CSE2209: Digital Electronics and Pulse Techniques

Course Conducted By:

Nowshin Nawar Arony

Lecturer, Dept of CSE, AUST

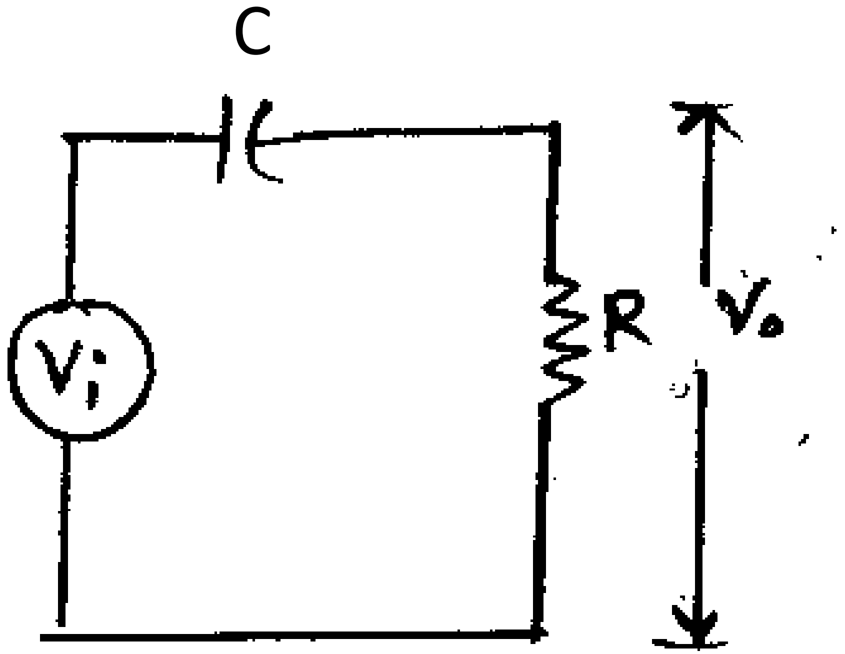
High Pass RC Circuit

$$V_{\dot{o}} = V_i - V_c$$

$$V_{\dot{o}}(t) = V_i(t) - V_c(t)$$

$$V_i(t) = V_{\dot{o}}(t) + V_c(t)$$

$$= V_R(t) + V_c(t) \quad \text{———} \quad (1)$$



$$i_c(t) = C\frac{d}{dt}V_c(t)$$ [As, $V_c = \frac{Q}{C}$ and i = $\frac{Q}{t}$]

$$V_R(t) = i\,(t) * R$$

$$\boxed{V_i(t) = V_c(t) + RC\frac{d}{dt}V_c(t)}$$

$$V_i(t) = V_c(t) + RC\frac{d}{dt}V_c(t)$$

1) $\mathcal{L}\left[\frac{d}{dt}f(x)\right] = sf(s) - f(0)$

2) $\mathcal{L}[f(x)] = f(s)$

Now using Laplace transformation

$$V_i(S) = V_c(S) + RC\,[S\,V_c(S) - V_c(0)]$$

$$V_i(S) = V_c(S) + SRC\,V_c(S) - RC\,V_c(0)$$

$$V_i(S) = [1 + SRC]\,V_c(S) - RC\,V_c(0) \qquad \text{———— (2)}$$

Here, $V_c(0)$ is the initial capacitor voltage

When initial capacitor voltage is zero, i.e. $V_c(0)$ = 0

Equation (2)

$$V_i(S) = [1 + SRC]\, V_c(S) - 0$$

$$V_c(S) = \frac{V_i(S)}{1 + SRC}$$

Again we know,

$$V_i(t) = V_c(t) + V_R(t)$$

$$= V_c(t) + V_0(t)$$

Applying Laplace Transformation

$$V_i(S) = V_c(S) + V_0(S) \quad \text{———} \quad (3)$$

$$V_0(S) = V_i(S) - V_c(S)$$

$$V_0(S) = V_i(S) - \frac{V_i(S)}{1 + SRC}$$

$$= \left[1 - \frac{1}{1 + SRC}\right] V_i(S)$$

$$V_0(S) = \left[\frac{1 + SRC - 1}{1 + SRC}\right] V_i(S)$$

$$= \left[\frac{SRC}{1 + SRC}\right] V_i(S)$$

Dividing by SRC,

$$\boxed{V_0(S) = \left[\frac{1}{1 + \frac{1}{SRC}}\right] V_i(S)}$$

When initial capacitor voltage is not zero, i.e. $V_c(0) = V'$

From Equation (2)

$$V_i(S) = (1 + SRC)\, V_c(S) - RCV'$$

$$V_c(S) = \frac{V_i(S) + RCV'}{1 + SRC}$$

From Equation (3)

