Data Structure
Graph

Graphs

- A data structure that consists of a set of nodes (*vertices*) and a set of edges that relate the nodes to each other
- The set of edges describes relationships among the vertices .
- A graph G is defined as follows:

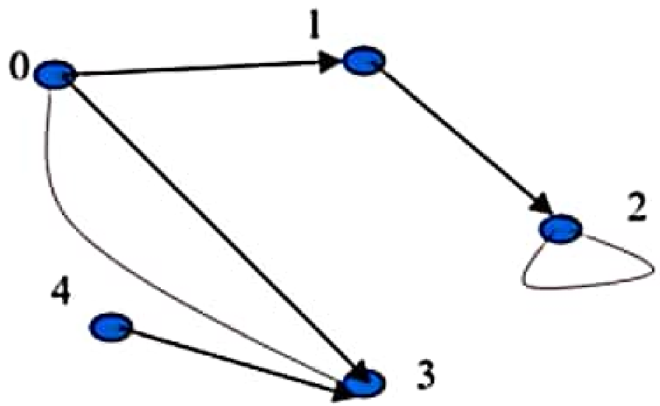
$$G=(V,E)$$

$V(G)$: a finite, nonempty set of vertices

$E(G)$: a set of edges (pairs of vertices)

Examples of Graphs

- V={0,1,2,3,4}
- E={(0,1), (1,2), (0,3), (3,0), (2,2), (4,3)}



When (x,y) is an edge,
we say that x is *adjacent to* y, and y is *adjacent from* x.

1 is adjacent to 1.
2is not adjacent to 0.
3 is adjacent from 1.

Directed vs. Undirected Graphs

- Undirected edge has no orientation (no arrow head)
- Directed edge has an orientation (has an arrow head)
- Undirected graph – all edges are undirected
- Directed graph – all edges are directed

u —— v

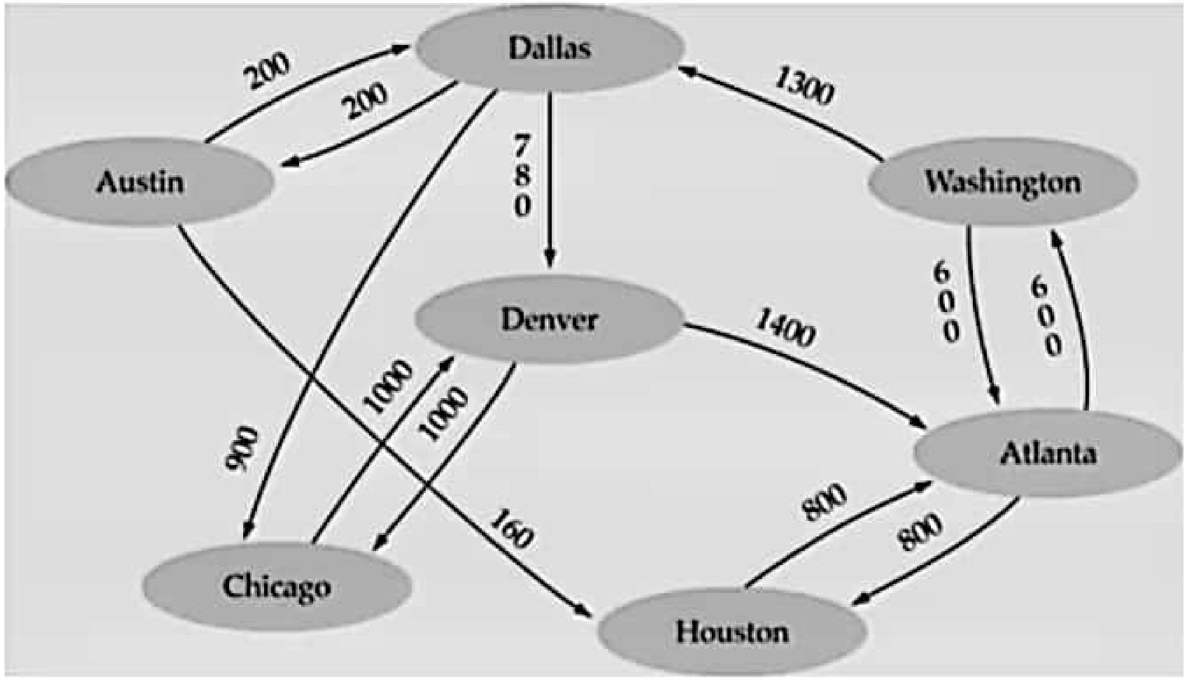
undirected edge

$u \longrightarrow v$

directed edge

Weighted graph:

