Chapter-3

⊛ Alpha Compositing:

$$c = \alpha C_f + (1-\alpha) C_b$$

Here, c = final color/mixed color

C_f = forground color

C_b = background color

① Propose an alpha compositing formula for blending the colors of three objects c_1, c_2 and c_3, where c_1 is the forground of c_2 and c_2 is the forground of c_3.

Ans: Let,

α = Alpha compositing parameter to blend c_1 and c_2

β = " " " " " c_2 " c_3

We know,

$$c = \alpha C_f + (1-\alpha) C_b$$

Here,
C_f - Foreground color
C_b - background color
C - mixed/final color

$$\therefore c = \alpha c_1 + (1-\alpha) c_2$$

$$= \alpha c_1 + (1-\alpha) \boxed{\{\beta c_2 + (1-\beta) c_3\}}$$

$$= \alpha c_1 + \beta c_2 - \alpha\beta c_2 + (1-\beta) c_3 - \alpha(1-\beta) c_3$$

$$= \alpha c_1 + (1-\alpha)\beta c_2 + (1-\alpha)(1-\beta) c_3$$

∴ This is the required alpha compositing formula.

② Given that, $C_f = 1.0$, $C_b = 0.2$ and $C = 0.8$, where C_f, C_b and C are the foreground, background and composite intensities respectively. Determine the alpha (α) value to perform this composition. (5)

Ans: Given,

$$C_f = 1.0$$
$$C_b = 0.2$$
$$C = 0.8$$

$\therefore \alpha = ?$

We know,

$$C = \alpha C_f + (1-\alpha) C_b$$

$$\Rightarrow 0.8 = \alpha * 1.0 + (1-\alpha) * 0.2$$

$$\Rightarrow 0.8 = \alpha + (1-\alpha) * 0.2$$

$$\Rightarrow 0.8 = \alpha + 0.2 - 0.2\alpha$$

$$\Rightarrow 0.8 - 0.2 = 0.8\alpha$$

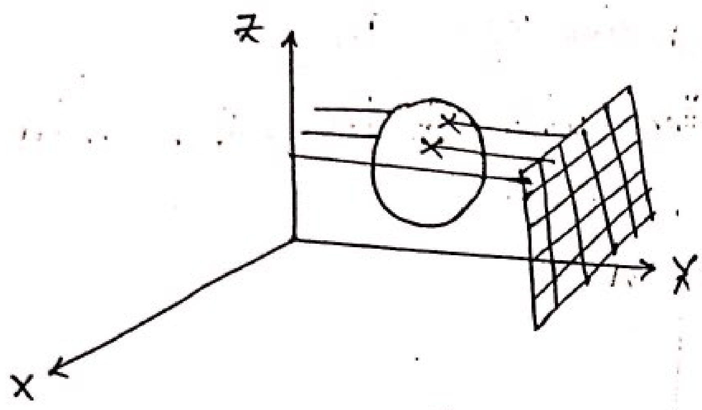
$$\Rightarrow 0.6 = 0.8\alpha$$

$$\therefore \alpha = \frac{0.6}{0.8} = 0.75$$

** $\begin{bmatrix} r_c \\ g_c \\ b_c \end{bmatrix} = \alpha \begin{bmatrix} r_f \\ g_f \\ b_f \end{bmatrix} + (1-\alpha) \begin{bmatrix} r_b \\ g_b \\ b_b \end{bmatrix}$; vector operation

(*) Practice Problems:

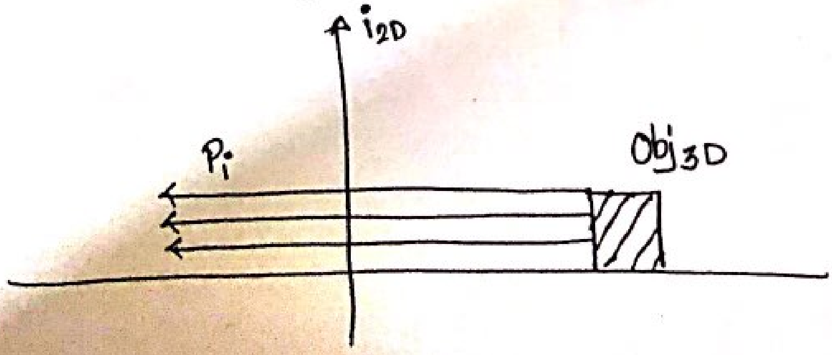
① Is the projected image on the image plane in the given example perspective?



Ans: No. The projected image on the image plane in the given example is not perspective. It is parallel projection on the image plane.

In parallel projection —

1. Projectors are parallel
2. Projectors are meet at infinity
3. The size of an object remains same on the image plane.



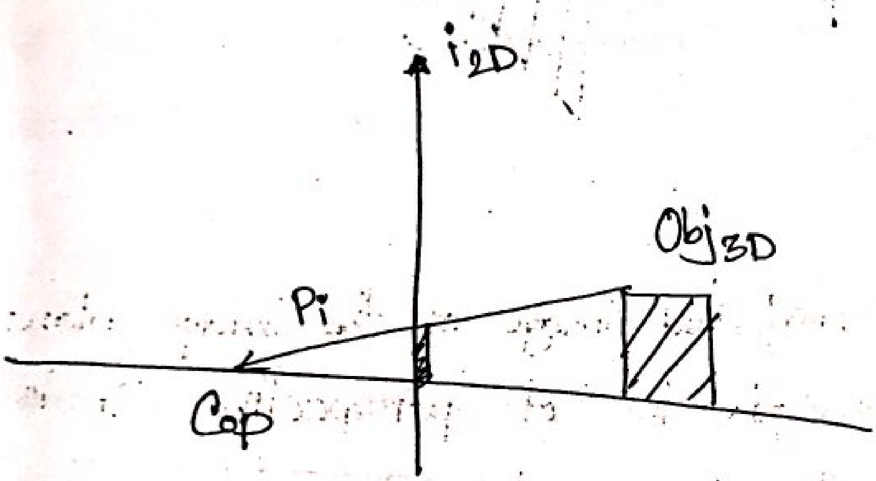
All the features remain same in the given example, so, this is a parrallel projection.

On the other hand, the perspective projection —

① Projectors are not parallel.

② Projectors are meet at one point. That point is called central of projection (Cop)

③ Objects look smaller when it is farther away and vice versa.



The features are not same with the given example. So, this is not a perspective projection.

② Consider the following setup :

Image plane : Situated at $y=13$, parallel to ZX plane, (Resolution : 11×11), M is the corresponding array and Y-axis goes through $(6,6)$.

Sphere: Center at $(0,0,0)$, Radius $=6$

Light: at $(0,15,0)$.

Now,

ⓐ Draw the ray tracing setup showing two viewing rays (one hitting, another missing).

ⓑ Fill up the array (pixel) with 1 (for hitting) and 0 (for missing). Show the hitting/missing mathematically for at least one pixel.

ⓒ Fill up the array (pixel) with angles between surface normal and viewing ray. Show the angle calculation for at least one pixel.

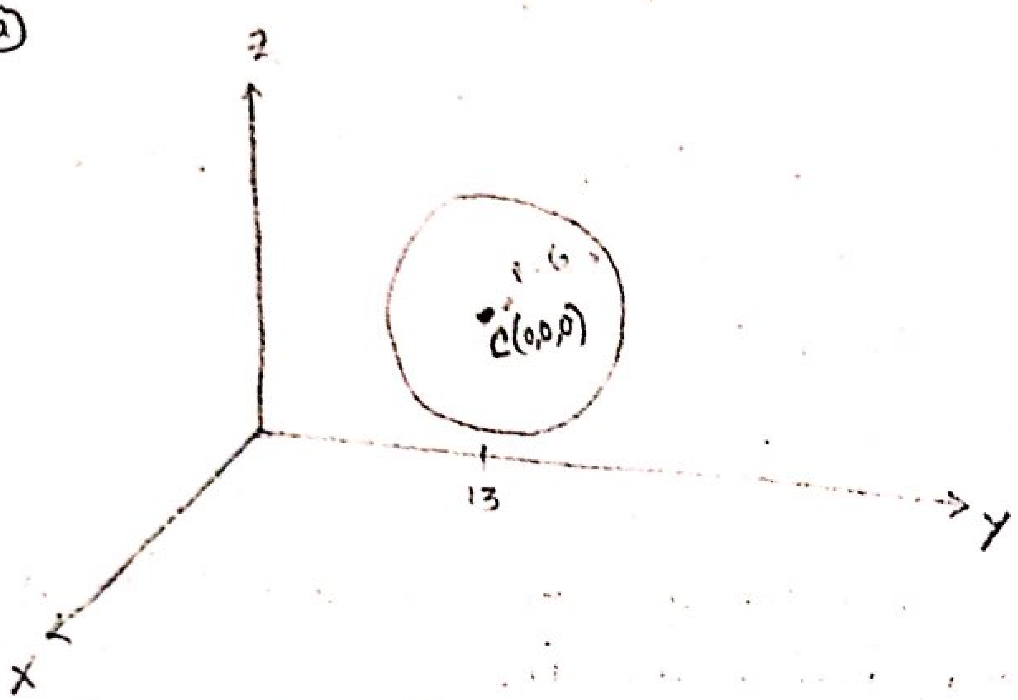
Ans: Given,

$n_x = 11$, $n_y = 11$

$L(0,15,0)$, $x_L = 0$, $y_L = 15$, $z_L = 0$

$R = 6$

(a)



Part-C

Formula:

① ray origin $= e + uU + vV$; e : viewport

u : right

v = up

w = back

② ray direction $= -W$

③ $u = l + (r-l)(i+0.5)/n_x$

④ $v = b + (t-b)(j+0.5)/n_y$

⑤ ray end, $s = e + w(-W)$

⑥ intersecting point, $p = e + t(s-e)$

$= e + td$

⑦ direction, $d = s - e$

⑧ $At^2 + Bt + C = 0$

Here, $A = d \cdot d$

$B = 2 \cdot d \cdot (e-c)$

$C = (e-c) \cdot (e-c)$

⑨ $t = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Here, $B^2 - 4AC$ is called the discriminant.

[i] if it is negative, the line and sphere donot intersect

[ii] " " " positive, there are 2 solutions -

ⓐ one sol^n where the ray enters the sphere

ⓑ one where it leaves.

[iii] if it is zero, the line touches the sphere at exactly one point.

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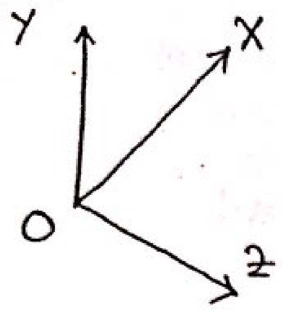
(*) Practice Problems:

① 

Camera Frame (orthographic):

$e = [4,4,6]$; $u = [1,0,0]$; $v = [0,1,0]$;
$w = [0,0,1]$

ⓐ – plot the camera frame on the given axis.

Viewing Ray:

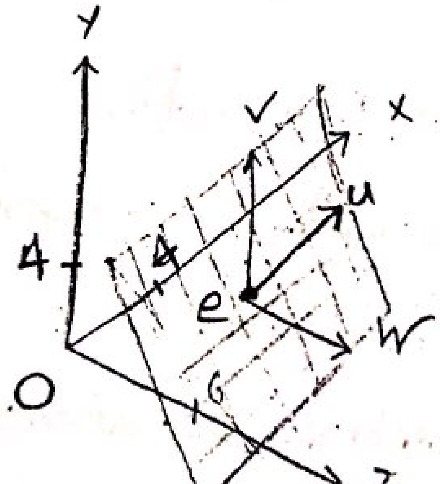
- $ray_1 \cdot origin = e + 2u + 2v$; $ray_1 \cdot end = [6,6,0]$
- $ray_2 \cdot origin = e - 1u + 1v$; $ray_2 \cdot end = [4,4,0]$

ⓑ – plot the origins for ray_1 and ray_2.

Sphere: $F(x,y,z) = x^2 + y^2 + z^2 - (4)^2 = 0$

1. What are the intersecting points for ray_1 and ray_2 ?
2. Plot the intersecting points.

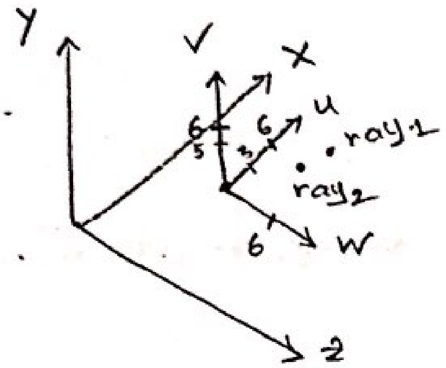
Ans:

ⓐ 

e: viewport
u = right
v = up
w = back

(b) For ray_1, origin, $e = E + 2U + 2V$
$= [4,4,6] + 2[1,0,0] + 2[0,1,0]$
$= [6,6,6]$

For ray2, origin, $e = E - 1U + 1V$
$= [4,4,6] - 1[1,0,0] + 1[0,1,0]$
$= [3,5,6]$



1.ll Here,
$R = 4$
$C = [0,0,0]^T$

For, ray_1, origin, $e = E + 2U + 2V$
$= [4,4,6] + 2[1,0,0] + 2[0,1,0]$
$= [4,4,6] + [2,0,0] + [0,2,0]$
$= [6,6,6]$

end, $s = [6,6,0]$

$\therefore d = s - e$
$= [6,6,0] - [6,6,6] = [0,0,-6]$

$A = d \cdot d = [0,0,-6] \cdot [0,0,-6] = 36$

$B = 2 \cdot d \cdot (e - c) = 2 \cdot [0,0,-6] \cdot ([6,6,6] - [0,0,0])$
$= 2 \cdot [0,0,-6] \cdot [6,6,6] = 2 * (-6) * 6 = -72$

$$C = (e-c)\cdot(e-c) - R^2$$
$$= [6,6,6]\cdot[6,6,6] - 4^2$$
$$= 108 - 16$$
$$= 92$$

$$D = B^2 - 4AC$$
$$= (-72)^2 - 4 * 36 * 92$$
$$= -8064$$

As, D is negative, ray_1 is not intersect with the sphere.

