

Probability Distributions



Chapter 6

GOALS

- Define the terms probability distribution and random variable.
- Distinguish between discrete and continuous probability distributions.
- Calculate the mean, variance, and standard deviation of a discrete probability distribution.
- Describe the characteristics of and compute probabilities using the binomial probability distribution.
- Describe the characteristics of and compute probabilities using the hypergeometric probability distribution.
- Describe the characteristics of and compute probabilities using the Poisson



What is a Probability Distribution?

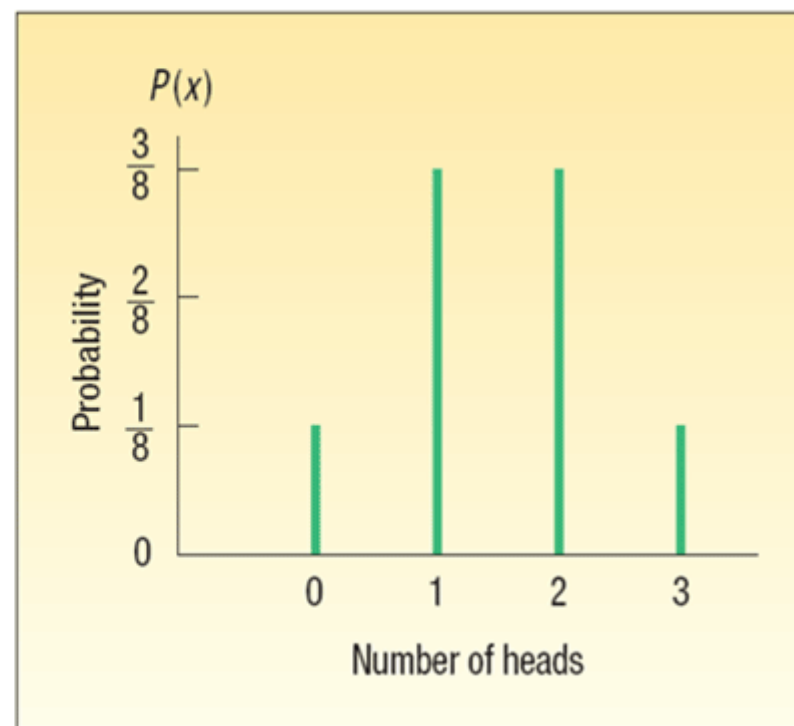
PROBABILITY DISTRIBUTION A listing of all the outcomes of an experiment and the probability associated with each outcome.

Experiment: Toss a coin three times. Observe the number of heads. The possible results are: zero heads, one head, two heads, and three heads. What is the probability distribution for the number of heads?

Possible Result	Coin Toss			Number of Heads
	First	Second	Third	
1	T	T	T	0
2	T	T	H	1
3	T	H	T	1
4	T	H	H	2
5	H	T	T	1
6	H	T	H	2
7	H	H	T	2
8	H	H	H	3

Probability Distribution of Number of Heads Observed in 3 Tosses of a Coin

Number of Heads, x	Probability of Outcome, $P(x)$
0	$\frac{1}{8} = .125$
1	$\frac{3}{8} = .375$
2	$\frac{3}{8} = .375$
3	$\frac{1}{8} = .125$
Total	$\frac{8}{8} = 1.000$



Characteristics of a Probability Distribution

CHARACTERISTICS OF A PROBABILITY DISTRIBUTION

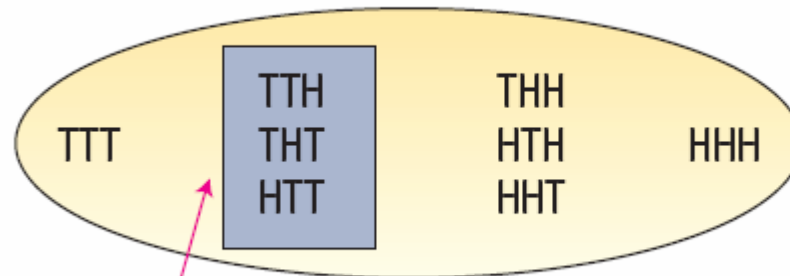
1. The probability of a particular outcome is between 0 and 1 inclusive.
2. The outcomes are mutually exclusive events.
3. The list is exhaustive. So the sum of the probabilities of the various events is equal to 1.



Random Variables

Random variable - a quantity resulting from an experiment that, by chance, can assume different values.

Possible *outcomes* for three coin tosses



The *event* {one head} occurs and the *random variable* $x = 1$.

Types of Random Variables

- **Discrete Random Variable** can assume only certain clearly separated values. It is usually the result of counting something
- **Continuous Random Variable** can assume an infinite number of values within a given range. It is usually the result of some type of measurement



Discrete Random Variables - Examples

- The number of students in a class.
- The number of children in a family.
- The number of cars entering a carwash in a hour.
- Number of home mortgages approved by Coastal Federal Bank last week.



