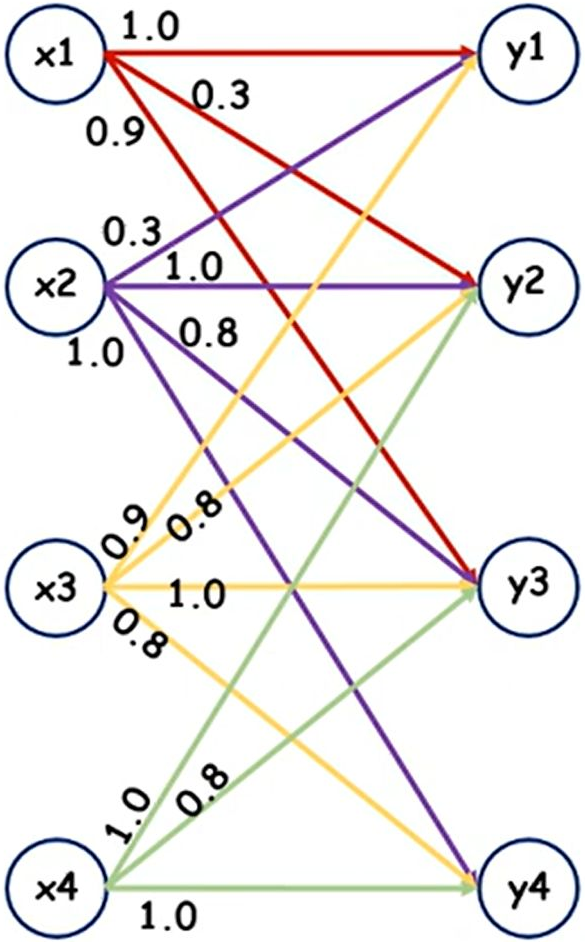
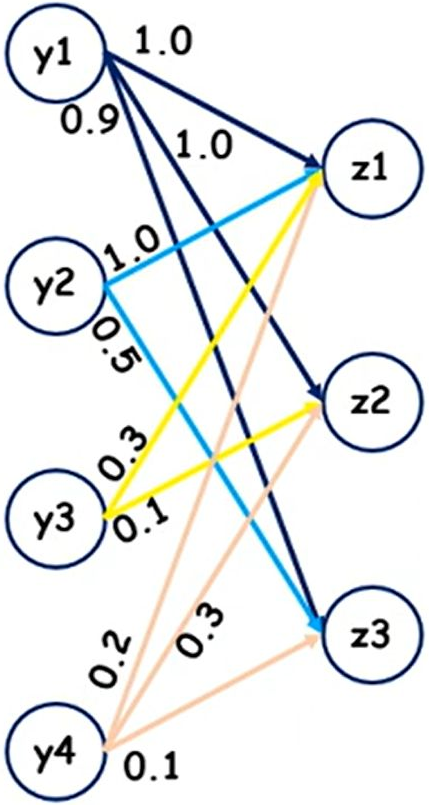
Composition of Fuzzy Relations

