Chapter : 01 [0 marks]

CG areas : metaphor
- → modeling (co-ordinate নিয়ে mathematical কাজ)
- → rendering (color করা + shadow create করা)
- → animation (image এ motion add করা)

Graphics API : OpenGL, WebGL, Direct 3D

A graphics API is a set of functions that perform basic operations such as drawing images and 3D surfaces into windows on 2D screen.

gL.triangle লিখলে points connect করে automatically triangle create করে

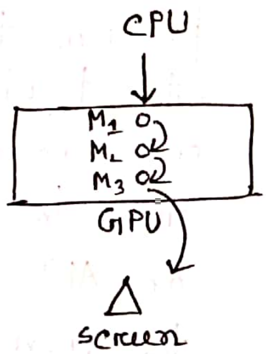
API : An API (Application Programming Interface) is a set of functions that allows applications to access data and interact with external software components, operating systems or microservices. An API is a software intermediary that allows two applications to talk to each other. In other words, an API is the messenger that delivers your request to the provider that you are requesting it from and then deliverse the response back to you. Example : কোন website বানালে login signup এর ক্ষেত্রে verification এর কাজটা করা tough। Google এ already থাকা API use করলে সেই verification এর কাজটা easy হয়ে যায়। যেমন : Maps।

CPU (Central Processing Unit) টা graphics এর কাজ বাদে সব কাজ করে যেমন ALU related works। GPU handles all the tasks on screen। We can't talk directly with GPU (Graphics Processing Unit) because there's a complicated machine related codes (low level languages that need to be handled) which is tough. That's why there's a layer on top of GPU which is API. This API don't have any implementation। কোন কাজ দরকার হলে API কে বলা লাগে। API সেটা GPU কে inform করে। Then GPU কাজটা করে। API acts like invoker। CPU আর GPU এর connection build করে API।

Direct 3D শুধুমাত্র windows এর জন্য, not a cross platform। But openGL can be used in windows operating system, IOS, embedded systems। Graphics cards: AMD, Nvidia

2 types of graphics program
- Graphics card এর সাথে যোগাযোগ করে: Graphics API
- User input এর সাথে যোগাযোগ করে: User Interface API

কোন ১টা triangle যদি screen এ show করাতে চাই তখন CPU থেকে GPU তে instruction যায়। GPU এর ভেতর module থাকে অনেক যেগুলো complete করে then screen এ triangle দেখি। module গুলোকে একসাথে pipeline বলে। Module গুলোর ১টা হতে পারে vertex shader or fragment shader (color) (coordinate নিয়ে কাজ করে)



Graphics pipeline এ ২টা state:

→ Programmer এর code gL এ transfer হলে সেটা programmable। যেমন: vertex shader, fragment shader.

→ built in যেটা মানে code করে change করা যাবেনা সেটা non-programmable। যেমন: triangle assembly (এমনভাবে points connect করে যেন overlap না করে), Rasterization (pixel select করে)

Graphics Pipeline is collection of stages through which our commands passes through and ultimately shows something on the screen। Graphics Pipeline is a special software or hardware subsystem that maps the 3D vertex locations to 2D screen.

Graphics object create করতে যে software use করি: Blender, Maya

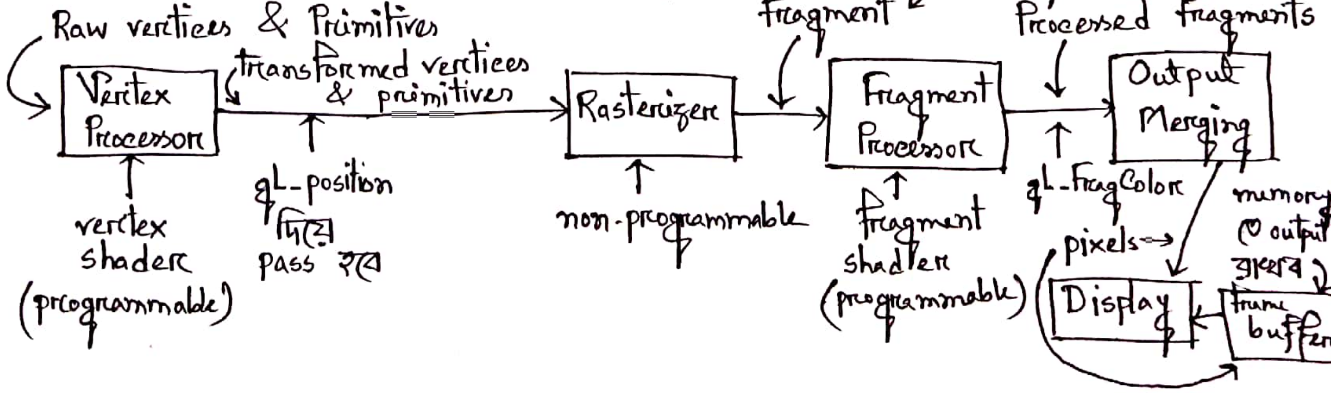
Modeling is nothing but a data structure which means organizing all the data as a triangle. This is the main target of graphics pipeline.

২টা point দিয়ে surface বানানো যায়না। Minimum ৩টা point লাগে ১টা surface বানানোর জন্য। ৪টা point দিয়ে area create করতে গেলে overlap হতে পারে, optimized surface না-ও হতে পারে। অর্থাৎ triangle এর surface টা unit surface.

ques: why triangle? ans: It's the simplest universal surface element. It's the convex hull of three points. A line or a point are even simpler but don't create surfaces. It isn't possible to use only a finite of them without having cracks.

