') + a'b'$$

$$= a'b' + c' \quad [\because 1+b' = 1]$$

2.4

$$\underline{\text{a) }} A'C' + ABC + AC'$$

$$= C'(A+A') + ABC$$

$$= C' \cdot 1 + ABC \quad [\text{Complement Law}]$$

$$= C' + ABC$$

$$= C' + AB \quad [\because A+A'B = A+B]$$

$$\underline{\text{b) }} (x'y' + z)' + z + xy + wz$$

$$= (x+y) \cdot z' + z + xy + wz$$

$$= (z + (x+y)) + xy + wz \quad [\because A+A'B = A+B]$$

$$= x+y + xy + z(1+w)$$

$$= x+y + xy + z \cdot 1 \quad [\text{Complement Law}]$$

$$= x+y(1+x) + z$$

$$= x+y+z$$

$$\underline{c)} \quad A'B (0' + C'D) + B (A + A'C'D)$$

$$= B (A'D' + A'C'D + A + A'C'D)$$

$$= B (A'D' + A'D (C + C') + A)$$

$$= B (A' (0 + D') + A)$$

$$= B (A' \cdot 1 + A) \quad [\text{Complement Law}]$$

$$= B (A' + A) \quad [\text{Identity Law}]$$

$$= B \cdot 1 \quad [\text{Complement Law}]$$

$$= B$$

$$\underline{d)} \quad (A' + C) (A' + C') (A + B + C'D)$$

$$= (A'A' + A'C' + A'C + CC') (A + B + C'D)$$

$$= (A' + A' (C + C') + 0) (A + B + C'D)$$

$$= (A' + A') (A + B + C'D)$$

$$= A' (A + B + C'D) \quad [\text{Idempotent Law}]$$

$$= AA' + A'B + A'C'D$$

$$= 0 + A' (B + C'D) \quad [\text{Complement Law}]$$

$$= A' (B + C'D)$$

$$\underline{e)} \quad ABC'D + A'BD + ABCD$$

$$= ABD(C + C') + A'BD$$

$$= ABD \cdot 1 + A'BD \quad [\text{Complement Law}]$$

$$= BD(A + A')$$

$$= BD \cdot 1 \quad [\text{Complement Law}]$$

$$= BD$$

2.22

$$\underline{o)} \quad (u + xw)(x + u'v)$$

$$= (ux + u \cdot u'v + xxw + xwuw'v)$$

$$= (ux + 0 + xw + xwuw'v)$$

$$= xw(1 + u'v) + ux$$

$$= xw + ux \quad [\text{Complement Law}]$$

$$= u(w + u)$$

$$\underline{b)} \quad x' + x(x+y')(y+z')$$

$$= x' + x(xy + xz' + yy' + y'z')$$

$$= x' + (xxy + xxz' + xyy' + xy'z')$$

$$= x' + xy + xz' + yz' \quad [\text{Idempotent Law}]$$

$$= x' + xy + xz' (1 + y')$$

$$= x' + xy + xz' \quad [\text{Complement Law}]$$

$$= x' + x(y+z')$$

$$= x' \{ 1 + (y+z') \} + x(y+z')$$

$$= x' + x'(y+z') + x(y+z')$$

$$= x' + (y+z')(x+x')$$

$$= x' + y + z'$$

3.1

(a) $F(x, y, z) = \Sigma(0, 2, 4, 5)$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1			1
x	1	1		

$$F = \bar{x}\bar{z} + xy$$

(b) $F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1			1
x	1	1		1

$$F = \bar{z} + xy$$

(c) $F(x, y, z) = \Sigma(0, 1, 2, 3, 5)$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1	1	1	1
x		1		

$$F = \bar{x} + \bar{y}z$$

$$(d) \quad F(x, y, z) = \Sigma (1, 2, 3, 7)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}		1	1	1
x			1	

$$F = \bar{x}z + \bar{x}y + yz$$

3.2

$$(e) \quad F(x, y, z) = \Sigma (0, 1, 5, 7)$$

	yz	$\bar{y}z$	yz	$\bar{y}z$
\bar{x}	1	1		
x		1	1	

$$F = \bar{x}\bar{y} + xz$$

$$(b) \quad F(x, y, z) = \Sigma (1, 2, 3, 6, 7)$$

	yz	$\bar{y}z$	yz	$\bar{y}z$
\bar{x}		1	1	1
x			1	1

$$F = \bar{x}z + y$$

$$(c) F(x,y,z) = \Sigma(2,3,4,5)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}			1	1
x	1	1		

