One Sample Tests of Hypothesis

Chapter 10

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GOALS

- Define a hypothesis and hypothesis testing.
- Describe the five-step hypothesis-testing procedure.
- Distinguish between a one-tailed and a two-tailed test of hypothesis.
- Conduct a test of hypothesis about a population mean.
- Conduct a test of hypothesis about a population proportion.
- Define Type I and Type II errors.
- Compute the probability of a Type II error.

What is a Hypothesis?

A Hypothesis is a statement about the value of a population parameter developed for the purpose of testing. Examples of hypotheses made about a population parameter are:

- The mean monthly income for systems analysts is $3,625.
- Twenty percent of all customers at Bovine's Chop House return for another meal within a month.

What is Hypothesis Testing?

Hypothesis testing is a procedure, based on sample evidence and probability theory, used to determine whether the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected.

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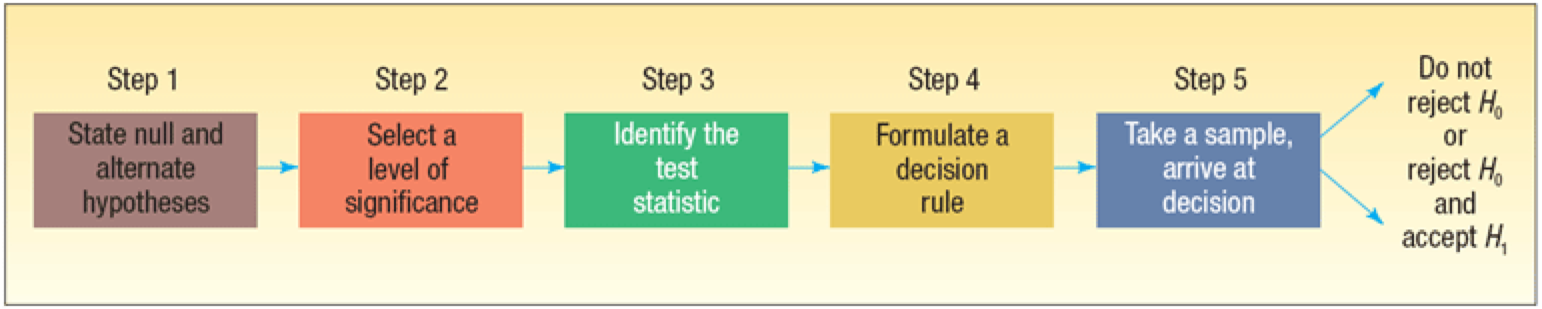
Hypothesis Testing Steps



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Important Things to Remember about H_0 and H_1

- H_0: null hypothesis and H_1: alternate hypothesis
- H_0 and H_1 are mutually exclusive and collectively exhaustive
- H_0 is always presumed to be true
- H_1 has the burden of proof
- A random sample (n) is used to "*reject H_0*"
- If we conclude 'do not reject H_0', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not sufficient evidence to reject H_0; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of H_0 (e.g. "$=$" , "$\geq$" , "$\leq$").
- "$\neq$" "$<$" and "$>$" always part of H_1

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How to Set Up a Claim as Hypothesis

- In actual practice, the status quo is set up as H_0
- If the claim is "boastful" the claim is set up as H_1 (we apply the Missouri rule – "show me"). Remember, H_1 has the burden of proof
- In problem solving, look for **key words** and convert them into symbols. Some key words include: "*improved, better than, as effective as, different from, has changed*, etc."

Left-tail or Right-tail Test?

