CSE2209: Digital Electronics and Pulse Techniques

Course Conducted By:

Nowshin Nawar Arony

Lecturer, Dept of CSE, AUST

Prove that, for square wave (symmetric) input and T<< 2RC

i) $V_1 = \frac{V}{2}(1+\frac{T}{4RC})$

ii) $V_1{}' = \frac{V}{2}(1-\frac{T}{4RC})$

i) We know,

$$V_1 = \frac{V}{\left(1+ e^{-\frac{T}{2Rc}}\right)}$$

$$\therefore V_1 = \frac{V}{1+\left(1-\frac{T}{2Rc}+\frac{T^2}{2!\,(2Rc)^2}-\cdots\cdots\right)} \qquad \left[e^{-x} = 1 - x + \frac{x^2}{2!} - \cdots\ \cdots\right]$$

$$= \frac{V}{1+1-\frac{T}{2Rc}}$$ [Neglecting higher terms since T<<2RC]

$$= \frac{V}{2\left(1-\frac{T}{4Rc}\right)}$$

$$= \frac{V}{2}\left\{1+\frac{T}{4Rc}+\frac{T^2}{(4Rc)^2}+\cdots\ \cdots\right\} \qquad \left[(1-x)^{-1} = 1 + x + x^2 + \cdots\ \cdots\right]$$

$$\therefore V_1 = \frac{V}{2}\left(1+\frac{T}{4Rc}\right)$$ [Neglecting higher terms since T<<2RC]

[Proved]

ii) We know, $$V_1' = \frac{V}{\left(1 + e^{\frac{T}{2Rc}}\right)}$$

$$\therefore V_1' = \frac{V}{1+\left(1+\frac{T}{2Rc}+\frac{T^2}{2!\,(2Rc)^2}+\cdots\cdots\right)}$$ $$\left[e^x = 1 + x + \frac{x^2}{2!} + \cdots\ \cdots\right]$$

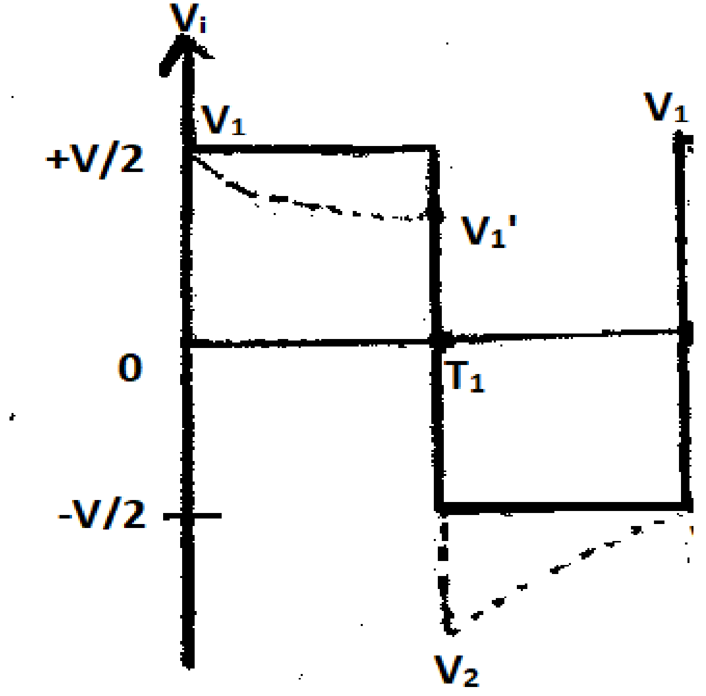
$$= \frac{V}{1+1+\frac{T}{2Rc}}$$ [Neglecting higher terms since T<<2RC]

$$= \frac{V}{2\left(1+\frac{T}{4Rc}\right)} \qquad = \frac{V}{2}\left(1 + \frac{T}{4Rc}\right)^{-1}$$

$$= \frac{V}{2}\left\{1 - \frac{T}{4Rc} + \frac{T^2}{(4Rc)^2} + \cdots\ \cdots\right\}$$ $$\left[(1 + x)^{-1} = 1 - x + x^2 - \cdots\ \cdots\right]$$

$$\therefore V_1' = \frac{V}{2}\left(1 - \frac{T}{4Rc}\right)$$ [Neglecting higher terms since T<<2RC] [Proved]