$$F = \bar{x}y + x\bar{y}$$

$$(d) F(x,y,z) = \Sigma(1,2,3,5,6,7)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}		1	1	1
x		1	1	1

$$F = y + z$$

$$(e) F(x,y,z) = \Sigma(0,2,4,6)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1			1
x	1			1

$$F = \bar{z}$$

$$(f) F(x,y,z) = \Sigma(3,4,5,6,7)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}			1	
x	1	1	1	1

$$F = x + yz$$

$$(a) \quad \underline{xy} + \underline{x'y'z'} + \underline{x'yz'}$$

$$= \underline{xy} (z + z') + \underline{x'y'z'} + \underline{x'yz'}$$

$$= \underline{xy}z + \underline{xy}z' + \underline{x'y'z'} + \underline{x'yz'}$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1			1
x			1	1

$$\therefore F = \underline{xy} + \bar{x}\bar{z}$$

$$(b) \quad \underline{x'y'} + \underline{yz} + \underline{x'yz'}$$

$$= \underline{x'y'} (z + z') + \underline{yz} (x + x') + \underline{x'yz'}$$

$$= \underline{x'y'z} + \underline{x'y'z'} + \underline{xy}z + \underline{x'yz} + \underline{x'yz'}$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1	1	1	1
x			1	

$$\therefore F = \bar{x} + yz$$

$$(c) \quad \underline{x'y + yz' + y'z'}$$

$$= x'y(z+z') + yz'(x+x') + y'z'(x+x')$$

$$= x'yz + x'yz' + xy'z + x'y'z + xy'z' + x'y'z'$$

$$= x'yz + x'yz' + xy'z + xy'z' + x'y'z'$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1		1	1
x	1			1

$$\therefore F = \bar{z} + \bar{x}y$$

$$(d) \quad F(x, y, z) = x'yz + xy'z' + xy'z$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}			1	
x	1	1		

$$\therefore F = x\bar{y} + \bar{x}yz$$

3.4

(a) $F(x, y, z) = \Sigma(2, 3, 6, 7)$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}			1	1
x			1	1

$$F = y$$

(b) $F(A, B, C, D) = \Sigma(4, 6, 7, 15)$

$\bar{C}D$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}B$				
$\bar{A}B$	1		1	1
AB			1	
$A\bar{B}$				

$$F = BCD + A'BD'$$

(c) $F(A, B, C, D) = \Sigma(3, 7, 11, 13, 14, 15)$

$\bar{C}D$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}B$			1	
$\bar{A}B$			1	
AB		1	1	1
$A\bar{B}$			1	

$$F = CD + ABD + ABC$$

$$(d) F(w, x, y, z) = \Sigma (2, 3, 12, 13, 14, 15)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$			1	1
$\bar{w}x$				
$w\bar{x}$	1	1	1	1
wx				

$$F = \bar{w}\bar{x}y + wx$$

$$(e) F(w, x, y, z) = \Sigma (11, 12, 13, 14, 15)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$				
$w\bar{x}$	1	1	1	1
wx			1	

$$F = wx + wyz$$

$$(5) \quad F(w, x, y, z) = \Sigma (8, 10, 12, 13, 14)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$				
$w\bar{x}$	1	1		1
wx	1			1

$$F = w\bar{x}\bar{y} + w\bar{z}$$

3.5

$$(6) \quad F(w, x, y, z) = \Sigma (1, 4, 5, 6, 12, 14, 15)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$		1		
$\bar{w}x$	1	1		1
$w\bar{x}$	1		1	1
wx				

$$F = x\bar{z} + \bar{w}\bar{y}z + w\bar{x}y$$

$$(b) \quad F(A, B, C, D) = \Sigma (2, 3, 6, 7, 12, 13, 14)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				1 1
$\bar{A}B$			1 1	
AB	1 1			1
$A\bar{B}$				

$$F = \bar{A}C + AB\bar{C} + A\bar{B}\bar{D}$$

$$(c) \quad F(w, x, y, z) = \Sigma (1, 3, 4, 5, 6, 7, 9, 11, 13, 15)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
$\bar{w}\bar{x}$		1 1		
$\bar{w}x$	1 1 1 1			
$w\bar{x}$		1 1		
wx				

$$F = z + \bar{w}x$$

$$\underline{(d)} \quad F(A, B, C, D) = \{0, 2, 4, 5, 6, 7, 8, 10, 12, 15\}$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$	1	1	1	1
AB		1	1	
$A\bar{B}$	1			1