- The direction of the test involving claims that use the words "*has improved*", "*is better than*", and the like will depend upon the variable being measured.
- For instance, if the variable involves *time for a certain medication to take effect*, the words "better" "improve" or more effective" are translated as "<" (less than, i.e. faster relief).
- On the other hand, if the variable refers to a *test score*, then the words "better" "improve" or more effective" are translated as ">" (greater than, i.e. higher test scores)

Keywords	Inequality Symbol	Part of:
Larger (or more) than	$>$	H_1
Smaller (or less)	$<$	H_1
No more than	$\leq$	H_0
At least	$\geq$	H_0
Has increased	$>$	H_1
Is there difference?	$\neq$	H_1
Has not changed	$=$	H_0
Has "improved", "is better than". "is more effective"	See right	H_1

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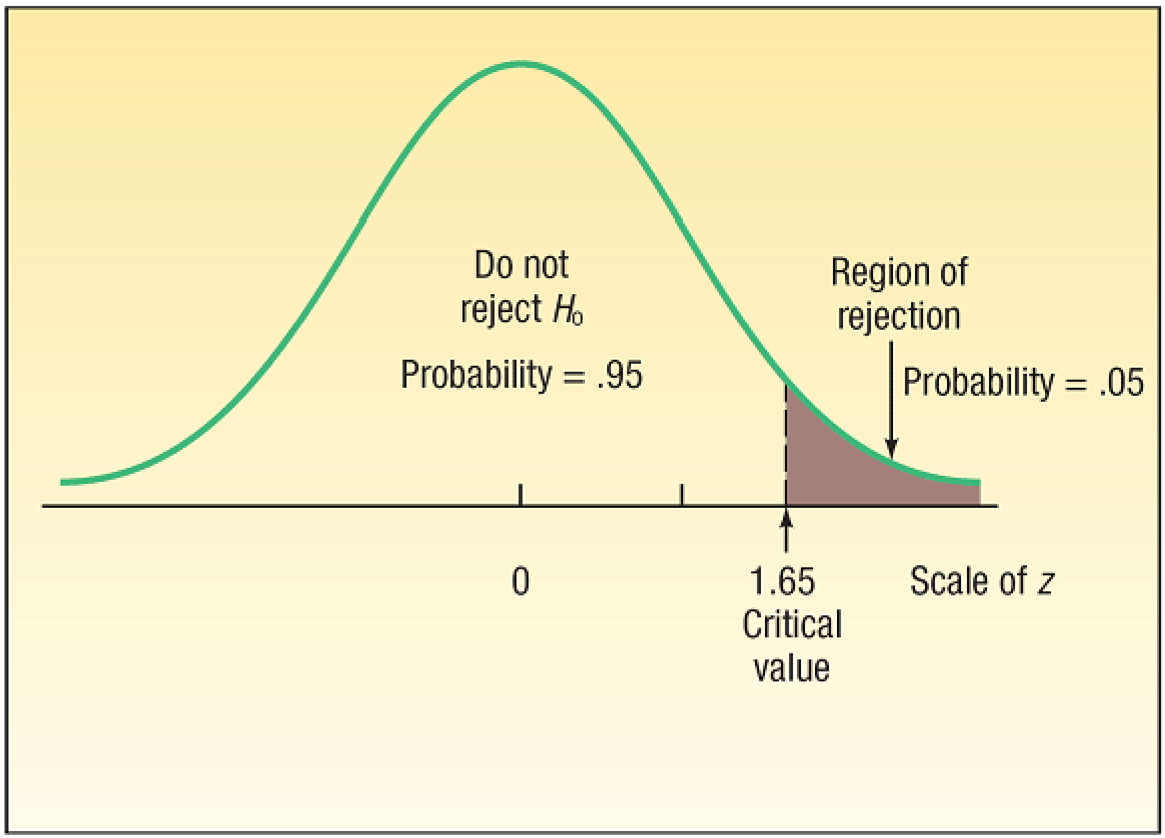
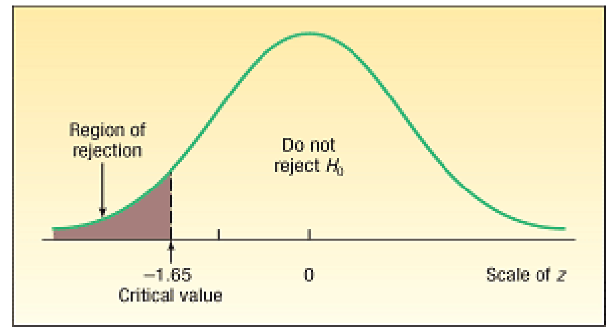
Null Hypothesis	Researcher: Does Not Reject H_0	Researcher: Rejects H_0
H_0 is true	Correct decision	Type I error
H_0 is false	Type II error	Correct decision