Continuous Random Variables - Examples

- The distance students travel to class.
- The time it takes an executive to drive to work.
- The length of an afternoon nap.
- The length of time of a particular phone call.



Features of a Discrete Distribution

The main features of a discrete probability distribution are:

- The sum of the probabilities of the various outcomes is 1.00.
- The probability of a particular outcome is between 0 and 1.00.
- The outcomes are mutually exclusive.



The Mean of a Probability Distribution

MEAN

- The mean is a typical value used to represent the central location of a probability distribution.
- The mean of a probability distribution is also referred to as its **expected value**.

MEAN OF A PROBABILITY DISTRIBUTION

$$\mu = \sum [xP(x)]$$

[6-1]



The Variance, and Standard Deviation of a Probability Distribution

Variance and Standard Deviation

- Measures the amount of spread in a distribution
- The computational steps are:
 1. Subtract the mean from each value, and square this difference.
 2. Multiply each squared difference by its probability.
 3. Sum the resulting products to arrive at the variance.The standard deviation is found by taking the positive square root of the variance.

VARIANCE OF A PROBABILITY DISTRIBUTION

$$\sigma^2 = \sum [(x - \mu)^2 P(x)]$$

[6-2]



Mean, Variance, and Standard Deviation of a Probability Distribution - Example



John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

Number of Cars Sold, x	Probability, $P(x)$
0	.10
1	.20
2	.30
3	.30
4	.10
Total	1.00



Mean of a Probability Distribution - Example

$$\begin{aligned}\mu &= \Sigma[xP(x)] \\ &= 0(.10) + 1(.20) + 2(.30) + 3(.30) + 4(.10) \\ &= 2.1\end{aligned}$$

Number of Cars Sold, x	Probability, $P(x)$	$x \cdot P(x)$
0	.10	0.00
1	.20	0.20
2	.30	0.60
3	.30	0.90
4	.10	0.40
Total	1.00	$\mu = 2.10$



Variance and Standard Deviation of a Probability Distribution - Example

Number of Cars Sold, x	Probability, $P(x)$	$(x - \mu)$	$(x - \mu)^2$	$(x - \mu)^2 P(x)$
0	.10	$0 - 2.1$	4.41	0.441
1	.20	$1 - 2.1$	1.21	0.242
2	.30	$2 - 2.1$	0.01	0.003
3	.30	$3 - 2.1$	0.81	0.243
4	.10	$4 - 2.1$	3.61	0.361
				$\sigma^2 = 1.290$



Binomial Probability Distribution

Characteristics of a Binomial Probability Distribution

- There are only two possible outcomes on a particular trial of an experiment.
- The outcomes are mutually exclusive,
- The random variable is the result of counts.
- Each trial is *independent* of any other trial



Binomial Probability Formula

BINOMIAL PROBABILITY FORMULA

$$P(x) = {}_nC_x \pi^x (1 - \pi)^{n - x}$$

[6-3]

where:

C denotes a combination.

n is the number of trials.

x is the random variable defined as the number of successes.

π is the probability of a success on each trial.