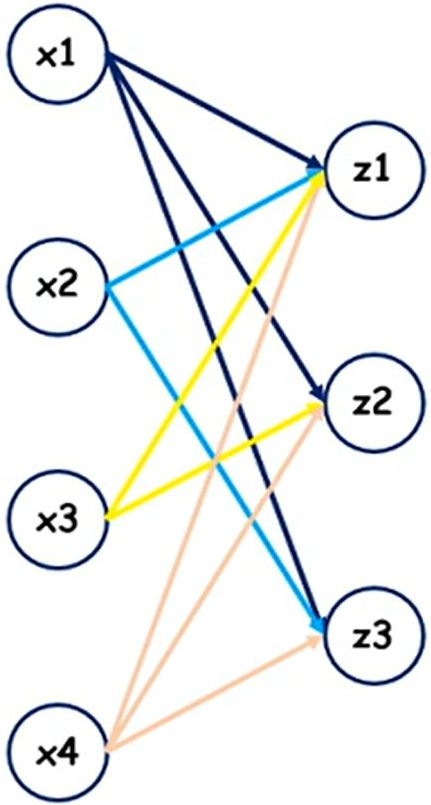
Soft Computing
CSE 4237



X x Y

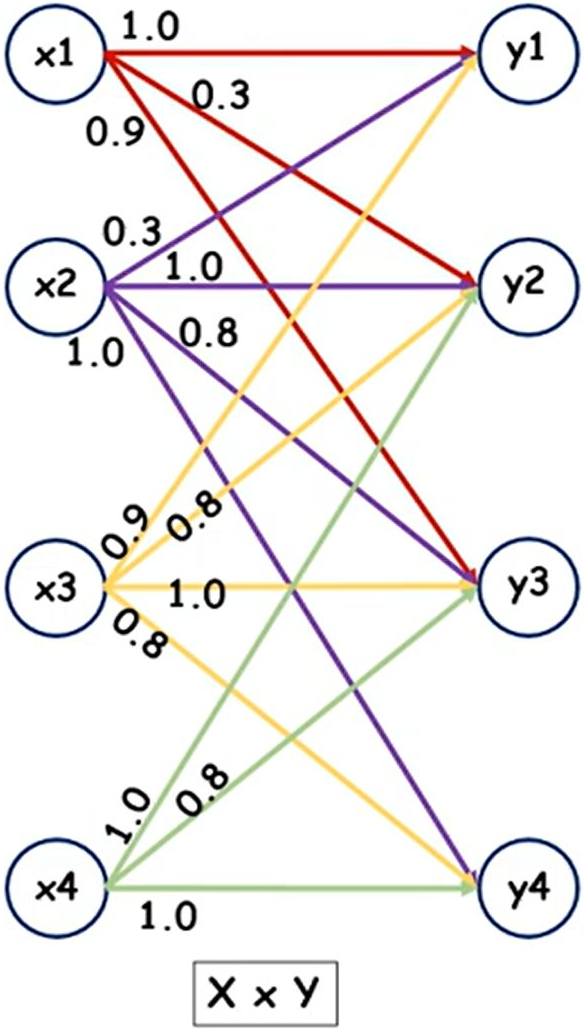


Y x Z



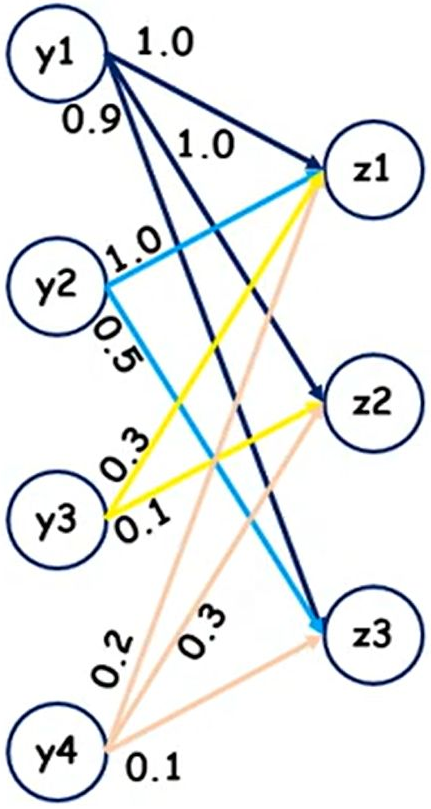
X x Z

Figure: The composition of two fuzzy relations is a new relation directly associating elements from X and Z.



$$R_1 = \begin{bmatrix} \mu_{R_1}(x_1, y_1) & \mu_{R_1}(x_1, y_2) & \mu_{R_1}(x_1, y_3) & \mu_{R_1}(x_1, y_4) \\ \mu_{R_1}(x_2, y_1) & \mu_{R_1}(x_2, y_2) & \mu_{R_1}(x_2, y_3) & \mu_{R_1}(x_2, y_4) \\ \mu_{R_1}(x_3, y_1) & \mu_{R_1}(x_3, y_2) & \mu_{R_1}(x_3, y_3) & \mu_{R_1}(x_3, y_4) \\ \mu_{R_1}(x_4, y_1) & \mu_{R_1}(x_4, y_2) & \mu_{R_1}(x_4, y_3) & \mu_{R_1}(x_4, y_4) \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix}$$



Y x Z

$$R_2 = \begin{bmatrix} \mu_{R_2}(y_1, z_1) & \mu_{R_2}(y_1, z_2) & \mu_{R_2}(y_1, z_3) \\ \mu_{R_2}(y_2, z_1) & \mu_{R_2}(y_2, z_2) & \mu_{R_2}(y_2, z_3) \\ \mu_{R_2}(y_3, z_1) & \mu_{R_2}(y_3, z_2) & \mu_{R_2}(y_3, z_3) \\ \mu_{R_2}(y_4, z_1) & \mu_{R_2}(y_4, z_2) & \mu_{R_2}(y_4, z_3) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