$$F = BD + \bar{A}\bar{B} + \bar{B}\bar{D}$$

3.6

$$\underline{(a)} \quad A'B'C'D' + AC'D' + B'CD' + A'BCD + BC'D$$

$$= A'B'C'D' + AC'D'(B+B') + B'CD'(A+A') + A'BCD + BC'D(A+A')$$

$$= A'B'C'D' + ABC'D' + AB'C'D' + AB'CD' + A'B'CD' + A'BCD +$$

$$ABC'D + A'BC'D$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$		1	1	
AB	1	1		
$A\bar{B}$	1			1

$$\therefore F = \bar{B}\bar{D} + \bar{A}BD + ABC$$

$$(b) x'z + w'xy' + w(x'y + xy')$$

$$= x'z(y+y') (w+w') + w'xy' (z+z') + wx'y (z+z') + wxy' (z+z')$$

$$= wx'yz + wx'y'z + w'x'yz + w'x'y'z + w'xy'z + w'xy'z' +$$

$$wx'yz + wx'y'z' + wx'y'z + wx'y'z'$$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
$\bar{w}x$		1	1	
$\bar{w}x$	1	1		
wx	1	1		
$w\bar{x}$		1	1	1

$$F = x\bar{y} + \bar{x}z + w\bar{x}y$$

$$(c) A'B'C'D + AB'D + A'BC' + ABCD + AB'C$$

$$= A'B'C'D + AB'D(c+c') + A'BC'(D+D') + ABCD +$$

$$AB'C(D+D')$$

$$= A'B'C'D + AB'CD + AB'C'D + A'BC'D + A'BC'D' + ABCD +$$

$$AB'CD + AB'CD'$$

	$\bar{c}\bar{d}$	$\bar{c}d$	$c\bar{d}$	cd
$\bar{a}\bar{b}$		1		
$\bar{a}b$	1	1		
$a\bar{b}$			1	
ab		1	1	1

$$F = \bar{a}\bar{b}\bar{c} + \bar{b}\bar{c}d + a\bar{c}d + a\bar{b}c$$

$$(d) \quad A'B'C'D' + BC'D + A'C'D + A'BCD + ACD'$$

$$= A'B'C'D' + (A+A')BC'D + A'C'D(B+B') + A'BCD + ACD'(B+B')$$

$$= A'B'C'D' + ABC'D + A'BC'D + A'BC'D + A'B'C'D + A'BCD +$$

$$ABC'D' + AB'C'D'$$

	$\bar{c}\bar{d}$	$\bar{c}d$	$c\bar{d}$	cd
$\bar{a}\bar{b}$	1	1		
$\bar{a}b$		1	1	
$a\bar{b}$		1		1
ab				1

$$F = \bar{a}\bar{b}\bar{c} + \bar{a}BD + B\bar{c}D + A\bar{c}\bar{d}$$

3.7

(a) $w'z + xz + x'y + wx'z$

$$= w'z(x+x') (y+y') + xz(w+w') (y+y') + x'y (w+w') (z+z') + wx'z (y+y')$$

$$= w'xyz + w'xy'z + w'x'y'z + w'x'y'z + wxyz + wx'y'z$$

$$+ w'xyz + w'xy'z + wx'yz + wx'y'z + w'x'y'z + w'x'y'z + wx'yz + wx'y'z$$

$$= w'xyz + w'xy'z + w'x'y'z + w'x'y'z + wxyz + wx'y'z + wx'yz + wx'y'z + w'x'y'z + w'x'y'z + wx'yz + wx'y'z$$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
$\bar{w}\bar{x}$		1	1	1
$\bar{w}x$		1	1	
$w\bar{x}$		1	1	
wx		1	1	1

