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AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Arts and Sciences

Program: B. Sc. in Computer Science and Engineering

Semester Final Examination: Spring 2020

Year: Second	**Course No: Math 2203**
Semester: Second	**Course Name: Mathematics IV**
Time: 3 (Three) hours	**Full marks: 60**

Instructions:	i)	Answer script should be hand written and should be written in A4 white paper. You must submit the hard copy of this answer script to the Department when the university reopens.
	ii)	Write down Student ID, Course number and put your signature on top of every single page of the answer script.
	iii)	Write down page number at the bottom of every page of the answer script.
	iv)	Upload the scan copy of your answer script in PDF format at the respective site of the course at **google form** using institutional email within the allocated time. Uploading clear and readable scan copy is your responsibility and must be covered the full page of your answer script.
	v)	Before uploading rename the PDF file as **CourseNo_StudentID.pdf** e.g. Math2203_180204001.pdf
	vi)	You must avoid **plagiarism,** maintain **academic integrity, and ethics**. You are not allowed to take any help from another individual and if taken so can result in stern disciplinary actions from the university authority.
	vii)	Marks allotted are indicated in the **right margin.**
	viii)	Necessary **charts/tables** are attached at the end of the question paper.
	xi)	Assume any reasonable data if needed.
	x)	Symbols and characters have their usual meaning.
	xi)	There are **7 (Seven) Questions in Group A, Group B and Group C. Answer any 5 (Five) taking at least 1 (One) from each Group.**

Group A

Question 1. [Marks: 12]		
a)	Find whether the set of vectors $\{(2,\ 1, -1\ ),\ (1, -2,\ 1),\ (7,\ -4,\ 1)\}$ is linearly dependent or independent.	**[6]**
b)	Find the directional derivative of $\varphi\ (x, y, z) = \frac{\mathrm{y}}{x-\mathrm{z}}$ at $P(2, 1, -1)$ in the direction from P to $Q(-1, -2, -2)$. Find also the greatest rate of change of φ .	**[6]**
Question 2. [Marks: 12]		
a)	Find whether the function $\vec{A} = (y^2 - 2xyz^3)\ \hat{\imath} + (3 + 2xy - x^2z^3)\ \hat{\jmath} + (6z^3 - 3x^2yz^2)\ \hat{k}$ is irrotational. If $\vec{A}$ is irrotational then find a scalar function φ, such that $\vec{A} = \nabla\varphi$.	**[6]**
b)	Evaluate $\oint_C \vec{F} \cdot \overrightarrow{dr}$ for $\vec{F} = (2x^2 + y)\ \hat{\imath} + (3x - 4y)\ \hat{\jmath}$, where C is the closed region bounded by the triangle with vertices (1, 0), (4, 0) and (4, 3).	**[6]**
Question 3. [Marks: 12]		
a)	Verify Green's theorem for $\oint_C\ [\ (x^2 + y^2)\ dx + xy^2\ dy]$, where C is the boundary of the closed region bounded by $y = -x$ and $x = y^2$.	**[6]**
b)	Use Divergence theorem to evaluate $\iint_S [(2xy + y^2z)\ dx\ dy\ + (z^2x)\ dy\ dz + (x^2y - z^3)\ dz\ dx]$, where S is the surface enclosing a region bounded by hemisphere $x^2 + y^2 + z^2 = 16$ above XY – plane.	**[6]**

Group B

Question 4. [Marks: 12]		
a)	Find the inverse of the following matrix by using row canonical form. $$A = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$	**[6]**
b)	Find the rank and eigenvalues of the following matrix: $$B = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 5 \end{pmatrix}.$$	**[6]**
Question 5. [Marks: 12]		
a)	Find the row reduced echelon form of the following matrix:	**[6]**

	$C = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}.$	
b)	Find the normal form of the following matrix: $D = \begin{pmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{pmatrix}.$	**[6]**

Group C

Question 6. [Marks: 12]		
a)	Find the graph of the following function and its corresponding Fourier series. $f(x) = \begin{cases} 2x & when \quad 0 < x < 4 \\ 7 - x & when \quad 4 < x < 8 \end{cases}$.	**[7]**
b)	Find the inverse Fourier cosine transform of $\frac{e^{-7\|S\|}}{S}$.	**[5]**
Question 7. [Marks: 12]		
a)	Find the Fourier cosine integral of $f(x) = e^{-4\|x\|}$.	**[4]**
b)	Solve the equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$, subject to the following conditions: (i) $u = 1 \quad when \quad x = 0,\ t > 0$ (ii) $u = \begin{cases} x & if \quad 0 < x < 2 \\ 0 & if \quad x \geq 2 \end{cases}$ when $t = 0$ (iii) $u(x, t)$ is bounded.	**[8]**