Matrices

Definition of matrix: A rectangular array of numbers enclosed by a pair of brackets such as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

and subject to certain rules of operations is called a matrix. The numbers a_{ij} are called the elements of the matrix.

Different types of matrices:

(i) Row matrix: If a matrix has only one row and any number of columns, is called a row matrix. As for example,

$\begin{bmatrix} 2 & 7 & 3 & 9 \end{bmatrix}$ is a row matrix.

(ii) Column matrix: A matrix, having one column and any number of rows, is called a column matrix. As for example,

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a column matrix.

(iii) Null matrix or zero matrix: Any matrix, in which all the elements are zeros, is called a zero matrix or null matrix.

As for example, $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is a null matrix.

(iv) Square matrix: A matrix, in which ~~all the elements~~ the number of rows is equal to the number of columns, is called a square matrix. As for example,

$$\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$$ is a square matrix.

(v) Diagonal matrix: A square matrix is called a diagonal matrix, if all its non-diagonal elements are zero. As for example,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$ is a diagonal matrix.

(vi) Unit or Identity matrix: A square matrix is called a unit matrix if all the diagonal elements are unity and non-diagonal elements are zero. As for example

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ is an identity matrix.

... equal to its transpose.

(vii) Symmetric matrix: A square matrix ~~is called~~ will be called symmetric, if for all values of i and j, that is $a_{ij} = a_{ji}$ or $A' = A$. As for example,

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & e \end{bmatrix}$$ is a symmetric matrix.

(viii) Transpose of a matrix: If in a given matrix A, we interchange the rows and the corresponding columns, the new matrix obtained is called the transpose of the matrix A and is denoted by A' or A^T. As for example.

If a matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ then the transpose of the matrix A is $A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$.

$A \cdot A' = I$

if $|A| = 1$, matrix A is proper.

Addition of Matrices:

Addition of matrices is defined only for the matrices having same number of rows and the same number of columns. Let A and B be two matrices having m rows and n columns.

That is, $A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ and

$$B = [b_{ij}] = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

Then the sum of A and B is

$$A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

Scalar multiplication of matrices:

The product of an $(m \times n)$ matrix A by a number k is denoted by kA or Ak, that is

$$kA = \begin{bmatrix} ka_{11} & ka_{12} & \cdots\cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots\cdots & ka_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ ka_{m1} & ka_{m2} & \cdots\cdots & ka_{mn} \end{bmatrix}$$

Properties:

(i) $A+B = B+A$ (Commutative law)

(ii) $(A+B)+C = A+(B+C)$ (Associative law)

(iii) $A+O = O+A = A$

(iv) $k(A+B) = kA+kB = (A+B)k$

where O is the zero matrix of the same order.

Matrix Multiplication.

Two matrices A and B are conformable for multiplication if the number of columns in A is equal to the number of rows in B.

~~As for example, let A =~~ If $A=[a_{ij}]$ is a $m\times p$ matrix and $B=[b_{ij}]$ is a $p\times n$ matrix, then AB is the $m\times n$ matrix $C=[c_{ij}]$.

As for example, let $A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & 0 & -1 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 1 & -1 \\ -2 & 4 \\ 3 & 0 \end{bmatrix}_{3\times2}$

then $AB = \begin{bmatrix} 1\cdot1+(-3)(-2)+5\cdot3 & 1(-1)+(-3)(4)+5\cdot0 \\ 2\cdot1+(0)(-2)+(-1)(3) & 2\cdot(-1)+0\cdot4+(-1)\cdot0 \end{bmatrix}$

$$= \begin{bmatrix} 1+6+15 & -1-12 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & -13 \\ -1 & -2 \end{bmatrix}_{2\times2}$$