$$V_0(S) = V_i(S) - V_c(S)$$

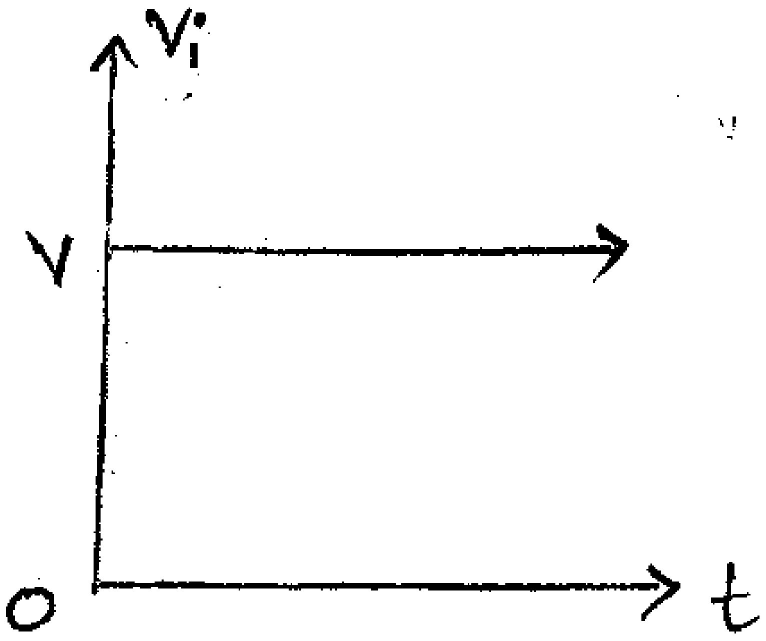
$$V_0(S) = V_i(S) - \frac{V_i(S)}{1+SRC} - \frac{RCV'}{1+SRC}$$

$$= \left[\frac{1}{1+\frac{1}{SRC}}\right] V_i(S) - \frac{\frac{RCV'}{RC}}{\frac{1}{RC}+\frac{SRC}{RC}}$$

$$\boxed{V_0(S) = \left[\frac{1}{1+\frac{1}{SRC}}\right] V_i(S) - \frac{V'}{S+\frac{1}{RC}}}$$

Step Voltage Input

$$V = \begin{cases} 0, if\ t < 0 \\ V, if\ t \geq 0 \end{cases}$$

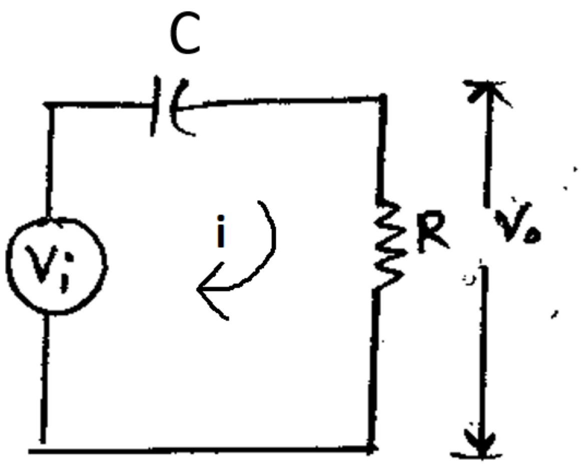


$Here,$

$V_i(t) = V$

$\mathcal{L}\{V_i(t)\} = V\mathcal{L}\{1\}$

$V_i(S) = V\frac{1}{S} \qquad As, \mathcal{L}\{1\} = \frac{1}{S}$



We know, for High Pass RC Circuit

$$V_0(S) = \left[\frac{1}{1+\frac{1}{SRC}}\right] V_i(S) \quad = \left[\frac{1}{1+\frac{1}{SRC}}\right] * \frac{V}{S}$$

$$\therefore \quad V_0(S) = \frac{V}{S+\frac{1}{RC}}$$

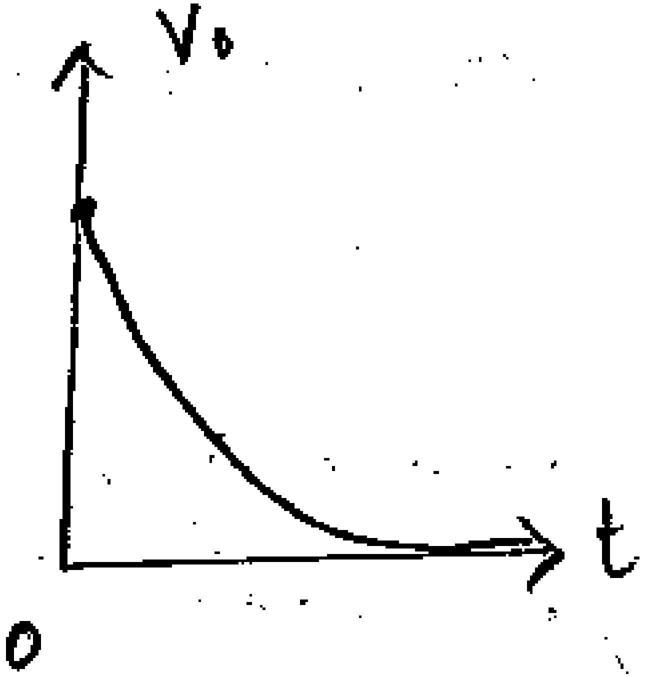
Applying Inverse Laplace transformation

$$V_0(t) = V \cdot \mathrm{e}^{-\frac{t}{Rc}} \qquad \left[\mathcal{L}\left\{\frac{1}{S+a}\right\} = \mathrm{e}^{-at}\right]$$

$$V_0(t) = v \cdot e^{-\frac{t}{Rc}}$$

Output:

In the above obtained equation



When $t = 0, V_0(t) =$ V

When $t = \infty, V_0(t) = 0$

Example 1

R= 1kΩ, C = 20 μF, V = 5V. Find $V_0(5\mu S)$

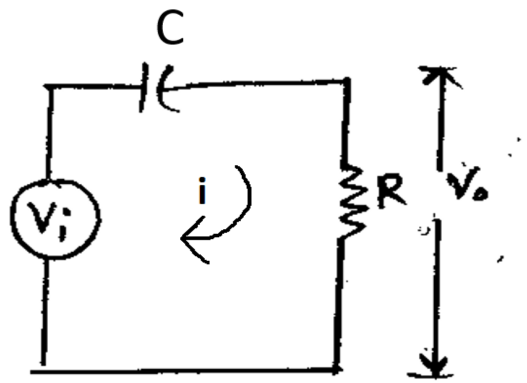
We know,

$$V_0(t) = \ v \cdot \mathrm{e}^{-\frac{t}{Rc}}$$

$$V_0(5\mu S) = 5 \cdot \mathrm{e}^{-\frac{5\ x\ 10^{-6}}{1\ x\ 10^{3}\ x\ 20\ x\ 10^{-6}}}$$

$$= 4.99875\ V$$

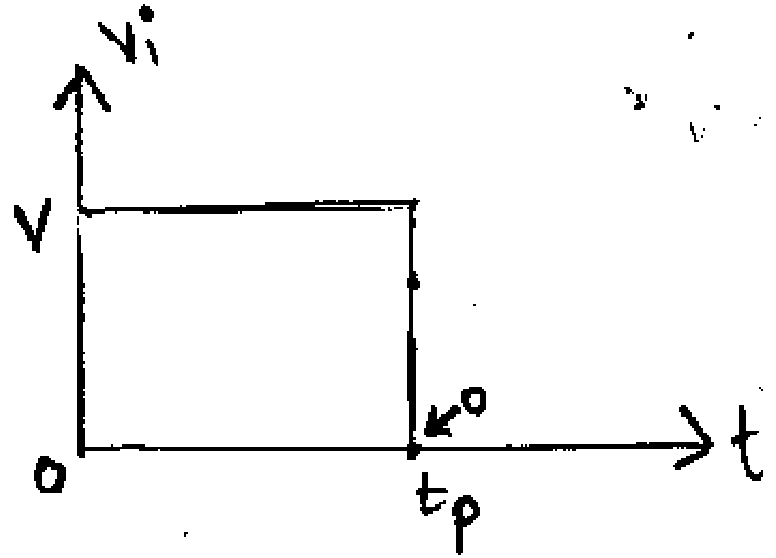
Pulse Input



Facts

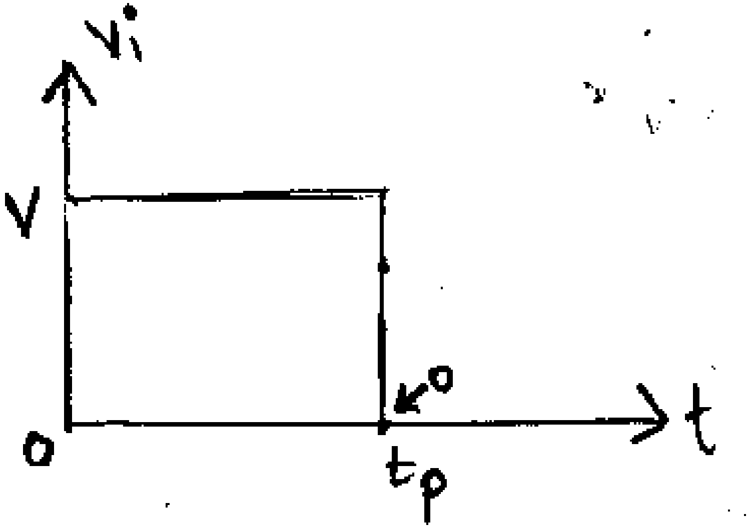
$$V = \begin{cases} V, \ 0 < t < t_p \\ 0, \ Otherwise \end{cases}$$

1) Any discontinuous change in input causes equal amount of change in the entire portion of the circuit except the capacitor.