Mesh is the data structure of computer graphics. A polygon mesh is a collection of vertices, edges and faces that defines the shape of a polyhedral object. Example: quad mesh, triangle mesh. অনেকগুলো triangle এর সমষ্টিকে triangle mesh বলে।

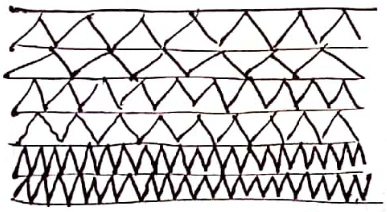
Graphics Pipeline:



LoD (Level of Detail) : FPS (Frame per Second)

60 FPS means 1 second এ 60টা frame চোখের সামনে দিয়ে যাবে এবং transition টা আমরা দেখতে পারবনা।

১টা dolphin screen এ show করতে চাইলে সেটার 1 million triangle GPU process করতে দেরি করে ফলে speed কমবে rendering এর। আবার triangle কমালে image quality drop করবে। আগের দিনের game এ human/ball/elements clear না কারন কম triangle use করতো। but now it's smooth because of trade off



কাছের element গুলোতে triangle বেশি রাখা হয়েছে যেন front এর elements গুলো more detailed এ বুঝা যায়। দূরের গুলোতে কম triangle use হয়েছে। এটাই LoD। Example : Google Map

Chapter : 06 [19 marks]

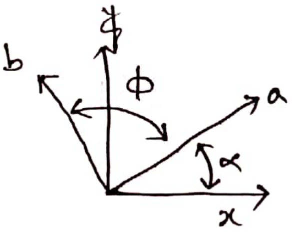
Linear Transformation : Operation of taking a vector and produces another vector by a simple matrix multiplication.

Scaling : $\text{scale}(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

Shearing : $\text{shear-}x(1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$; $\text{shear-}y(1) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Rotation :

$$x_a = r\cos\alpha$$
$$y_a = r\sin\alpha$$
$$x_b = r\cos(\alpha+\phi)$$
$$= r\cos\alpha \cdot \cos\phi - r\sin\alpha \cdot \sin\phi$$
$$y_b = r\sin(\alpha+\phi)$$
$$= r\sin\alpha \cdot \cos\phi + r\cos\alpha \cdot \sin\phi$$



$$x_b = x_a\cos\phi - y_a\sin\phi$$
$$y_b = y_a\cos\phi + x_a\sin\phi$$

$$\therefore \text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \qquad \therefore \begin{bmatrix} x_b \\ y_b \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_a \\ y_a \end{bmatrix}$$

clockwise rotate করলে angle = (-ve) [↻]

anti-clockwise rotate করলে angle = (+ve) [↺]

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \xrightarrow[\text{negative করলে or transpose করলে}]{} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

positive rotation / anti-clockwise rotation

negative rotation / anti-clockwise rotation

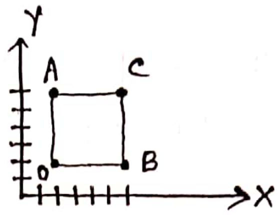
So, $R(\phi)$ কে negative করলে : $R(-\phi) = R^T(\phi)$

Reflection : $\text{reflect-}x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; $\text{reflect-}y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

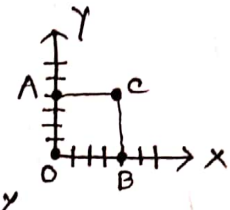
Spring : 2019 :

3) b) Initial position :

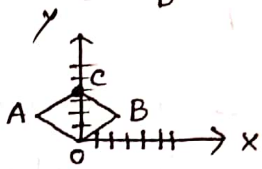
O(2,2), A (2,6),
C(6,6), B (6,2)



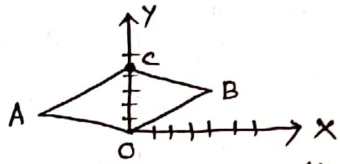
Step : 01 : translating by (-2,-2)



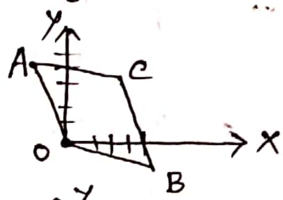
Step : 02 : rotating by (+45°)



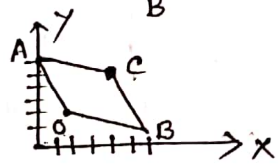
Step : 03 : scaling by (1.5, 1)



Step : 04 : rotating by (-45°)



Step : 05 : translating by (2,2)



Translate(-2,-2) → Rotate(+45°) → Scale(1.5,1) → Rotate(-45°) → Translate(2,2)

$\therefore$ Matrix, $M = T(2,2)\,R(-45°)\,S(1.5,1)\,R(+45°)\,T(-2,-2)$

$= T_1 R^T S R T_2$

Here,

$M = T_1 R^T S R T_2$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(45°) & \sin(45°) & 0 \\ -\sin(45°) & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.707 & 0 \\ 0.7071 & 0.7071 & -2.828 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.0606 & -1.06 & 0 \\ 0.7071 & 0.7071 & -2.828 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.25 & -0.25 & -2 \\ -0.25 & 1.25 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.25 & -0.25 & 0 \\ -0.25 & 1.25 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

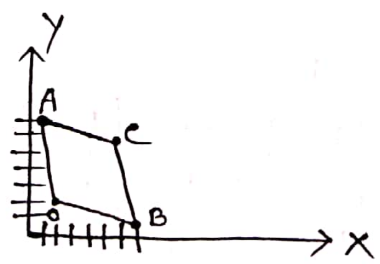
$\therefore$ Final vertices, $V' = M \times V$

$$= \begin{bmatrix} 1.25 & -0.25 & 0 \\ -0.25 & 1.25 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 6 & 6 \\ 2 & 6 & 6 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 6 & 7 \\ 2 & 7 & 6 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ New vertices : O (2,2), A (1,7), C (6,6), B (7,1)

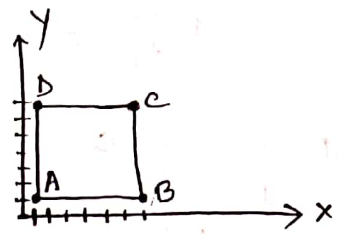
Final position after composite transformations:



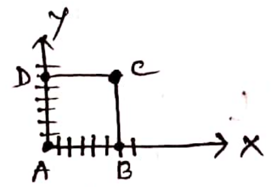
Spring : 2019 :

3) a) Initial position :

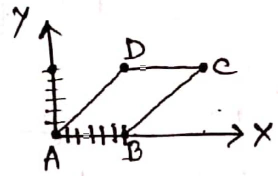
A (1,1) , B (7,1),
C (7,7) , D (1,7)



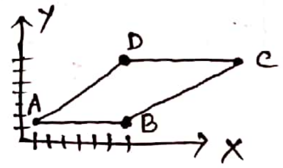
step : 01 : translating by (-1,-1)



step : 02 : shearing x axis by 1



step : 03 : translating by (1,1)