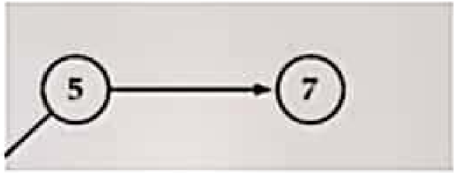
-a graph in which each edge carries a value



Graph

Graph terminology

- Adjacent nodes: two nodes are adjacent if they are connected by an edge

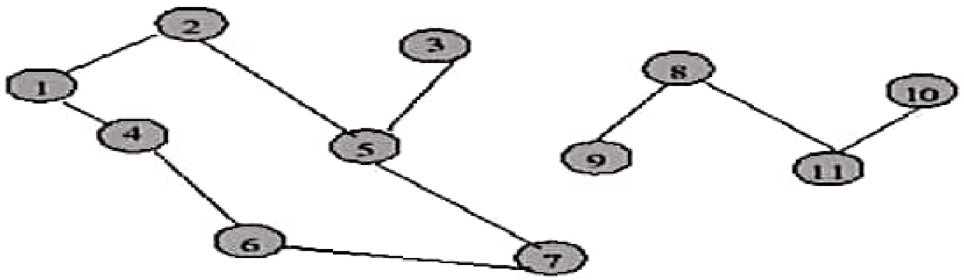


5 is adjacent to 7
7 is adjacent from

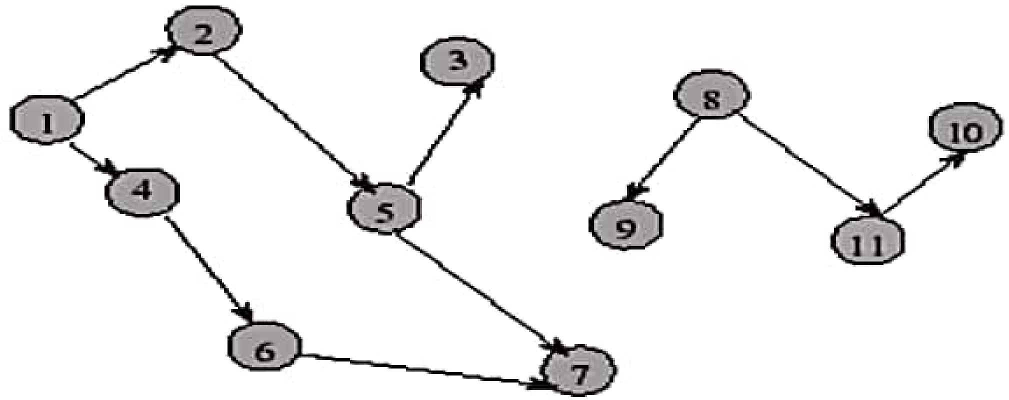
- Path: a sequence of vertices th at connect two nodes in a graph
- A simple path is a path in which all vertices, except possibly in the first and last, are different.
- Complete graph: a graph in which every vertex is directly connected to every other vertex

Continued...

