

**Note:**

2 sets of problems are given for this online lab test.

Set A: All Possible Roots by Modified Bisection Method

Set B: Multiple Roots of Polynomial using Newtons Method.

- If you choose Set A you will get 20% penalty and for choosing Set B you will get no penalty.
- After completing your code you must upload you code and output in the given Google form link.
- Allocated time for Set A is 30 minutes and for Set C is 40 Minutes

**Set A**

**Problem Statement:** Determine the all possible real roots of the equation:  $f(x) = x^3 - 7x^2 + 15x - 9 = 0$  using Modified Bisection Method. Employ initial guesses of  $x_{lower} = 0$  and  $x_{upper} = 4$  and iterate until the estimated error  $\epsilon_a$  falls below a level of  $\epsilon_s = 0.001$

**Algorithm:**

1. Enter lower limit  $x_{lower}$  and upper limit  $x_{upper}$  of the interval covering all the roots.
2. Decide the size of the increment interval  $\Delta x = 0.1$
3. set  $x_1 = x_{lower}$  and  $x_2 = x_{lower} + \Delta x$
4. Compute  $f_1 = f(x_1)$  and  $f_2 = f(x_2)$
5. If  $(f_1 * f_2) > 0$ ,  
then the interval does not bracket any root and go to step 9
6. Compute  $x_0 = (x_1 + x_2)/2$  and  $f_0 = f(x_0)$
7. If  $(f_1 * f_2) < 0$   
then set  $x_2 = x_0$   
Else set  $x_1 = x_0$  and  $f_1 = f_0$
8. If  $|(x_2 - x_1) / x_2| < E$ , then  
root =  $(x_1 + x_2) / 2$   
write the value of root  
go to step 9  
Else  
go to step 6
9. If  $x_2 < x_{upper}$ , then set  $x_{lower} = x_2$  and go to step 3
10. Stop.

### Tasks:

1. Write a program using Modified Bisection Method to locate the approximate roots of the function  $f(x) = x^3 - 7x^2 + 15x - 9 = 0$  with initial guesses  $[0, 4]$ .
2. Use Horner's rule to perform all iterations of the Modified Bisection Method until the estimated error  $\epsilon_a$  falls below a level of  $\epsilon_s = 0.0001$
3. Use appropriate functions from math header file.
4. Show the table with number of root, approximate value; number of iterations where the root is found and relative error found on that iteration. (You can count the step by adding 1 to a counting variable  $i$  in the loop of the program).

**Sample Input/output:**

Enter the highest degree of the equation: 3

Enter values of coefficients:

Coefficient  $x[3] = 1$

Coefficient  $x[2] = -7$

Coefficient  $x[1] = 15$

Coefficient  $x[0] = -9$

Enter the lower and upper limit:

2

4

Number of Root	Approximate Root	Number of Iteration	Relative Error
1	0.999994	9	0.000006
2	2.999682	29	0.000008
3	3.000341	30	0.000008

1

2

3

## Set B

**Problem Statement:** Determine the **multiple** real roots of the equation:  $f(x) = x^3 - 4x^2 + x + 6 = 0$  using Newton's Method. Employ initial guess of  $x_1 = 5$  and iterate until the estimated error  $\epsilon_a$  falls below a level of  $\epsilon_s = 0.001$

Algorithm:

1. Obtain degree and co-efficient of polynomial (n and  $a_i$ ).
2. Decide an initial estimate for the first root ( $x_0$ ) and error criterion, E.  
*Do while*  $n > 1$
3. Find the root using Newton-Raphson algorithm  
$$x_r = x_0 - f(x_0) / f'(x_0)$$
4. Root (n) =  $x_r$
5. Deflate the polynomial using synthetic division algorithm and make the factor polynomial as the new polynomial of order n-1.
6. Set  $x_0 = x_r$  [Initial value of the new root]  
*End of Do*
7. Root (1) =  $-a_0 / a_1$
8. Stop

**Tasks:**

1. Write a program using Newton's Method to locate the approximate roots of the function  $(x) = x^3 - 4x^2 + x + 6 = 0$  with initial guess 5.
2. Use Horner's rule to perform all iterations of the Newton's Method until the estimated error  $\epsilon_a$  falls below a level of  $\epsilon_s = 0.001$
3. Use synthetic division to deflate the polynomial at lower degree.

Algorithm for Synthetic Division:

$$b_{i-1} = a_i + x_r b_i \quad ; \text{for } i = n, n-1, \dots, 0$$
$$b_n = 0$$

Where a is the coefficient at degree n and b is the coefficient at degree n - 1

4. Use appropriate functions from math header file.
5. Print the degree of the polynomial, polynomial and roots found at each degree.

### Sample Input/output:

```
Enter values of coefficients:
Coefficient x[3] = 1

Coefficient x[2] = -4

Coefficient x[1] = 1

Coefficient x[0] = 6

Enter the intial value:5
The 3 order polynomial is:    1.00x^3+-4.00x^2+1.00x^1+6.00x^0
At order 3 the Root is 3.000000

The 2 order polynomial is:    1.00x^2+-1.00x^1+-2.00x^0
At order 2 the Root is 2.000000

The 1 order polynomial is:    1.00x^1+1.00x^0
At order 1 the Root is -1.000000

There are 3 Roots for the given polynomial
```