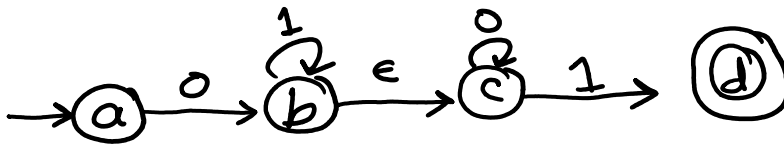


## Nondeterministic Finite Automaton with Epsilon-Transitions

- Another extension of Finite Automaton.
- The new “feature”: transition on  $\epsilon$  (empty string) is allowed.
- In effect, an NFA is allowed to make a transition spontaneously, without receiving an input symbol.
- Note that, the new capability does not expand the class of languages that can be accepted by finite automata, but it does give some added “programming convenience”.

Example:



### Formal Definition:

An  $\epsilon$ -NFA is a quintuple (5-tuple), that is, a system which consists of 5 elements. We describe an  $\epsilon$ -NFA, A as follows:

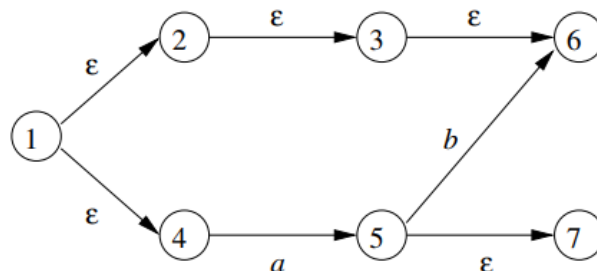
$$A = (Q, \Sigma, \delta, q_0, F)$$

where

- $Q$  - finite nonempty set of states;
- $\Sigma$  - finite nonempty set of input symbols, input alphabet;
- $\delta$  - transition function,  $\delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$ ;
- $q_0$  - initial state,  $q_0 \in Q$ ;
- $F$  - set of final or accepting states,  $F \subseteq Q$ .

### Epsilon-Closures ( $\epsilon^*$ ):

All the states that can be reached from a particular state only by seeing the  $\epsilon$  symbol.



$$\text{ECLOSE}(3) = \{3, 6\}$$

$$\text{ECLOSE}(2) = \{2, 3, 6\}$$

$$\text{ECLOSE}(1) = \{1, 2, 4, 3, 6\}$$

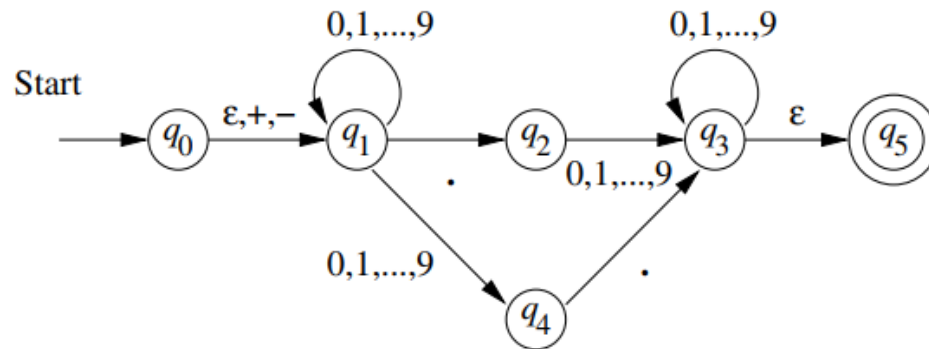
**Eliminating  $\epsilon$ -Transitions ( $\epsilon$ -NFA to DFA):**

Given any  $\epsilon$ -NFA  $E$ , we can find a DFA  $D$  that accepts the same language as  $E$ .

Let  $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$ , then the equivalent DFA can be found by defining

$$D = (Q_D, \Sigma, \delta_D, q_D, F_D).$$

Example:



The Equivalent DFA,