Translate $(-1,-1) \rightarrow$ Shear-x $(1) \longrightarrow$ Translate $(1,1)$

$\therefore$ Matrix, $M = T(1,1)\,Sh\text{-}x(1)\,T(-1,-1)$

$= T_1 \, Sh\text{-}x \, T_2$

Here, $M = T_1 \, Sh\text{-}x \, T_2$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

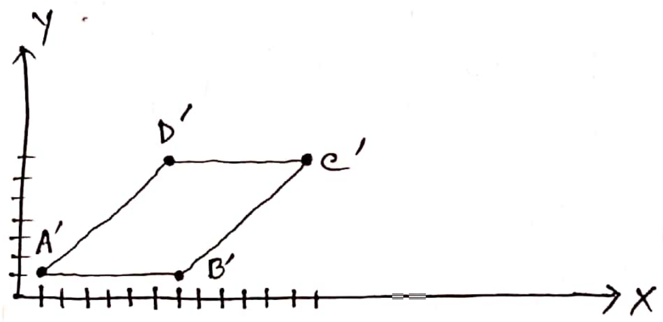
$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Final vertices, $V' = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 7 & 7 & 1 \\ 1 & 1 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 7 & 13 & 7 \\ 1 & 1 & 7 & 7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ New vertices: $A'(1,1)$, $B'(7,1)$, $C'(13,7)$, $D'(7,7)$



Spring : 2020:

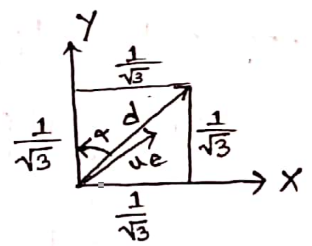
1) a) <u>step : 01</u>: translate by $(-1,-1,-1)$

$$\therefore T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>step : 02</u>: $u_e = \dfrac{B-A}{|B-A|} = \dfrac{2,2,2}{\sqrt{2^2+2^2+2^2}} = \dfrac{2,2,2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}$

$$\therefore d = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\sqrt{6}}{3}$$

$$\cos\alpha = \frac{\frac{1}{\sqrt{3}}}{d} = \frac{1/\sqrt{3}}{\sqrt{6}/3} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$



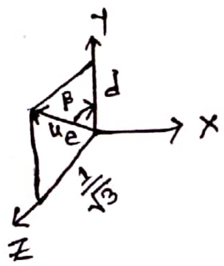
$\therefore \alpha = 45^\circ$

$$\therefore R_z(\alpha) = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{[along z-axis]}$$

<u>step : 03</u>:

$$\cos\beta = \frac{d}{u_e} = \frac{\sqrt{6}/3}{1} = \sqrt{6}/3$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{\sqrt{6}}{3}\right) = 35.26438968$$



$$\therefore R_x(-\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 35.26^\circ & \sin 35.26^\circ & 0 \\ 0 & -\sin 35.26^\circ & \cos 35.26^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{[along x-axis]}$$

step:04:

$$R_y(\theta) = \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [\text{along } y\text{-axis}]$$

step:05:

$$R_x(-\beta)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 35.26^\circ & -\sin 35.26^\circ & 0 \\ 0 & \sin 35.26^\circ & \cos 35.26^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

step:06:

$$R_z(\alpha)^{-1} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

step:07: translating by $(1,1,1)$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore M = T^{-1} R_z(\alpha)^{-1} R_x(-\beta)^{-1} R_y(\theta) R_x(-\beta) R_z(\alpha)\, T$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 0 \\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.82 & -0.58 & 0 \\ 0 & 0.58 & 0.82 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.82 & 0.58 & 0 \\ 0 & -0.58 & 0.82 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.33 & -0.24 & 0.91 & 0 \\ 0.91 & 0.33 & -0.24 & 0 \\ -0.24 & 0.91 & 0.33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Rotated Point, $P' = MP$

$$= \begin{bmatrix} 0.33 & -0.24 & 0.91 & 0 \\ 0.91 & 0.33 & -0.24 & 0 \\ -0.24 & 0.91 & 0.33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.8 \\ 1.5 \\ 2.6 \\ 1 \end{bmatrix}$$

$\therefore P' = (3.8, 1.5, 2.6)$ (ans:)

<u>chapter : 08</u> [23 marks]

1. Draw line from (1,1) to (-4,-7)

<u>Solution</u>: start point $(x_0, y_0) = (1,1)$

end point $(x_n, y_n) = (-4,-7)$

$$\therefore m = \frac{y_n - y_0}{x_n - x_0} = \frac{-7-1}{-4-1} = \frac{-8}{-5} = 1.6$$

As $y_n < y_0$ and $1 < m < \alpha$, it's in 6^{th} octant

<u>step :01</u>: $(x_0, y_0) = (1,1)$

After negating,

$(x_0, y_0) = (-1,-1)$; $(x_n, y_n) = (4,7)$

After swapping,

$(x_0, y_0) = (-1,-1)$; $(x_n, y_n) = (7,4)$

$\therefore dy = y_n - y_0 = 4-(-1) = 4+1 = 5$

$\therefore d_x = x_n - x_0 = 7-(-1) = 7+1 = 8$

$\therefore d = 2dy - dx = 2\times5-8 = 10-8 = 2$

$\therefore E = 2dy = 2\times5 = 10$

$\therefore NE = 2(dy-dx) = 2(5-8) = 2\times(-3) = -6$

<u>step : 02</u> : $d>0$, we will move to NE

$\therefore x_1 = -1+1 = 0$

$\therefore y_1 = -1+1 = 0$

$\therefore x_1, y_1 = (0,0)$

$\therefore x_1, y_1 = (-0,-0)$ [negating]

$\therefore x_1, y_1 = (-0,-0)$ [swapping]

$\therefore d = d + NE = 2+(-6) = 2-6 = -4$

<u>step : 03</u> : $d<0$, we will move to E

$\therefore x_2 = 0+1 = 1$

$\therefore y_2 = 0$

$\therefore (x_2, y_2) = (1, 0)$

$\therefore (x_2, y_2) = (-1, -0)$ [negating]

$\therefore (x_2, y_2) = (-0, -1)$ [swapping]