Binomial Probability - Example

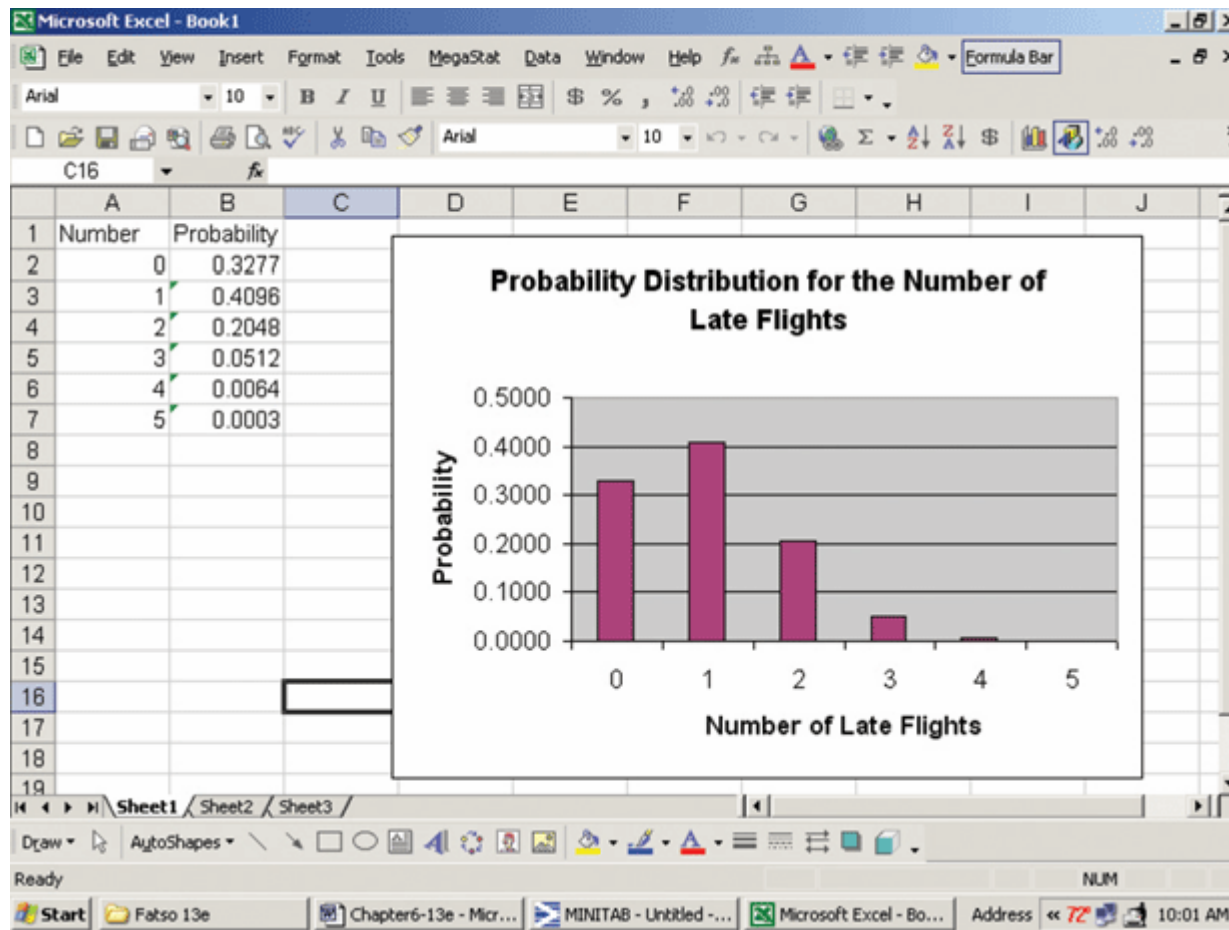
There are five flights daily from Pittsburgh via US Airways into the Bradford, Pennsylvania, Regional Airport. Suppose the probability that any flight arrives late is .20.

What is the probability that none of the flights are late today?

$$\begin{aligned}P(x = 0) &= {}_n C_x \pi^x (1 - \pi)^{n-x} \\&= {}_5 C_0 (.20)^0 (1 - .20)^{5-0} \\&= (1)(1)(.3277) \\&= 0.3277\end{aligned}$$



Binomial Probability - Excel



Binomial Dist. – Mean and Variance

MEAN OF A BINOMIAL DISTRIBUTION

$$\mu = n\pi$$

[6-4]

VARIANCE OF A BINOMIAL DISTRIBUTION

$$\sigma^2 = n\pi(1 - \pi)$$

[6-5]



Binomial Dist. – Mean and Variance: Example

For the example
regarding the number
of late flights, recall
that $\pi = .20$ and $n = 5$.

What is the average
number of late flights?

What is the variance of
the number of late
flights?

$$\begin{aligned}\mu &= n\pi \\ &= (5)(0.20) = 1.0\end{aligned}$$

$$\begin{aligned}\sigma^2 &= n\pi(1 - \pi) \\ &= (5)(0.20)(1 - 0.20) \\ &= (5)(0.20)(0.80) \\ &= 0.80\end{aligned}$$



Binomial Dist. – Mean and Variance: Another Solution

Number of Late Flights, x	$P(x)$	$xP(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2P(x)$
0	0.3277	0.0000	-1	1	0.3277
1	0.4096	0.4096	0	0	0
2	0.2048	0.4096	1	1	0.2048
3	0.0512	0.1536	2	4	0.2048
4	0.0064	0.0256	3	9	0.0576
5	0.0003	0.0015	4	16	0.0048
		$\mu = 1.0000$			
					$\sigma^2 = 0.7997$



Binomial Distribution - Table

Five percent of the worm gears produced by an automatic, high-speed Carter-Bell milling machine are defective. What is the probability that out of six gears selected at random none will be defective? Exactly one? Exactly two? Exactly three? Exactly four? Exactly five? Exactly six out of six?