Parts of a Distribution in Hypothesis Testing

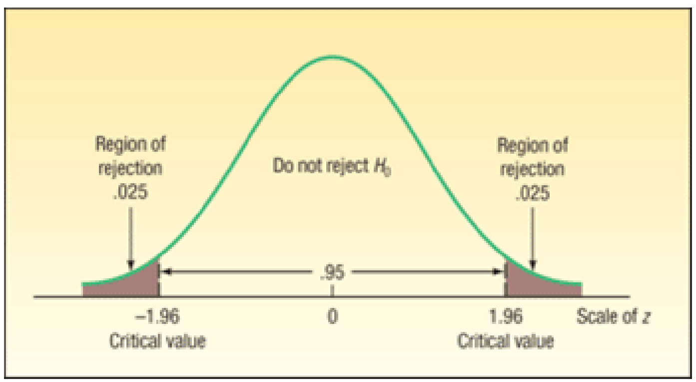


One-tail vs. Two-tail Test

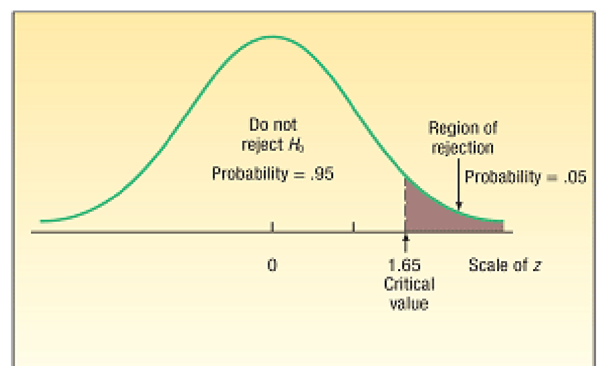
One-tail, Left Tail Test



Two-tail or Non-directional Test



One-tail, Right Tail Test



Hypothesis Setups for Testing a Mean (μ)

H_0: μ = value H_1: $\mu \neq$ value	H_0: $\mu \geq$ value H_1: $\mu <$ value	H_0: $\mu \leq$ value H_1: $\mu >$ value
Reject H_0 if: $\lvert Z \rvert > Z_{\alpha/2}$ $\lvert t \rvert > t_{\alpha/2,\, n-1}$	Reject H_0 if: $Z < -Z_{\alpha}$ $t < -t_{\alpha,\, n-1}$	Reject H_0 if: $Z > Z_{\alpha}$ $t > t_{\alpha,\, n-1}$

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}} \qquad \boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

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Hypothesis Setups for Testing a Proportion (π)

H_0: π = value
H_1: $\pi \neq$ value
Reject H_0 if:
$|Z| > Z_{\alpha/2}$

H_0: $\pi \geq$ value
H_1: $\pi <$ value
Reject H_0 if:
$Z < -Z_{\alpha}$

H_0: $\pi \leq$ value
H_1: $\pi >$ value
Reject H_0 if:
$Z > Z_{\alpha}$

$$\boxed{z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}}$$

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Testing for a Population Mean with a Known Population Standard Deviation- Example

Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in western New York State. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, because of market expansion, new production methods have been introduced and new employees hired. The vice president of manufacturing would like to investigate whether there has been a *change* in the weekly production of the Model A325 desk.

Testing for a Population Mean with a Known Population Standard Deviation- Example

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: $\mu = 200$

H_1: $\mu \neq 200$

(note: keyword in the problem "has changed")

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since σ is known

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Testing for a Population Mean with a Known Population Standard Deviation- Example

Step 4: Formulate the decision rule.