Percentage Tilt:



$$\%\text{tilt} = \frac{V_1 - V_1'}{\frac{V}{2}} \; x \; 100\%$$

$$= \frac{\frac{V}{2}\left(1+\frac{T}{4Rc}\right) - \frac{V}{2}\left(1-\frac{T}{4Rc}\right)}{\frac{V}{2}} \; x \; 100\%$$

$$= \left(1 + \frac{T}{4Rc} - 1 + \frac{T}{4Rc}\right) x \; 100\%$$

$$= \frac{T}{2Rc} \, x \; 100 \; \%$$

$$= \frac{T \cdot \pi}{2\pi Rc} \, x \; 100 \; \%$$

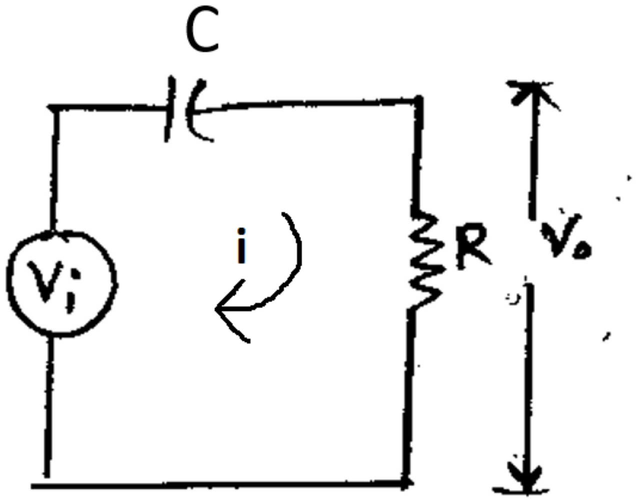
$$\therefore \%\text{tilt} = \mathrm{T}\pi f_1 x \; 100 \; \%$$

$$[\because \frac{1}{2\pi Rc} = f_1 \text{ (Linear frequency)}]$$

Prove that, for High Pass RC circuit amplification $|A| = 0.707$

$$Z_C = \frac{1}{jwc}$$

$$Z_C = -\frac{j}{wc} \qquad [\because \frac{1}{j} = -j]$$



$$i = \frac{V_i}{Z_C + R}$$

$$V_0 = iR = \frac{V_i}{Z_C + R} R$$

Here, j is imaginary coefficient, j = $\sqrt{-1}$

w = angular frequency = $2\pi f$

$$A = \frac{V_0}{V_i} \qquad = \frac{V_i \cdot \mathrm{R}}{Z_C + R} \cdot \frac{1}{V_i} \qquad [V_0 = \frac{V_i}{Z_C + R}\ \mathrm{R}]$$

$$= \frac{\mathrm{R}}{Z_C + R}$$

$$= \frac{1}{\frac{Z_C}{R} + 1}$$

$$= \frac{1}{\frac{-j}{wRc} + 1} \qquad = \frac{1}{1 - \frac{j}{wRc}}$$

$$A = \frac{1}{1-\frac{j}{wRc}}$$

$$= \frac{\left(1+\frac{j}{wRc}\right)}{\left(1-\frac{j}{wRc}\right)\left(1+\frac{j}{wRc}\right)} \quad = \frac{\left(1+\frac{j}{wRc}\right)}{1^2-\left(\frac{j}{wRc}\right)^2}$$

$$= \frac{\left(1+\frac{j}{wRc}\right)}{1+\frac{1}{w^2R^2c^2}}$$

$$\mathrm{A} = \frac{1}{1+\frac{1}{w^2R^2c^2}} + j\,\frac{\frac{1}{wRc}}{1+\frac{1}{w^2R^2c^2}}$$

$$\left(\begin{array}{l} \mathrm{r} = a + ib \\ \therefore |r| = \sqrt{a^2 + b^2} \end{array}\right)$$

$$\therefore |A| = \sqrt{\left(\frac{1}{1+\frac{1}{w^2R^2c^2}}\right)^2 + \left(\frac{\frac{1}{wRc}}{1+\frac{1}{w^2R^2c^2}}\right)^2} = \sqrt{\frac{1+\frac{1}{w^2R^2c^2}}{\left(1+\frac{1}{w^2R^2c^2}\right)^2}}$$