- A cycle is a simple path with the same start and end vertex.
- The **degree** of vertex *i* is the no. of edges incident on vertex *i*.



e.g., degree(2) = 2, degree(5) = 3, degree(3) = 1



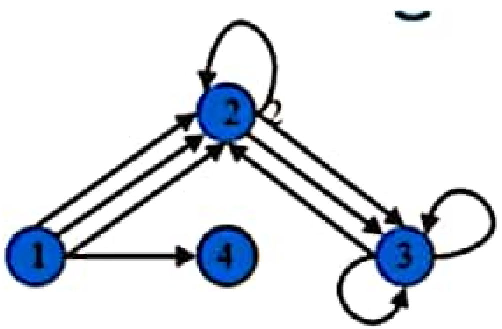
- In-degree of vertex *i* is the number of edges incident to *i* (i.e., the number of incoming edges).

  e.g., indegree(2) = 1, indegree(8) = 0
- Out-degree of vertex *i* is the number of edges incident from *i* (i.e., the number of outgoing edges).

  e.g., outdegree(2) = 1, outdegree(8) = 2

Continued...

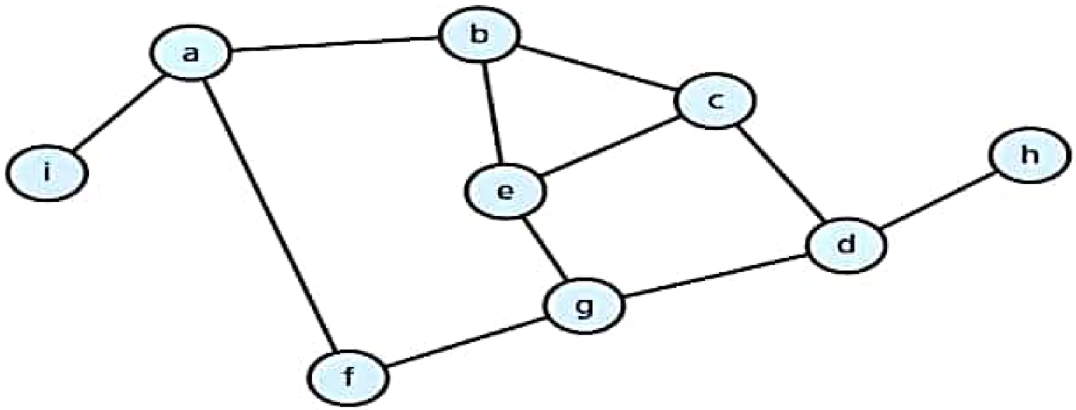
- *Loops*: edges that connect a vertex to itself
- *Multiple Edges*: two nodes may be connected by >1 edge
- *Simple Graphs*: have no loops and no multiple edges



Graph Properties

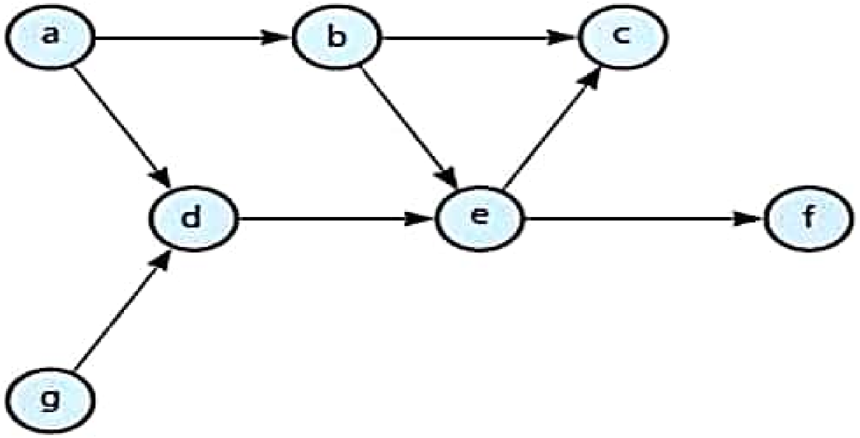
Number of Edges – Undirected Graph

- The no. of possible pairs in an n vertex graph is n*(n-1)
- Since edge *(u,v)* is **the same** as edge *(v,u)*, the maximum number of edges in an undirected graph is n*(n- 1)/2.