$$F = z + \bar{x}y$$

$$\underline{(b)} \quad AD' + B'C'D + BCD' + BC'D$$

$$= AD'(B+B')(C+C') + B'C'D(A+A') + BCD'(A+A') + BC'D(A+A')$$

$$= ABCD' + ABC'C'D' + AB'C'D' + AB'C'D' + AB'C'D + A'B'C'D +$$

$$ABCD' + A'BCD' + ABC'D + A'BC'D$$

$$= ABCD' + ABC'D' + AB'C'D' + AB'C'D' + AB'C'D + A'B'C'D +$$

$$A'BCD' + ABC'D + A'BC'D$$

	CD	CD	CD	CD
AB		1		
AB		1		1
AB	1	1		1
AB	1	1		1

$$F = \overline{CD} + A\overline{D} + B\overline{C}\overline{D}$$

$$\begin{aligned}
 \text{(c)} \quad & AB'C + B'C'D' + BCD + ACD' + A'B'C + A'BC'D \\
 & = AB'C (D + D') + (A + A') B'C'D' + (A + A') BCD + ACD' (D + B') + \\
 & \quad A'B'C (D + D') + A'BC'D \\
 & = AB'CD + AB'CD' + AB'C'D + A'B'C'D' + ABCD + A'BCD + ABCD' + \\
 & \quad AB'CD' + A'B'CD + A'B'CD' + A'BC'D \\
 & = AB'CD + AB'CD' + AB'C'D' + A'B'C'D' + ABCD + A'BCD + ABCD' \\
 & \quad + A'B'CD + A'B'CD' + A'BC'D
 \end{aligned}$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1		1	1
$\bar{A}B$		1	1	
$A\bar{B}$			1	1
AB	1		1	1

$$F = CD + AC + \bar{BD} + \bar{AB}D$$

$$(D) \quad \underline{wxy} + xz + \underline{wx'z} + \underline{w'x}$$

$$= \underline{wxy}(z+z') + xz(w+w') (y+y') + \underline{wx'z}(y+y') + \underline{w'x}(y+y')(z+z')$$

$$= \underline{wxyz} + \underline{wxyz'} + \underline{wx'yz} + \underline{w'xyz} + \underline{wx'y'z} + \underline{w'xy'z} +$$

$$w'x'y'z + \underline{w'xyz} + \underline{w'xyz'} + \underline{w'xy'z} + \underline{w'xy'z'}$$

$$= \underline{wxyz} + \underline{wxyz'} + \underline{w'xyz} + \underline{wxy'z} + \underline{w'xy'z} + \underline{wx'yz} +$$

$$+ \underline{w'xyz'} + \underline{w'xy'z'}$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$	1	1	1	1
$w\bar{x}$		1	1	1
wx		1	1	

$$F = \bar{w}x + w\bar{z} + \underline{xy}$$

$$(a) F(w, x, y, z) = \{0, 2, 4, 5, 6, 7, 8, 10, 13, 15\}$$

$$0 \rightarrow 0000$$

$$7 \rightarrow 0111$$

$$2 \rightarrow 0010$$

$$8 \rightarrow 1000$$

$$4 \rightarrow 0100$$

$$10 \rightarrow 1010$$

$$5 \rightarrow 0101$$

$$13 \rightarrow 1101$$

$$6 \rightarrow 0110$$

$$15 \rightarrow 1111$$

	w	x	y	z	w	x	y	z		
0	0	0	0	0	0,2	0	0	-0	0,2,4,6	0 - - 0
1	0	0	1	0	0,4	0	-0	0	0,2,8,10	-0 - 0
2	0	1	0	0	0,8	-	0	0	0,4,2,6	0 - - 0
3	1	0	0	0	2,6	0	-1	0	0,8,2,10	-0 - 0
4	0	1	0	1	2,10	-	0	1	4,6,5,7	0 1 - -
5	0	1	1	0	4,5	0	1	0	4,5,6,7	0 1 - -
6	0	1	1	0	4,6	0	1	-0	5,7,13,15	-1 - 1
7	1	0	1	0	8,10	1	0	-0	5,13,7,15	-1 - 1
8	0	1	1	1	5,7	0	1	-1		
9	1	1	0	1	6,7	0	1	1		
10	1	1	1	0	5,13	-	1	0		
11	1	1	1	1	7,15	-	1	1		
12	1	1	1	1	13,15	1	1	-1		

$$\therefore F = \bar{w}\bar{z} + \bar{x}\bar{z} + \bar{w}x + x\bar{z}$$

	0	2	4	5	6	7	8	10	13	15
(0, 2, 4, 6) $\bar{w}\bar{z}$	x	x	x		x					
(0, 2, 8, 10) $\bar{x}\bar{z}$	x	x					x	x		
(4, 5, 6, 7) $\bar{w}x$			x	x	x	x				
(5, 7, 13, 15) $x\bar{z}$				x		x			x	x
	✓	✓	□	✓	□	✓	✓	✓	✓	✓