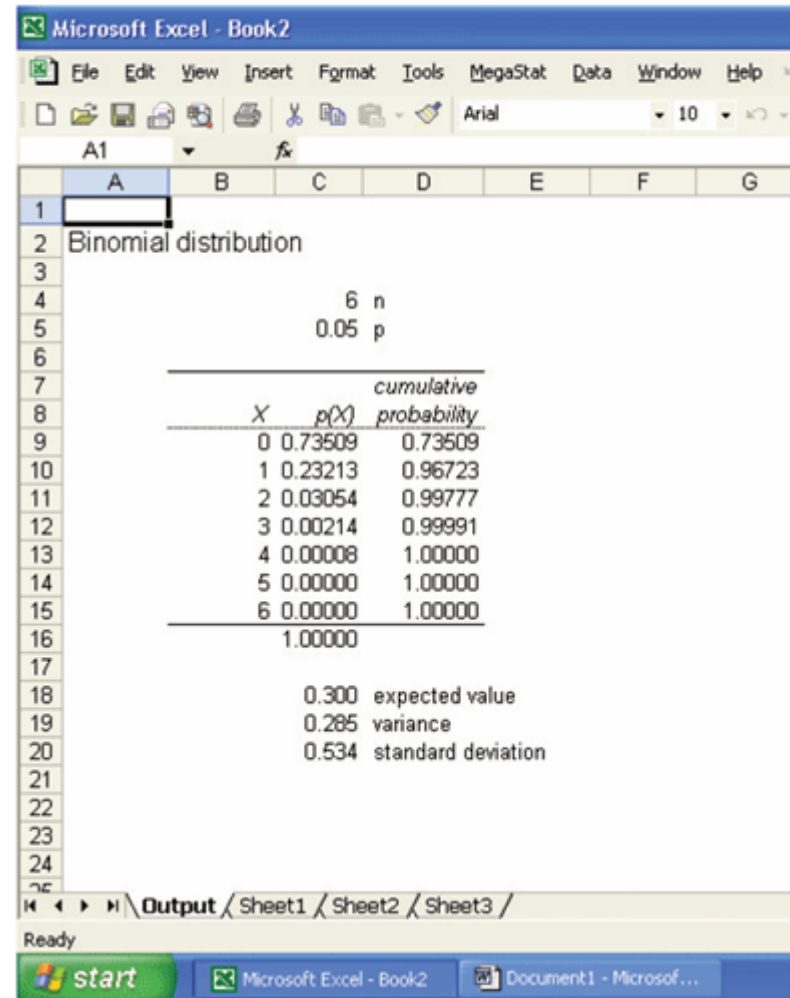
TABLE 6-2 Binomial Probabilities for $n = 6$ and Selecte Values of π

$n = 6$ Probability											
$x \backslash \pi$.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95
0	.735	.531	.262	.118	.047	.016	.004	.001	.000	.000	.000
1	.232	.354	.393	.303	.187	.094	.037	.010	.002	.000	.000
2	.031	.098	.246	.324	.311	.234	.138	.060	.015	.001	.000
3	.002	.015	.082	.185	.276	.313	.276	.185	.082	.015	.002
4	.000	.001	.015	.060	.138	.234	.311	.324	.246	.098	.031
5	.000	.000	.002	.010	.037	.094	.187	.303	.393	.354	.232
6	.000	.000	.000	.001	.004	.016	.047	.118	.262	.531	.735



Binomial Distribution - MegaStat

Five percent of the worm gears produced by an automatic, high-speed Carter-Bell milling machine are defective. What is the probability that out of six gears selected at random none will be defective? Exactly one? Exactly two? Exactly three? Exactly four? Exactly five? Exactly six out of six?



The screenshot shows the MegaStat output for a binomial distribution with $n = 6$ and $p = 0.05$. The output table includes the probability mass function $p(X)$ and the cumulative probability. Below the table, the expected value, variance, and standard deviation are calculated.

X	$p(X)$	cumulative probability
0	0.73509	0.73509
1	0.23213	0.96723
2	0.03054	0.99777
3	0.00214	0.99991
4	0.00008	1.00000
5	0.00000	1.00000
6	0.00000	1.00000