For, ray_2, origin, $e = E - 1U + 1V$
$$= E - 1[1,0,0] + 1[0,1,0]$$
$$= [4,4,6] - [1,0,0] + [0,1,0]$$
$$= [3,4,6] + [0,1,0]$$
$$= [3,5,6]$$

end, $s = [4,4,0]$

$$d = s - e$$
$$= [4,4,0] - [3,5,6] = [1,-1,-6]$$

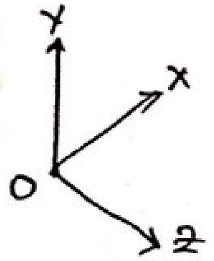
$$A = d\cdot d = [1,-1,-6]\cdot[1,-1,-6] = 38$$
$$B = 2\cdot d\cdot(e-c) = 2\cdot[1,-1,-6]\,([3,5,6]-[0,0,0])$$
$$= 2\,[1,-1,-6][3,5,6]$$
$$= 2 * (-38)$$
$$= -76$$

$$C = (e-c)\cdot(e-c) - R^2$$
$$= [3,5,6][3,5,6] - 4^2$$
$$= 70 - 16$$
$$= 54$$

$$D = B^2 - 4AC$$
$$= (-76)^2 - 4 * 38 * 54$$
$$= -2432$$

As, D is negative, ray_2 is not intersect with the sphere.

② Problems :



Camera frame (Orthographic):

$e = [4,4,8]$; $u = [1,0,0]$; $v = [0,1,0]$; $w = [0,1,0]$

Image plane:

left: $u = -5$; right: $u = 5$; top: $v = 4$;
bottom: $v = -4$

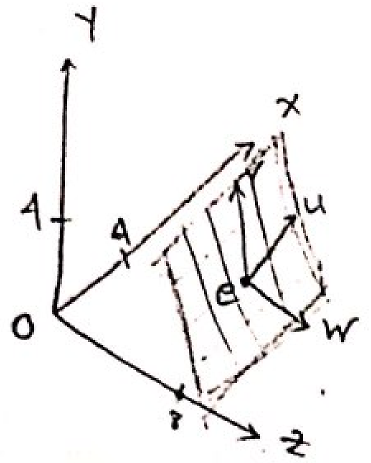
[1] Plot the image plane on the given axis.

[2] For a 10X10 image matrix M, what is the position on the image plane for the ray origin at M(4,3)?

[3] Will it intersect $F(x,y,z) = x^2 + y^2 + z^2 - 5^2 = 0$?

Ans:

$\boxed{1}$



$\boxed{2}$ Here,

$n_x = 10$, $n_y = 10$, $l = -5$, $r = 5$, $t = 4$, $b = -4$

$i = 4$, $j = 3$

$$u = l + \frac{(r-l)(i+0.5)}{n_x}$$

$$= -5 + \frac{(5+5)(4+0.5)}{10} = -0.5$$

$$v = b + \frac{(t-b)(j+0.5)}{n_y}$$

$$= -4 + \frac{(4+4)(3+0.5)}{10} = -1.2$$

$\therefore$ Origin, $e = E + uU + vV$

$$= [4,4,8] - 0.5[1,0,0] - 1.2[0,1,0]$$

$$= [4,4,8] - [0.5,0,0] - [0,1.2,0]$$

$$= [3.5, 2.8, 8]$$

$\boxed{3}$ Here, $R=5$, $C=[0, 0, 0]$

ending point,

$$S = \hat{e} + w(-W)$$

$$\cancel{= [1, 1, 8] + w[0, -1, 0]}$$

$$= [3.5, 2.8, 8]$$

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③ **Problem:** Consider the following parameters for an orthographic ray-tracing.

Camera Frame:

$E = [-2, 7, 17]^T$, $U = [1,0,0]^T$, $V = [0,1,0]^T$, $W = [0,0,1]^T$

Image Plane:

$l = -15$, $r = 15$, $t = 10$, $b = -10$

Raster image resolution: 13×11

Sphere: $(x+3)^2 + (y-5)^2 + (z-3)^2 = 64$

Determine the ray-sphere intersection points for a ray (with length = 25) at the center of the raster image. Drawing figures is NOT mandatory.

Ans: Given,

$n_x = 13$, $n_y = 11$; $i = \frac{n_x}{2} = 6$; $j = \frac{n_y}{2} = 5$

From sphere, $R^2 = 64$, $\therefore R = 8$

$$C = [-3, 5, 3]^T$$

$$\therefore u = l + \frac{(r-l)(i+0.5)}{n_x}$$

$$= -15 + \frac{(15+15)(6+0.5)}{13} = 0$$

$$v = b + \frac{(t-b)(j+0.5)}{11}$$

$$= -10 + \frac{(10+10)(5+0.5)}{11} = 0$$

$\therefore$ The starting point of the ray,

$$e = E + uU + vV$$
$$= [-2, 7, 17]^T + 0[1,0,0]^T + 0[0,1,0]^T$$
$$= [-2, 7, 17]^T$$

Ray length, $w = 25$

As, we look through $-w$ from the camera frame, we add -25 to the last element (z) of e.

So, we get,

Ending point of the ray, $s = e + w(-W)$

$$s = [-2, 7, (17-25)]^T$$
$$= [-2, 7, -8]^T$$

$\therefore$ Direction, $d = s - e$

$$= [-2, 7, -8]^T - [-2, 7, 17]^T$$
$$= [0, 0, -25]^T$$

$\therefore$ $A = d \cdot d = [0, 0, -25]^T \cdot [0, 0, -25]^T = 625$

$$B = 2 \cdot d \cdot (e - c)$$
$$= 2 \cdot [0, 0, -25]^T \left([-2, 7, 17]^T - [-3, 5, 3]^T\right)$$
$$= 2 \cdot [0, 0, -25]^T \cdot [1, 2, 14]^T$$
$$= 2 * (-25) * 14$$
$$= -700$$

$$C = (e-c)\cdot(e-c) - R^2$$
$$= [1, 2, 14]^T \cdot [1, 2, 14]^T - 64$$
$$= 1+4+196-64$$
$$= 137$$

$$D = B^2 - 4AC$$
$$= (-700)^2 - 4 * 625 * 137$$
$$= 147500$$

As, D is positive, there are 2 intersection points –
– one soln where the ray enters the sphere
– where it leaves.

Hence, t parameters for intersected points are –

$$t_1 = \left(\frac{-B+\sqrt{D}}{2A}\right)$$
$$= \left(\frac{+700+\sqrt{147500}}{2*625}\right)$$
$$= 0.86$$

$$t_2 = \left(\frac{-B-\sqrt{D}}{2A}\right)$$
$$= \left(\frac{+700-\sqrt{147500}}{2*625}\right)$$
$$= 0.25$$

We can obtain the intersecting points—

$$P_1 = e + t_1 (s-e)$$

$$= [-2, 7, 17]^T + 0.86[0, 0, -25]^T$$

$$= [-2, 7, 17]^T + [0, 0, -21.5]^T$$

$$= [-2, 7, -4.5]^T$$

$$P_2 = e + t_2 (s-e)$$

$$= [-2, 7, 17]^T + 0.25[0, 0, -25]^T$$

$$= [-2, 7, 17]^T + [0, 0, -6.25]^T$$

$$= [-2, 7, 10.75]^T$$

④ Consider the following parameters for an orthographic ray-tracing:

Camera frame:

$E=[-3,7,17]^T$, $U=[1,0,0]^T$, $V=[0,1,0]^T$, $W=[0,0,1]^T$

Image Plane: $l=-10$, $r=10$, $t=15$, $b=-15$

Raster image resolution: 11×13

Sphere: $(x+5)^2+(y-4)^2+(z-3)^2=64$

Determine the ray-sphere intersection point(s) for a ray (with length = 25) at the center of the raster image.

Ans: Given,

$n_x = 11$, $n_y = 13$, $i = \frac{n_x}{2} = \frac{11}{2} = 5$, $j = \frac{n_y}{2} = 6$

$R^2 = 64$ $\therefore R = 8$

$\therefore C = [-5, 4, 3]^T$

$$\therefore u = l + \frac{(r-l)(i+0.5)}{n_x}$$

$$= -10 + \frac{(10+10)(5+0.5)}{11} = 0$$

$$\therefore v = b + \frac{(t-b)(j+0.5)}{n_y}$$

$$= -15 + \frac{(15+15)(6+0.5)}{13} = 0$$

starting point,

$$e = E + uU + vV$$

$$= [-3, 7, 17]^T + 0 + 0$$

$$= [-3, 7, 17]^T$$

length, $w = 25$

$\therefore$ ending point, $s = [-3, 7, (17-25)]^T = [-3, 7, -8]^T$,

$\therefore$ direction, $d = s - e$

$= [-3, 7, -8] - [-3, 7, 17]^T$

$= [0, 0, -25]^T$

$\therefore A = d \cdot d = [0, 0, -25]^T \cdot [0, 0, -25]^T = 625$

$B = 2 \cdot d \cdot (e - c)$

$= 2 [0, 0, -25]^T \cdot ([-3, 7, 17]^T - [-5, 4, 3]^T)$

$= 2 \cdot [0, 0, -25]^T \cdot [2, 3, 14]^T$

$= 2 * (-25) * 14$

$= -700$

$C = (e - c) \cdot (e - c) - R^2$

$= [2, 3, 14]^T \cdot [2, 3, 14]^T - 64$

$= [2 \times 2 + 3 \times 3 + 14 \times 14] - 64$

$= 145$

$D = B^2 - 4AC$

$= (-700)^2 - 4 * 625 * 145$

$= 127500$

As, D is positive, there is two intersection points,

$\therefore t_1 = \dfrac{-B + \sqrt{D}}{2A} = \dfrac{700 + \sqrt{127500}}{2 * 625} = 0.85$

$t_2 = \dfrac{-B - \sqrt{D}}{2A} = \dfrac{700 - \sqrt{127500}}{2 * 625} = 0.27$

$\therefore$ Intersecting points,

$$P_1 = e + t_1(s-e)$$
$$= [-3, 7, 17]^T + 0.85\,[0, 0, -25]^T$$
$$= [-3, 7, 17]^T + [0, 0, -21.25]^T$$
$$= [-3, 7, -4.25]^T$$

$$P_2 = e + t_2(s-e)$$
$$= [-3, 7, 17]^T + 0.27\,[0, 0, -25]^T.$$
$$= [-3, 7, 17]^T + [0, 0, -6.75]^T$$
$$= [-3, 7, 10.25]^T$$

Quiz - Set A

⑤ Consider the following parameters for an orthographic ray-tracing:

Camera-frame: $E = [-2, 7, 17]^T$, $U = [1, 0, 0]^T$, $V = [0, 1, 0]^T$, $W = [0, 0, 1]^T$

Image plane: $l = -15$, $r = 15$, $t = 10$, $b = -10$

Raster image matrix resolution: $7 * 9$

Sphere: $(x+3)^2 + (y-5)^2 + (z-3)^2 = 9$

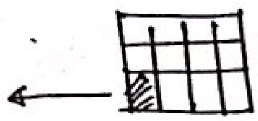
Will there be any ray-sphere intersection for the lower left pixel in raster image matrix [Yes/No]? Assume, ray length = 25. Show your calculations. Drawing figures is NOT mandatory.

Ans: Given,

$$n_x = 7, \quad n_y = 9, \quad R^2 = 9 \quad \therefore R = 3$$

$$C = [-3, 5, 3]^T$$

Lower left pixel, So, $i = 0$; $j = 0$



$$\therefore u = l + \frac{(r-l)(i+0.5)}{n_x}$$

$$= -15 + \frac{(15+15)(0+0.5)}{7} = -12.85$$

$$\therefore v = b + \frac{(t-b)(j+0.5)}{n_y}$$

$$= -10 + \frac{(10+10)(0+0.5)}{9} = -8.89$$

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starting point, $e = E + uU + vV$

$= [-2, 7, 17]^T - 12.86[1,0,0]^T - 8.89[0,1,0]^T$

$= [-2, 7, 17]^T - [12.86, 0, 0]^T - [0, 8.89, 0]^T$

$= [-14.86, -1.89, 17]^T$

length $= 25 = w$

$\therefore$ ending point, $s = [-14.86, -1.89, (17-25)]^T$

$= [-14.86, -1.89, -8]^T$

$\therefore$ direction, $d = s - e$

$= [-14.86, -1.89, -8]^T - [-14.86, -1.89, 17]^T$

$= [0, 0, -25]^T$

$\therefore A = d \cdot d = [0, 0, -25]^T \cdot [0, 0, -25]^T = 625$

$B = 2 \cdot d \cdot (e - c)$

$= 2 \cdot [0, 0, -25]^T \left([-14.86, -1.89, 17]^T - [-3, 5, 3]^T\right)$

$= 2 \cdot [0, 0, -25]^T [-11.86, -6.89, 14]^T$

$= 2 * (-25) * 14$

$= -700$

$C = (e-c) \cdot (e-c) - R^2$

$= [-11.86, -6.89, 14]^T \cdot [-11.86, -6.89, 14]^T - 9$

$= 384.1317 - 9$

$= 375.1317$

$D = B^2 - 4AC = (-700)^2 - 4 * 625 * 375.1317 = -447829.25$

As, D is negative, So, there will be no intersection.

SetB

(6) Consider the following parameters for an orthographic ray-tracing:

Camera-frame: $E=[-3,7,17]^T$, $U=[1,0,0]^T$, $V=[0,1,0]^T$, $W=[0,0,1]^T$

Image plane: $l=-10$, $r=10$, $t=15$, $b=-15$

Raster image matrix resolution: $9*7$

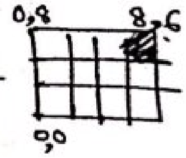
Sphere: $(x+3)^2+(y-5)^2+(z-3)^2=9$

Will there be any ray-sphere intersection for the upper right pixel in raster image matrix [Yes/No]? Assume, ray length =25. Show your calculations. Drawing figures is NOT mandatory.