2) For any finite time interval, if the input remains constant then the output decays (more to 0) exponentially.

$i)\ For\ \ 0 < t < t_p$



$$V_0(t) = V \cdot e^{-\frac{t}{Rc}}$$

$$V_0\left(t_p{}^{-}\right) = V \cdot e^{-\frac{t_p{}^{-}}{Rc}} \quad = V_1 \text{ (assuming)}$$

$ii)\ For\ \ t = t_p$

Changes in input = 0 - V

= - V

= Changes in output

$$V_0(t_p) = V_1 - V$$

$$= V \cdot e^{-\frac{t_p^{-}}{Rc}} - V$$

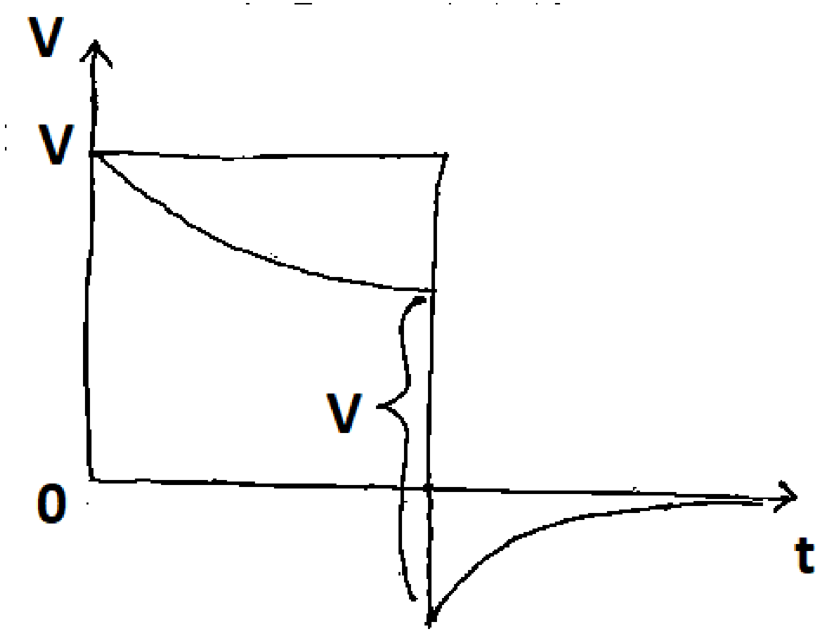
$iii)\ For\ \ t < t_p < \infty$

$$V_0(t) = [V_0(t_p)] \cdot e^{-\frac{t - t_p}{Rc}}$$

$$= [V \cdot e^{-\frac{t_p^{-}}{Rc}} - V] \cdot e^{-\frac{t - t_p}{Rc}}$$

$$= V(e^{-\frac{t_p^{-}}{Rc}} - 1) \cdot e^{-\frac{t - t_p}{Rc}}$$

Output



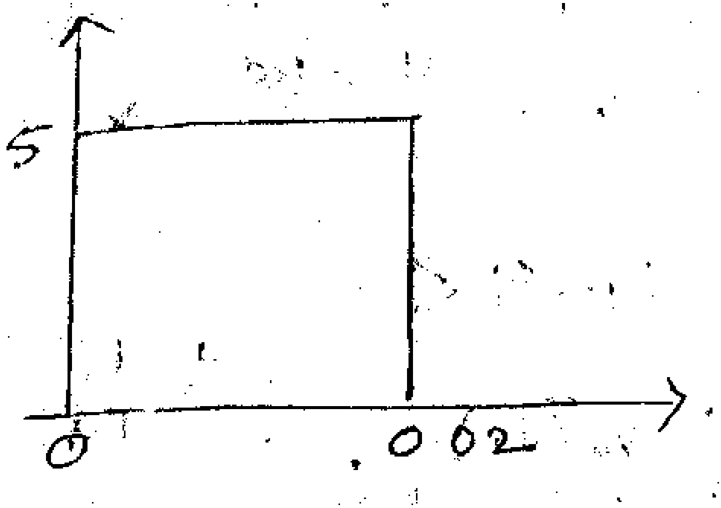
Example 2

R= 2kΩ, C = 10 μF, V = 5V, t = 0.02s Find V_t

$i)\ at\ \ t = 0$

$$V_0(0) = V \cdot \mathrm{e}^{-\frac{0}{Rc}}$$

$$V_0(0) = 5 \cdot 1 = 5\mathrm{V}$$



$ii)\ at\ \ t = {t_p}^{-}$

$$V_0({t_p}^{-}) = V \cdot \mathrm{e}^{-\frac{{t_p}^{-}}{Rc}}$$

$$V_0\left({t_p}^-\right) = 5 \cdot \mathrm{e}^{-\frac{0.02}{2\,x\,10^3\,x\,10\,x\,10^{-6}}}$$