Max-min composition of Fuzzy Relations

$$R_1 \circ R_2 = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

Max-min composition of Fuzzy Relations

$$R_1 \circ R_2 = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 \\ 1.0 \\ 0.3 \\ 0.2 \end{bmatrix}$$

$$= [1.0 \wedge 1.0] \vee [0.3 \wedge 1.0] \vee [0.9 \wedge 0.3] \vee [0.0 \wedge 0.2]$$

Max-min composition of Fuzzy Relations

$$R_1 \circ R_2 = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 \\ 1.0 \\ 0.3 \\ 0.2 \end{bmatrix}$$

$$= [1.0 \wedge 1.0] \vee [0.3 \wedge 1.0] \vee [0.9 \wedge 0.3] \vee [0.0 \wedge 0.2]$$

$$= 1.0 \vee 0.3 \vee 0.3 \vee 0.0 \quad = 1.0$$

Max-min composition of Fuzzy Relations

$$R_1 \circ R_2 = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \end{bmatrix} \circ \begin{bmatrix} 1.0 \\ 1.0 \\ 0.3 \\ 0.2 \end{bmatrix}$$

$$R_1 \circ R_2 = \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.3 & 0.5 \\ 0.9 & 0.9 & 0.9 \\ 1.0 & 0.3 & 0.5 \end{bmatrix}$$

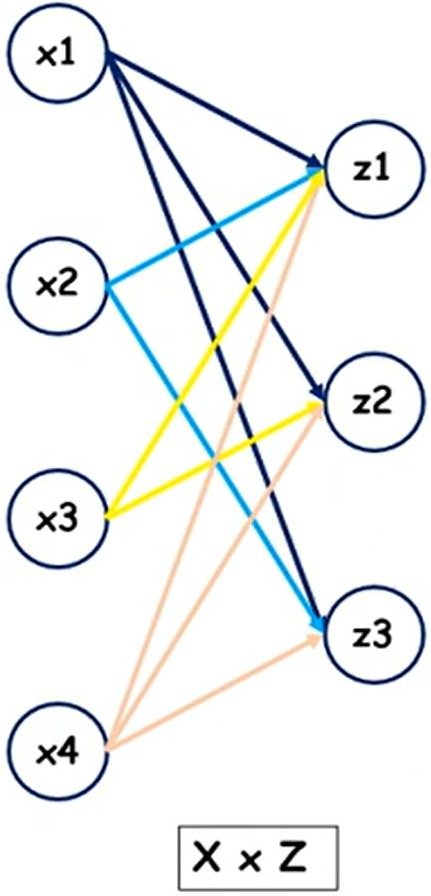
$$= [1.0 \wedge 1.0] \vee [0.3 \wedge 1.0] \vee [0.9 \wedge 0.3] \vee [0.0 \wedge 0.2]$$

$$= 1.0 \vee 0.3 \vee 0.3 \vee 0.0 \quad = 1.0$$

Max-min composition of Fuzzy Relations

$$R_1 \circ R_2 = \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.3 & 0.5 \\ 0.9 & 0.9 & 0.9 \\ 1.0 & 0.3 & 0.5 \end{bmatrix}$$

$$\begin{aligned} R_1 \circ R_2 = {} & 1.0/(x_1, z_1) + 1.0/(x_1, z_2) + 0.9/(x_1, z_3) \\ & + 1.0/(x_2, z_1) + 0.3/(x_2, z_2) + 0.5/(x_2, z_3) \\ & + 0.9/(x_3, z_1) + 0.9/(x_3, z_2) + 0.9/(x_3, z_3) \\ & + 1.0/(x_4, z_1) + 0.3/(x_4, z_2) + 0.5/(x_4, z_3) \end{aligned}$$



Max-product composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

Max-product composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

$$\mu_{R_1}(x_1, y_1) \cdot \mu_{R_2}(y_1, z_1) = 0.1 \times 0.9 = 0.09$$
$$\mu_{R_1}(x_1, y_2) \cdot \mu_{R_2}(y_2, z_1) = 0.2 \times 0.2 = 0.04$$
$$\mu_{R_1}(x_1, y_3) \cdot \mu_{R_2}(y_3, z_1) = 0.0 \times 0.8 = 0.0$$
$$\mu_{R_1}(x_1, y_4) \cdot \mu_{R_2}(y_4, z_1) = 1.0 \times 0.4 = 0.4$$
$$\mu_{R_1}(x_1, y_5) \cdot \mu_{R_2}(y_5, z_1) = 0.7 \times 0.0 = 0.0$$