Essential implicants : $F = \bar{x}\bar{z} + x\bar{z} + \bar{w}x$

$$(b) F(A, B, C, D) = \Sigma (0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$$

$$0 \rightarrow 0000$$

$$8 \rightarrow 1000$$

$$2 \rightarrow 0010$$

$$10 \rightarrow 1010$$

$$3 \rightarrow 0011$$

$$11 \rightarrow 1011$$

$$5 \rightarrow 0101$$

$$14 \rightarrow 1110$$

$$7 \rightarrow 0111$$

$$15 \rightarrow 1111$$

	W	X	Y	Z		W	X	Y	Z		W	X	Y	Z
$\checkmark 0$	0	0	0	0	$\checkmark(0,2)$	0	0	-	0	$\checkmark(0,2,8,10)$	-	0	-	0
$\checkmark 2$	0	0	1	0	$\checkmark(0,8)$	-	0	0	0	$\checkmark(0,8,2,10)$	-	0	-	0
$\checkmark 8$	1	0	0	0	$\checkmark(2,3)$	0	0	1	-	$\checkmark(2,10,3,11)$	-	0	1	-
$\checkmark 3$	0	0	1	1	$\checkmark(2,10)$	-	0	1	0	$\checkmark(2,3,10,11)$	-	0	1	-
$\checkmark 5$	0	1	0	1	$\checkmark(8,10)$	1	0	-	0					
$\checkmark 10$	1	0	1	0	$\checkmark(3,7)$	0	-	1	1	$\checkmark(3,7,11,15)$	-	-	1	1
$\checkmark 7$	0	1	1	1	$\checkmark(3,11)$	-	0	1	1	$\checkmark(3,11,7,15)$	-	-	1	1
$\checkmark 11$	1	0	1	1	$\checkmark(5,7)$	0	1	-	1	$\checkmark(10,14,11,15)$	1	-	1	-
$\checkmark 14$	1	1	1	0	$\checkmark(10,14)$	1	-	1	0	$\checkmark(10,11,14,15)$	1	-	1	-
$\checkmark 15$	1	1	1	1	$\checkmark(10,11)$	1	0	1	-	$\checkmark(7,15)$	-	1	1	1
					$\checkmark(7,15)$	-	1	1	1	$\checkmark(11,15)$	1	-	1	1
					$\checkmark(11,15)$	1	-	1	1	$\checkmark(14,15)$	1	1	1	-

$$F = \overline{ABD} + \overline{BD} + \overline{BC} + CD + AC$$

	0	2	3	5	7	8	10	11	14	15
(5,7) $\bar{A}BD$				⊗	x					
(0, 2, 8, 10) $\bar{B}\bar{D}$	⊗	x				⊗	x			
(2, 3, 10, 11) $\bar{B}C$		x	x				x	x		
(3, 7, 11, 15) CD			x		x			x	x	x
(10, 11, 14, 15) AC							x	x	⊗	x
	✓	✓	□	✓	✓	✓	✓	✓	✓	✓

∴ Essential Implicants, $F = \bar{A}BD + \bar{B}\bar{D} + AC + \bar{B}C$

$$(c) \quad F(A, B, C, D) = \Sigma (2, 3, 4, 5, 6, 7, 9, 11, 12, 13)$$

$$\begin{array}{ll}
 2 \rightarrow 0010 & 7 \rightarrow 0111 \\
 3 \rightarrow 0011 & 9 \rightarrow 1001 \\
 4 \rightarrow 0100 & 11 \rightarrow 1011 \\
 5 \rightarrow 0101 & 12 \rightarrow 1100 \\
 6 \rightarrow 0110 & 13 \rightarrow 1101
 \end{array}$$

.	A	B	C	D	A	B	C	D	
✓2	0	0	1	0	(2,3)	0	0	1	-
✓4	0	1	0	0	(2,6)	0	-	1	0
✓3	0	0	1	1	(4,5)	0	1	0	-
✓5	0	1	0	1	(4,6)	0	1	-	0
✓6	0	1	1	0	(4,12)	-	1	0	0
✓9	1	0	0	1	(3,7)	0	-	1	1
✓12	1	1	0	0	(3,11)	-	0	1	1
✓7	0	1	1	1	(5,7)	0	1	-	1
✓11	1	0	1	1	(5,13)	-	1	0	1
✓13	1	1	0	1	(6,7)	0	1	1	-
					(9,11)	1	0	-	1
					(9,13)	1	-	0	1
					(12,13)	1	1	0	-

$$F = \overline{BCD} + A\overline{BD} + A\overline{CD} + \overline{AC} + \overline{AB} + B\overline{C}$$