Ans: Given,

$n_x=9$, $n_y=7$, $R^2=9$, $R=3$, $C=[-3,5,3]^T$

Upper right pixel, So, $i=8$, $j=6$



$$\therefore u = l + \frac{(r-l)(i+0.5)}{n_x}$$

$$= -10 + \frac{(10+10)(8+0.5)}{9} = 8.89$$

$$v = b + \frac{(t-b)(j+0.5)}{n_y}$$

$$= -15 + \frac{(15+15)(6+0.5)}{7} = 12.86$$

Starting point,

$$e = E + uU + vV$$

$$= [-3,7,17]^T + 8.89[1,0,0]^T + 12.86[0,1,0]^T$$

$$= [-3,7,17]^T + [8.89,0,0]^T + [0,12.86,0]^T$$

$$= [5.89, 19.86, 17]^T$$

length $= 25 = W$

$\therefore$ ending point, $s = [5.89, 19.86, (17-25)]^T$

$= [5.89, 19.86, -8]^T$

$\therefore$ direction, $d = s - e$

$= [5.89, 19.86, -8]^T - [5.89, 19.86, 17]^T$

$= [0, 0, -25]^T$

$A = d \cdot d = [0, 0, -25]^T \cdot [0, 0, -25]^T = 625$

$B = 2 \cdot d \cdot (e - c)$

$= 2 \cdot [0, 0, -25]^T ([5.89, 19.86, 17]^T - [-3, 5, 3]^T)$

$= 2 \cdot [0, 0, -25]^T \cdot [8.89, 14.86, 14]^T$

$= 2 * (-25) * 14$

$= -700$

$C = (e-c)(e-c) - R^2$

$= [8.89, 14.86, 14]^T \cdot [8.89, 14.86, 14]^T - 9$

$= 495.8517 - 9$

$= 486.8517$

$D = B^2 - 4AC$

$= (-700)^2 - 4 * 625 * 486.8517$

$= -727129.25$

As, D is negative, So, there will be no intersection.

① What are the ray parameters of the intersection points between ray $(1,1,1)+t(-1,-1,-1)$ and the sphere centered at the origin with radius 1?

Ans: Given,

$C = [0,0,0]$, $R = 1$

$p = (1,1,1) + t(-1,-1,-1)$

We know, $p = e + t(s-e) = e + td$

$\therefore e = [1,1,1]$ and, $d = [-1,-1,-1]$

$$\boxed{\begin{aligned} s-e &= [-1,-1,-1] \\ \Rightarrow s-[1,1,1] &= [-1,-1,-1] \\ \Rightarrow s &= [1,1,1]+[-1,-1,-1] \\ &= [0,0,0] \end{aligned}}$$

$\therefore A = d \cdot d = [-1,-1,-1]\cdot[-1,-1,-1] = 3$

$B = 2 \cdot d \cdot (e-c)$

$= 2 \cdot [-1,-1,-1]\left([1,1,1]-[0,0,0]\right)$

$= 2[-1,-1,-1][1,1,1]$

$= -6$

$C = (e-c)(e-c) - R^2$

$= [1,1,1][1,1,1] \; 1$

$= 2$

$D = B^2 - 4AC$

$= (-6)^2 - 4*3*2$

$= 12$

$\therefore t_1 = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{6+\sqrt{12}}{2*3} = 1.58$

$t_2 = \dfrac{-B-\sqrt{D}}{2A} = \dfrac{6-\sqrt{12}}{2*3} = 0.42$

$\therefore P_1 = e + t_1 d$

$= [1,1,1] + 1.58[-1,-1,-1]$

$= [-0.58, -0.58, -0.58]$

$\therefore P_2 = e + t_2 d$

$= [1,1,1] + 0.42[-1,-1,-1]$

$= [0.58, 0.58; 0.58]$

Chapter-6

Transformation Matrices (Part-A)

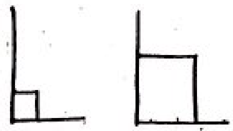
⊛ 2D Linear Transformation :

$v' = Mv$; Here, v' = Output vector
M = Matrix
v = input vector

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

1 Scaling :

$$\text{Scale}(S_x, S_y) = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$



$$\therefore \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} S_x x \\ S_y y \end{bmatrix}$$

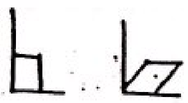
; Here, S_x, S_y can be 0.1, 0.5, 2 etc

2 Shearing :

$$\text{Shear-}x\,(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

; Here s can be 1, 0.5 etc

$$\text{Shear-}y\,(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$



3 Rotation :

$$x_a = r\cos\alpha$$
$$y_a = r\sin\alpha$$

$$x_b = r\cos(\alpha + \phi) = r\cos\alpha\cos\phi - r\sin\alpha\sin\phi = x_a\cos\phi - y_a\sin\phi$$

$$y_b = r\sin(\alpha + \phi) = r\sin\alpha\cos\phi + r\cos\alpha\sin\phi = y_a\cos\phi + x_a\sin\phi$$

$$\text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

; ϕ can be $\pi/4$, 30° etc

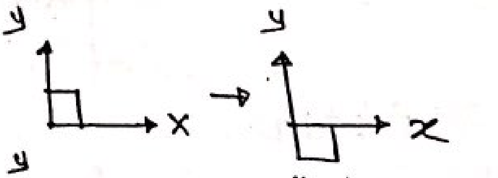
ϕ = negative হলে, Traverse

$$rotate(\phi) = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

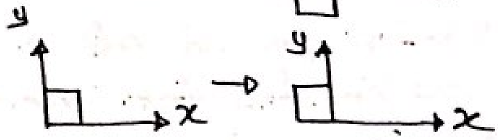
$$\therefore \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = V'$$

4 Reflection:

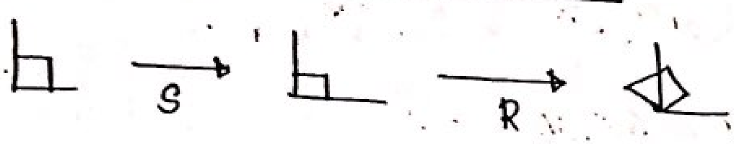
$$reflect{-x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \longrightarrow$$



$$reflect{-y} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow$$



5 Composition of Transformation:



1. $V_2 = SV_1$
2. $V_3 = RV_2$
3. $V_3 = R(SV_1)$
4. $V_3 = (RS)V_1$
5. $V_3 = MV_1$; $M = RS$

Maintain the order of multiplication

* $M = RS$!= $M = SR$
* $M = T_n * T_{n-1} * \ldots * T_1$

(Part-B)

⊛ 3D Transformation:

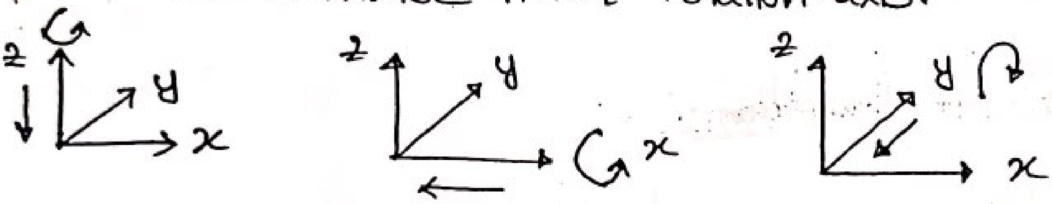
$V' = MV$

[1] 3D Scaling:

$$Scale(S_x, S_y, S_z) = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix}$$

[2] 3D Rotation:

- Rotation around axis
- counter clockwise w.r.t rotation axis.



$$rotate\text{-}z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

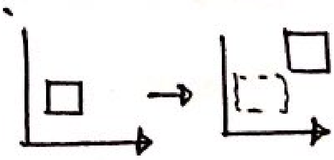
$$rotate\text{-}x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$rotate\text{-}y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

⊛ 2D Translation:

$x' = x + t_x$

$y' = y + t_y$



$\therefore V' = V + t$

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

⊛ Affine Transformation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & x_t \\ m_{21} & m_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

** homogeneous coordinates: implementing affine transformati by adding an extra dimension is called h.. c...

⊛ 2D Affine Transformation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & x_t \\ m_{21} & m_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

1 Translation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2 Scaling:

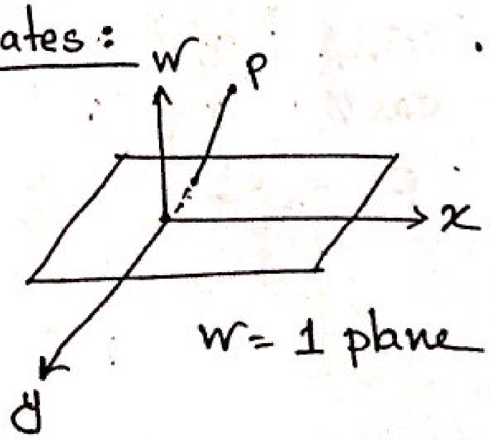
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3 Rotation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

⊛ Homogeneous Coordinates:

Geometric intuition



(*) 3D Affine Transformation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

[1] Translation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

[2] Scaling:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

[3] Rotation:

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse of Transformation Matrices:

Let, Scale : $S(S_x, S_y, S_z)$

Translate : $T(t_x, t_y, t_z)$

Rotate : $R_x(d)$, $R_y(d)$, $R_z(d)$

$\therefore$ The inverses are –

$$S^{-1} = S\left(\frac{1}{S_x}, \frac{1}{S_y}, \frac{1}{S_z}\right)$$

$$T^{-1} = T(-t_x, -t_y, -t_z)$$

$$R^{-1} = R(-d) = R^T$$

$$R_x^{-1} = R_x^T \; ; \; R_y^{-1} = R_y^T \; ; \; R_z^{-1} = R_z^T$$

Ex: T

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

T^{-1}

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -p \\ 0 & 1 & 0 & -q \\ 0 & 0 & 1 & -r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

S

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

S^{-1}

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1/p & 0 & 0 & 0 \\ 0 & 1/q & 0 & 0 \\ 0 & 0 & 1/r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

$\underline{T}$ $\qquad\qquad$ $\underline{T^{-1}}$

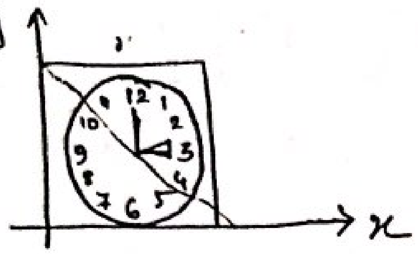
$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \; ; \quad Q_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

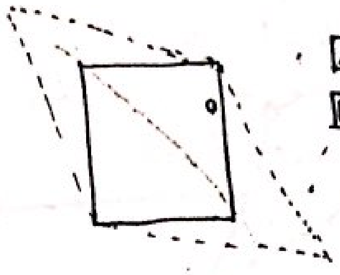
$$R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \; ; \quad Q_y(\phi) = \begin{bmatrix} \cos\phi & 1 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \; ; \quad Q_z(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(*) Practice Problem :

① Stretch the clock by 50% along one of its diagonals - so that 8:00 through 1:00 move to the northwest and 2:00 through 7:00 move to the southeast.

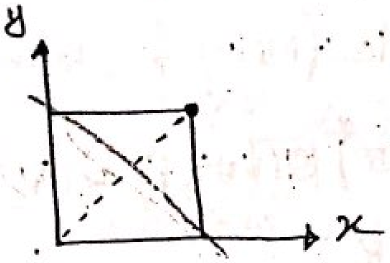




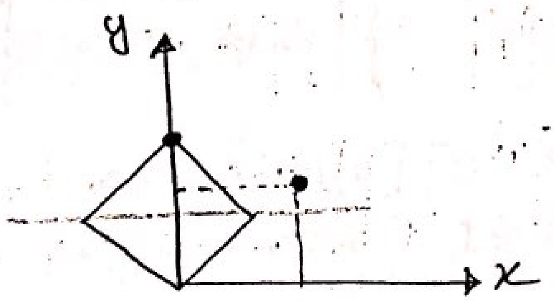
A Draw the steps
B Calculate the matrix

Ans:

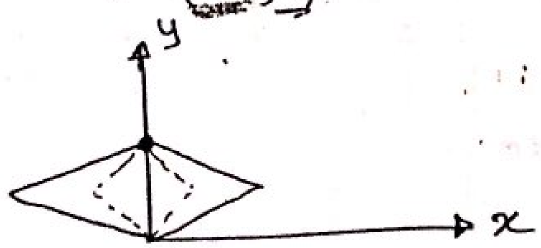
Initial phase:



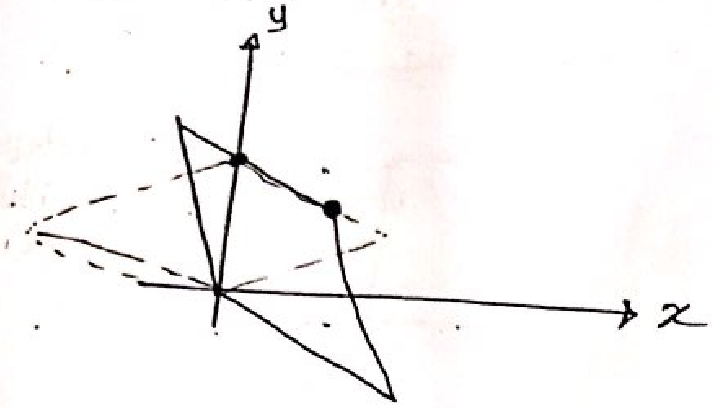
Step 1: Rotate by factor $R = (+45^\circ)$, putting the diagonal in principal axis.



Step 2: Scaling by factor through the x-axis, $S = (1.5, 1)$



Step 3: Now, Rotate (-45°), so we will transpose the rotating factor R^T.



$\therefore$ Rotate $(45^\circ) \to$ Scale $(1.5, 1) \to$ Rotate (-45°)

$\therefore$ Matrix, $M = R(-45^\circ)\, S(1.5, 1)\, R(45^\circ)$

$= R^T S R$

$\therefore M = R^T S R$

$$= \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) \\ \sin(-45^\circ) & \cos(-45^\circ) \end{bmatrix} \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix} \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0605 & 0.707 \\ -1.06 & 0.707 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

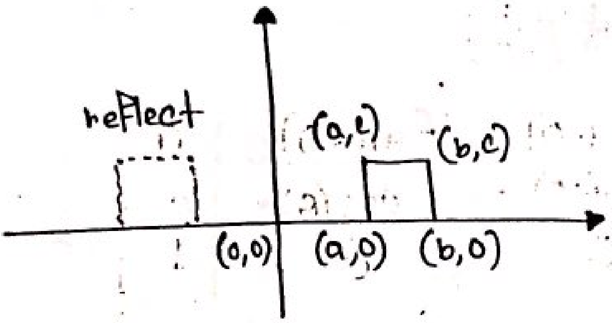
$$= \begin{bmatrix} 1.2496 & 1.2496 \\ -0.249 & -0.249 \end{bmatrix}$$

w.r.t y-axis

w.r.t arbitrary line

Ans:

[i]



w.r.t y-axis

We know that,

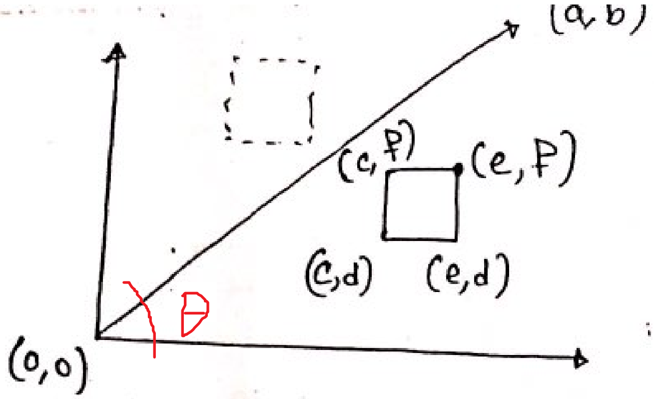
$$\text{reflect\_y} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & a & b \\ 0 & 0 & c & c \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -a & -b & -a & -b \\ 0 & 0 & c & c \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

?? why homogeneous?