Max-product composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

$$\mu_{R_1}(x_1, y_1) \cdot \mu_{R_2}(y_1, z_1) = 0.1 \times 0.9 = 0.09$$
$$\mu_{R_1}(x_1, y_2) \cdot \mu_{R_2}(y_2, z_1) = 0.2 \times 0.2 = 0.04$$
$$\mu_{R_1}(x_1, y_3) \cdot \mu_{R_2}(y_3, z_1) = 0.0 \times 0.8 = 0.0$$
$$\mu_{R_1}(x_1, y_4) \cdot \mu_{R_2}(y_4, z_1) = 1.0 \times 0.4 = 0.4$$
$$\mu_{R_1}(x_1, y_5) \cdot \mu_{R_2}(y_5, z_1) = 0.7 \times 0.0 = 0.0$$

$$\mu_{R_1 \cdot R_2}(x_1, z_1)$$
$$= 0.09 \vee 0.04 \vee 0.0 \vee 0.4 \vee 0.0 = 0.4$$

Max-product composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

Max-product composition $R_1 \cdot R_2$

	z_1	z_2	z_3	z_4
x_1	0.4	0.7	0.3	0.56
x_2	0.27	1.0	0.4	0.8
x_3	0.8	0.3	0.7	1.0

$$\mu_{R_1 \cdot R_2}(x_1, z_1)$$
$$= 0.09 \vee 0.04 \vee 0.0 \vee 0.4 \vee 0.0 = 0.4$$

Max-average composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

$$\mu_{R_1}(x_1, y_1) + \mu_{R_2}(y_1, z_1) = 0.1 + 0.9 = 1.0$$
$$\mu_{R_1}(x_1, y_2) + \mu_{R_2}(y_2, z_1) = 0.2 + 0.2 = 0.4$$
$$\mu_{R_1}(x_1, y_3) + \mu_{R_2}(y_3, z_1) = 0.0 + 0.8 = 0.8$$
$$\mu_{R_1}(x_1, y_4) + \mu_{R_2}(y_4, z_1) = 1.0 + 0.4 = 1.4$$
$$\mu_{R_1}(x_1, y_5) + \mu_{R_2}(y_5, z_1) = 0.7 + 0.0 = 0.7$$

Max-average composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

$$\mu_{R_1}(x_1, y_1) + \mu_{R_2}(y_1, z_1) = 0.1 + 0.9 = 1.0$$
$$\mu_{R_1}(x_1, y_2) + \mu_{R_2}(y_2, z_1) = 0.2 + 0.2 = 0.4$$
$$\mu_{R_1}(x_1, y_3) + \mu_{R_2}(y_3, z_1) = 0.0 + 0.8 = 0.8$$
$$\mu_{R_1}(x_1, y_4) + \mu_{R_2}(y_4, z_1) = 1.0 + 0.4 = 1.4$$
$$\mu_{R_1}(x_1, y_5) + \mu_{R_2}(y_5, z_1) = 0.7 + 0.0 = 0.7$$

$$\mu_{R_1\langle+\rangle R_2}(x_1, z_1)$$

$$= \frac{1}{2}\left[1.0 \vee 0.4 \vee 0.8 \vee 1.4 \vee 0.7\right] = 0.7$$

Max-average composition of Fuzzy Relations

R_1:

	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0.0	1.0	0.7
x_2	0.3	0.5	0.0	0.2	1.0
x_3	0.8	0.0	1.0	0.4	0.3

R_2:

	z_1	z_2	z_3	z_4
y_1	0.9	0.0	0.3	0.4
y_2	0.2	1.0	0.8	0.0
y_3	0.8	0.0	0.7	1.0
y_4	0.4	0.2	0.3	0.0
y_5	0.0	1.0	0.0	0.8

	z_1	z_2	z_3	z_4
x_1	0.7	0.85	0.65	0.75
x_2	0.6	1.0	0.65	0.9
x_3	0.9	0.65	0.85	1.0

$$\mu_{R_1\langle+\rangle R_2}(x_1, z_1)$$

$$= \frac{1}{2}\left[1.0 \vee 0.4 \vee 0.8 \vee 1.4 \vee 0.7\right] = 0.7$$