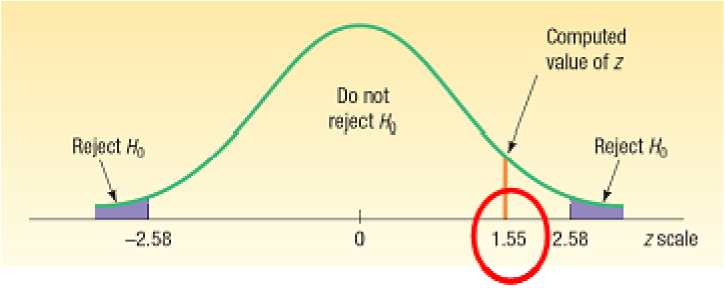
Reject H_0 if $|Z| > Z_{\alpha/2}$

$$|Z| > Z_{\alpha/2}$$

$$\left|\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\right| > Z_{\alpha/2}$$

$$\left|\frac{203.5-200}{16/\sqrt{50}}\right| > Z_{.01/2}$$

$$1.55 \text{ is not} > 2.58$$



Step 5: Make a decision and interpret the result.

Because 1.55 does not fall in the rejection region, H_0 is not rejected. We conclude that the population mean is not different from 200. So we would report to the vice president of manufacturing that the sample evidence does not show that the production rate at the Fredonia Plant has changed from 200 per week.

Testing for a Population Mean with a Known Population Standard Deviation- Another Example

Suppose in the previous problem the vice president wants to know whether there has been an *increase* in the number of units assembled. To put it another way, can we conclude, because of the improved production methods, that the mean number of desks assembled in the last 50 weeks was more than 200?

Recall: $\sigma=16$, $n=200$, $\alpha=.01$

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Testing for a Population Mean with a Known Population Standard Deviation- Example

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: $\mu \leq 200$

H_1: $\mu > 200$

(note: keyword in the problem "an *increase*")

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since σ is known

Testing for a Population Mean with a Known Population Standard Deviation- Example

Step 4: Formulate the decision rule.

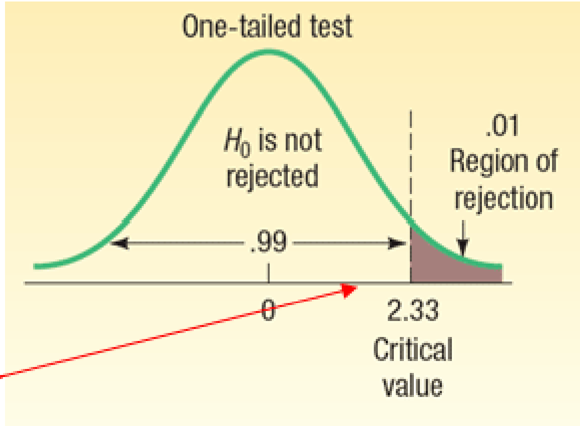
Reject H_0 if $Z > Z_\alpha$

$$Z > Z_\alpha$$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > Z_\alpha$$

$$\frac{203.5-200}{16/\sqrt{50}} > Z_{.01}$$

$$1.55 \text{ is not } > 2.33$$



Step 5: Make a decision and interpret the result.
Because 1.55 does not fall in the rejection region, H_0 is not rejected. We conclude that the average number of desks assembled in the last 50 weeks is not more than 200

Type of Errors in Hypothesis Testing

- Type I Error -
  - Defined as the probability of rejecting the null hypothesis when it is actually true.
  - This is denoted by the Greek letter "α"
  - Also known as the significance level of a test

- Type II Error:
  - Defined as the probability of "accepting" the null hypothesis when it is actually false.
  - This is denoted by the Greek letter "β"

p-Value in Hypothesis Testing

- ***p*-VALUE** is the probability of observing a sample value as extreme as, or more extreme than, the value observed, given that the null hypothesis is true.

- In testing a hypothesis, we can also compare the *p*-value to with the significance level (α).

- If the *p*-value < significance level, H_0 is rejected, else H_0 is not rejected.