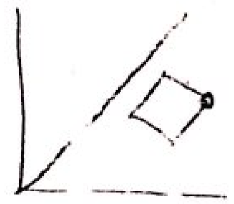
ii



We know,
$\tan\theta = \frac{\text{লম্ব}}{\text{ভূমি}}$
$\Rightarrow \tan\theta = \frac{b}{a}$
$\therefore \theta = \tan^{-1}\left(\frac{b}{a}\right)$
Say

Here, slope, $m = \frac{b}{a}$

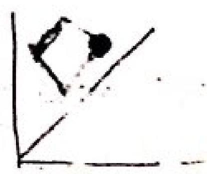
$\therefore \theta = \tan^{-1}\left(\frac{b}{a}\right)$



step 1:

$$\therefore R(+\theta) = \begin{bmatrix} \cos(+\theta) & -\sin(+\theta) & 0 \\ \sin(+\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

step 2:

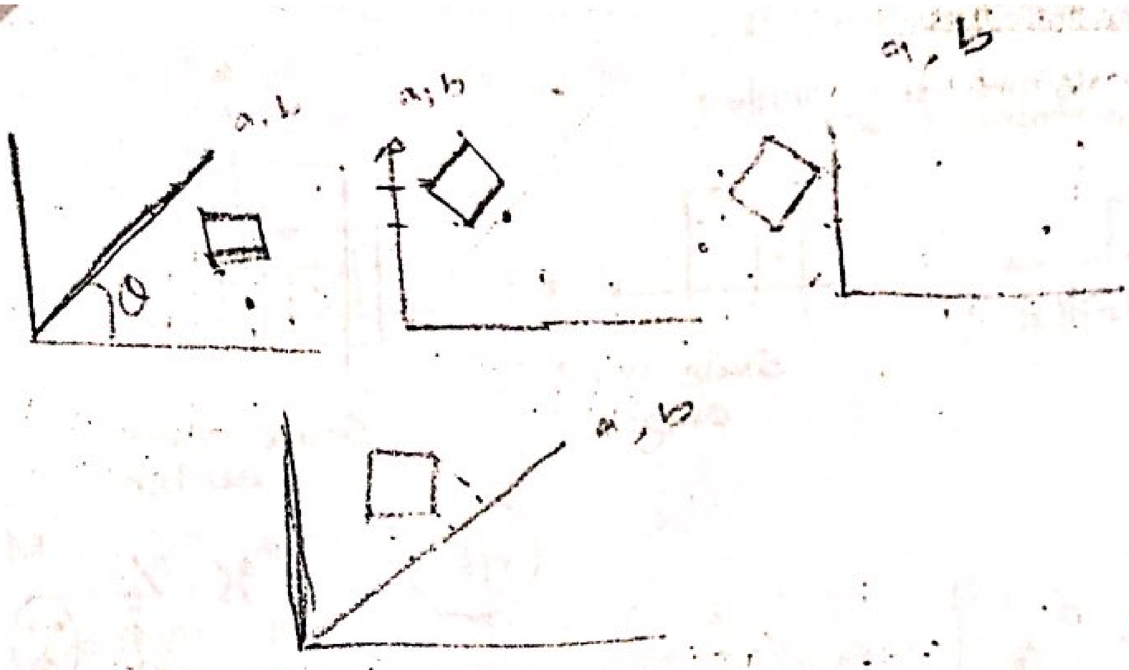


$$\text{Reflect-}x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

step 3:

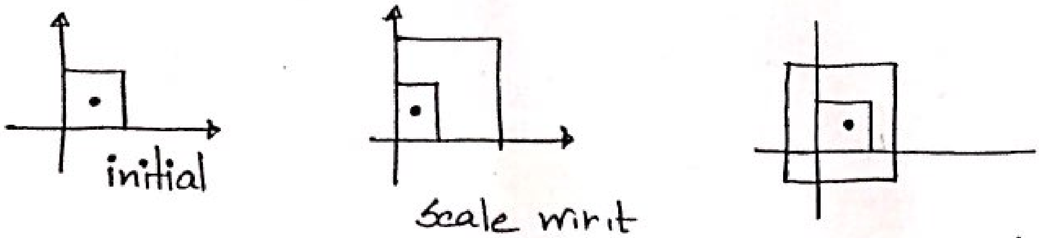
$$R(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore M = R(-\theta) * \text{Reflect-}x * R(+\theta)$$


a, b
θ
a, b
a, b
a, b

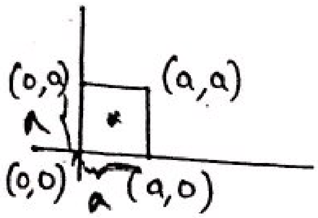
⊛ Practice Problem (Part-B):

① Scale w.r.t the center



Ans:

Initial:



$a/2$ ✓ $\quad a/2, a/2$

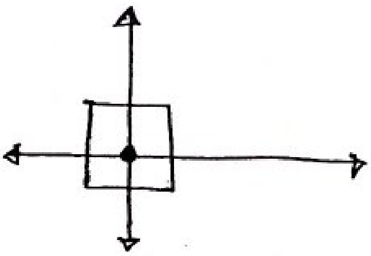
$\boxed{\text{Center} = \frac{\sqrt{2}a}{2}}$?

$\therefore$ center point $= \left(\frac{a\sqrt{2}}{2}, \frac{a\sqrt{2}}{2}\right)$

Steps:

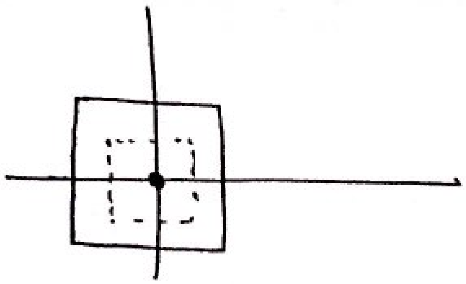
1. shifting by / Translation by $\left(\frac{-a\sqrt{2}}{2}, \frac{-a\sqrt{2}}{2}\right)$ $\quad -\frac{a}{2}, -\frac{a}{2}$
2. Scaling by (S_x, S_y)
3. Shifting by / Translation by $\left(\frac{a\sqrt{2}}{2}, \frac{a\sqrt{2}}{2}\right)$

Step-1:



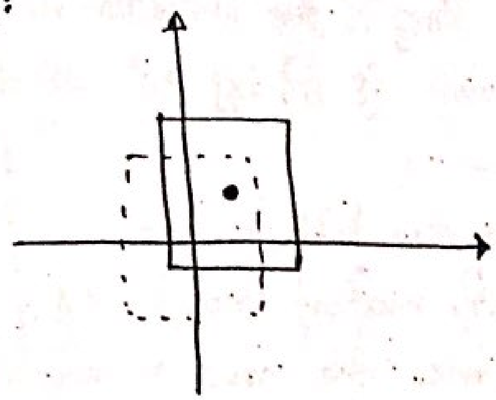
Translation (T)

Step-2:



Scaling (S)

step-3:



Inverse Translation = T^{-1}

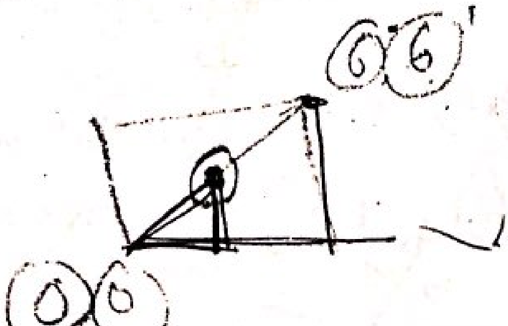
$\therefore M = T^{-1} * S * T$

$$X = \begin{bmatrix} 1 & 0 & 0 & -p \\ 0 & 1 & 0 & -q \\ 0 & 0 & 1 & -r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

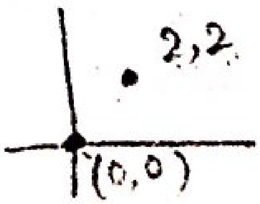
$$= \begin{bmatrix} 1 & 0 & a\sqrt{2}/2 \\ 0 & 1 & a\sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a\sqrt{2}/2 \\ 0 & 1 & -a\sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix}$$

? :(

when we use what ?

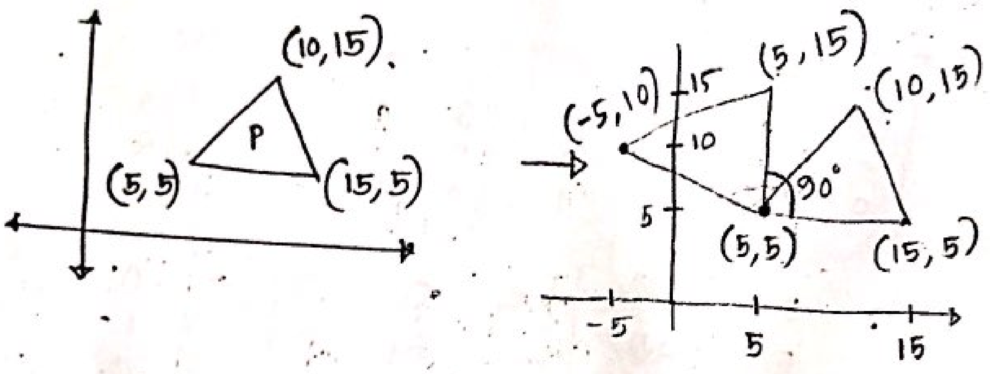


$\frac{\sqrt{2}\,a}{2}$ $\frac{\sqrt{2}\,a}{2}$



② In a movie, "The last Egyptian King", the director needs to rotate a pyramid P about point (5,5) by 90°. As a VFX team member, you have to –

- Mention the steps to perform the task.
- Determine the composite transformation matrix M.
- Multiply M with P and determine the new coordinates P′.
- Plot P and P′ on the same axis to show the rotation.



Ans: Here, $\theta = 90°$

Steps:

1. Translation by (−5, −5)
2. Rotation by 90°
3. Inverse Translation by (5, 5)

* origin এ নাই triangle টা, So, rotate করলে সেটা origin থেকে দূরে সরে যাবে। তাই আগে translation করতে হবে, origin এ আনতে হবে।

∴ Composite Transformation matrix,

$$M = T^{-1} * R(90°) * T$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

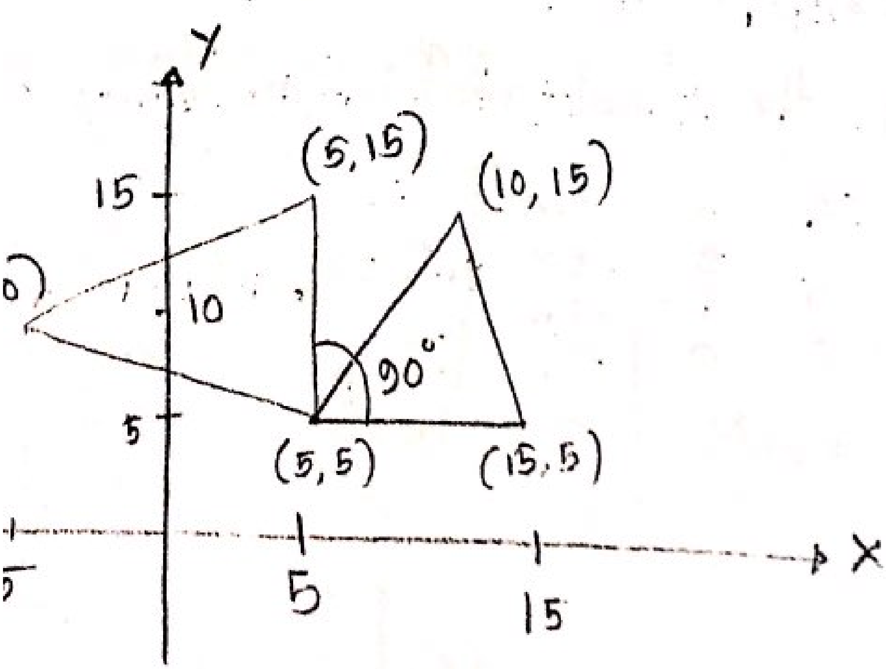
$$\begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

M * P

$$\begin{bmatrix} 0 & -1 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 15 & 10 \\ 5 & 5 & 15 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 5 & -5 \\ 5 & 15 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$



(*) TextBook Exercise — 1–6, 8, 9 :

(1) Ex-1: Show that the inverse of a rotation matrix is its transpose.

Ans:

$$\text{Rotation Matrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

$$R(\phi)^{-1} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

$$= R^T$$ [Showed]

(2) Ex-2: Write down the 4x4 (4 by 4) 3D matrix to move by (x_m, y_m, z_m).

Ans: Let, $M_{4\times4}$ = 4x4 3D matrix

After moving the final matrix becomes,

$$M'_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & x_m \\ 0 & 1 & 0 & y_m \\ 0 & 0 & 1 & z_m \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot M_{4\times4}$$

② Write down the 4X4 3D matrix to rotate by an angle θ about the y-axis.

Ans: let, M_{4x4} = 4X4 3D matrix

After rotation, the final matrix becomes,

$$M'_{4x4} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot M_{4x4}$$

③ Write down the 4X4 3D matrix to scale an object by 50% in all directions.