$\therefore d = d + E = -4 + 10 = 6$

<u>step : 04</u> : $d>0$, we will move to NE

$\therefore x_3 = 1+1 = 2$

$\therefore y_3 = 0+1 = 1$

$\therefore (x_3, y_3) = (2, 1)$

$\therefore (x_3, y_3) = (-2, -1)$ [negating]

$\therefore (x_3, y_3) = (-1, -2)$ [swapping]

$\therefore d = d + NE = 6 + (-6) = 6 - 6 = 0$

<u>step : 05</u> : $d = 0$, we will move to E

$\therefore x_4 = 2+1 = 3$

$\therefore y_4 = 1$

$\therefore (x_4, y_4) = (3, 1)$

$\therefore (x_4, y_4) = (-3, -1)$ [negating]

$\therefore (x_4, y_4) = (-1, -3)$ [swapping]

$\therefore d = d + E = 0 + 10 = 10$

<u>step : 06</u> : $d>0$, we will move to NE

$\therefore x_5 = 3 + 01 = 4$

$\therefore y_5 = 1+1 = 2$

$\therefore (x_5, y_5) = (4, 2)$

$\therefore (x_5, y_5) = (-4, -2)$ [negating]

$\therefore (x_5, y_5) = (-2, -4)$ [swapping]

$\therefore d = d + NE = 10 + (-6) = 10 - 6 = 4$

<u>step : 07</u> : $d > 0$, we will move to NE

$\therefore x_6 = 4 + 1 = 5$

$\therefore y_6 = 2 + 1 = 3$

$\therefore (x_6 , y_6) = (5, 3)$

$\therefore (x_6 , y_6) = (-5, -3)$ [negating]

$\therefore (x_6 , y_6) = (-3, -5)$ [swapping]

$\therefore d = d + NE = 4 + (-6) = 4 - 6 = -2$

<u>step : 08</u> : $d < 0$, we will move to E

$\therefore x_7 = 5 + 1 = 6$

$\therefore y_7 = 3$

$\therefore (x_7 , y_7) = (6, 3)$

$\therefore (x_7 , y_7) = (-6, -3)$ [negating]

$\therefore (x_7 , y_7) = (-3, -6)$ [swapping]

$\therefore d = d + E = -2 + 10 = 8$

<u>step : 09</u> : $d > 0$, we will move to NE

$\therefore x_8 = 6 + 1 = 7$

$\therefore y_8 = 3 + 1 = 4$

$\therefore (x_8 , y_8) = (7, 4)$

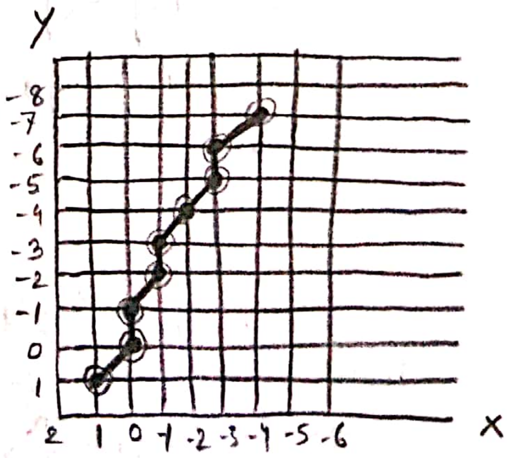
$\therefore (x_8 , y_8) = (-7, -4)$ [negating]

$\therefore (x_8 , y_8) = (-4, -7)$ [swapping]

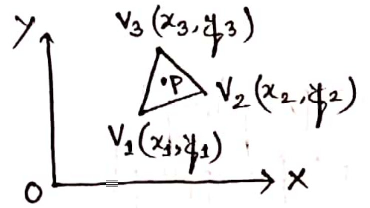
$\therefore d = d + NE = 8 + (-6) = 8 - 6 = 2$

<u>Summary</u> :

Moves	x	y	d
initial	1	1	2
NE	-0	-0	-4
E	-0	-1	6
NE	-1	-2	0
E	-1	-3	10
NE	-2	-4	4
NE	-3	-5	-2
E	-3	-6	8
NE	-4	-7	2



2. Discuss the mechanism to determine point P's barycentric coordinates (α, β, γ) based on the triangle $V_1V_2V_3$ in the following scenerio.

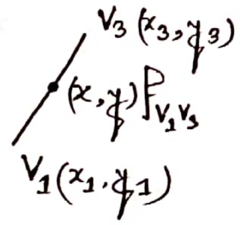


<u>Solution</u> : In a barycentric co-ordinate system, location of a point is specified by reference to a triangle for points in a plane. Here,

$$P(\alpha, \beta, \gamma) = V_1 + \beta(V_2 - V_1) + \gamma(V_3 - V_1)$$
$$= V_1 + \beta V_2 - \beta V_1 + \gamma V_3 - \gamma V_1$$
$$= V_1(1 - \beta - \gamma) + V_2\beta + V_3\gamma$$
$$= \alpha V_1 + \beta V_2 + \gamma V_3 \quad [1 - \beta - \gamma = \alpha]$$

$\therefore \alpha + \beta + \gamma = 1$ where $0 < \alpha < 1,\ 0 < \beta < 1,\ 0 < \gamma < 1$

Suppose, a point (x, y) lies on V_1V_3 line.



Now, $y = mx + b$ [line equation]

$\Rightarrow y - mx - b = 0$

$\Rightarrow y - \dfrac{y_3 - y_1}{x_3 - x_1}x - b = 0$ [m = slope]

$\Rightarrow (x_3 - x_1)y - (y_3 - y_1)x - (x_3 - x_1)b = 0$

$\Rightarrow (x_3 - x_1)y - (y_3 - y_1)x - c = 0$ [b is a constant from beginning and x_1, x_3 is known. So, the whole part can be considered as a constant c]

$\Rightarrow (y_1 - y_3)x + (x_3 - x_1)y - c = 0$ ... ...①

From equation ①,

$$c = (y_1 - y_3)x_1 + (x_3 - x_1)y_1$$

$$\Rightarrow c = x_1y_1 - x_1y_3 + x_3y_1 - x_1y_1$$

$$\therefore c = x_3y_1 - x_1y_3$$

Putting the value of c in equation ①,

$$f_{V_1V_3}(x,y) = (y_1-y_3)x + (x_3-x_1)y - x_3y_1 + x_1y_3 = 0$$

$$\therefore f_{V_1V_3}(x_2,y_2) = (y_1-y_3)x_2 + (x_3-x_1)y_2 - x_3y_1 + x_1y_3 = 0$$

$$\therefore \beta = \frac{f_{V_1V_3}(x,y)}{f_{V_1V_3}(x_2,y_2)}$$

if $\beta = 1$ then the point (x,y) is the point $V_2(x_2,y_2)$
if $\beta = 0$ then the point (x,y) lies on the line V_1V_3
if $\beta > 0$ or $\beta < 1$ then the point (x,y) is inside the triangle
if $\beta < 0$ or $\beta > 1$ then the point (x,y) is outside the triangle