$$= \sqrt{\frac{1}{1+\frac{1}{w^2R^2c^2}}}$$

$$\frac{1}{wRc} = \frac{1}{2\pi f Rc} = \frac{1}{f} \cdot \frac{1}{2\pi Rc} = \frac{1}{f} \cdot f_1 \qquad [\because \frac{1}{2\pi Rc} = f_1]$$

$$= \frac{f_1}{f}$$

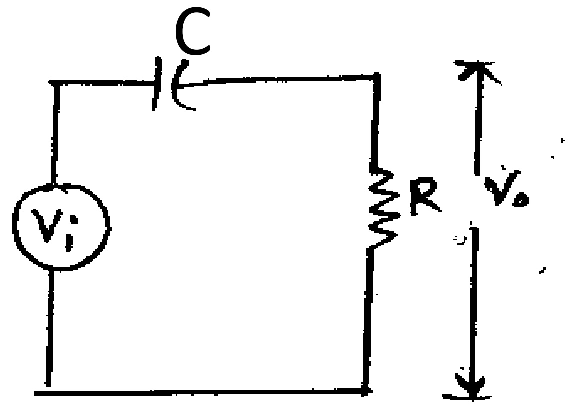
$$\therefore |A| = \sqrt{\frac{1}{1 + \left(\frac{f_1}{f}\right)^2}}$$

If $f_1 = f$, then $|A| = \sqrt{\frac{1}{1+1}} = \sqrt{\frac{1}{2}} = 0.707$

[Proved]

High Pass RC Circuit as Differentiator

When time constant (T) is too small i.e. R and C are very small in comparison with the time required for i/p signal to make an appreciable change, then the circuit acts like a differentiator. This name arises from the fact that under this circumstances, the voltage drop across R will be very small compared to the drop across C.



Hence, we may consider that the total i/p V_i appears across C

$$V_i(t) = V_{\dot{c}}(t) + V_R(t)$$

$V_i(t) \approx V_c(t)$ [∵ R is too small]

$$i\,(t) = \frac{V_R(t)}{R}$$

Again,

$i_c(t) = C\,\dfrac{d}{dt}V_c(t)$ [As, V_c = $\frac{Q}{C}$ and i = $\frac{Q}{t}$]

$$\frac{V_R(t)}{R} = C\,\frac{d}{dt}V_c(t)$$

$$V_R(t) = RC\frac{d}{dt}V_c(t)$$

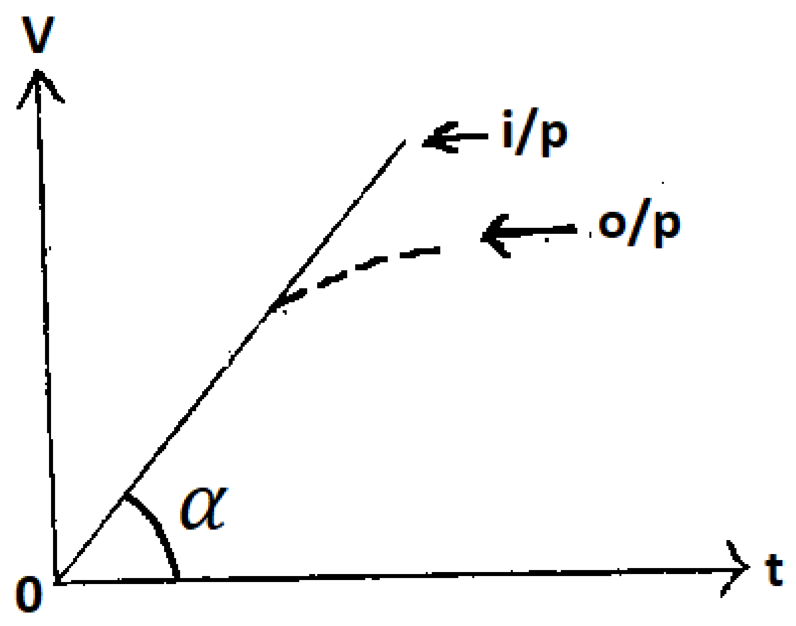
$$V_0(t) = RC\frac{d}{dt}V_i(t) \quad [V_i(t) \approx V_c(t)]$$

$$V_0(t) \propto \frac{d}{dt}V_i(t)$$

So, the output is proportional to the derivative of the input signal.