	2	3	4	5	6	7	9	11	12	13
(3, 11) $\overline{B}CD$		x							x	
(9, 11) $A\overline{B}D$							x	x		
(9, 13) $A\overline{C}D$							x			x
(3, 5, 6, 7) $\overline{A}C$	⊗	x			x	x				
(4, 5, 6, 7) $\overline{A}B$			x	x	x	x				
(4, 5, 12, 13) $B\overline{C}$			x	x					⊗	x
	✓	✓	✓	✓	✓	✓	□	□	✓	✓

∴ Essential Implicants, $F = \overline{A}C + B\overline{C} + A\overline{B}D$

$$(d) \quad F(w, x, y, z) = \Sigma (1, 3, 6, 7, 8, 9, 12, 13, 14, 15)$$

$$\begin{array}{ll}
 1 \rightarrow 0001 & 9 \rightarrow 1001 \\
 3 \rightarrow 0011 & 12 \rightarrow 1100 \\
 6 \rightarrow 0110 & 13 \rightarrow 1101 \\
 7 \rightarrow 0111 & 14 \rightarrow 1110 \\
 8 \rightarrow 1000 & 15 \rightarrow 1111
 \end{array}$$

	w	x	y	z	w	x	y	z	w	x	y	z		
1	0	0	0	1	(1,3)	0	0	-1	(8,9,12,13)	1	-	0	-	
8	1	0	0	0	(1,9)	-	0	0	1	(8,12,9,13)	1	-	0	-
3	0	0	-1	1	(8,9)	1	0	0	-	(6,7,14,15)	-	1	1	-
6	0	1	-1	0	(8,12)	1	-	0	0	(6,14,7,15)	-	1	1	-
9	1	0	0	1	(3,7)	0	-	1	1	(12,13,14,15)	1	1	-	-
12	1	1	0	0	(6,7)	0	1	1	-	(12,14,13,15)	1	1	-	-
7	0	1	-1	1	(6,14)	-	1	1	0					
13	1	1	0	1	(9,13)	1	-	0	1					
14	1	1	-1	0	(12,13)	1	1	0	-					
15	1	1	-1	1	(12,14)	1	1	-	0					
					(7,15)	-	1	1	1					
					(13,15)	1	1	-	1					
					(14,15)	1	1	1	-					

$$F = \bar{w}\bar{x}z + \bar{x}\bar{y}z + \bar{w}yz + w\bar{y} + xy + wx$$

	1	3	6	7	8	9	12	13	14	15
(1,3) $\bar{W}\bar{X}Z$	x	x								
(1,9) $\bar{X}\bar{Y}Z$	x					x				
(3,7) $\bar{W}Y\bar{Z}$		x		x						
(8,9,12,13) $W\bar{Y}$					(x)	x	x	x		
(6,7,14,15) XY			(x)	x					x	x
(12,13,14,15) $W\bar{X}$						x	x	x	x	

□ □ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

\therefore Essential Implicants, $F = W\bar{Y} + XY + \bar{W}\bar{X}Z$

$$(e) F(A, B, C, D) = \Sigma (0, 4, 2, 5, 7, 8, 9, 10, 13, 15)$$

$$\begin{array}{ll}
 0 \rightarrow 0000 & 8 \rightarrow 1000 \\
 1 \rightarrow 0001 & 9 \rightarrow 1001 \\
 2 \rightarrow 0010 & 10 \rightarrow 1010 \\
 5 \rightarrow 0101 & 13 \rightarrow 1101 \\
 7 \rightarrow 0111 & 15 \rightarrow 1111
 \end{array}$$

	A	B	C	D		A	B	C	D	
✓0	0	0	0	0	(0,1)	0	0	0	-	(0,1,8,9) - 0 0 -
✓1	0	0	0	1	(0,2)	0	0	-0	0	(0,2,8,10) - 0 - 0
✓2	0	0	1	0	(0,8)	-	0	0	0	(0,8,1,9) - 0 0 -
✓8	1	0	0	0	(1,5)	0	-	0	1	(0,8,2,10) - 0 - 0
✓5	0	1	0	1	(1,9)	-	0	0	1	(1,5,9,13) - - 0 1
✓9	1	0	0	1	(2,10)	-	0	1	0	(1,9,5,13) - - 0 1
✓10	1	0	1	0	(8,9)	1	0	0	-	(5,7,13,15) - 1 - 1
✓7	0	1	1	1	(8,10)	1	0	-	0	(5,13,7,15) - 1 - 1
✓13	1	1	0	1	(5,7)	0	1	-	1	
✓15	1	1	1	1	(5,13)	-	1	0	1	
					(7,15)	-	1	1	1	
					(13,15)	1	1	-	1	