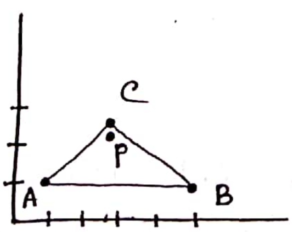
Similarly $\gamma = \dfrac{f_{V_1V_2}(x,y)}{f_{V_1V_2}(x_3,y_3)} = \dfrac{(y_1-y_2)x + (x_2-x_1)y - x_2y_1 + x_1y_2}{(y_1-y_2)x_3 + (x_2-x_1)y_3 - x_2y_1 + x_1y_2}$

$$\therefore \alpha = 1 - \beta - \gamma$$

$$\therefore P(x,y) \longrightarrow P(\alpha, \beta, \gamma)$$

3. Determine the barycentric coordinates of a 2D point $P(2.5, 2.25)$ with respect to a 2D triangle with vertices $A(1,1)$, $B(5,1)$, $C(2.5, 2.5)$.

Solution:



$(x_1,y_1) = (1,1)$

$(x_2,y_2) = (5,1)$

$(x_3,y_3) = (2.5, 2.5)$

$(x,y) = (2.5, 2.25)$

Here,

$$\beta = \frac{f_{AC}(2.5, 2.25)}{f_{AC}(5, 1)}$$

$$= \frac{(y_1 - y_3)x + (x_3 - x_1)y - x_3 y_1 + x_1 y_3}{(y_1 - y_3)x_2 + (x_3 - x_1)y_2 - x_3 y_1 + x_1 y_3}$$

$$= \frac{(1-2.5)2.5 + (2.5-1)2.25 - (2.5 \times 1) + (1 \times 2.5)}{(1-2.5)5 + (2.5-1)1 - (2.5 \times 1) + (1 \times 2.5)}$$

$$= \frac{(-1.5) \times 2.5 + (1.5 \times 2.25) - 2.5 + 2.5}{(-1.5) \times 5 + (1.5 \times 1) - 2.5 + 2.5}$$

$$= \frac{-3.75 + 3.375}{-7.5 + 1.5}$$

$$= \frac{-0.375}{-6}$$

$$= 0.0625$$

$$\gamma = \frac{f_{AB}(x, y)}{f_{AB}(x_3, y_3)}$$

$$= \frac{(y_1 - y_2)x + (x_2 - x_1)y - x_2 y_1 + y_2 x_1}{(y_1 - y_2)x_3 + (x_2 - x_1)y_3 - x_2 y_1 + x_1 y_2}$$

$$= \frac{(1-1)2.5 + (5-1)2.25 - (5 \times 1) + (1 \times 1)}{(1-1)2.5 + (5-1)2.5 - (5 \times 1) + (1 \times 1)}$$

$$= \frac{0 + (4 \times 2.25) - 5 + 1}{0 + (4 \times 2.5) - 5 + 1}$$

$$= \frac{9-4}{10-4}$$

$$= \frac{5}{6}$$

$$\approx 0.833$$

$$\alpha = 1 - \beta - \gamma = 1 - 0.0625 - 0.833 = 0.1041666667 \approx 0.1042$$

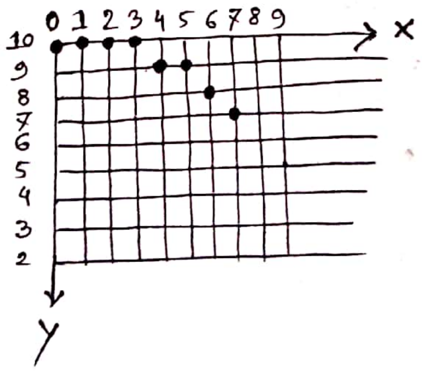
$$\therefore P(2.5, 2.25) \longrightarrow P(0.1042, 0.0625, 0.833)$$

(ans:)

4. Draw circle for radius 10

Solution:

x	y	2x	2y	h	Moves
0	10	2X0 = 0	2X10 = 20	1-R = 1-10 = -9	h<0, E
0+1 = 1	10	2X1 = 2	2X10 = 20	h+2x+3 = -9+0+3 = -6	h<0, E
1+1 = 2	10	2X2 = 4	2X10 = 20	h+2x+3 = -6+2+3 = -1	h<0, E
2+1 = 3	10	2X3 = 6	2X10 = 20	h+2x+3 = -1+4+3 = 6	h>0, SE
3+1 = 4	10-1 = 9	2X4 = 8	2X9 = 18	h+2x-2y+5 = 6+6-20+5 = -3	h<0, E
4+1 = 5	9	2X5 = 10	2X9 = 18	h+2x+3 = -3+8+3 = 8	h>0, SE
5+1 = 6	9-1 = 8	2X6 = 12	2X8 = 16	h+2x-2y+5 = 8+10-18+5 = 5	h>0, SE
6+1 = 7	8-1 = 7	2X7 = 14	2X7 = 14	h+2x-2y+5 = 5+12-16+5 = 6	h>0, SE



