Vector Integration (cont....)

Ex. If $\bar{A} = (2y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}$, then evaluate $\int_c \bar{A}\cdot d\bar{r}$ along the straight line joining $(0,0,0)$ and $(2,1,1)$.

Soln: The straight line joining $(0,0,0)$ and $(2,1,1)$ is given in parametric form by $x = 2t,\ y = t,\ z = t$

$$\therefore dx = 2dt,\ dy = dt,\ dz = dt$$

when $x=0,\ t=0$	$x=2,\ t=1$
$y=0,\ t=0$	$y=1,\ t=1$
$z=0,\ t=0$	$z=1,\ t=1$

Then $$\int_c \bar{A}\cdot d\bar{r} = \int_c \left[(2y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}\right]\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= \int_c \left[(2y+3)\,dx + xz\,dy + (yz-x)\,dz\right]$$

$$= \int_{t=0}^{1} \left[(2t+3)\cdot 2dt + 2t\cdot t\,dt + (t\cdot t - 2t)\,dt\right]$$

$$= \int_{t=0}^{1} (4t + 6 + 2t^2 + t^2 - 2t)\,dt$$

$$= \int_{t=0}^{1} (3t^2 + 2t + 6)\,dt$$

$$= \left[3 \cdot \frac{t^3}{3} + 2 \cdot \frac{t^2}{2} + 6t\right]_0^1$$

$$= \left[t^3 + t^2 + 6t\right]_0^1$$

$$= \left[(1+1+6) - (0+0+0)\right] = 8. \text{ (Ans).}$$