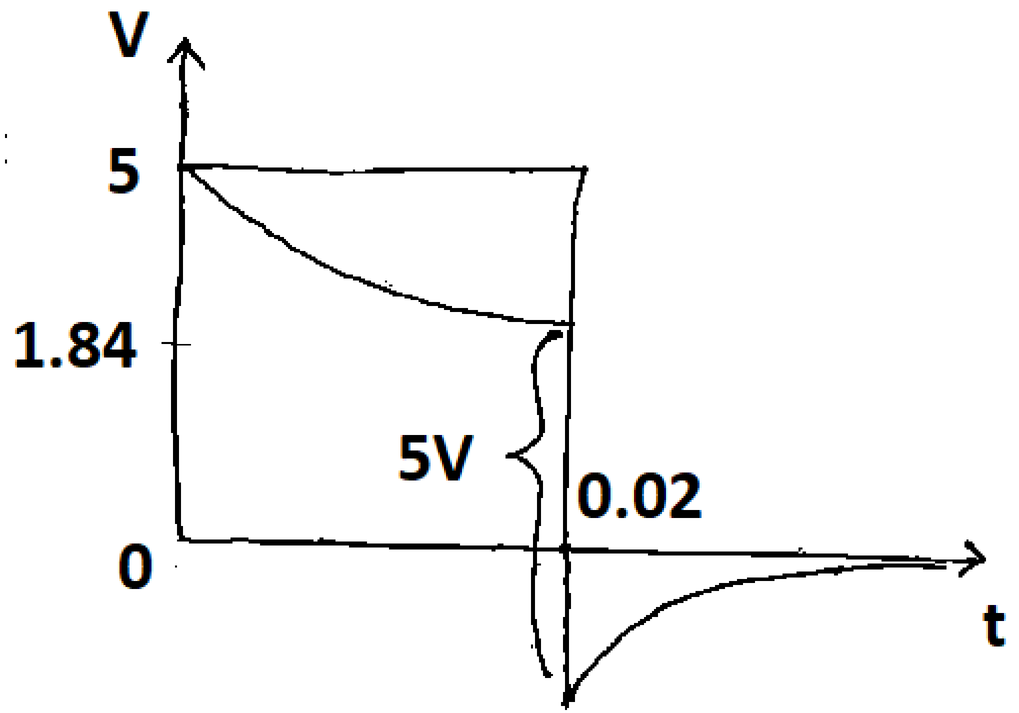
$$= 1.84\ \ V = V_1 \text{ (assuming)}$$