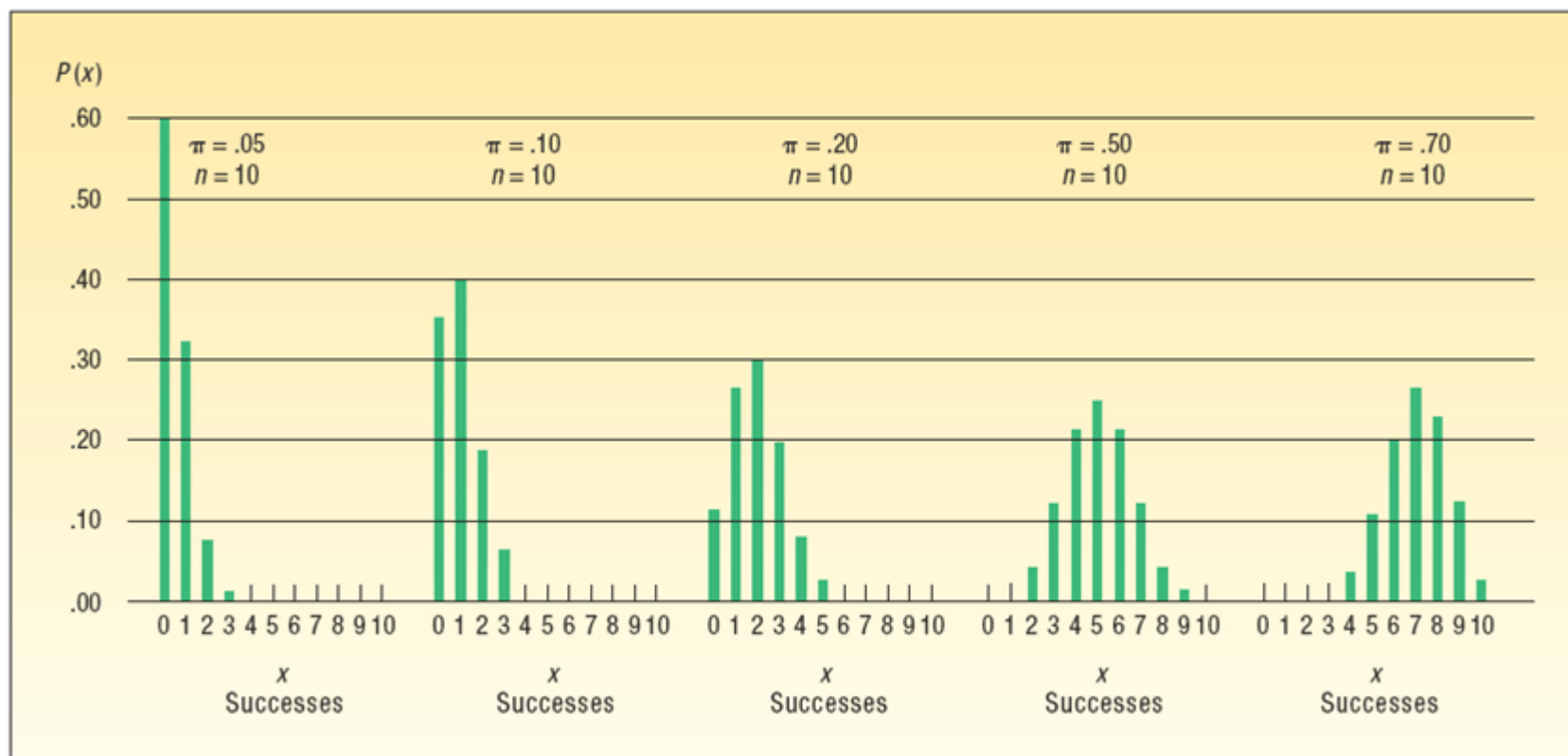
1.00000

0.300 expected value
0.285 variance
0.534 standard deviation

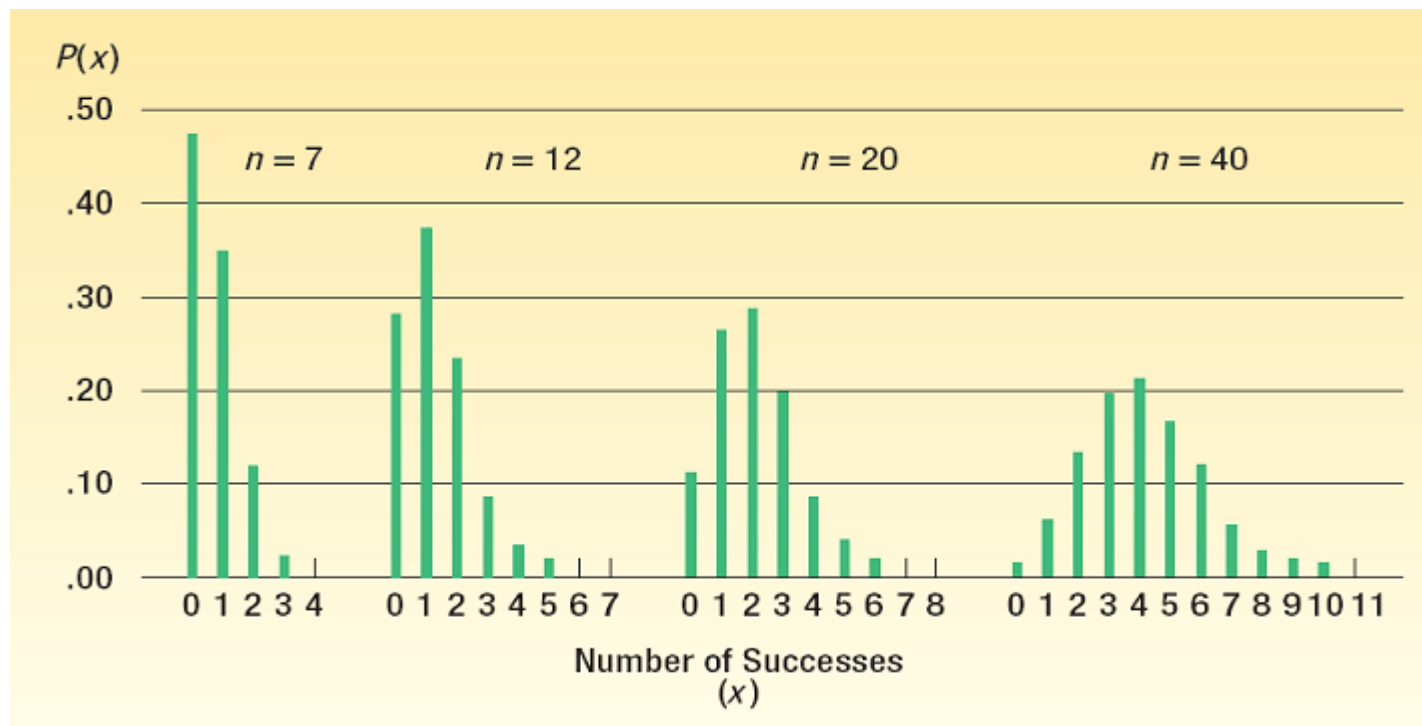


Binomial – Shapes for Varying π (n constant)