Ans: let, M_{4x4} = 4X4 cos(-90) 3D matrix

After scaling by 50%, the final matrix becomes,

$$M'_{4x4} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot M_{4x4}$$

④ Write the 2D rotation matrix that rotates by $\boxed{90° \text{ clockwise.}}$

Ans: For 2D rotation,

let, the matrix be 3X3

M_{3x3} = 2D matrix

After 90° clockwise rotation,

$$M'_{3x3} = \begin{bmatrix} \overset{\cos(-90)}{\cos 90°} & \overset{-\sin(-90)}{-\sin 90°} & 0 \\ \underset{\sin(-90)}{\sin 90°} & \underset{\cos(-90)}{\cos 90°} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot M_{3x3}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot M_{3x3}$$

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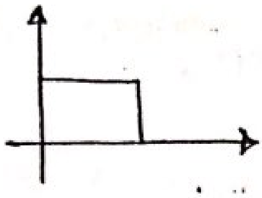
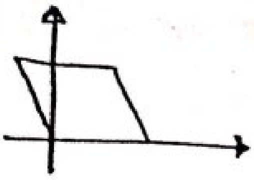
⑤ Write the matrix from exercise 4 as a product of 3 shear matrices. ⑩

Ans:

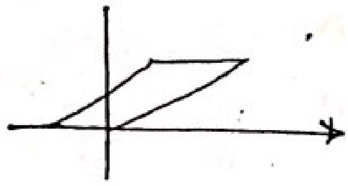
Initial:



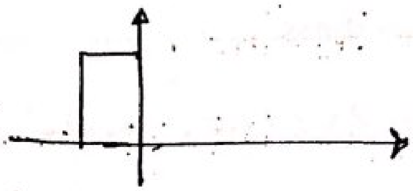
step-1: shear_x(-θ)



step-2: shear_y(θ)



step-3: shear_x(-θ)



$\therefore$ M = shear_x(-θ) * shear_y(θ) * shear_x(-θ)

$$= \begin{bmatrix} 1 & \tan(-\theta) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \tan\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan(-\theta) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After shearing 3 times

= M. 2D Matrix

⑥ Find the inverse of rigid body transformation:

$$\begin{bmatrix} & R & & t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where R is a 3x3 rotation matrix and t is 3 vector.

<u>Ans:</u> let, $t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$ $\therefore t^{-1} = \begin{bmatrix} -t_x \\ -t_y \\ -t_z \end{bmatrix}$

$\therefore t^{-1} = -t$

here, R is a 3x3 rotation matrix.

$\therefore R^{-1} = R^T$; as we know inverse of a rotation matrix is a transpose of that matrix.

After inversion of rigid body transformation, we get

$$\begin{bmatrix} & R^T & & -t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

** Rigid body → translate + Rotate
(not scaling)
actual shape ঠিক থাকে।
just location change হওয়া।

⑧ Describe in words what this 2D transform matrix does: $\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Step 2:

Ans: if $\theta = 90°$,

Then, $\text{Rotate-}z(90°) = \begin{bmatrix} \cos(90°) & -\sin 90° & 0 \\ \sin(90°) & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = M$$

Now, Translation for $(1,1)$; M becomes

$$M' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

That means, firstly rotation is done and then translation is done on it.

⑨ Write down the 3X3 matrix that rotates a 2D point by angle θ about a point $p = (x_p, y_p)$.

Ans:

Step 1: As the point is $P = (x_p, y_p)$; we have to translate it.

$$T = \begin{bmatrix} 1 & 0 & -x_p \\ 0 & 1 & -y_p \\ 0 & 0 & 1 \end{bmatrix}$$

step 2: Rotation

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

step 3: Inverse Translation

$$T^{-1} = \begin{bmatrix} 1 & 0 & x_p \\ 0 & 1 & y_p \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Transformation Matrix, $M = T^{-1} * R * T$

$$= \begin{bmatrix} 1 & 0 & x_p \\ 0 & 1 & y_p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_p \\ 0 & 1 & -y_p \\ 0 & 0 & 1 \end{bmatrix}$$

⊛ Previous Year Quiz Question:

(1) Reflect a 2D point $P(-2,-2)$ along the diagonal AB of a square. Where the four vertices of the square are: $O(0,0)$, $A(0,6)$, $C(6,6)$ and $B(6,0)$. You must -

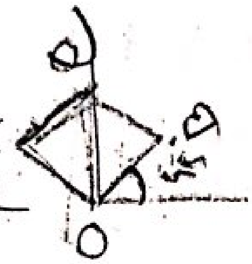
[a] Mention the steps.

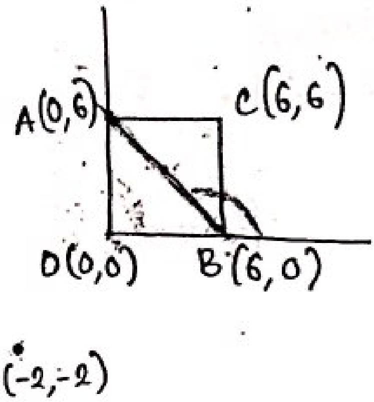
[b] Determine the composite transformation matrix.

Ans:

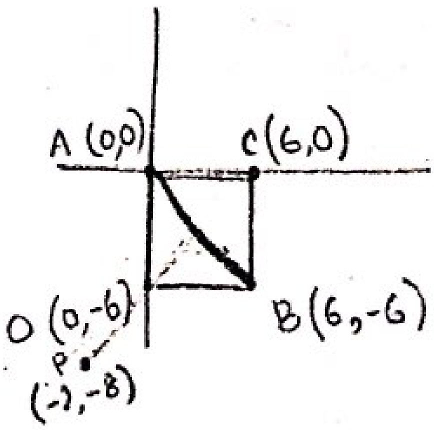
[a] steps:

1. Translate by $(0,-6)$
2. Rotate by 45 degree
3. Reflect along X-axis
4. Undo step-2
5. Undo step-1 $(0,+6)$

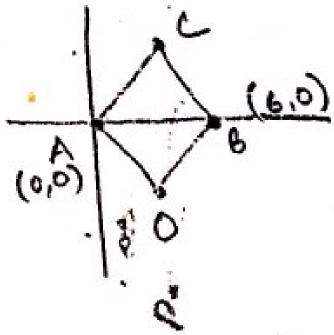




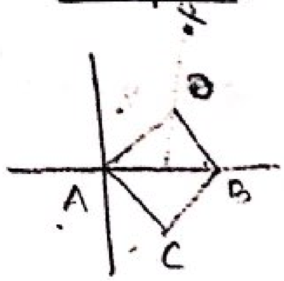
step 1:

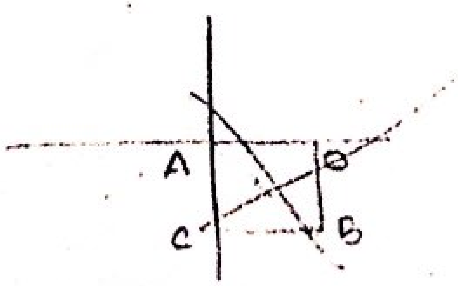


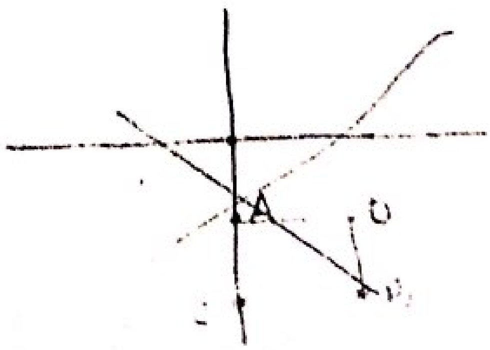
step 2:

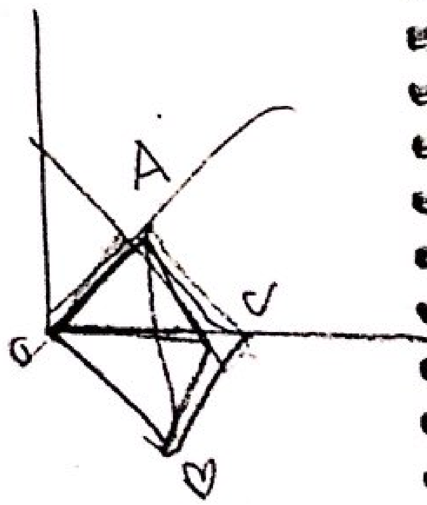


step 3:









(b) Transformation matrix:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(45^\circ) = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Reflect}(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^{-1}(45^\circ) = \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

(MATA × MATB) × MATC

∴ Composite transformation matrix,

$$M = T^{-1} * R^{-1} * \text{Ref-}x * R * T.$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & -0.707 & 0 \\ -0.707 & -0.707 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

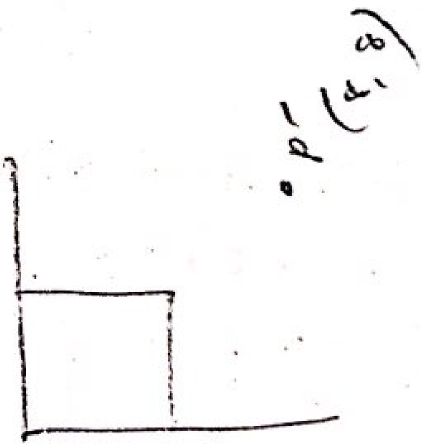
$$= \begin{bmatrix} 0 & -0.999 & 0 \\ -0.999 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.999 & 5.994 \\ -0.999 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 6 \\ -1 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & -1 & 6 \\ -1 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 8 \\ 1 \end{bmatrix}$$



② Write
0 abov

② Write down the 4X4 3D matrix to rotate by an angle θ about x-axis.

Ans: Let, $M_{4\times4}$ = 4X4 3D matrix

After rotating it by θ angle about x-axis,

$$M'_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix} \cdot M_{4\times4}$$

→ 3D rot-x

③ . . . . . . along z-axis

Ans: . . . .

$$M'_{4\times4} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot M_{4\times4}$$

① Reflect a 2D point $P(-2,2)$ along a line $y=3$.
You must —

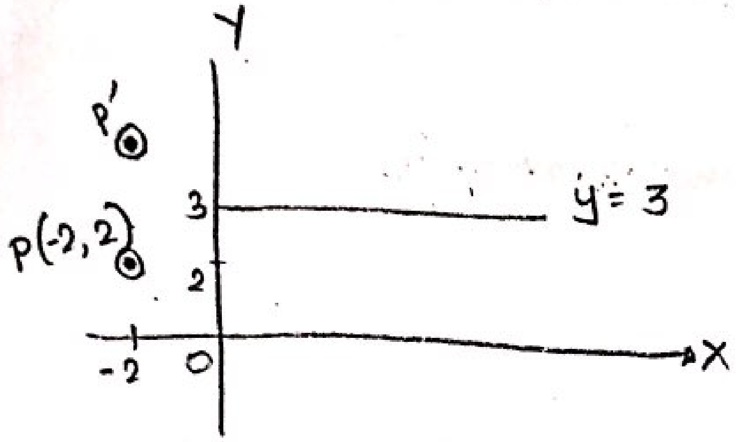
a Mention the steps.

b Determine the composite transformation matrix.

c Perform multiplication to determine the reflected point P'.

d Plot P and P'.

Ans:

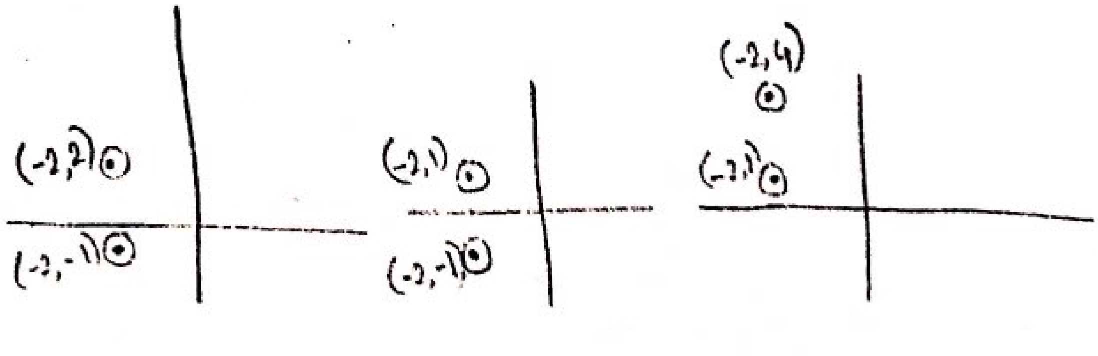


steps:

① For $y=3$, $x=0$, Translate by $(0,-3)$

② Reflect along x-axis

③ Inverse translate by $(0,3)$



(b) Composite transformation matrix,

$$M = T^{-1} * \text{Reflec-}x * T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

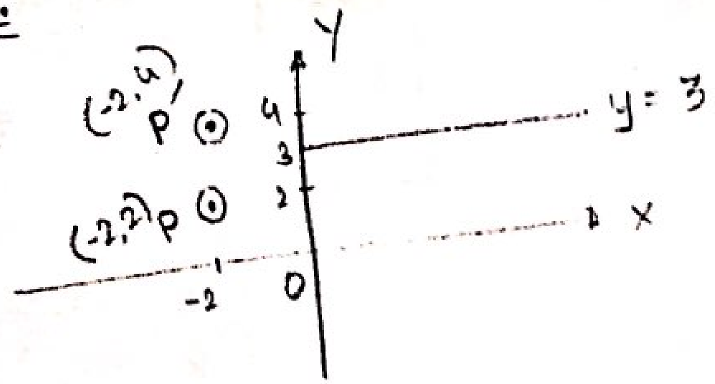
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) For $P(-2, 2)$ point,

Reflected point $P' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$$

(d) Plot:



(*) Quiz Question:

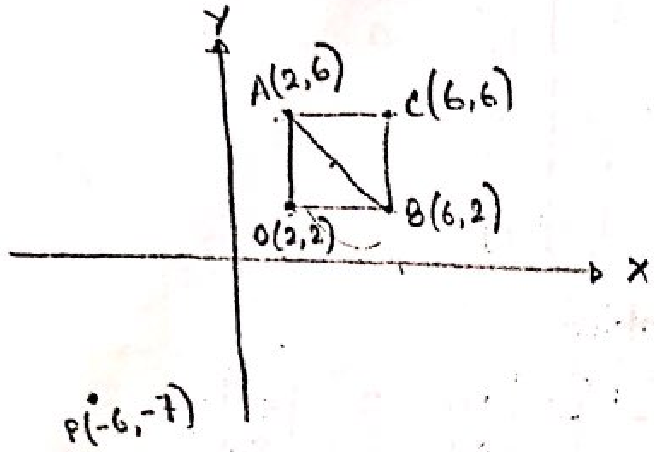
① Reflect a 2D point P(-6,-7) along the diagonal AB of a square OACB; four vertices of the square are: O(2,2), A(2,6), C(6,6) and B(6,2). You must —

[a] Mention the steps.