$iii)\ at\ \ t = t_p$

$$V_0(t_p) = \ V_1 - V$$

$$= 1.84\ \ - 5$$

$$= -3.16\,V$$



$iv)\ Let\ us\ test\ for\ any\ random\ value.$

Let,

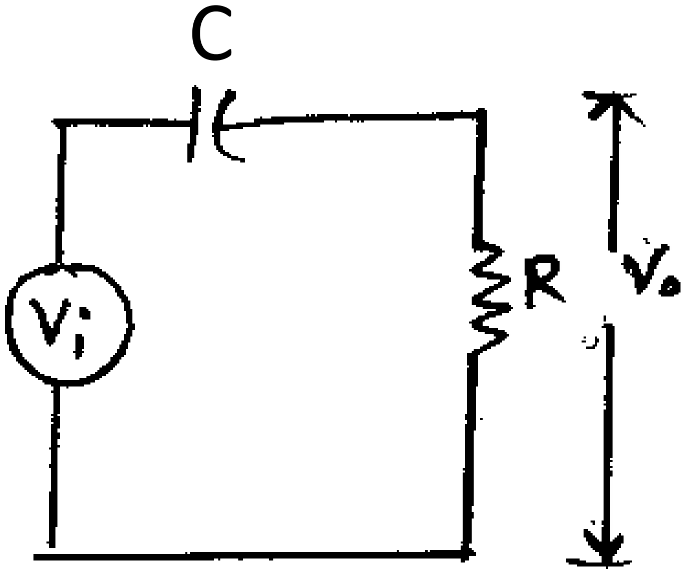
At t = 20s

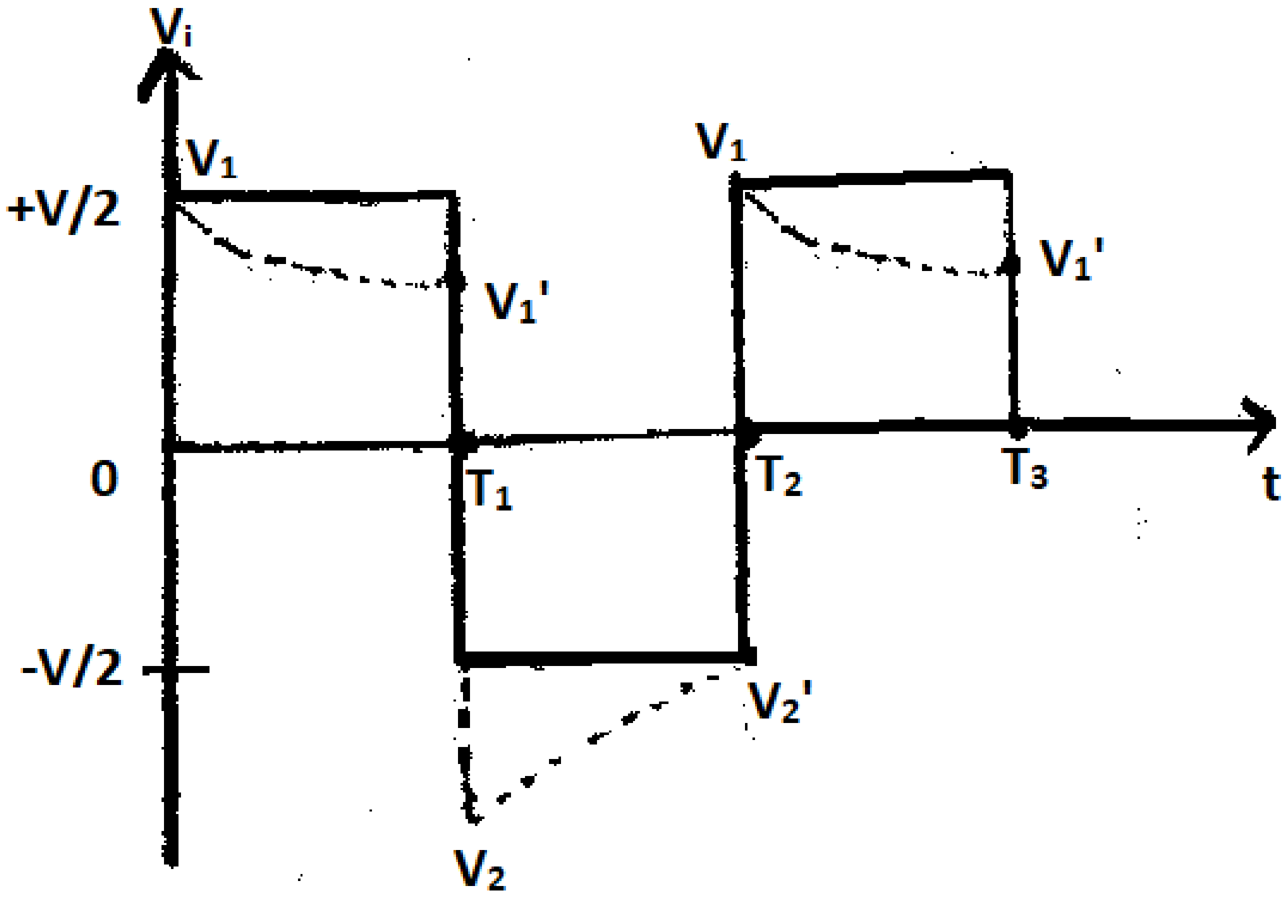
$$V_0(20s) = -3.16 \cdot \mathrm{e}^{-\frac{20-0.02}{2\ x\ 10^3\ x\ 10\ x\ 10^{-6}}}$$

$$= 0\ \ V$$

Square Wave Input

1) $Asymmetric$





$$V_1{}' = V_1 \cdot \mathrm{e}^{-\frac{T_1}{Rc}} \quad \text{——— (i)}$$

$$V_2{}' = V_2 \cdot \mathrm{e}^{-\frac{T_2}{Rc}} \quad \text{——— (ii)}$$

$$V_1{}' - V_2 = V \quad \text{——— (iii)}$$

$$V_1 - V_2{}' = V \quad \text{——— (iv)}$$

$Let, T_1 \neq T_2$ [asymmetric]

$From\ (iii)$

$$V_1{}' = V + V_2$$

$$V_1 \cdot e^{-\frac{T_1}{Rc}} = V + V_2{}' \cdot e^{\frac{T_2}{Rc}} \quad \text{[Using (i) and (ii)]}$$

$$V_1 \cdot e^{-\frac{T_1}{Rc}} = V + (V_1 - V) \cdot e^{\frac{T_2}{Rc}} \quad \text{[Using (iv)]}$$

$$V_1 \cdot (e^{-\frac{T_1}{Rc}} - e^{\frac{T_2}{Rc}}) = V(1 - e^{\frac{T_2}{Rc}})$$

$$\boxed{\therefore V_1 = \frac{V(1-e^{\frac{T_2}{Rc}})}{e^{-\frac{T_1}{Rc}}-e^{\frac{T_2}{Rc}}}}$$

$$\boxed{\therefore V_1{}' = \frac{V(1-e^{\frac{T_2}{Rc}})}{e^{-\frac{T_1}{Rc}}-e^{\frac{T_2}{Rc}}} \cdot e^{-\frac{T_1}{Rc}} \quad \text{[Using (i)]}}$$

