Ex. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$.

Sol^n: The Fourier cosine transform of $f(x)$ is given by

$$F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx$$

Putting the value of $f(x)$ we get

$$F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty \left(e^{-2x} + 4e^{-3x}\right) \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-2x} \cos sx \, dx + \sqrt{\frac{2}{\pi}} \int_0^\infty 4e^{-3x} \cos sx \, dx$$

$$\left[\because \int e^{-ax} \cos bx \, dx = \frac{e^{-ax}}{a^2+b^2}(b \sin bx - a \cos bx)\right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{e^{-2x}}{4+s^2}(s \sin sx - 2 \cos sx)\right]_0^\infty + 4\sqrt{\frac{2}{\pi}} \left[\frac{e^{-3x}}{9+s^2}(s \sin sx - 3 \cos sx)\right]_0^\infty$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{2}{s^2+4} + 4 \cdot \sqrt{\frac{2}{\pi}} \cdot \frac{3}{s^2+9} = 2 \cdot \sqrt{\frac{2}{\pi}} \left[\frac{1}{s^2+4} + \frac{6}{s^2+9}\right] \text{ (Ans)}$$

Ex. Find the Fourier cosine transform of the following function:

$$f(x) = x \quad \text{for } 0 < x < \tfrac{1}{2}$$
$$= 1 - x \quad \text{for } \tfrac{1}{2} < x < 1$$
$$= 0 \quad \text{for } x > 1$$

Sol^n. The Fourier cosine transform of $f(x)$ is given by

$$F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cdot \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\frac{1}{2}} x \cos sx \, dx + \sqrt{\frac{2}{\pi}} \int_{\frac{1}{2}}^1 (1-x) \cos sx \, dx + 0$$

$$\int u \cdot v \, dx = u \int v \, dx - \int \left\{\frac{du}{dx} \int v \, dx\right\} dx$$

Scanned with CamScanner

$$=\sqrt{\frac{2}{\pi}}\left[x\cdot\frac{\sin sx}{s}-\int 1\cdot\left(\frac{\sin sx}{s}\right)dx\right]_0^{1/2}+\sqrt{\frac{2}{\pi}}\left[(1-x)\cdot\frac{\sin sx}{s}-\int(-1)\cdot\frac{\sin sx}{s}\,dx\right]_{1/2}^{1}$$

$$=\sqrt{\frac{2}{\pi}}\left[x\cdot\frac{\sin sx}{s}+\frac{\cos sx}{s^2}\right]_0^{1/2}+\sqrt{\frac{2}{\pi}}\left[(1-x)\cdot\frac{\sin sx}{s}-\frac{\cos sx}{s^2}\right]_{1/2}^{1}$$

$$=\sqrt{\frac{2}{\pi}}\left[\frac{1}{2}\cdot\frac{\sin\frac{s}{2}}{s}+\frac{\cos\frac{s}{2}}{s^2}-\frac{1}{s^2}\right]+\sqrt{\frac{2}{\pi}}\left[0-\frac{\cos s}{s^2}-\frac{1}{2}\cdot\frac{\sin\frac{s}{2}}{s}+\frac{\cos\frac{s}{2}}{s^2}\right]$$

$$=\sqrt{\frac{2}{\pi}}\left[-\frac{\cos s}{s^2}+\frac{2\cos\frac{s}{2}}{s^2}-\frac{1}{s^2}\right].\ (\text{Ans})$$