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p-Value in Hypothesis Testing - Example

Recall the last problem where the hypothesis and decision rules were set up as:

$H_0: \mu \leq 200$

$H_1: \mu > 200$

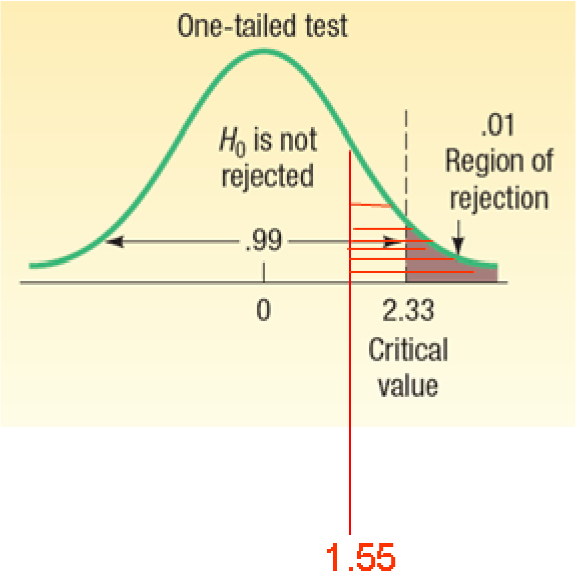
Reject H_0 if $Z > Z_\alpha$

where $Z = 1.55$ and $Z_\alpha = 2.33$

Reject H_0 if p-value $< \alpha$

0.0606 is not < 0.01

Conclude: Fail to reject H_0



P(Z>1.55)=.5000 -.4394

P-value = .0606

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What does it mean when p-value < α?

(a) .10, we have some evidence that H_0 is not true.

(b) .05, we have strong evidence that H_0 is not true.

(c) .01, we have very strong evidence that H_0 is not true.

(d) .001, we have extremely strong evidence that H_0 is not true.

Testing for the Population Mean: Population Standard Deviation Unknown

- When the population standard deviation (σ) is unknown, the sample standard deviation (s) is used in its place
- The *t*-distribution is used as test statistic, which is computed using the formula:

TESTING A MEAN, σ UNKNOWN

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} \qquad [10\text{–}2]$$

with $n - 1$ degrees of freedom, where:

$\overline{X}$ is the sample mean.
μ is the hypothesized population mean.
s is the sample standard deviation.
n is the number of observations in the sample.

Testing for the Population Mean: Population Standard Deviation Unknown - Example

The McFarland Insurance Company Claims Department reports the mean cost to process a claim is $60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of 26 claims processed last month. The sample information is reported below.

At the .01 significance level is it reasonable a claim is *now less than $60*?

$45	$49	$62	$40	$43	$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				

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Testing for a Population Mean with a Known Population Standard Deviation- Example

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: $\mu \geq \$60$

H_1: $\mu < \$60$

(note: keyword in the problem "*now less than*")

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use *t*-distribution since σ is unknown