From (iv), $V_2{}' = V_1 - V$

$$V_2 \cdot \mathrm{e}^{-\frac{T_2}{Rc}} = V_1{}' \cdot \mathrm{e}^{\frac{T_1}{Rc}} - V \qquad \text{[Using (i) and (ii)]}$$

$$V_2 \cdot \mathrm{e}^{-\frac{T_2}{Rc}} = (V + V_2) \cdot \mathrm{e}^{\frac{T_1}{Rc}} - V \qquad \text{[Using (iii)]}$$

$$V_2 \cdot (\mathrm{e}^{-\frac{T_2}{Rc}} - \mathrm{e}^{\frac{T_1}{Rc}}) = V(\mathrm{e}^{\frac{T_1}{Rc}} - 1)$$

$$\boxed{\therefore V_2 = \frac{V(\mathrm{e}^{\frac{T_1}{Rc}} - 1)}{\mathrm{e}^{-\frac{T_2}{Rc}} - \mathrm{e}^{\frac{T_1}{Rc}}}}$$

$$\boxed{\therefore V_2{}' = \frac{V(\mathrm{e}^{\frac{T_1}{Rc}} - 1)}{\mathrm{e}^{-\frac{T_2}{Rc}} - \mathrm{e}^{\frac{T_1}{Rc}}} \cdot \mathrm{e}^{-\frac{T_2}{Rc}} \qquad \text{[Using (ii)]}}$$

2) $Symmetric$

$If\ the\ square\ wave\ is\ symmetric, T_1 = T_2$

Let, $T = T_1 + T_2$

$\therefore T_1 = T_2 = \frac{T}{2}$

$$V_1{}' = V_1 \cdot e^{-\frac{T}{2Rc}} \quad \text{——— (i)}$$

$$V_2{}' = V_2 \cdot e^{-\frac{T}{2Rc}} \quad \text{——— (ii)}$$

$$V_1{}' - V_2 = V \quad \text{——— (iii)}$$

$$V_1 - V_2{}' = V \quad \text{——— (iv)}$$

$From\ (iii),$

$$V_1{}' = V + V_2$$

$$V_1 \cdot \mathrm{e}^{-\frac{T}{2Rc}} = V + V_2{}' \cdot \mathrm{e}^{\frac{T}{2Rc}} \text{ [Using (i) and (ii)]}$$

$$V_1 \cdot \mathrm{e}^{-\frac{T}{2Rc}} = V + (V_1 - V) \cdot \mathrm{e}^{\frac{T}{2Rc}} \text{ [Using (iv)]}$$

$$V_1 \cdot (\mathrm{e}^{-\frac{T}{2Rc}} - \mathrm{e}^{\frac{T}{2Rc}}) = V(1 - \mathrm{e}^{\frac{T}{2Rc}})$$

$$\therefore V_1 = \frac{V(1-\mathrm{e}^{\frac{T}{2Rc}})}{\mathrm{e}^{-\frac{T}{2Rc}} - \mathrm{e}^{\frac{T}{2Rc}}}$$

$$V_1 = \frac{V\left(1-e^{\frac{T}{2Rc}}\right)}{e^{-\frac{T}{2Rc}} - e^{\frac{T}{2Rc}}} = \frac{V\left(1-e^{\frac{T}{2Rc}}\right)}{\left(e^{-\frac{T}{2Rc}} - e^{\frac{T}{2Rc}}\right) - (1-1)}$$

$$= \frac{V\left(1-e^{\frac{T}{2Rc}}\right)}{\left(1-e^{\frac{T}{2Rc}}\right) + e^{-\frac{T}{2Rc}} - 1}$$

$$= \frac{V\left(1-e^{\frac{T}{2Rc}}\right)}{\left(1-e^{\frac{T}{2Rc}}\right) + e^{-\frac{T}{2Rc}} - \frac{e^{-\frac{T}{2Rc}}}{e^{-\frac{T}{2Rc}}}}$$

$$= \frac{V(1-e^{\frac{T}{2Rc}})}{\left(1-e^{\frac{T}{2Rc}}\right) + e^{-\frac{T}{2Rc}}\left(1-\frac{1}{e^{-\frac{T}{2Rc}}}\right)}$$

$$= \frac{V(1-e^{\frac{T}{2Rc}})}{\left(1-e^{\frac{T}{2Rc}}\right) + e^{-\frac{T}{2Rc}}\left(1-e^{\frac{T}{2Rc}}\right)}$$

$$= \frac{V(1-e^{\frac{T}{2Rc}})}{\left(1-e^{\frac{T}{2Rc}}\right)\left(1+e^{-\frac{T}{2Rc}}\right)}$$

$$\boxed{\therefore V_1 = \frac{V}{\left(1+e^{-\frac{T}{2Rc}}\right)}}$$

$$\therefore V_1{}' = \frac{V}{\left(1+ \mathrm{e}^{-\frac{T}{2Rc}}\right)} \cdot \mathrm{e}^{-\frac{T}{2Rc}}$$

$$= \frac{V\mathrm{e}^{-\frac{T}{2Rc}}}{\left(\frac{\mathrm{e}^{-\frac{T}{2Rc}}}{\mathrm{e}^{-\frac{T}{2Rc}}} + \mathrm{e}^{-\frac{T}{2Rc}}\right)} = \frac{V\mathrm{e}^{-\frac{T}{2Rc}}}{\mathrm{e}^{-\frac{T}{2Rc}}\left(1 + \frac{1}{\mathrm{e}^{-\frac{T}{2Rc}}}\right)}$$

$$\boxed{\therefore V_1{}' = \frac{V}{\left(1 + \mathrm{e}^{\frac{T}{2Rc}}\right)}}$$

