Example on Error

Problem 1: Find The Chopping And Symmetric Round Off Error for the number 118.68, Using Four Digit Mantissa.

- **Solution:** (A) True value $= 118 \cdot 68$

Chopping: True value = approximate value + error

$$= \left(f_x \times 10^{E} + g_x \times 10^{E-d} \right)$$

$$118 \cdot 68 = 0 \cdot 11868 \times 10^{3}$$

$$= (\cdot 1186 + 0 \cdot 00008) \times 10^{3}$$

$$= (0 \cdot 1186 \times 10^{3}) + (0 \cdot 8 \times 10^{-4}) \times 10^{3}$$

$$= (0 \cdot 1186 \times 10^{3}) + 0 \cdot 8 \times 10^{-1}$$

$$\therefore \text{error} = 0 \cdot 8 \times 10^{-1} = 0 \cdot 08$$

(A) symmetric round off

$$\text{error} = (g_x - 1) \times 10^{E-d}$$

$$g_x > 0.5$$

$$\text{error} = (0.8 - 1) \times 10^{E-d}$$

$$= -0.2 \times 10^{3-4}$$

$$= -0.02$$

Find the absolute and relative errors for the arithmetic operation 87.26 + 31.42.

Since the number x & y are stored in 4 digit mantissa system, therefore

$$|e_{rx}| = \frac{1}{2} \times 10^{-d+1} = \frac{1}{2} \times 10^{-4+1} = \frac{1}{2} \times 10^{-3}$$

$$= 0.5 \times 10^{-3}$$

$$|e_{ry}| = \frac{1}{2} \times 10^{-3} = 0.5 \times 10^{-3}$$

Then absolute error $|e_x| = |x| * e_{rx}$

$$= |0.8726 \times 10^2| * 0.5 \times 10^{-3}$$

$$= 0.4363$$

$$|e_y| = |y| \times e_{ry} = |0.3142 \times 10^2| * 0.5 \times 10^{-3}$$

$$= 0.5934$$

$$|e_z| = |e_x| + |e_y|$$
$$= |0.4363| + |0.1571|$$
$$= 0.5934$$

$$|e_{rz}| = \frac{\text{absolute error}}{x+y}$$
$$= \frac{0.5934}{|118.68|} = 0.005$$

* Error in result is bigger than error in individual number.

* Error can be calculate using the concept of differential calculus.

* Example: $\omega = x^n$

$\Delta w = n x^{n-1} \Delta x$ | Δw = Error

therefor relative error $e_{rw} = n * \frac{\Delta x}{x}$

$= n * e_{rx}$

- If u = 5xy2/z3 (3) Δ x = Δ y = Δ z = 1 and x = y = z = 1, find the value of relative error.

$u = 5xy^2z^3$

$\frac{du}{dx} - \frac{5y^2}{z^3} = 5$ [putting the value of x, y, z]

$\frac{du}{dy} - \frac{10xy}{z^3} = 10$

$\frac{dy}{dz} = \frac{-15xy^2}{z^4} = -15$

$\Delta u = \left|\frac{du}{dx}\Delta x\right| + \left|\frac{du}{dy}\Delta y\right| + \left|\frac{dy}{dz}\Delta z\right|$

$= 5*1 + 10*1 + 15*1 = 30$

$E_R = \frac{\Delta u}{u}$

$= \frac{30}{5}$

$= 6$

$x_a = 2.35$

absolute error $e_x = |x_a| \varepsilon = |2.35| * \frac{1}{2} \times 10^{-d+1} = 2.35 * \frac{1}{2} * 10^{-3}$

$2.35 * 0.005 = 0.01175$

$e_y = |y| \varepsilon = |6.74| * \frac{1}{2} \times 10^{-3} = 6.75 * 0.005 = 0.03370$

$e_z = |z| \varepsilon = |3.45| * \frac{1}{2} * 10^{-3} = 3.45 * 0.005 = 0.01725$

$e_{xy} = |x_a| e_y + |y_a| e_x$ [From total error due to multiplication]

$= 2.35 * 0.03370 + 6.74 * 0\ 0175 = 0\ 15839$

$e_w = |e_{xy}| + |e_z| = 0.15839 + 0.01725 = 0.17564$

- Find the absolute error in w=xy +z, if x = 2.35, y = 6.74 and z = 3.45 stored as 4 digit mantissa.

absolute error in $x = e_x = |x|\varepsilon$

$e_x = |x_a|\varepsilon = |2.35| \times \frac{1}{2} \times 10^{-4+1}$

$= |2.35| * 0.005$

$= 0.01175$

$e_y = |y_a|\varepsilon = |6.74| \times \frac{1}{2} \times 10^{-4+1}$

$= |6.74| * 0.005$

$= 0.03370$

$e_z = |z_a|\varepsilon = |3.45| * \frac{1}{2} \times 10^{-4+1}$

$= |3.45| * 0.005$

$= 0.01725$

Total error due to multiplication

$e_{xy} = |x_a| e_y + |y_a| e_x$

$= 2.35 * 0.03370$
$+ 6.74 * 0.01175$

$= 0.15839$

$e_w = e_{xy} + e_z$

$= 0.15839 + 0.01725$

$= 0.7564$