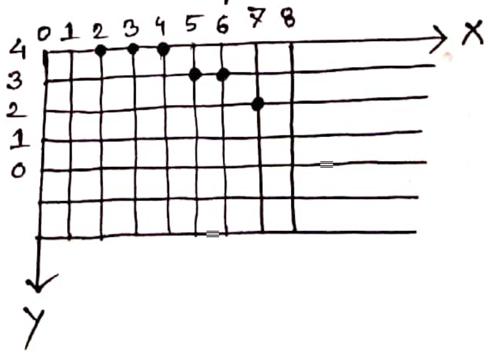
Practice Problem: center $(2,-3)$ and radius 7

x	y	$2x$	$2y$	h	Moves
0	7	0×2 $=0$	7×2 $=14$	$1-R$ $=1-7$ $=-6$	$h<0$, E
$0+1$ $=1$	7	2×1 $=2$	2×7 $=14$	$h+2x+3$ $=-6+0+3$ $=-3$	$h<0$, E
$1+1$ $=2$	7	2×2 $=4$	2×7 $=14$	$h+2x+3$ $=-3+2+3$ $=2$	$h>0$, SE
$2+1$ $=3$	$7-1$ $=6$	2×3 $=6$	2×6 $=12$	$h+2x-2y+5$ $=2+4-14+5$ $=-3$	$h<0$, E
$3+1$ $=4$	6	2×4 $=8$	2×6 $=12$	$h+2x+3$ $=-3+6+3$ $=6$	$h>0$, SE
$4+1$ $=5$	$6-1$ $=5$	2×5 $=10$	2×5 $=10$	$h+2x-2y+5$ $=6+8-12+5$ $=7$	$h>0$, SE

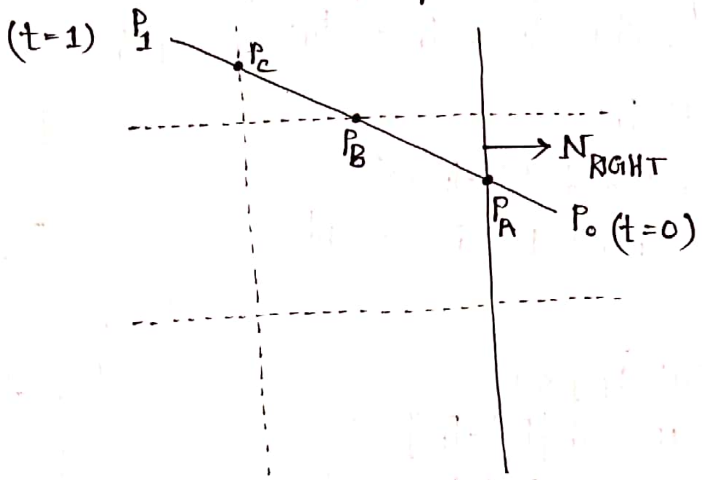
Summary:

Moves	x	y	h	$x+x_c$	$y+y_c$
initial	0	7	-6	2	4
E	1	7	-3	3	4
E	2	7	2	4	4
SE	3	6	-3	5	3
E	4	6	6	6	3
SE	5	5	7	7	2

Here, $x_c = 2$, $y_c = -3$



5. Consider the line P_0P_1 with a clipping rectangle (in figure) where N_{RIGHT} is normal to the right clipping edge. Answer the questions from (i) to (iii) based on the Cyrus-Beck Line clipping algorithm.



i) Derive the formula to determine parameter t for finding P_0P_1's intersection point with the right edge P_A.

ii) How to decide the line P_0P_1 is parallel to right edge or not?

iii) How to determine potentially entering and potentially leaving interaction points? How can we select the true intersection points?

Solution:

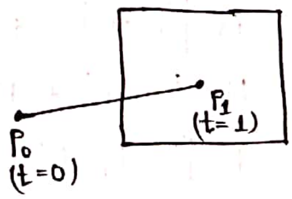
i) $N_{RIGHT} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ = outward normal to the right edge.

Parametric equation of a line : $P(t) = P_0 + t\,(P_1 - P_0)$ ... ...①

where,

$p(t)$ = any point on line

t = parameter



if $t=0$, $P(t) = P_0$

if $t=1$, $P(t) = P_1$

which means the value of t is in between 0 to 1, so that it traverse the whole line P_0P_1.

equation ① can be written as:

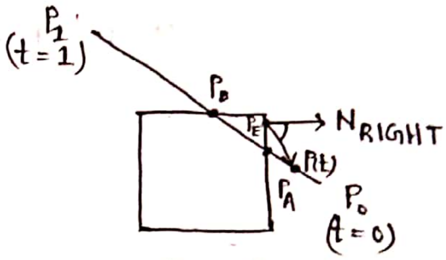
$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \end{bmatrix}$$

$$\therefore x(t) = x_0 + t(x_1 - x_0)$$

$$\therefore y(t) = y_0 + t(y_1 - y_0)$$

Suppose, P_E = any point on the right edge

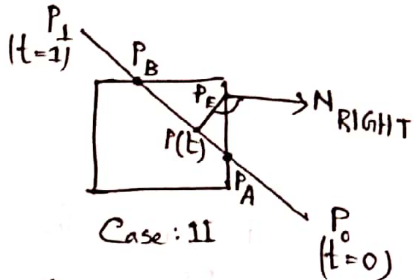
$P(t) - P_E$ = vector from P_E to $P(t)$



Case : I

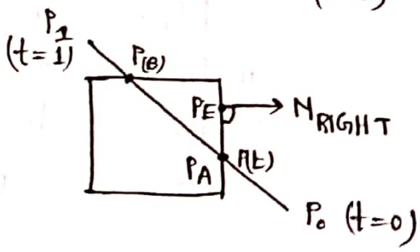
<u>Case : I:</u> $N.[P(t) - P_E] > 0$

$\therefore$ angle between N and $[P(t) - P_E] < 90°$ which means the point $P(t)$ is outside the rectangle.



<u>Case : II:</u> $N.[P(t) - P_E] < 0$

$\therefore$ angle between N and $[P(t) - P_E] > 90°$ which means the point $P(t)$ is inside the rectangle.



<u>Case : III:</u> $N.[P(t) - P_E] = 0$

$\therefore$ angle between N and $[P(t) - P_E] = 90°$ which means $P(t)$ point is the intersection point of edge and line.

