EXAMPLE OF BISECTION AND FALSE POSITION METHODS

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BISECTION METHODS

Bisection method Steps (Rule)	
Step-1:	Find points a and b such that $a<b$ and $f(a)\cdot f(b)<0$.
Step-2:	Take the interval [a, b] and find next value $x0= (a+b)/2$
Step-3:	If $f(x0)=0$ then x0 is an exact root, else if $f(a)\cdot f(x0)<0$ then $b=x0$, else if $f(x0)\cdot f(b)<0$ then $a=x0$.
Step-4:	Repeat steps 2 & 3 until $f(xi)=0$ or $\vert f(xi)\vert \leq$Accuracy

EXAMPLE OF BISECTION METHOD

Find a root of an equation $f(x) = x^3 - x - 1$ using Bisection method for the interval [0,2] and given predefined relative error $\in_r$ = 0.01

Solution:

1st iteration :

Here $f(1) = -1 < 0 \; and \; f(2) = 5 > 0$

∴ Now, Root lies between 1 and 2

$$x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = f(1.5) = 0.875 > 0$

relative error = $\left|\frac{1.5-1}{1.5}\right|$ = 0.33

2nd iteration :

$$Here\ f(1) = -1 < 0\ and\ f(1.5) = 0.875 > 0$$

∴ Now, Root lies between 1 and 1.5

$x_1 = \frac{1+1.5}{2}$ = 1.25

$$f(x_1) = f(1.25) = -0.29688 < 0$$

relative error $=\left|\frac{1.25-1.5}{1.25}\right|$= 0.2

3rd iteration :

$$Here\ f(1.25) = -0.29688 < 0\ and\ f(1.5) = 0.875 > 0$$

∴ Now, Root lies between 1.25 and 1.5

$x_2 = \frac{1.25+1.5}{2}$ =1.375

$$f(x_2) = f(1.375) = 0.22461 > 0$$

Relative error $=\left|\frac{1.375-1.25}{1.375}\right|$= 0.09

4^{th} iteration :

$Here\ f(1.25) = -0.29688 < 0 and\ f(1.375) = 0.22461 > 0$

∴ Now, Root lies between 1.25 and 1.375

$$x_3 = \frac{1.25 + 1.375}{2} = 1.3125$$

$$f(x_3) = f(1.3125) = -0.05151 < 0$$

Relative error = $\left|\frac{1.3125- \quad .375}{1.3125}\right|$= 0.04

5^{th} iteration :

$Here\ f(1.3125) = -0.05151 < 0\ and\ f(1.375) = 0.22461 > 0$

∴ Now, Root lies between 1.3125 and 1.375

$$x_4 = \frac{1.3125+1.375}{2} =1.34375$$

$$f(x_4) = f(1.34375) = 0.08261 > 0$$

Relative error = $\left|\frac{1.34375 \quad .3125}{1.34375}\right|$ = 0.02

6^{th} iteration :

$Here\ f(1.3125) = -0.05151 < 0\ and\ f(1.34375) = 0.08261 > 0$

∴ Now, Root lies between 1.3125 and 1.34375

$x_5 = \frac{1.3125+1.34375}{2}$=1.32812

$f(x_5)$=$f(1.32812) = 0.01458 > 0$

Relative error = $\left|\frac{1.32812\quad\quad .3437}{1.32812}\right|$= 0.01

7^{th} iteration :

$Here\ f(1.3125) = -0.05151 < 0\ and\ f(1.32812) = 0.01458 > 0$

∴ Now, Root lies between 1.3125 and 1.32812

$x_6 = \frac{1.3125+1.32812}{2}$ = 1.32031

$f(x_6) = f(1.32031) = -0.01871 < 0$

Relative error = $\left|\frac{1.32031-1.32812}{1.32031}\right|$ = 0.005

Relative error is <0.01 after 7th iteration, so approximation root is: 1.32031

Iteration Table for Bisection Method:

itr	*a*	*f(a)*	*b*	*f(b)*	$c = \frac{a+b}{2}$	*f(c)*	**Update**	**Relative error**
1	1	-1	2	5	1.5	0.875	*b*=c	0.33
2	1	-1	1.5	0.875	1.25	-0.29688	*a*=c	0.2
3	1.25	-0.29688	1.5	0.875	1.375	0.22461	*b*=c	0.09
4	1.25	-0.29688	1.375	0.22461	1.3125	-0.05151	*a*=c	0.04
5	1.3125	-0.05151	1.375	0.22461	1.34375	0.08261	*b*=c	0.02
6	1.3125	-0.05151	1.34375	0.08261	1.32812	0.01458	*b*=c	0.01
7	1.3125	-0.05151	1.32812	0.01458	1.32031	-0.01871	*a*=c	0.005