Ramp Input:

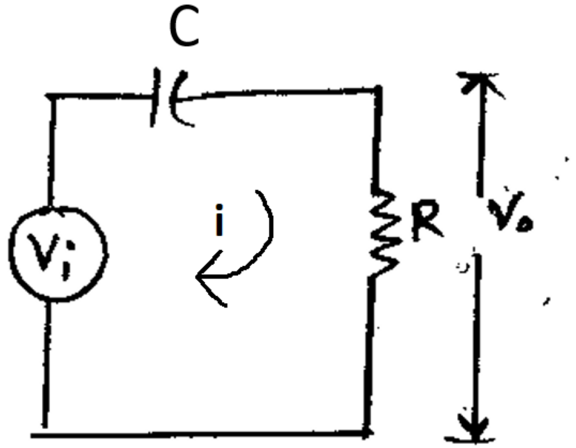
$$V_i(t) = \begin{cases} \alpha t, & if\ t \geq 0 \\ 0, & Otherwise \end{cases}$$



$$V_i(t) = \alpha t$$

$$\mathcal{L}\{V_i(t)\} = \alpha\mathcal{L}\{t\}$$

$$\therefore V_i(S) = \alpha\frac{1}{S^2} \qquad \left[\mathcal{L}\{t\} = \frac{1}{S^2}\right]$$



We know, for High Pass RC Circuit

$$V_0(S) = \left[\frac{1}{1 + \frac{1}{SRC}}\right] V_i(S) \quad = \left[\frac{1}{1 + \frac{1}{SRC}}\right] * \frac{\alpha}{S^2}$$

$$= \frac{\alpha}{S^2\left(1 + \frac{1}{SRC}\right)}$$

$$\therefore \; V_0(S) = \frac{\alpha}{S(S + \frac{1}{RC})}$$

$$V_0(S) = \frac{\alpha}{S(S + \frac{1}{RC})}$$

Do all the partial fraction steps yourself

$$\frac{px+q}{(x-a)(x-b)}, a \neq b \qquad \frac{A}{x-a} + \frac{B}{x-b}$$

Using this

Now using partial fraction,

$$V_0(S) = \alpha RC \left(\frac{1}{S} - \frac{1}{S + \frac{1}{RC}} \right)$$

$$V_0(S) = \alpha RC\left(\frac{1}{S} - \frac{1}{S + \frac{1}{RC}}\right)$$

Applying Inverse Laplace transformation

$$V_0(t) = \alpha RC \cdot \left(1 - \mathrm{e}^{-\frac{t}{Rc}}\right) \qquad \left[\mathcal{L}\left\{\frac{1}{S+a}\right\} = \mathrm{e}^{-at}\right]$$

Now we have to find the transmission error for Ramp input.

Transmission Error

Now, if $\frac{t}{Rc} \ll 1$, then $\qquad [e^{-x} = 1 - x + \frac{x^2}{2!} - \cdots \cdots]$

$$V_0(t) = \alpha RC \left\{1 - \left(1 \ - \frac{t}{Rc} + \frac{t^2}{2!\,(Rc)^2} - \cdots \ \cdots\right)\right\}$$

$$= \alpha RC \ \left(\frac{t}{Rc} - \frac{t^2}{2\ R^2 c^2}\right)$$ [Neglecting higher terms]

$$= \alpha \ \left(t - \frac{t^2}{2\ RC}\right)$$

Now transmission error at time t = T

$$e_i(T) = \frac{V_i(T) - V_0(T)}{V_i(T)}$$

$$= \frac{\alpha\, T - \alpha\left(T - \frac{T^2}{2\,RC}\right)}{\alpha\, T}$$

$$= 1 - \left(1 - \frac{T}{2\,RC}\right)$$

$$\therefore e_i(T) = \frac{T}{2\,RC}$$

Again, if $\frac{t}{Rc} \gg 1$, then $e^{-\frac{t}{Rc}} = e^{-\frac{\infty}{Rc}} = 0$

$$V_0(t) = \alpha RC \cdot (1-0) \quad = \alpha RC$$

Now transmission error at time t = T

$$e_i(T) = \frac{V_i(T) - V_0(T)}{V_i(T)} \qquad = \frac{\alpha\, T - \alpha RC}{\alpha\, T}$$

$$\therefore e_i(T) = 1 - \frac{RC}{T}$$