From the three cases we observed that for intersection,

$$N.[P(t) - P_E] = 0$$

$$\Rightarrow N.[P_0 + t(P_1 - P_0) - P_E] = 0 \quad [\text{From equation } ①]$$

$$\Rightarrow N.[(P_0 - P_E) + t(P_1 - P_0)] = 0$$

$$\Rightarrow N.(P_0 - P_E) + N.[t(P_1 - P_0)] = 0$$

$$\Rightarrow N.[t(P_1 - P_0)] = -N.[P_0 - P_E]$$

$$\Rightarrow t(P_1 - P_0) = \frac{-N.[P_0 - P_E]}{N}$$

$$\Rightarrow t = \frac{N.[P_0 - P_E]}{-N.[P_1 - P_0]}$$

$$\therefore t = \frac{N.[P_0 - P_E]}{-N.D} \quad [\text{where } D = P_1 - P_0]$$

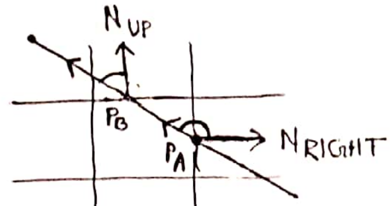
(ii) To determine the line P_0P_1 is parallel to right edge or not we have to calculate the dot product of N_{RIGHT} and $P_0P_1(D)$

$$N.D = 0$$
$$\Rightarrow ND\cos\theta = 0$$
$$\Rightarrow \cos\theta = 0$$
$$\Rightarrow \cos\theta = \cos 90^\circ$$
$$\therefore \theta = 90^\circ$$

which means N_{RIGHT} and P_0P_1 are in perpendicular

if $N_{RIGHT} \cdot D \neq 0$ then the right edge and the line P_0P_1 is parallel. For that, $N \neq 0$, $D \neq 0$ $[P_1 \neq P_0]$

iii) If $N.D < 0$ then the point is entering
If $N.D > 0$ then the point is leaving
which means, for entering the angle between line and edge is greater than 90° and for leaving the angle between line and edge is less than 90°.



P_A = entering
P_B = leaving

From all the entering points, the maximum entering point is the intersection point of line whereas the minimum leaving point from all the leaving points is the true intersection-point.

Theory

Bresenham's Midpoint Algorithm :

Given: Start Point (x_0, y_0)
End Point (x_1, y_1)

Initialization: $x = x_0$, $y = y_0$;

$dx = x_1 - x_0$; $dy = y_1 - y_0$;

$d = 2dy - dx$;

$\Delta E = 2dy$; $\Delta NE = 2(dy - dx)$;

PlotPoint (x, y);

Loop :

```
while (x <= x1)
    if d <= 0
        d = d + ΔE
    else
        y = y + 1
        d = d + ΔNE
    Endif
    x = x + 1
    PlotPoint (x, y);
end while
```

Fall-2020 :

1) b) In midpoint line drawing algorithm,

$dx = x_1 - x_0$

$dy = y_1 - y_0$

$\Delta NE = 2(dy - dx)$

$= dy - dx$ [2 can be denied as it's added to remove floating point]

$= (y_1 - y_0) - (x_1 - x_0)$

$= y_1 - y_0 - x_1 + x_0$

$= (y_1 - x_1) - (y_0 - x_0)$

during decision variable update,

$d = d + \Delta NE$ [if $d > 0$]

$\therefore d = d + \{(y_1 - x_1) - (y_0 - x_0)\}$

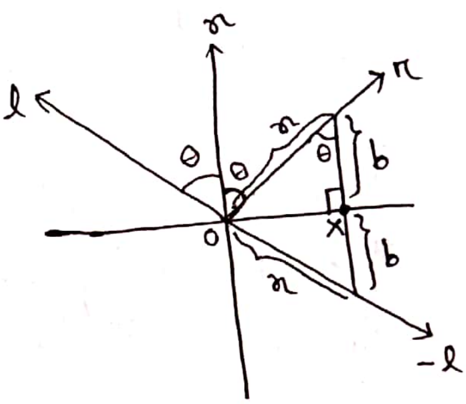
So, we can successively update decision variable by adding $(y_1 - x_1) - (y_0 - x_0)$ for each selection of northeast pixel.

[showed]

Midpoint Algorithm for circle:

```
voidMidpointCircle (int radius)
{
    int x=0;
    int y= radius;
    int d= 1-radius;
    CirclePoints (x,y);
    while (y>x)
    {
        if (d<0)
            d = d+ (2*x+3);
        else
        { d= d+ 2*(x-y) +5 ;
          y = y-1;
        }
    x=x+1;
    CirclePoints (x,y);
    }
}
```

Fall : 2020

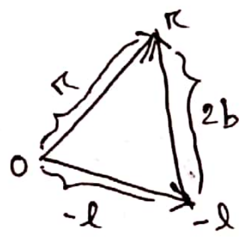


3) b) In ΔOrX,

$$\cos\theta = \frac{b}{r} \quad [\angle X = 90^\circ]$$

$$\Rightarrow b = r\cos\theta$$

$$\therefore b = n\cos\theta \quad [r = n = \text{unit vector}]$$



From $\Delta Or(-l)$,

$$r = -l + 2b$$

$$\Rightarrow r = -l + 2n\cos\theta \quad \ldots \ldots (1)$$

Here, $n \cdot l = |n||l| \cos\theta$

$$\Rightarrow n \cdot l = 1 \times \cos\theta \quad [n = l = \text{unit vector}]$$

$$\Rightarrow n \cdot l = \cos\theta$$

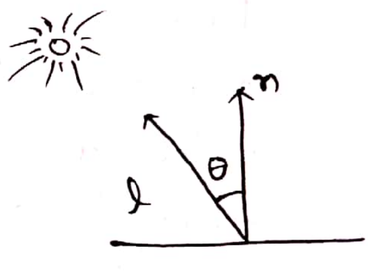
putting the value of $\cos\theta$ in equation (1),

$$r = -l + 2n\,(n \cdot l)$$

$$\therefore r = 2\,(l \cdot n)\,n - l \quad \text{[showed]}$$

Spring : 2020 :

4) b) From Lambertian Shading model,



$$c = c_r c_l \,(n \cdot l)$$

$$\Rightarrow \begin{bmatrix} R_c \\ G_c \\ B_c \end{bmatrix} = \begin{bmatrix} R_r \\ G_r \\ B_r \end{bmatrix} \begin{bmatrix} R_l \\ G_l \\ B_l \end{bmatrix} \times (n \cdot l)$$

Here, c_r and c_l value will be from $[0, 1]$.
But the value of dot product $(n \cdot l)$ can be negative.
which is a problem as c can't be negative.