Number of Edges - Directed Graph

- The no. of possible pairs in an n vertex graph is n*(n-1)
- Since edge *(u,v)* is **not the same** as edge *(v,u)*, the number of edges in a directed graph is n*(n-1)
- Thus, the number of edges in a directed graph is **≤ n*(n-1)**

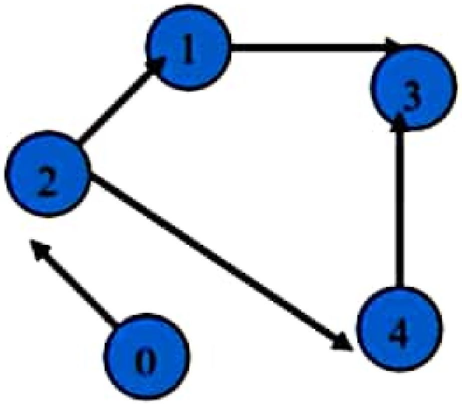


Graph Representation

- For graphs to be computationally useful, they have to be conveniently represented in programs
- There are two computer representations of graphs:
  - Adjacency matrix representation
  - Adjacency lists representation

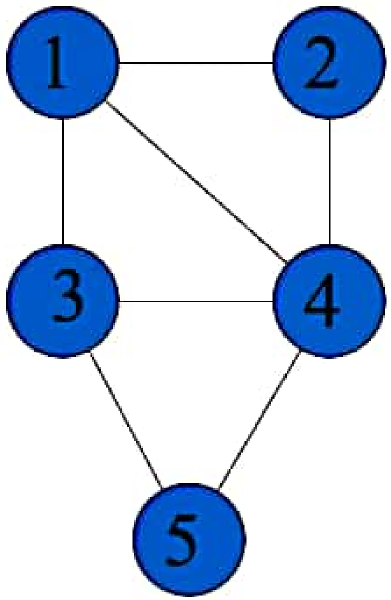
- *Adjacency Matrix*
  - A square grid of boolean values
  - If the graph contains N vertices, then the grid contains N rows and N columns
  - For two vertices numbered I and J, the element at row I and column J is true if there is an edge from I to J, otherwise false

Adjacency Matrix



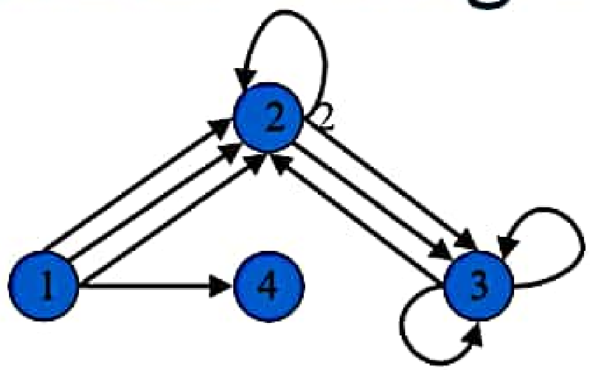
$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & \text{false} & \text{false} & \text{true} & \text{false} & \text{false} \\ 1 & \text{false} & \text{false} & \text{false} & \text{true} & \text{false} \\ 2 & \text{false} & \text{true} & \text{false} & \text{false} & \text{true} \\ 3 & \text{false} & \text{false} & \text{false} & \text{false} & \text{false} \\ 4 & \text{false} & \text{false} & \text{false} & \text{true} & \text{false} \end{array}$$

Adjacency Matrix



	1	2	3	4	5
1	0	1	1	1	0
2	1	0	0	1	0
3	1	0	0	1	1
4	1	1	1	0	1
5	0	0	1	1	0

Adjacency Matrix
-Directed Multigraphs



A:

$$\begin{pmatrix} 0 & 3 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency Lists Representation

- A graph of n nodes is represented by a one-dimensional array L of linked lists, where
  - L[i] is the linked list containing all the nodes adjacent from node i.
  - The nodes in the list L[i] are in no particular order

Graphs: Adjacency List

- Adjacency list: for each vertex $v \in V$, store a list of vertices adjacent to v
- Example:
  - Adj[1] = {2,3}
  - Adj[2] = {3}
  - Adj[3] = {}
  - Adj[4] = {3}