[b] Determine the composite transformation matrix.

[c] Calculate and plot the reflected point P′.

Ans:



যেকোনো point কে origin এ আনবো, Then x/y axis বরাবর ।

[a] Steps:

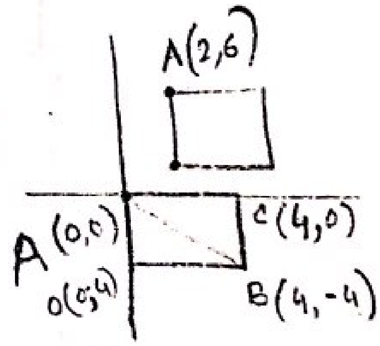
① Translate by (-2,-6)

② Rotate by 45 degrees

③ Reflect along X-axis

④ Undo step-2

⑤ Undo step-1



[b] Composite Transformation matrix,

$$M = T^{-1} * R^{-1} * R_{fx} * R * T$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(45^\circ) & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1. \end{bmatrix}$$

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 & 2 \\ -0.707 & 0.707 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & -0.707 & 2 \\ -0.707 & -0.707 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.999 & 2 \\ -0.999 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.999 & 7.994 \\ -0.999 & 0 & 7.998 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 8 \\ -1 & 0 & 8 \\ 0 & 0 & 1 \end{bmatrix}$$

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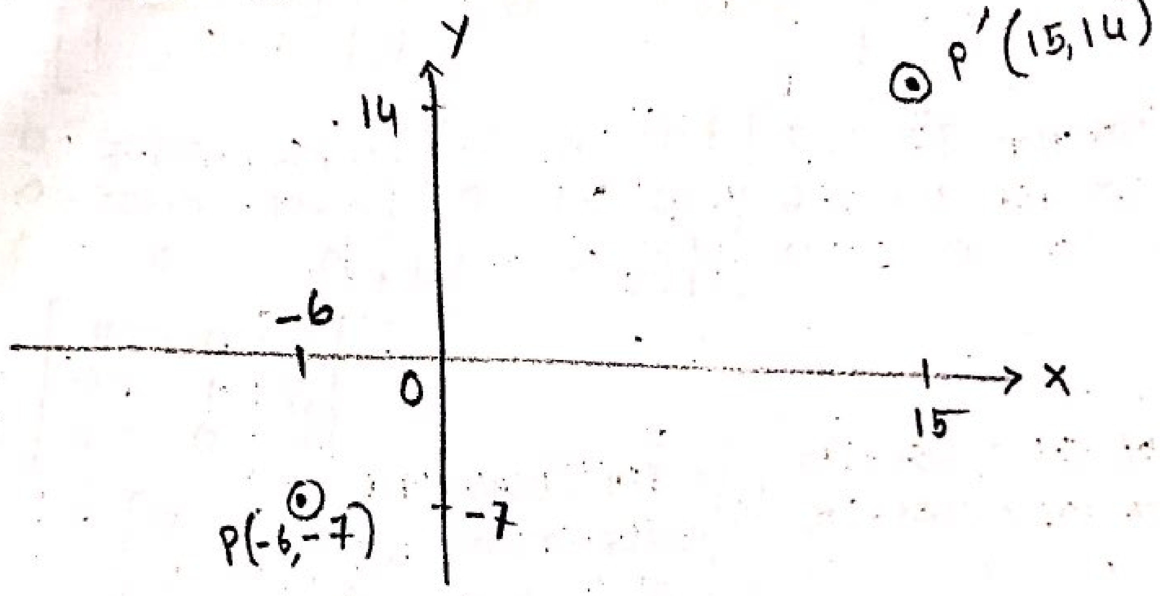
ⓒ $P' = M * P$

$$= \begin{bmatrix} 0 & -1 & 8 \\ -1 & 0 & 8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -6 \\ -7 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 14 \\ 1 \end{bmatrix}$$



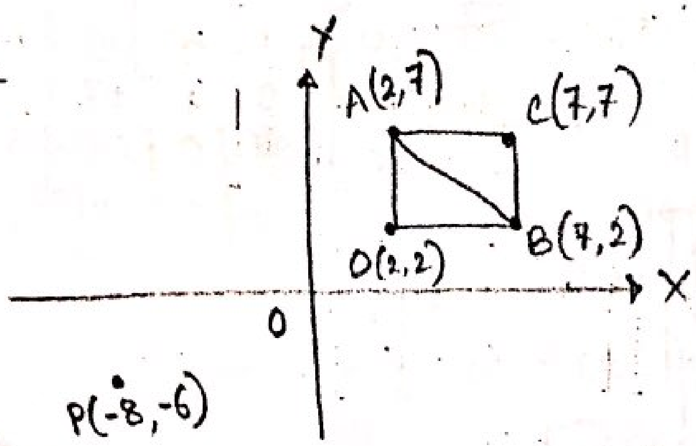
② Reflect a 2D point $P(-8,-6)$ along the diagonal AB of a square OACB; Four vertices of the square are: $O(2,2)$, $A(2,7)$, $C(7,7)$, and $B(7,2)$. You must –

[a] Mention the steps.

[b] Determine the composite transformation matrix.

[c] Calculate and plot the reflected point P'.

Ans:



[a] steps:

1. Translate by $(-2,-7)$
2. Rotate by 45 degrees
3. Reflect along X-axis
4. Undo step-2
5. Undo step-1

[b] Composite transformation matrix,

$$M = T^{-1} * R^{-1} * Ref\text{-}x * R * T$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 & 2 \\ -0.707 & 0.707 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & -0.707 & 2 \\ -0.707 & -0.707 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

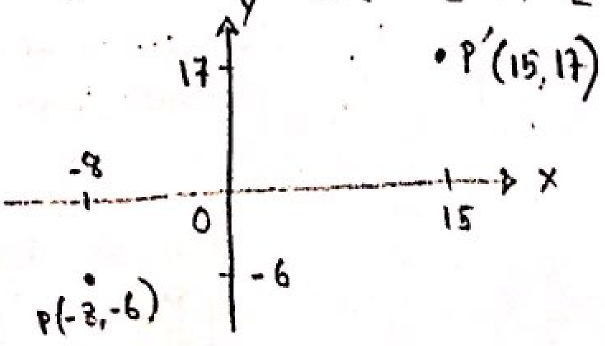
$$= \begin{bmatrix} 0 & -0.999 & 2 \\ -0.999 & 0 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.999 & 8.9978 \\ -0.999 & 0 & 8.9993 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ -1 & 0 & 9 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{c}$ $P' = M * P$

$$= \begin{bmatrix} 0 & -1 & 9 \\ -1 & 0 & 9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -8 \\ -6 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 17 \\ 1 \end{bmatrix}$$



Chapter-7

Viewing (Part A)

⊛ **Viewport Transformation:** (Canonical to Screen)

$$M_{vp} = T(-\tfrac{1}{2}, -\tfrac{1}{2}) * S(n_x/2, n_y/2) * T(1,1)$$

2D

$$= \begin{bmatrix} n_x/2 & 0 & \frac{n_x-1}{2} \\ 0 & n_y/2 & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_{pixel} \\ y_{pixel} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canonical} \\ y_{canonical} \\ 1 \end{bmatrix}$$

3D

$$M_{vp} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{canonical} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{canonical} \\ y_{canonical} \\ z_{canonical} \\ 1 \end{bmatrix}$$

⊛ **Orthographic to Canonical view volume:**

$$M_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Orthographic $\rightarrow$ Canonical $\rightarrow$ Screen

$$\begin{bmatrix} x_{pixel} \\ y_{pixel} \\ z_{canonical} \\ 1 \end{bmatrix} = M_{vp} M_{orth} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(Part B)

⊛ Coordinate Transformation :

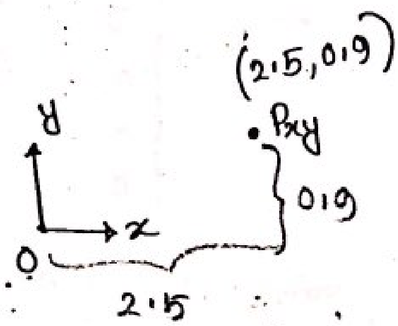
$\boxed{a}$ In a frame $\rightarrow$ origin **O** and basis $\{x, y\}$

Canonical

coordinates (x, y) describe the point :

$$O + xX + yY$$

$$P_{xy} = (x_p, y_p) = O + x_p X + y_p Y$$



$$\Rightarrow \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 2.5 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.9 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

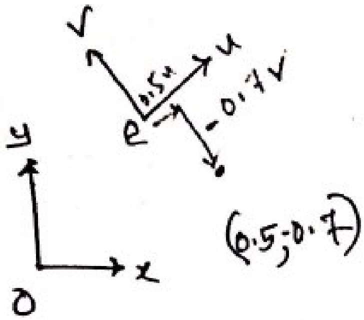
$$= \begin{bmatrix} 2.5 \\ 0.9 \end{bmatrix}$$

$\boxed{b}$ Another coordinate system (frame):

Frame

origin **e** and orthonormal basis vectors $\{u, v\}$

$$P_{uv} = (u_p, v_p) = \mathbf{e} + u_p \mathbf{U} + v_p V$$

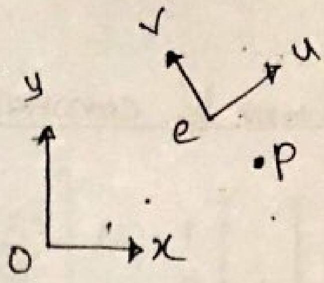


$$\Rightarrow \begin{bmatrix} u_p \\ v_p \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix} + 0.5 \begin{bmatrix} x_u \\ y_u \end{bmatrix} + (-0.7) \begin{bmatrix} x_v \\ y_v \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 \\ -0.7 \end{bmatrix}$$

⊛ $P_{xy} = (x_p, y_p) = O + x_p X + y_p Y$

$P_{uv} = (u_p, v_p) = e + u_p U + v_p V$



$P_{xy} \leftrightarrow P_{uv}$

$P_{xy} = e + u_p U + v_p V$

$$\Rightarrow \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix} + u_p \begin{bmatrix} x_u \\ y_u \end{bmatrix} + v_p \begin{bmatrix} x_v \\ y_v \end{bmatrix} \quad ; 2D$$

$$\Rightarrow \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_e \\ 0 & 1 & y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & x_v & 0 \\ y_u & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} \quad ; 3D$$

$$\boxed{\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & x_v & x_e \\ y_u & y_v & y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}}$$

$$\boxed{\therefore P_{xy} = \begin{bmatrix} u & v & e \\ 0 & 0 & 1 \end{bmatrix} P_{uv}}$$ frame to canonical matrix (2D)

Again,

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

$$\boxed{\therefore P_{uv} = \begin{bmatrix} u & v & e \\ 0 & 0 & 1 \end{bmatrix}^{-1} P_{xy}}$$ canonical to frame matrix (2D

* 3D :

[a] Frame to canonical Matrix :

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x_e \\ 0 & 1 & 0 & y_e \\ 0 & 0 & 1 & z_e \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & x_v & x_w & 0 \\ y_u & y_v & y_w & 0 \\ z_u & z_v & z_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ w_p \\ 1 \end{bmatrix}$$

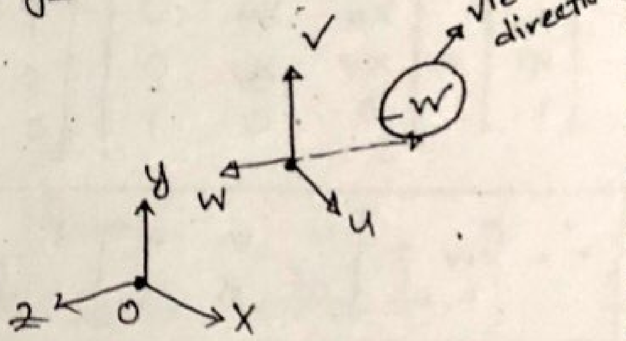
$$\therefore P_{xyz} = \begin{bmatrix} u & v & w & e \\ 0 & 0 & 0 & 1 \end{bmatrix} P_{uvw}$$

[b] Canonical to frame Matrix :

$$\begin{bmatrix} u_p \\ v_p \\ w_p \\ 1 \end{bmatrix} = \overbrace{\begin{bmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{M_{cam}} \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$

$$\therefore P_{uvw} = \begin{bmatrix} u & v & w & e \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} P_{xyz}$$

⊛ Camera Transformation :



e : camera/eye position

$-w$: viewing direction

v : top

u : right

** From xyz-coordinates into uvw-coordinates

$$P_{uvw} = \begin{bmatrix} u & v & w & e \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} P_{xyz}$$

* Canonical-to-basis matrix:

$$M_{cam} = \begin{bmatrix} u & v & w & e \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

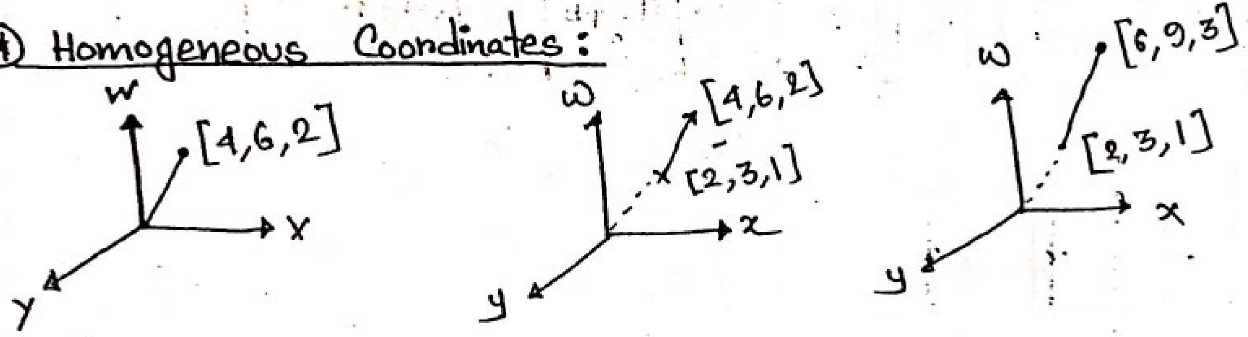
$$= \begin{bmatrix} x_u & y_u & z_u & 0 \\ x_v & y_v & z_v & 0 \\ x_w & y_w & z_w & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