FALSE POSITION/REGULA FALSI METHOD

False Position method (regula falsi method) Steps	
Step-1:	Find points $x0$ and $x1$ such that $x0<x1$ and $f(x0)\cdot f(x1)<0$.
Step-2:	Take the interval $[x0, x1]$ and find next value $x2 = x0 - (f(x0)\ (x1 - x0)\ ) / (f\ (x1) - f\ (x0))$
Step-3:	If $f(x2)=0$ then $x2$ is an exact root, else if $f(x0)\cdot f(x2)<0$ then $x1=x2$, else if $f(x2)\cdot f(x1)<0$ then $x0=x2$.
Step-4:	Repeat steps 2 & 3 until $f(xi)=0$ or $\lvert f(xi)\rvert \leq$Accuracy

EXAMPLE OF FALSE POSITION METHOD

Find a root of an equation $f(x) = x^3 - x - 1$ using False Position method for the interval [0,2] and given predefined relative error $\in_r$ = 0.01

Solution:

1^{st} iteration :
Here $f(1) = -1 < 0\ and\ f(2) = 5 > 0$

∴ Now, Root lies between $x0$=1 and $x1$=2

$$x2 = x0 - f(x0) \cdot (x1 - x0)/f(x1) - f(x0)$$

$$x2 = 1 - (-1) \cdot 2 - 15 - (-1)$$

$$x2 = 1.16667$$

$$f(x2) = f(1.16667) = -0.5787 < 0$$

Relative error = $\left|\frac{1.16667-1}{1.16667}\right|$ = 0.14

2nd iteration :

$Here\ f(1.16667) = -0.5787 < 0\ and\ f(2) = 5 > 0$
$\therefore\ Now, Root\ lies\ between\ x0 = 1.16667\ and\ x1 = 2$

$x3 = x0 - f(x0) \cdot (x1 - x0)/f(x1) - f(x0)$

$x3 = 1.16667 - (-0.5787) \cdot 2 - 1.166675 - (-0.5787)$

$x3 = 1.25311$

$f(x3) = f(1.25311) = -0.28536 < 0$

Relative error = $\left|\frac{1.25311-1.166675}{1.25311}\right|$ = 0.06

3rd iteration :

$Here\ f(1.25311) = -0.28536 < 0\ and\ f(2) = 5 > 0$

$\therefore\ Now, Root\ lies\ between\ x0 = 1.25311\ and\ x1 = 2$
$x4 = x0 - f(x0) \cdot (x1 - x0)/f(x1) - f(x0)$
$x4 = 1.25311 - (-0.28536) \cdot 2 - 1.253115 - (-0.28536)$

$x4 = 1.29344$
$f(x4) = f(1.29344) = -0.12954 < 0$

Relative error = $\left|\frac{1.29344 \quad .25311}{1.29344}\right|$ = 0.03

Relative error is <0.01 after 5^{th} iteration, so approximation root is: 1.31899

Iteration Table for False Position Method:

n	$x0$	$f(x0)$	$x1$	$f(x1)$	$x2$	$f(x2)$	**Update**	Relative Error
1	1	-1	2	5	1.16667	-0.5787	$x0=x2$	0.14
2	1.16667	-0.5787	2	5	1.25311	-0.28536	$x0=x2$	0.06
3	1.25311	-0.28536	2	5	1.29344	-0.12954	$x0=x2$	0.03
4	1.29344	-0.12954	2	5	1.31128	-0.05659	$x0=x2$	0.01
5	1.31128	-0.05659	2	5	1.31899	-0.0243	$x0=x2$	0.005

GRAPHICAL REPRESENTATION OF THE POLYNOMIAL $f(x) = x^3 - x - 1$