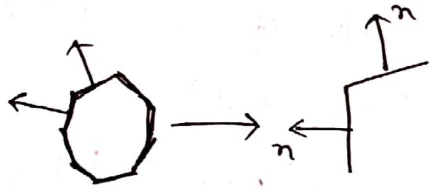
That's why max() function is introduced. If we write the equation as:

$$c = c_r c_l \max(0, n \cdot l)$$

Now, if the dot product gives negative value, it will be replaced by 0. As we are using $\max(0, n \cdot l)$, the value will be whether 0 or any positive value. That's how we will get proper value for c.

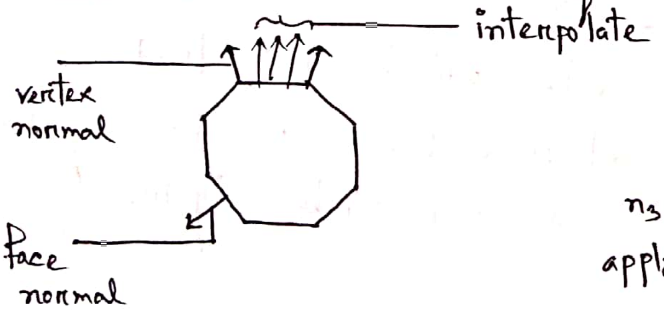
Spring : 2019:

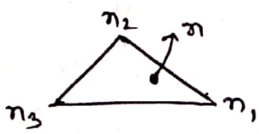
2) c) In an object, when the object don't look smooth or the triangles are visible, then it's called faceted appearance. It's because of the drastic changes of normals from surface to surface.



drastical change of normal

First of all we have to take all the face normal and find out the mean. The mean value will be vertex normal. Using barycentric coordinate, we can interpolate face normals. That's how sudden transitions won't be visible so easily.





applying barycentric coordinate

Beziere Curve [8 marks]

Fall-2019:

4) a) $V_0 = (-1,0)$; $V_1 = (-1, \frac{\sqrt{3}}{2})$; $V_2 = (1, \frac{\sqrt{3}}{2})$

$V_3 = (1,0)$; $V_4 = (1, -\frac{\sqrt{3}}{2})$; $V_5 = (-1, -\frac{\sqrt{3}}{2})$

We know,

$$Q(u) = \sum_{i=0}^{d} B_{i,d}(u) P_i \; [0 \le u \le 1]$$

$$B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$$

$$\binom{d}{i} = \frac{d!}{i!\,(d-i)!}$$

Here, $d = N-1 = 6-1 = 5$; $u = \frac{1}{2}$

$$\therefore Q_5\left(\tfrac{1}{2}\right) = B_{0,5}(u)P_0 + B_{1,5}(u)P_1 + B_{2,5}(u)P_2 + B_{3,5}(u)P_3 + B_{4,5}(u)P_4 + B_{5,5}(u)P_5$$

$$B_{0,5}(u) = \binom{5}{0} u^0 \cdot (1-u)^{5-0} = \frac{5!}{0!\,(5-0)!} \times 1 \times (1-\ )^5 = 1 \times 1 \times (1-u)^5 = (1-u)^5$$

$$B_{1,5}(u) = \binom{5}{1} u^1 \cdot (1-u)^{5-1} = \frac{5!}{1!(5-1)!} \times u \times (1-u)^4 = 5u(1-u)^4$$

$$B_{2,5}(u) = \binom{5}{2} u^2 (1-u)^{5-2} = \frac{5!}{2!(5-2)!} \times u^2 (1-u)^3 = 10u^2(1-u)^3$$

$$B_{3,5}(u) = \binom{5}{3} u^3 (1-u)^{5-3} = \frac{5!}{3!(5-3)!} \times u^3 \times (1-u)^2 = 10u^3(1-u)^2$$

$$B_{4,5}(u) = \binom{5}{4} u^4 (1-u)^{5-4} = \frac{5!}{4!(5-4)!} \times u^4 \times (1-u)^1 = 5u^4(1-u)$$

$$B_{5,5}(u) = \binom{5}{5} u^5 (1-u)^{5-5} = \frac{5!}{5!(5-5)!} \times u^5 \times (1-u)^0 = u^5$$

$$\therefore Q_5\left(\tfrac{1}{2}\right) = (1-u)^5 \begin{bmatrix} -1 \\ 0 \end{bmatrix} + 5u(1-u)^4 \begin{bmatrix} -1 \\ \sqrt{3}/2 \end{bmatrix} + 10u^2(1-u)^3 \begin{bmatrix} 1 \\ \sqrt{3}/2 \end{bmatrix} + 10u^3(1-u)^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 5u^4(1-u) \begin{bmatrix} 1 \\ -\sqrt{3}/2 \end{bmatrix} + u^5 \begin{bmatrix} -1 \\ -\sqrt{3}/2 \end{bmatrix}$$

$$= \frac{1}{32} \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \frac{5}{32} \begin{bmatrix} -1 \\ \sqrt{3}/2 \end{bmatrix} + \frac{5}{16} \begin{bmatrix} 1 \\ \frac{\sqrt{3}}{2} \end{bmatrix} + \frac{5}{16} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{5}{32} \begin{bmatrix} 1 \\ -\sqrt{3}/2 \end{bmatrix} + \frac{1}{32} \begin{bmatrix} -1 \\ -\sqrt{3}/2 \end{bmatrix}$$

[using $u = \frac{1}{2}$]

$$= \begin{bmatrix} -\frac{1}{32} \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{5}{32} \\ \frac{5\sqrt{3}}{64} \end{bmatrix} + \begin{bmatrix} \frac{5}{16} \\ \frac{5\sqrt{3}}{32} \end{bmatrix} + \begin{bmatrix} \frac{5}{16} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{5}{32} \\ -\frac{5\sqrt{3}}{64} \end{bmatrix} + \begin{bmatrix} -\frac{1}{32} \\ -\frac{\sqrt{3}}{64} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{32} - \frac{5}{32} + \frac{5}{16} + \frac{5}{16} + \frac{5}{32} - \frac{1}{32} \\ 0 + \frac{5\sqrt{3}}{64} + \frac{5\sqrt{3}}{32} + 0 - \frac{5\sqrt{3}}{64} - \frac{\sqrt{3}}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{16} \\ \frac{9\sqrt{3}}{64} \end{bmatrix}$$

$$\approx \begin{bmatrix} 0.563 \\ 0.244 \end{bmatrix}$$

$$\therefore Q_5\left(\frac{1}{2}\right) = (0.563, 0.244) \quad \text{(ans:)}$$