** Summary:

$$M = M_{vp} * M_{orth} * M_{cam}$$

Part(c)

* Homogeneous Coordinates:

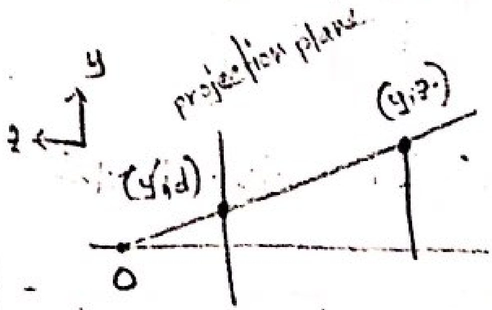


$[x, y, w] \rightarrow [4, 6, 2]$ $\qquad$ $[x, y, \omega] \rightarrow [x/\omega, y/\omega, 1]$

$$\begin{bmatrix} x \\ y \\ \omega \end{bmatrix} \sim \begin{bmatrix} x/\omega \\ y/\omega \\ \omega/\omega \end{bmatrix} = \begin{bmatrix} x/\omega \\ y/\omega \\ 1 \end{bmatrix}.$$

* Perspective Projection:

$\boxed{a}$ For 1D: $\quad \frac{y'}{d} = \frac{y}{z} \quad \therefore y' = dy/z$

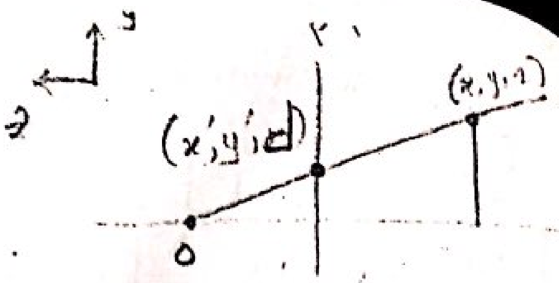


$$\begin{bmatrix} d & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dy \\ z \end{bmatrix} \sim \begin{bmatrix} dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} y' \\ 1 \end{bmatrix}.$$

b For 2D:

$$y' = \frac{dy}{z}$$

$$x' = \frac{dx}{z}$$

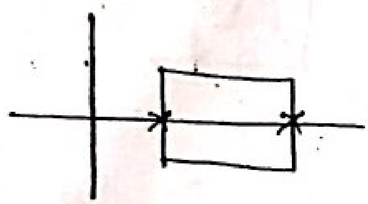


$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ z \end{bmatrix} \sim \begin{bmatrix} dx/z \\ dy/z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

c For 3D:

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ az+b \\ z \end{bmatrix} = \begin{bmatrix} dx/z \\ dy/z \\ (az+b)/z \\ 1 \end{bmatrix}$$

(*)



$$z = n \quad \begin{bmatrix} 0 \\ 0 \\ (an+b)/n \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ n \\ 1 \end{bmatrix} \quad ; \quad z = f \quad \begin{bmatrix} 0 \\ 0 \\ (af\ \ )/f \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ f \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} a &= (n+f) \\ b &= -fn \end{aligned}}$$

$\therefore$ Perspective Matrix, $P = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix}$

$$\boxed{P = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}} \quad ; \text{Perspective Matrix}$$

Here, $M_{per} = M_{orth} * P$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_{openGL} = \begin{bmatrix} \frac{2|n|}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2|n|}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{|n|+|f|}{|n|-|f|} & \frac{2|f||n|}{|n|-|f|} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$\boxed{***}$ Perspective Transformation Chain :

1. Modeling Transform : M_m
2. Camera " : M_{cam}
3. Perspective : P
4. Orthographic projection : M_{orth}
5. Viewport transform : M_{vp}

$$P_s = M_{vp} M_{orth} P M_{cam} M_m P_o$$

(*) Practice Problem :

① Transform a 3D line AB from an orthographic view volume to a viewport of size 128 x 96. Vertices of the line are A(-1, -3, 1) and B (2, 4, -1). The orthographic view volume has the following setup:

$l=-4,\ r=4,\ b=-4,\ t=4,\ n=-4,\ f=-8$

You must –

[a] Determine the transformation matrix M.

[b] Multiply M with the vertices of the line and determine the position of vertices on viewport.

Ans: Given,

$n_x = 128,\ n_y = 96$

$l=-4,\ r=4,\ b=-4,\ t=4,\ n=-4,\ f=-8$

$A(-1,-3,1),\ B(2,4,-1)$

[a] Transformation Matrix,

$M = M_{vp} * M_{orth}$

$$= \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{128}{2} & 0 & 0 & 127/2 \\ 0 & \frac{96}{2} & 0 & 95/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{4+4} & 0 & 0 & -\frac{4-4}{4+4} \\ 0 & \frac{2}{4+4} & 0 & -\frac{4-4}{4+4} \\ 0 & 0 & \frac{2}{-4+8} & -\frac{-4-8}{-4+8} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 64 & 0 & 0 & 63.5 \\ 0 & 48 & 0 & 47.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 0 & 0 & 63.5 \\ 0 & 12 & 0 & 47.5 \\ 0 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 0 & 0 & 127/2 \\ 0 & 12 & 0 & 95/2 \\ 0 & 0 & 1/2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{b}$ $A' = MA$

$$= \begin{bmatrix} 16 & 0 & 0 & 63.5 \\ 0 & 12 & 0 & 47.5 \\ 0 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 95/2 \\ 23/2 \\ 7/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 47.5 \\ 11.5 \\ 3.5 \\ 1 \end{bmatrix}$$

$B' = MB$

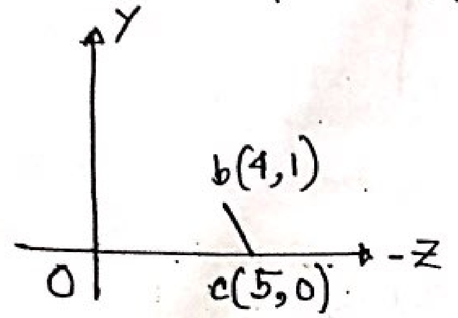
$$= \begin{bmatrix} 16 & 0 & 0 & 63.5 \\ 0 & 12 & 0 & 47.5 \\ 0 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 191/2 \\ 191/2 \\ 5/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 95.5 \\ 95.5 \\ 2.5 \\ 1 \end{bmatrix}$$

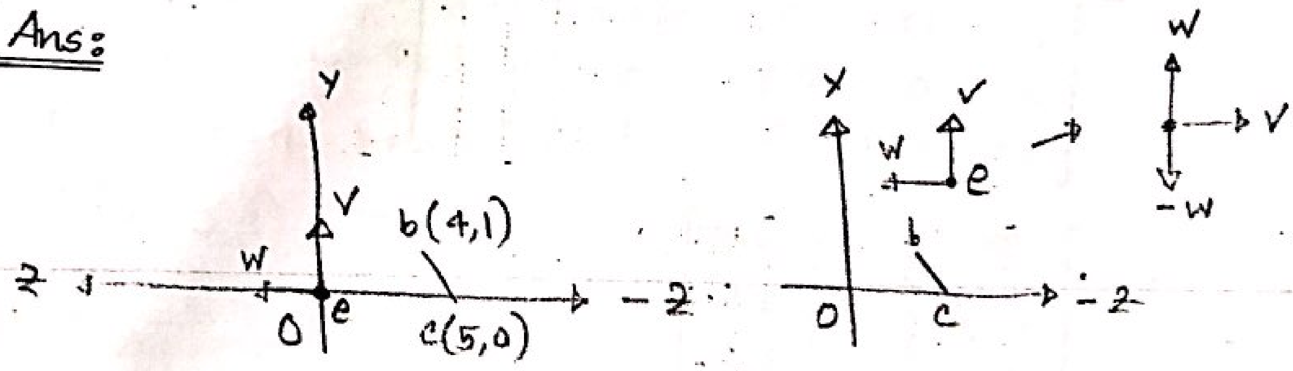
Imp ② Origin O and basis {y, z} construct a 2D canonical coordinate system. Within this, line bc is our model (P_{xy}). We want to view it from a new 2D camera (frame) with origin (e) looking downward. (e = 4, 8)

[a] Determine canonical-to-basis matrix

[b] Calculate and plot P_{wv}.



Ans:



Here, $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\therefore v' = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$; $w' = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $-w = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

Or,

$$\text{rotate}(-90^\circ) = \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 \\ \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{cc} w & v \\ \downarrow & \downarrow \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\therefore w' = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad v' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\boxed{a}$ Canonical-to-basis Matrix.

$$M_{cam} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -8 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

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$\boxed{b}$

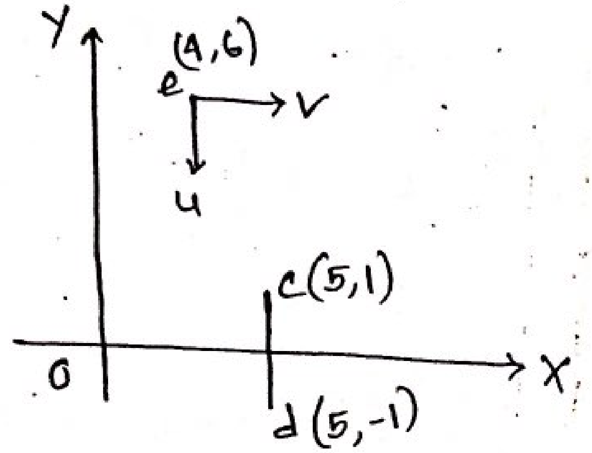
$$P_{uv} = M_{cam} * P_{xy}$$

$$= \begin{bmatrix} & \\ & \\ & \end{bmatrix} \begin{bmatrix} & \\ & \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

③ Here (in the figure), origin O and basis {x, y} construct a 2D canonical coordinate system. Within this, line cd is our model P_{xy}. Now we want to view the model from a new 2D camera with origin e and basis {u, v}, where u vector is perpendicular to the X-axis. Assume that, u is the viewing direction.

<u>Ans:</u>



$\boxed{a}$ Determine the canonical-to-basis matrix

$\boxed{b}$ Calculate and plot P_{uv}.

<u>Ans:</u>

u vector is perpendicular to the x-axis,

$\therefore x_u = 0$

u vector is pointing downward y-axis,

$\therefore y_u = -1$

v vector is pointing towards x-axis,

$\therefore x_v = 1, \quad y_v = 0$

$$\therefore u = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Given,

$xe = 4, \quad ye = 6$

a Canonical-to-basis matrix,

$$M_{cam} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -xe \\ 0 & 1 & -ye \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

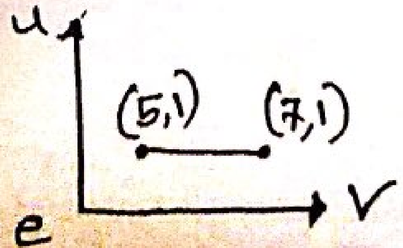
$$= \begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}.$$

b $$P_{xy} = \begin{bmatrix} 5 & 5 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\therefore P_{uv} = M_{cam} * P_{xy}$$

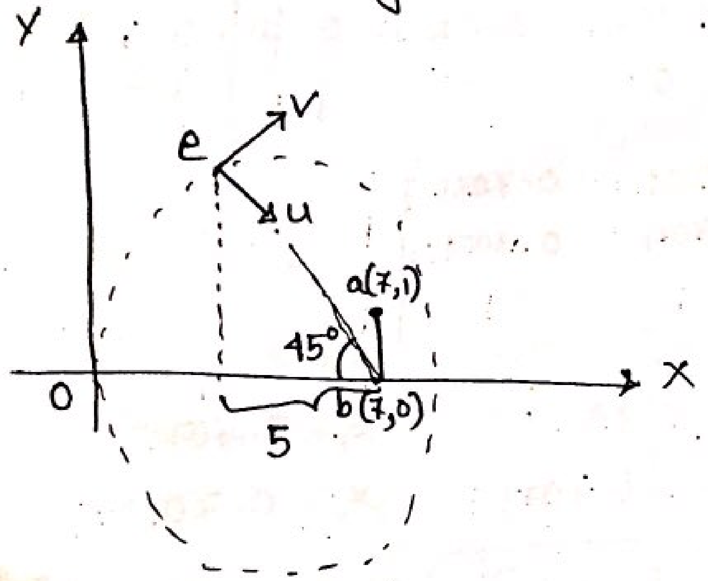
$$= \begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 5 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$



④ Here (in the figure), origin O and basis $\{x, y\}$ construct a 2D canonical coordinate system. Within this, line ab is our model (P_{xy}). Now, we want to view it from a new 2D camera with eye e and basis $\{u, v\}$; which is rotated by -45 degrees around b. Assume that, u is the viewing direction and v is the up vector.

Ans:



a Determine the canonical-to-basis matrix,

b Calculate and plot P_{uv}.

Ans: The basis vectors are rotated by $-45°$.