CHART 6-2 Graphing the Binomial Probability Distribution for a π of .05, .10, .20, .50, and .70 and an n of 10



Binomial – Shapes for Varying n (π constant)



Cumulative Binomial Probability Distributions

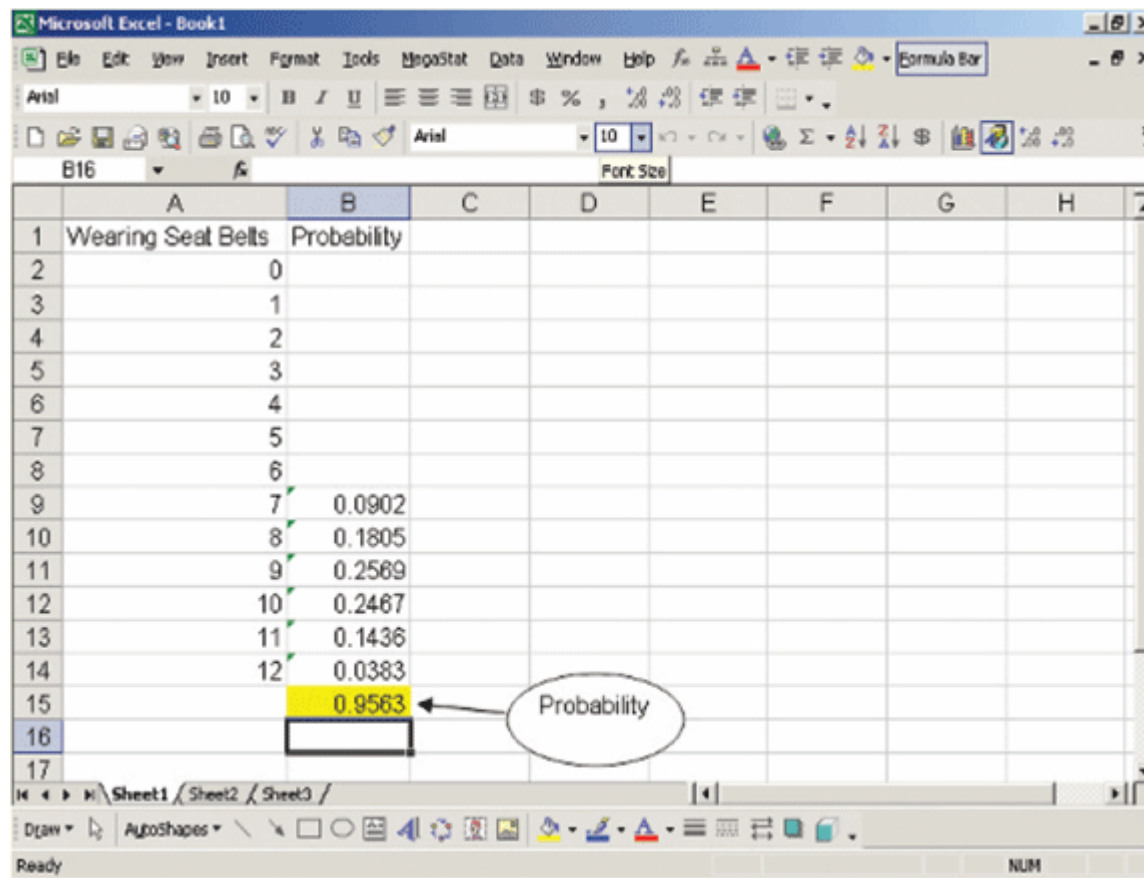
A study in June 2003 by the Illinois Department of Transportation concluded that 76.2 percent of front seat occupants used seat belts. A sample of 12 vehicles is selected. What is the probability the front seat occupants in at least 7 of the 12 vehicles are wearing seat belts?

$$\begin{aligned}P(X \geq 7 | n=12 \text{ and } \pi = .762) \\&= P(X=7) + P(X=8) + P(X=9) + P(X=10) + P(X=11) + P(X=12) \\&= .0902 + .1805 + .2569 + .2467 + .1436 + .0383 \\&= .9563\end{aligned}$$

$$\begin{aligned}P(x = 7 | n = 12 \text{ and } \pi = .762) \\&= {}_{12}C_7 (.762)^7 (1 - .762)^{12-7} = 792 (.149171) (.000764) = .0902\end{aligned}$$



