Date of Examination: 31-10-2021

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
Department/School: Computer Science and Engineering
Program: B.Sc. in Computer Science and Engineering
Semester Final Examination: Fall 2020
Year: 3rd Semester: 1st
Course Number: CSE 3101
Course Name: Mathematical Analysis for Computer Science

Time: 02(Two) Hours **Full Marks: 50**

Use single answer script

Instructions:	i)	Answer script should be hand written and should be written in A4 white paper. You must submit the hard copy of this answer script to the Department when the university reopens.
	ii)	You must write the following information at the top page of each answer script: **Department:** **Program:** **Course no:** **Course Title:** **Examination:** **Semester (Session):** **Student ID:** **Signature and Date:**
	iii)	Write down Student ID, Course number and put your signature on top of every single page of the answer script.
	iv)	Write down page number at the bottom of every page of the answer script.
	v)	Upload the scan copy of your answer script in PDF format through provided **google form** at the respective course site (i.e., **google classroom)** using institutional email within the allocated time. Uploading clear and readable scan copy (uncorrupted) is your responsibility and must cover the full page of your answer script. However, for clear and readable scan copy of the answer script student should use only one side of a page for answering the questions.
	vi)	You must avoid **plagiarism,** maintain **academic integrity, and ethics**. You are not allowed to take any help from another individual and if taken so can result in stern disciplinary actions from the university authority.
	vii)	Marks allotted are indicated in the **right margin.**
	viii)	Necessary **charts/tables** are attached at the end of the question paper. You may use graph papers where necessary.
	ix)	Assume any reasonable data if needed.
	x)	Symbols and characters have their usual meaning.
	xi)	Before uploading rename the PDF file as **CourseNo_StudentID.pdf** For example, CSE3101_180103001.pdf
	xii	The answer script **(one single pdf file)** must be uploaded at designated location in the provided **google form link** available in the google classroom.

There are 5 (Five) questions, from which you have to answer any 4 (Four) questions, including the Question no. 1 (ONE), which is mandatory for everyone.

Question 1. [Marks: 20]		
a)	For the Josephus problem, formulate the recurrence relation on $J(n)$ after establishing arguments on $J(2n)$ and $J(2n+1)$. From the solution pattern for small values of n, guess the general solution and prove its correctness using mathematical induction. Also, find the minimum value of n so that the person standing at the $(2X+11)$-th position finally survives, where X is the last digit (i.e., rightmost digit) of your AUST student ID.	[2+2+4 =8]
b)	"The minimum number of moves to solve the Double Tower of Hanoi problem with n pairs of disks is always twice the number of moves required to solve the Single Tower of Hanoi problem with n disks" – do you agree? Please justify your answer with the necessary derivations. Also, find the number of moves to solve the Triple Tower of Hanoi problem with 12 disks.	[3+3 =6]
c)	Provide the intuitive interpretations of symmetry identity and addition formula for binomial coefficients. Also, find the level no. of the Stern-Brocot tree that contains the fraction $(X+1) / 504$, where X is the last digit (i.e., the rightmost digit) of your AUST student ID.	[3+3 =6]
Question 2. [Marks: 10]		
For the Markovian single server exponential queueing system, derive the expressions for the following quantities. i) The probability that there are n customers in the system. ii) The average number of customers in the system. Also, make comment on the average queue length of the Shoeshine shop model with a high arrival rate and very low service rate at both the chairs.		[6+4 =10]
Question 3. [Marks: 10]		
Suppose that, whether or not it rains today depends on previous weather conditions through the last two days. If it has rained for the past two days, then it will NOT rain tomorrow with probability 0.2 If it rained today but not yesterday, then it will rain tomorrow with probability 0.5. If it rained yesterday but not today, then it will NOT rain tomorrow with probability 0.4. If it has not rained in the past two days, then it will rain tomorrow with probability 0.4. i) What is the probability that it will NOT rain on Friday, given that it already rained on both Tuesday and Wednesday? ii) What is the probability that it will rain on the 2nd March of the year 2000, given that it rained neither on 27th nor on 28th February of the same year?		[6+4 =10]

Question 4. [Marks: 10]	
Cross fertilizing a Red and a White flower produces White flowers 60% of the time. Now we cross fertilize five pairs of Red and White flowers and produce five offspring. Find the probability that there will be majority of Red flowered plants in the five offspring. Also, mention the names of two different random variables which have the memoryless property.	[6+4 =10]
Question 5. [Marks: 10]	
Fish experts have found that the length of Salmon and Sea Bass fishes follow Gaussian distribution with mean of 5 and 11 inches, respectively and variance of 0.64 and 2.25 $inch^2$, respectively. For the Bayes classifier, find the Decision Boundary between the Salmon and Sea Bass classes based on their length, when the ratio of Salmon to Sea bass fishes is 3 to 1. Also, mention the underlying assumption of the naïve Bayes classifier and identify the four components of the Bayes conditional probability formula on posterior probability.	[6+4 =10]