Do it yourself :

Find V_2 and V_2'

Hint:

From (iv), final answers will be -

$$V_2 = \frac{V}{-\left(1+\mathrm{e}^{-\frac{T}{2Rc}}\right)}$$

$$V_2' = \frac{V}{-\left(1+\mathrm{e}^{\frac{T}{2Rc}}\right)}$$

Example 3

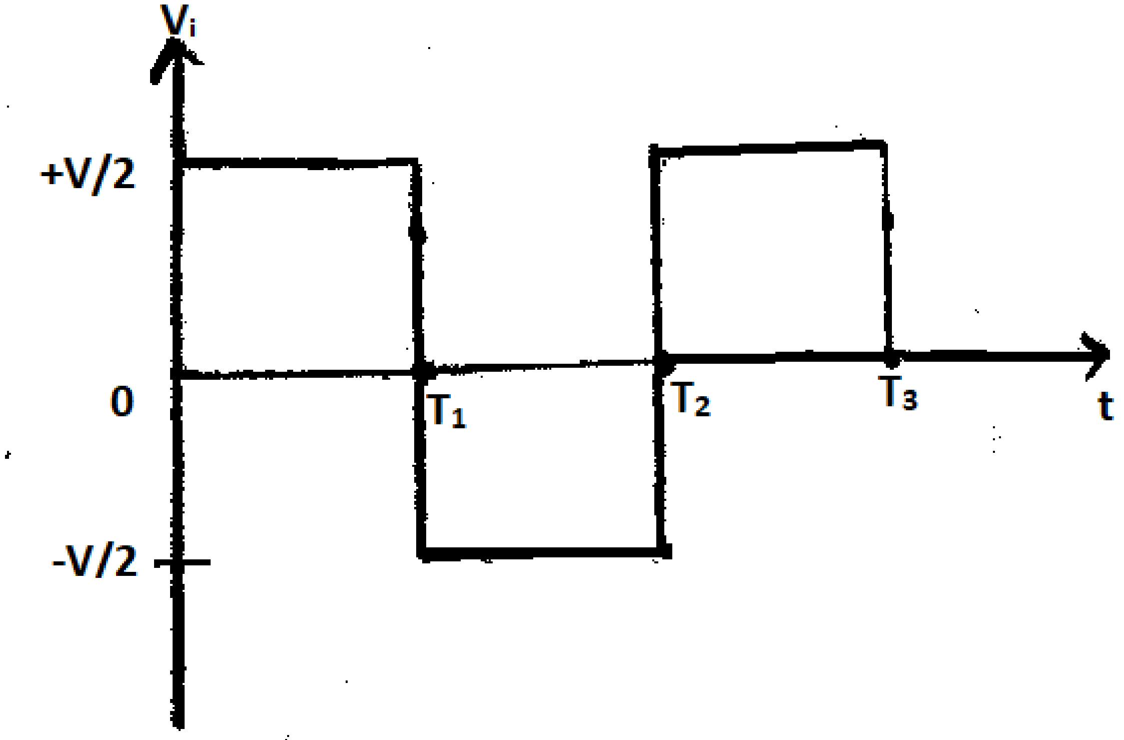
For symmetric square wave input, given R= 2kΩ, C = 10 μF, V = 5V, T = 2ms Find output.

We know,

$$V_1 = \frac{V}{\left(1 + e^{-\frac{T}{2Rc}}\right)}$$

$$= \frac{5}{\left(1 + e^{-\frac{2 \, x \, 10^{-3}}{2 \, x \, 2 \, x 10^{3} x \, 10 \, x 10^{-6}}}\right)}$$

$$= 2.56 \, V$$



$$V_1' = \frac{V}{\left(1 + e^{\frac{T}{2Rc}}\right)} = \frac{5}{\left(1 + e^{\frac{2\,x\,10^{-3}}{2\,x\,2\,x10^{3}x\,10\,x10^{-6}}}\right)} = 2.43\,V$$

$Also,$

$$V_1' - V_2 = V$$

$$V_2 = V_1' - V = 2.43 - 5 = -2.57\,V$$

$And,$

$$V_1 - V_2' = V$$

$$V_2' = V_1 - V = 2.56 - 5 = -2.44\,V$$

Output