Cumulative Binomial Probability Distributions - Excel



Microsoft Excel - Book1

File Edit View Insert Format Tools MegaStat Data Window Help

Arial 10

B16

Font Size

	A	B	C	D	E	F	G	H
1	Wearing Seat Belts	Probability						
2		0						
3		1						
4		2						
5		3						
6		4						
7		5						
8		6						
9		7	0.0902					
10		8	0.1805					
11		9	0.2589					
12		10	0.2487					
13		11	0.1436					
14		12	0.0383					
15			0.9563					
16								
17								

Sheet1 Sheet2 Sheet3

Draw AutoShapes

Ready

NUM

Finite Population

A **finite population** is a population consisting of a fixed number of known individuals, objects, or measurements. Examples include:

- The number of students in this class.
- The number of cars in the parking lot.
- The number of homes built in Blackmoor



Hypergeometric Distribution

The **hypergeometric distribution** has the following characteristics:

- There are only 2 possible outcomes.
- The probability of a success is not the same on each trial.
- It results from a count of the number of successes in a fixed number of trials.



Hypergeometric Distribution

- Use the **hypergeometric distribution** to find the probability of a specified number of successes or failures if:
- the sample is selected from a finite population without replacement
 - the size of the sample n is greater than 5% of the size of the population N (i.e. $n/N \geq .05$)



Hypergeometric Distribution

HYPERGEOMETRIC DISTRIBUTION

$$P(x) = \frac{{}_sC_x (N-s)C_{n-x}}{{}_NC_n}$$

[6-6]

where:

N is the size of the population.

S is the number of successes in the population.

x is the number of successes in the sample. It may be 0, 1, 2, 3,

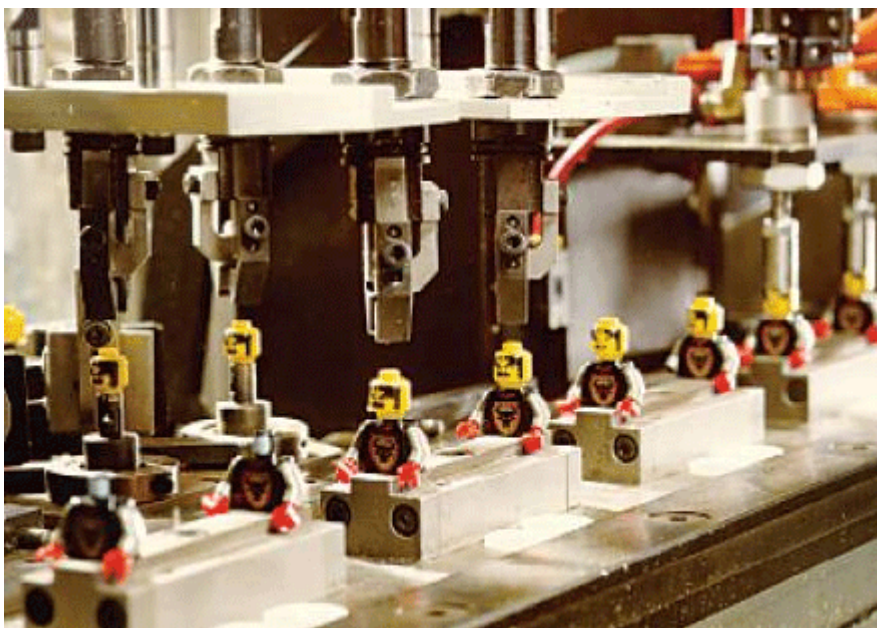
n is the size of the sample or the number of trials.

C is the symbol for a combination.



Hypergeometric Distribution - Example

PlayTime Toys, Inc., employs 50 people in the Assembly Department. Forty of the employees belong to a union and ten do not. Five employees are selected at random to form a committee to meet with management regarding shift starting times. What is the probability that four of the five selected for the committee belong to a union?



Hypergeometric Distribution - Example

N is 50, the number of employees.

S is 40, the number of union employees.