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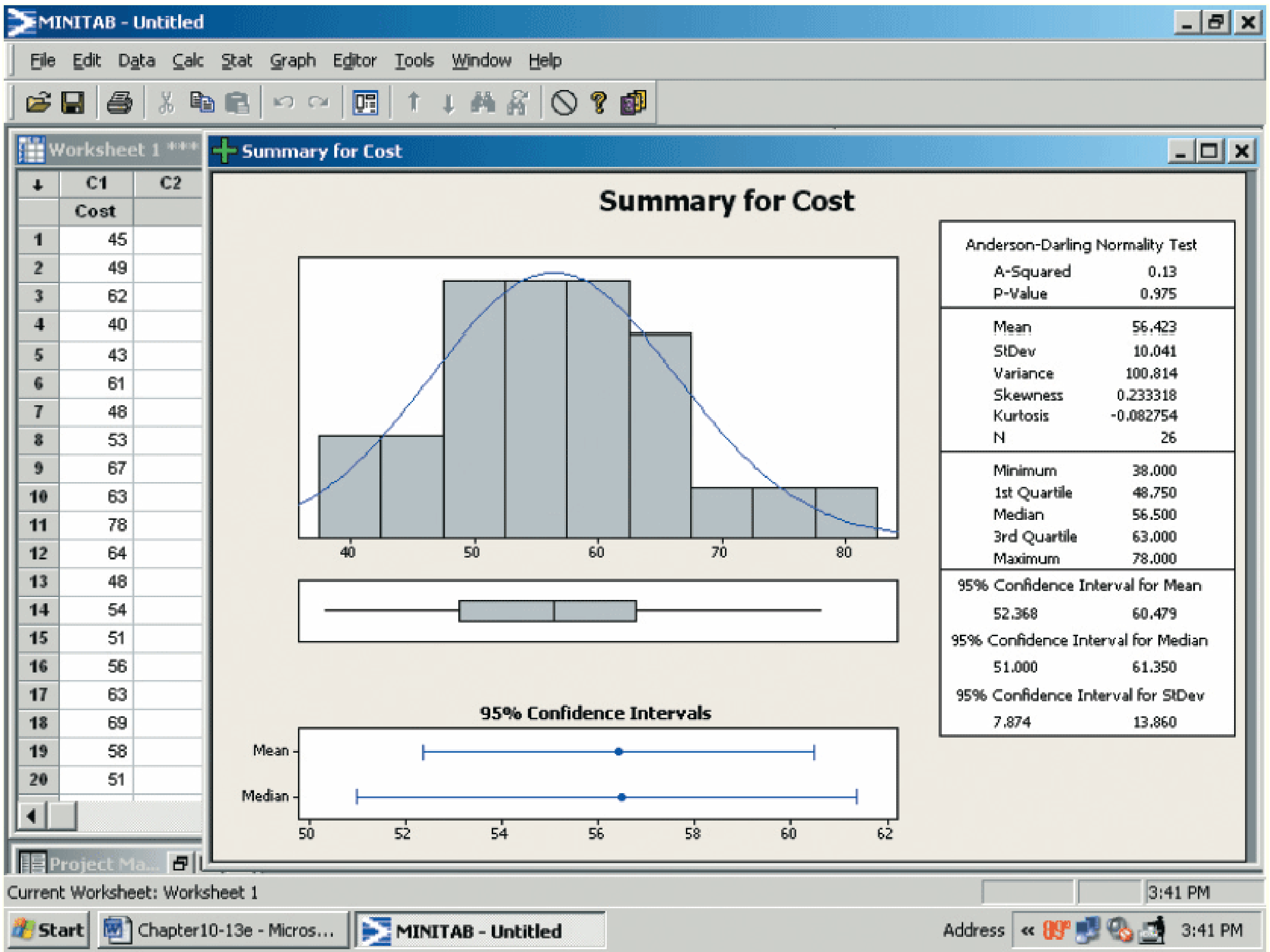
t-Distribution Table (portion)

TABLE 10–1 A Portion of the t Distribution Table

	Confidence Intervals					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test, α					
df	0.100	0.050	0.025	0.010	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
⋮	⋮	⋮	⋮	⋮	⋮	⋮
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	$\boxed{2.485}$	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646

Testing for the Population Mean: Population Standard Deviation Unknown – Minitab Solution



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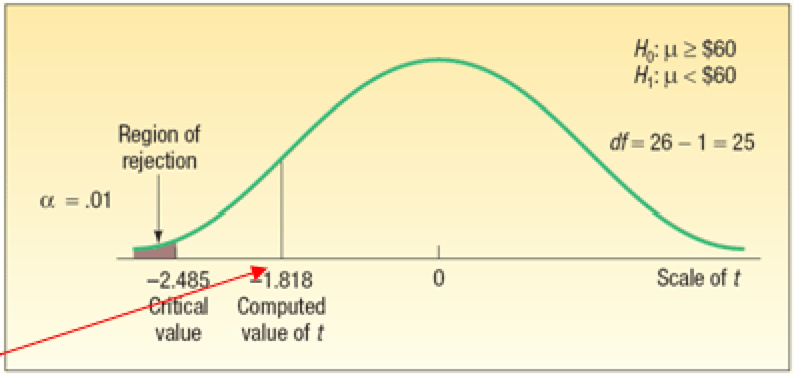
Testing for a Population Mean with a Known Population Standard Deviation- Example

Step 4: Formulate the decision rule.

