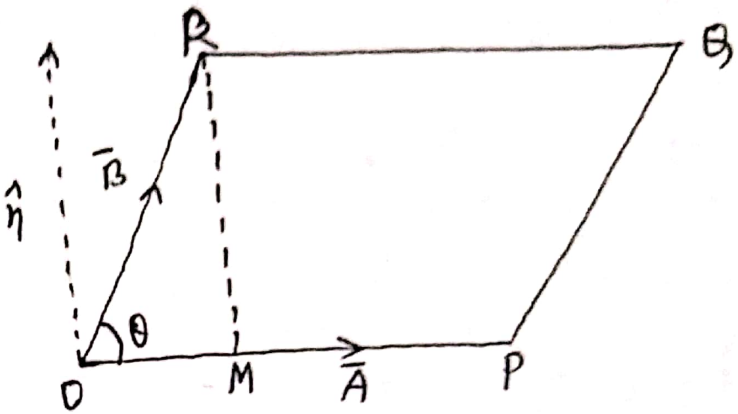
Geometrical Interpretation of $\bar{A}\times\bar{B}$:



$\bar{A}\times\bar{B}$ represents the vector area of the parallelogram whose adjacent sides are $\bar{A}$ and $\bar{B}$.

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$$\bar{A}\times\bar{B} = \vec{OP}\cdot OR\cdot \sin\theta\cdot\hat{n}$$

$$\sin\theta = \frac{RM}{OR}$$

$$= OP\cdot RM\cdot\hat{n}$$

$$= \text{Base}\cdot\text{height}\cdot\hat{n}$$

= vector area of parallelogram OPQR.

⊞ Vector product is not commutative $\bar{a}\times\bar{b} \neq \bar{b}\times\bar{a}$

<u>Proof:</u> $\bar{b}\times\bar{a} = |\bar{b}||\bar{a}|\sin(-\theta)\,\hat{n}$

$$= -|\bar{a}||\bar{b}|\sin\theta\cdot\hat{n}$$

$$= -\bar{a}\times\bar{b}$$

Formula : Work done = Force · Displacement

Ex. 2: Constant forces $\vec{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$ and $\vec{Q} = -\hat{i} + 2\hat{j} - \hat{k}$ act on particle. Determine the work done when the particle is displaced from A to B, the position vectors of A and B being $4\hat{i} - 3\hat{j} - 2\hat{k}$ and $6\hat{i} + \hat{j} - 3\hat{k}$ respectively.

Sol^n: Total force $= (2\hat{i} - 5\hat{j} + 6\hat{k}) + (-\hat{i} + 2\hat{j} - \hat{k})$

$$= \hat{i} - 3\hat{j} + 5\hat{k}$$

Displacement, $\vec{AB} = \bar{B} - \bar{A}$

$$= (6\hat{i} + \hat{j} - 3\hat{k}) - (4\hat{i} - 3\hat{j} - 2\hat{k})$$

$$= 2\hat{i} + 4\hat{j} - \hat{k}$$

$\therefore$ Work done = Force · Displacement

$$= (\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} - \hat{k})$$

$$= 1 \cdot 2 - 3 \cdot 4 - 5 \cdot 1$$

$$= 2 - 12 - 5 = -15$$

$$= 15. \text{ (Ans.)}$$

Triple Products

Scalar triple product:

Let $\bar{a}, \bar{b}, \bar{c}$ be three vectors then their dot product is written as $\bar{a}\cdot(\bar{b}\times\bar{c})$.

If $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

then
$$\bar{a}\cdot(\bar{b}\times\bar{c}) = \bar{a}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\cdot\left[\hat{i}(b_2c_3 - b_3c_2) - \hat{j}(b_1c_3 - b_3c_1) + \hat{k}(b_1c_2 - b_2c_1)\right]$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

* Similarly, $\bar{b}\cdot(\bar{c}\times\bar{a})$ and $\bar{c}\cdot(\bar{a}\times\bar{b})$ have the same value.

$\therefore\ \bar{a}\cdot(\bar{b}\times\bar{c}) = \bar{b}\cdot(\bar{c}\times\bar{a}) = \bar{c}\cdot(\bar{a}\times\bar{b})$

Note: 1. The value of the product changes if the order is non-cycle.

2. $\bar{a}\cdot(\bar{b}\cdot\bar{c})$, $\bar{a}\times(\bar{b}\cdot\bar{c})$ are meaningless.

Vector product of three vectors:

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors, then their vector product is written as $\vec{a} \times (\vec{b} \times \vec{c})$.

Let $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

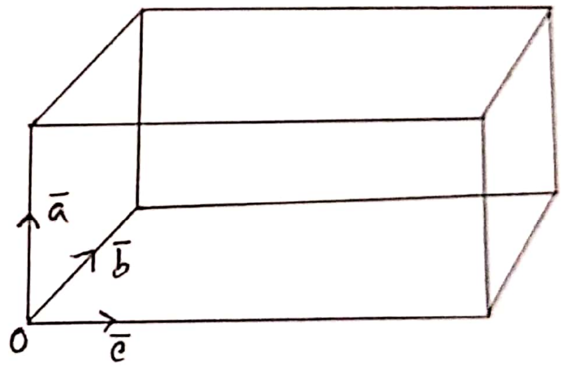
then $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$

Geometrical interpretation of $\bar{a}\cdot(\bar{b}\times\bar{c})$:

$\bar{a}\cdot(\bar{b}\times\bar{c})$ = Volume of the parallelopiped

Parallelopiped = A solid figure having six faces, all parallelograms; all opposite pairs of faces being similar and parallel.

In the figure, $\bar{a}$, $\bar{b}$ and $\bar{c}$ are the co-terminous edges or sides of the parallelopiped.



Note: If $\bar{a}\cdot(\bar{b}\times\bar{c}) = 0$, then $\bar{a}$, $\bar{b}$ and $\bar{c}$ are co-planer.

Ex: Find the volume of parallelopiped if $\bar{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\bar{b} = -3\hat{i} + 7\hat{j} - 3\hat{k}$ and $\bar{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are the three co-terminous edges of the parallelopiped.

Sol^n: Volume of the parallelopiped $= \bar{a}\cdot(\bar{b}\times\bar{c})$

Exercise: Determine λ and μ by using vectors, such that the points $(-1, 3, 2)$, $(-4, 2, -2)$ and $(5, \lambda, \mu)$ lie on a straight line.