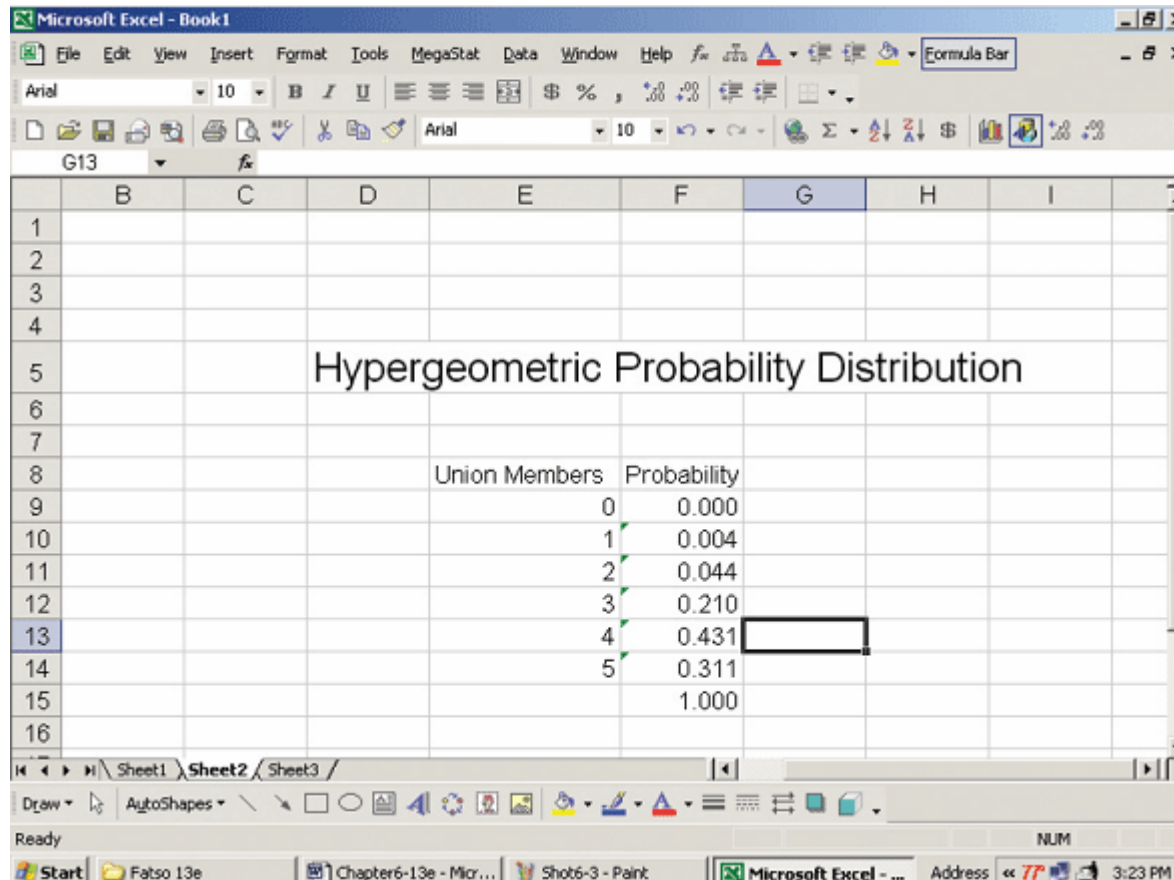
x is 4, the number of union employees selected.

n is 5, the number of employees selected.

$$P(4) = \frac{{}_{40}C_4 {}_{50-40}C_{5-4}}{{}_{50}C_5} = \frac{\left(\frac{40!}{4!36!}\right)\left(\frac{10!}{1!9!}\right)}{\frac{50!}{5!45!}} = \frac{(91,390)(10)}{2,118,760} = .431$$



Hypergeometric Distribution - Excel



The screenshot shows a Microsoft Excel window titled "Microsoft Excel - Book1". The worksheet is named "Sheet2" and contains a table titled "Hypergeometric Probability Distribution". The table lists the probability for different numbers of union members (0 to 5). The cell G13 is selected, and a small black rectangle is visible next to the probability value 0.431.

Union Members	Probability
0	0.000
1	0.004
2	0.044
3	0.210
4	0.431
5	0.311
	1.000

Poisson Probability Distribution

The **Poisson probability distribution**

describes the number of times some event occurs during a specified interval. The interval may be time, distance, area, or volume.

- **Assumptions of the Poisson Distribution**
 - (1) The probability is proportional to the length of the interval.
 - (2) The intervals are independent.



Poisson Probability Distribution

The **Poisson** distribution can be described mathematically using the formula:

POISSON DISTRIBUTION

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

[6-7]

where:

μ (μ) is the mean number of occurrences (successes) in a particular interval.


e is the constant 2.71828 (base of the Napierian logarithmic system).

x is the number of occurrences (successes).

$P(x)$ is the probability for a specified value of x .



Poisson Probability Distribution

- The mean number of successes can be determined in binomial situations by $n\pi$, where n is the number of trials and π the probability of a success. 
- The variance of the Poisson distribution is also equal to $n\pi$.



Poisson Probability Distribution - Example

Assume baggage is rarely lost by Northwest Airlines. Suppose a random sample of 1,000 flights shows a total of 300 bags were lost. Thus, the arithmetic mean number of lost bags per flight is 0.3 (300/1,000). If the number of lost bags per flight follows a Poisson distribution with $\mu = 0.3$, find the probability of not losing any bags.

$$P(0) = \frac{\mu^x e^{-\mu}}{x!} = \frac{0.3^0 e^{-0.3}}{0!} = .7408$$



Poisson Probability Distribution - Table

Assume baggage is rarely lost by Northwest Airlines. Suppose a random sample of 1,000 flights shows a total of 300 bags were lost. Thus, the arithmetic mean number of lost bags per flight is 0.3 (300/1,000). If the number of lost bags per flight follows a Poisson distribution with mean = 0.3, find the probability of not losing any bags

TABLE 6-6 Poisson Table for Various Values of μ (from Appendix B.5)

x	μ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000



End of Chapter 6