$$F = \overline{BC} + \overline{BD} + \overline{CD} + BD$$

	0	1	2	5	7	8	9	10	13	15
(0,1,8,9) $\bar{B}\bar{C}$	x	x				x	x			
(0,2,8,10) $\bar{B}\bar{D}$	x		(x)			x		(x)		
(1,5,9,13) $\bar{C}\bar{D}$		x		x			x		x	
(5,7,13,15) $B\bar{D}$				x	(x)				x	(x)
	✓	□	✓	✓	✓	✓	□	✓	✓	✓

$$\therefore \text{Essential Implicants, } F = \bar{B}\bar{D} + B\bar{D} + \bar{B}\bar{C}$$

$$\text{Q1} \quad F(w, x, y, z) = \Sigma (0, 1, 2, 5, 7, 8, 10, 15)$$

$$\begin{array}{ll}
 0 \rightarrow 0000 & 7 \rightarrow 0111 \\
 1 \rightarrow 0001 & 8 \rightarrow 1000 \\
 2 \rightarrow 0010 & 10 \rightarrow 1010 \\
 5 \rightarrow 0101 & 15 \rightarrow 1111
 \end{array}$$

	w x y z	w x y z	w x y z
$\sqrt{0}$	0 0 0 0	(0,1) 0 0 0 -	(0, 2, 8, 10) - 0 - 0
$\sqrt{1}$	0 0 0 1	(0,2) 0 0 - 0	(0, 8, 2, 10) - 0 - 0
$\sqrt{2}$	0 0 1 0	(0,8) - 0 0 0	
$\sqrt{8}$	1 0 0 0	(1,5) 0 - 0 1	
$\sqrt{5}$	0 1 0 1	(2,10) - 0 1 0	
$\sqrt{10}$	1 0 1 0	(8,10) 1 0 - 0	
$\sqrt{7}$	0 1 1 1	(5,7) 0 1 - 1	
$\sqrt{15}$	1 1 1 1	(7,15) - 1 1 1	

$$F = \bar{w}\bar{x}\bar{y} + \bar{w}\bar{y}z + \bar{w}xz + xy\bar{z} + \bar{x}\bar{z}$$

	0	1	2	5	7	8	10	15
(0,1) $\bar{W}\bar{X}\bar{Y}$	x	x						
(1,5) $\bar{W}\bar{Y}Z$		x			x			
(5,7) $\bar{W}XZ$					x	x		
(7,15) XYZ						x		(x)
(0,2,8,10) $\bar{X}\bar{Z}$	x		(x)				(x)	(x)
	✓			✓		✓	✓	✓

∴ Essential Implicants, $F = XYZ + \bar{X}\bar{Z} + \bar{W}\bar{Y}Z$

3.11

$$F(x, y, z) = \Sigma (0, 1, 2, 5, 8, 10, 13)$$

$$F = m_0 + m_1 + m_2 + m_5 + m_8 + m_{10} + m_{13}$$

$$F' = m_3 + m_4 + m_6 + m_7 + m_9 + m_{11} + m_{12} + m_{14} + m_{15}$$

$$(F')' = M_3 \cdot M_4 \cdot M_6 \cdot M_7 \cdot M_9 \cdot M_{11} \cdot M_{12} \cdot M_{14} \cdot M_{15}$$

$$= \Pi (3, 4, 6, 7, 9, 11, 12, 14, 15) \quad [\text{Product of sum}]$$

	$Y+Z$	$Y+\bar{Z}$	$\bar{Y}+Z$	$\bar{Y}+\bar{Z}$
$W+X$			0	
$W+\bar{X}$	0		0	0
$\bar{W}+\bar{X}$	0		0	0
$\bar{W}+X$		0	0	

$$\therefore F = (\bar{X}+Z) (\bar{Y}+\bar{Z}) (\bar{W}+X+\bar{Z})$$

3.13

(a) (1) $\Rightarrow X'Z' + Y'Z' + YZ' + XY$

	$Y'Z'$	YZ	YZ'	
X'	1			1
X	1		1	1

$$\therefore F = \Sigma (0, 2, 4, 6, 7)$$

$$= \bar{Z} + XY$$

(2) $\Rightarrow F = \pi (1, 3, 5)$

	$Y+Z$	$Y+\bar{Z}$	$\bar{Y}+\bar{Z}$	$\bar{Y}+Z$
X		0	0	
\bar{X}		0		

$$\therefore F = (X+\bar{Z}) (Y+\bar{Z})$$

(b) (2) $\Rightarrow ACD' + C'D + AB' + ABCD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1		
$\bar{A}B$		1		
$A\bar{B}$		1	1	1
AB	1	1	1	1

