Canonical Matrix

A canonical matrix is one in which all terms not of the principal diagonal are zero, all terms on the principal diagonal are zero or one, and all ones precedes all zeros. As for example,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is a canonical matrix.

<u>Ex.</u> Reduce the following matrix to the canonical form:

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

<u>Sol^n:</u> We will apply both elementary row and column operations to the matrix A for reducing it to the canonical form.

First apply $R_3' = R_3 - R_1$ and $R_4' = R_4 - R_3$ and get the equivalent

$$\text{matrix} \sim \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 2 & 4 & 1 \end{bmatrix}$$

Now apply $R_2' = R_2 - R_4$ and $R_3' = R_3 - R_4$ and get the equivalent matrix $\sim \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix}$

Now apply $c_1' = \frac{c_1}{2}$ and $R_3' = \frac{R_3}{2}$ and get

$$\sim \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix}$$

Now apply $R_3' = R_3 - R_2$ and get $\sim \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix}$

Now interchange R_3 and R_4 and get $\sim \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Now apply $c_2' = c_2 + c_1$, $c_3' = c_3 - 3c_1$ and $c_4' = c_4 - 4c_1$ and get

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now apply $c_2' = c_2 - 2c_4$ and $c_3' = c_3 - 4c_4$ and get

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now interchange e_3 and e_4 and get

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which is the required canonical form. (Ans.)