Before rotation,

$xu_0 = 1$, $yu_0 = 0$

$xv_0 = 0$ $yv_0 = 1$

$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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After rotation matrix to the basis vectors,

$$\begin{bmatrix} x_u & x_v \\ y_u & y_v \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{u_0} & x_{v_0} \\ y_{u_0} & y_{v_0} \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \\ 1 & 1 \end{bmatrix}$$

$\therefore$ We get, $x_u = 0.7071$ $\quad x_v = 0.7071$

$y_u = -0.7071$ $\quad y_v = 0.7071$

$$\boxed{u = \begin{bmatrix} 0.7071 \\ -0.7071 \end{bmatrix} \quad v = \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix}}$$

Now, $x_e = 7 - 5 = 2$

$y_e = 4.89 \approx 5$

$\tan\theta = \frac{\text{লম্ব}}{\text{ভূমি}}$

$\Rightarrow \tan 45^\circ = \frac{\text{লম্ব}}{5}$

$\Rightarrow$ লম্ব $= 5 * 1 = 5$

(a) Canonical-to-basis matrix,

$$M_{cam} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

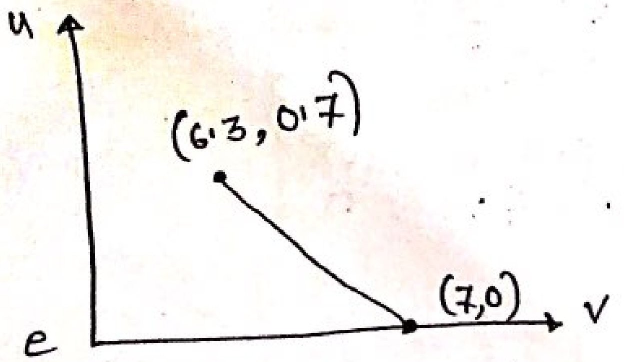
$$= \begin{bmatrix} 0.7071 & -0.7071 & 2.1213 \\ 0.7071 & 0.7071 & -4.949 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{b}$ Now, $P_{xy} = \begin{bmatrix} 7 & 7 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\therefore P_{uv} = M_{cam} * P_{xy}$

$$= \begin{bmatrix} 0.7071 & -0.7071 & 2.1213 \\ 0.7071 & 0.7071 & -4.949 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 & 7 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

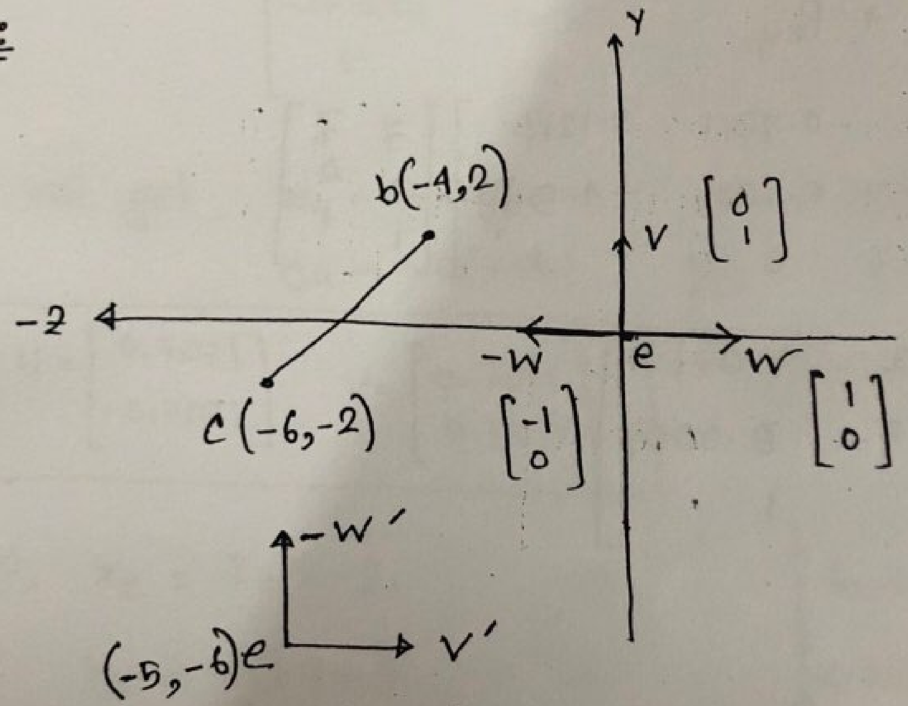
$$= \begin{bmatrix} 6.3639 & 7.071 \\ 0.7078 & 0.0007 \\ 1 & 1 \end{bmatrix}$$



5. Origin O and basis $\{z, y\}$ construct a 2D canonical coordinate system where $-z$ is the viewing direction. Within this, a line bc is our model (P_{xy}) where vertices b and c are $(-4, 2)$ and $(-6, -2)$ respectively. We want to gaze at it from a new 2D camera <u>looking upward</u> with origin $(-5, -6)$.

1. Determine the canonical-to-basis matrix.
2. Calculate and plot P_{wv}
3. Which vertex of P_{wv} will be closer to the viewer?

Ans:



Here, $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

From the diagram,

$v' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $w' = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

or,

Rotate $-90°$.

$$\begin{array}{ccc} & & \begin{array}{cc} w & v \\ \downarrow & \downarrow \end{array} \end{array}$$

$$\therefore \begin{bmatrix} x_{w'} & x_{v'} \\ y_{w'} & y_{v'} \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-90°) & -\sin(-90°) & 0 \\ \sin(-90°) & \cos(-90°) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w & x_v \\ y_w & y_v \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\therefore w' = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \; ; \; v' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Now, $e = \begin{bmatrix} -5 \\ -6 \end{bmatrix} \quad \therefore x_e = -5, \; y_e = -6$

$\boxed{a}$ Canonical-to-basis matrix,

$M_{cam} = \text{Basis} * E$

$$= \begin{bmatrix} x_{w'} & y_{w'} & 0 \\ x_{v'} & y_{v'} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

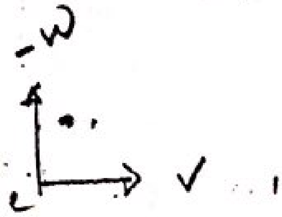
$$= \begin{bmatrix} 0 & -1 & -6 \\ 1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

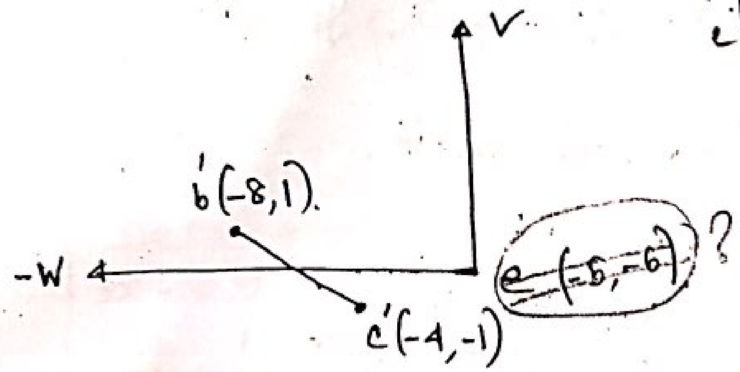
$\boxed{b}$ Now, $P_{xyz} = \begin{bmatrix} -4 & -6 \\ 2 & -2 \\ 1 & 1 \end{bmatrix}$

$\therefore P_{uvw} = M_{cam} * P_{xyz}$

$$= \begin{bmatrix} 0 & -1 & -6 \\ 1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -6 \\ 2 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -4 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$





$\boxed{c}$ c will be closer to the viewer.

The distance between e and b'

$$= \sqrt{(-5+8)^2+(-6-1)^2}$$

$$= \sqrt{9+49}$$

$$= \sqrt{58}$$

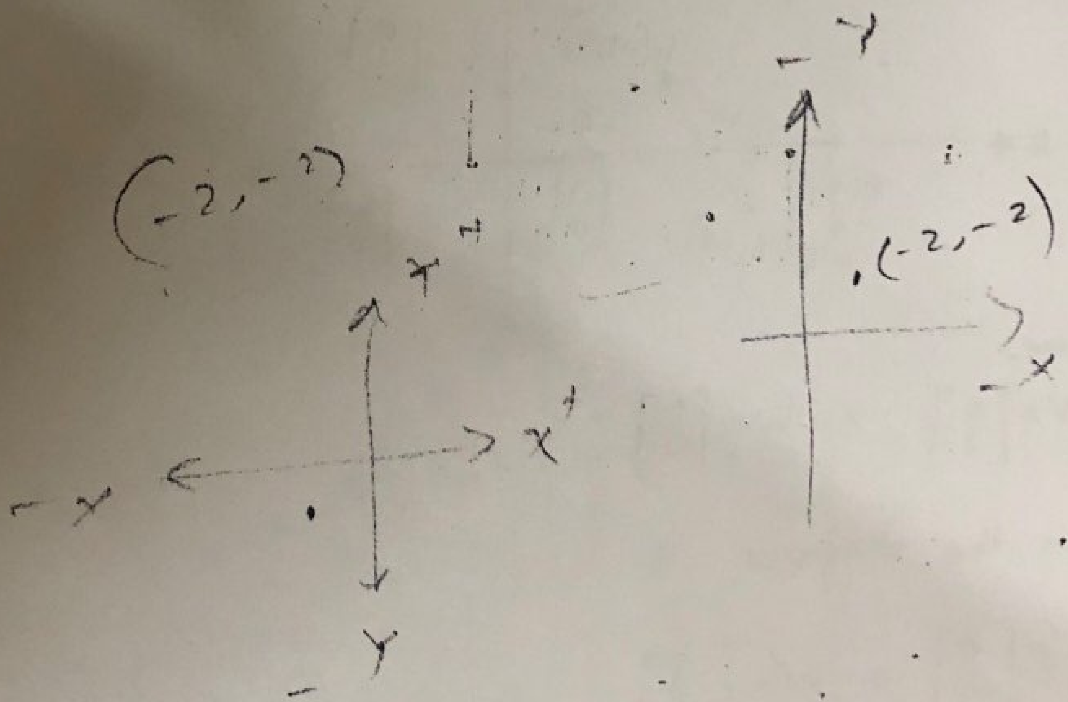
$$= 7.61577$$

The distance between e and c'

$$= \sqrt{(-5+4)^2+(-6+1)^2}$$

$$= \sqrt{1+25} = \sqrt{26} = 5.099$$

So, c' is closer to the viewer.



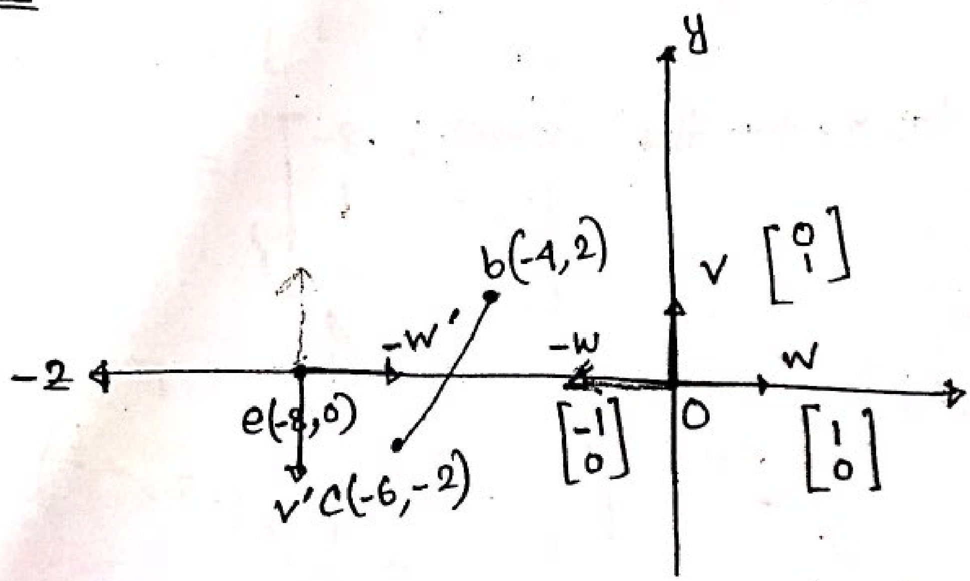
Quiz

⑥ Origin O and basis {x, y} construct a 2D canonical coordinate system where $-z$ is the viewing direction. Within this, a line bc is our model (P_{xy}) where vertices b and c are $(-4, 2)$ and $(-6, -2)$ respectively. We want to gaze at it from a new 2D camera looking backward with origin $(-8, 0)$.

1. Determine the canonical-to-basis matrix.
2. Calculate and plot P_{wv}.
3. Which vertex of P_{wv} will be closer to the viewer?

?

Ans:



Here,

$$v = \begin{bmatrix}0\\1\end{bmatrix}, \quad w = \begin{bmatrix}1\\0\end{bmatrix}$$

From the diagram,

$$v' = \begin{bmatrix}0\\-1\end{bmatrix}, \quad w' = \begin{bmatrix}-1\\0\end{bmatrix}$$

Or, Rotate 180°

$$\begin{bmatrix} x_w' & x_v' \\ y_w' & y_v' \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos 180° & -\sin 180° & 0 \\ \sin 180° & \cos 180° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cc} w & v \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\therefore w' = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad v' = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Now, $e = \begin{bmatrix} -8 \\ 0 \end{bmatrix}$ $\therefore x_e = -8,\ y_e = 0$

$\boxed{a}$ Canonical-to-basis matrix,

M_{cam} = Basis * Eye

$$= \begin{bmatrix} x_w' & y_w' & 0 \\ x_v' & y_v' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix}$$

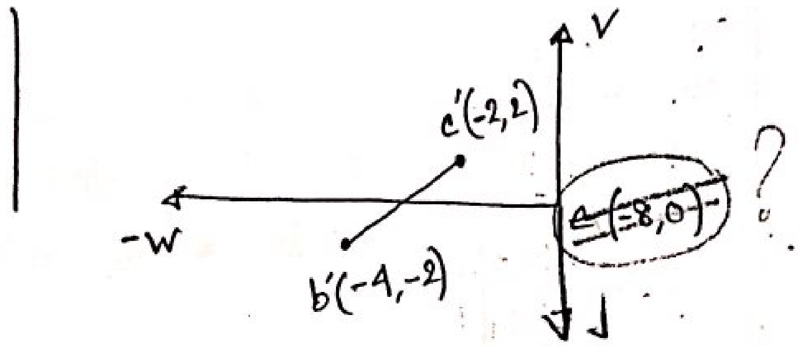
$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scanned with CamScanner

$\boxed{b}$ Now, $P_{xyz} = \begin{bmatrix} -4 & -6 \\ 2 & -2 \\ 1 & 1 \end{bmatrix}$

$\therefore P_{uvw} = M_{cam} * P_{xyz}$

$$= \begin{bmatrix} -1 & 0 & -8 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -6 \\ 2 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -2 & 2 \\ 1 & 1 \end{bmatrix}$$



$\boxed{c}$ The distance between e and b'

$$= \sqrt{(-8+4)^2 + (0+2)^2} = 4.47$$

The distance between e and c'

$$= \sqrt{(-8+2)^2 + (0-2)^2} = 6.32$$

So, b' is closer to the viewer.

?