$$F = \Sigma (1, 5, 8, 9, 10, 11, 13, 14, 15)$$

$$= A\bar{B} + \bar{C}D + AC$$

$$(2) \Rightarrow F = \pi (0, 2, 3, 4, 6, 7, 12)$$

	$C+D$	$C+\bar{D}$	$\bar{C}+D$	$\bar{C}+\bar{D}$
$A+B$	0		0	0
$A+\bar{B}$	0		0	0
$\bar{A}+\bar{B}$	0			
$\bar{A}+B$				

\therefore simplified Product of sum

$$F = (A+\bar{C}) (A+D) (\bar{B}+C+D)$$

(c)

$$(A'+B+D') (A'+B'+C') (A'+B'+C) (B'+C+D')$$

	$C+D$	$C+\bar{D}$	$\bar{C}+D$	$\bar{C}+\bar{D}$
$A+B$				
$A+\bar{B}$		0		
$\bar{A}+\bar{B}$	0	0	0	0
$\bar{A}+B$		0	0	

$$F = \pi (5, 9, 11, 12, 13, 14, 15)$$

\therefore Product of sum :

$$F = (\bar{A}+\bar{B}) (\bar{A}+\bar{D}) (\bar{B}+C+\bar{D})$$

$$F = \Sigma (0, 1, 2, 3, 4, 5, 7, 8, 10)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1		1	1
$A\bar{B}$				
AB	1			1

$$\therefore \text{sum of products} : F = \bar{A}\bar{B} + \bar{B}\bar{D} + \bar{A}\bar{D} + \bar{A}C$$

(d) $BCD' + ABC' + ACD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$				1
AB	1	1	1	1
$A\bar{B}$			1	

$$\therefore \text{simplified sum of products} :$$

$$F = AB + ACD + BCD$$

$$F = \Sigma (6, 11, 12, 13, 14, 15)$$

$$F = \Pi (0, 1, 2, 3, 4, 5, 7, 8, 9, 10)$$

	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	0	0	0	0
$A+\bar{B}$	0	0	0	
$\bar{A}+\bar{B}$				
$\bar{A}+B$	0	0	0	0

$$\therefore \text{simplified product of sum} : F = (A+C)(A+\bar{D})(B+\bar{D})(B+C)$$

3.15

(a) $F(x, y, z) = \Sigma (0, 1, 4, 5, 6)$

$$d(x, y, z) = \Sigma (2, 3, 7)$$

	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
\bar{X}	1	1	x	x
X	1	1	x	1

$$F = 1$$

$$F = \Sigma (0, 1, 2, 3, 4, 5, 6, 7)$$

(b) $F(A, B, C, D) = \Sigma (0, 6, 8, 13, 14)$

$$d(A, B, C, D) = \Sigma (2, 4, 10)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			\cancel{X}
$\bar{A}B$	\cancel{X}			1
AB		1		1
$A\bar{B}$	1			\cancel{X}

$$F = \bar{B}\bar{D} + \bar{C}\bar{D} + AB\bar{C}\bar{D}$$

$$F = \Sigma (0, 2, 6, 8, 10, 13, 14)$$

$$\underline{(c)} \quad F(A, B, C, D) = \Sigma (5, 6, 7, 12, 14, 15)$$

$$d(A, B, C, D) = \Sigma (3, 9, 11, 15)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}B$			X	
$\bar{A}B$		1	1	1
AB	1	1	X	1
$A\bar{B}$		X	X	

$$F = BC + BD + AB$$

$$F = \Sigma (5, 6, 7, 12, 13, 14, 15)$$

$$\underline{(d)} \quad F(A, B, C, D) = \Sigma (4, 12, 7, 2, 10)$$

$$d(A, B, C, D) = \Sigma (0, 6, 8)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(X)			1
$\bar{A}B$	1		1	X
AB	1			
$A\bar{B}$	(X)			1

$$F = \bar{B}\bar{D} + \bar{C}\bar{D} + \bar{A}BC$$

$$F = \Sigma (0, 2, 4, 6, 7, 8, 10, 12)$$