Reject H_0 if $t < -t_{\alpha,n-1}$



$$t < -t_{\alpha,n-1}$$

$$\frac{\bar{X}-\mu}{s/\sqrt{n}} < -t_{\alpha,n-1}$$

$$\frac{\$56.42-\$60}{\$10.04/\sqrt{26}} < -t_{.01,26-1}$$

$$-1.818 \text{ is not} < -2.485$$

Step 5: Make a decision and interpret the result.

Because -1.818 does not fall in the rejection region, H_0 is not rejected at the .01 significance level. We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than $60. The difference of $3.58 ($56.42 - $60) between the sample mean and the population mean could be due to sampling error.

Testing for a Population Mean with an Unknown Population Standard Deviation- Example

The current rate for producing 5 amp fuses at Neary Electric Co. is 250 per hour. A new machine has been purchased and installed that, according to the supplier, will increase the production rate. A sample of 10 randomly selected hours from last month revealed the mean hourly production on the new machine was 256 units, with a sample standard deviation of 6 per hour.

At the .05 significance level can Neary conclude that the new machine is faster?

Testing for a Population Mean with a Known Population Standard Deviation- Example *continued*

Step 1: State the null and the alternate hypothesis.

H_0: μ ≤ 250; H_1: μ > 250

Step 2: Select the level of significance.

It is .05.

Step 3: Find a test statistic. Use the *t* distribution because the population standard deviation is not known and the sample size is less than 30.

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Testing for a Population Mean with a Known Population Standard Deviation- Example *continued*

Step 4: State the decision rule.

There are 10 – 1 = 9 degrees of freedom. The null hypothesis is rejected if $t > 1.833$.

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{256 - 250}{6/\sqrt{10}} = 3.162$$

Step 5: Make a decision and interpret the results.

The null hypothesis is rejected. The mean number produced is more than 250 per hour.

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Tests Concerning Proportion

- A Proportion is the fraction or percentage that indicates the part of the population or sample having a particular trait of interest.
- The sample proportion is denoted by *p* and is found by *x/n*
- The test statistic is computed as follows:

TEST OF HYPOTHESIS, ONE PROPORTION

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \quad [10\text{–}3]$$

where:

π is the population proportion.
p is the sample proportion.
n is the sample size.

Assumptions in Testing a Population Proportion using the z-Distribution

- A random sample is chosen from the population.
- It is assumed that the binomial assumptions discussed in Chapter 6 are met:

  (1) the sample data collected are the result of counts;

  (2) the outcome of an experiment is classified into one of two mutually exclusive categories—a "success" or a "failure";

  (3) the probability of a success is the same for each trial; and
  (4) the trials are independent
- The test we will conduct shortly is appropriate when both $n\pi$ and $n(1-\pi)$ are at least 5.
- When the above conditions are met, the normal distribution can be used as an approximation to the binomial distribution

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Test Statistic for Testing a Single Population Proportion

Sample proportion

Hypothesized population proportion

$$z = \frac{p - \pi}{\sqrt{\dfrac{\pi(1-\pi)}{n}}}$$

Sample size

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Test Statistic for Testing a Single Population Proportion - Example

Suppose prior elections in a certain state indicated it is necessary for a candidate for governor to receive at least 80 percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office and plans to conduct a survey of 2,000 registered voters in the northern section of the state. Using the hypothesis-testing procedure, assess the governor's chances of reelection.

Test Statistic for Testing a Single Population Proportion - Example

Step 1: State the null hypothesis and the alternate hypothesis.

H_0: $\pi \geq .80$

H_1: $\pi < .80$

(note: keyword in the problem "*at least*")

Step 2: Select the level of significance.

α = 0.01 as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since the assumptions are met and $n\pi$ and $n(1-\pi) \geq 5$

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Testing for a Population Proportion - Example

Step 4: Formulate the decision rule.

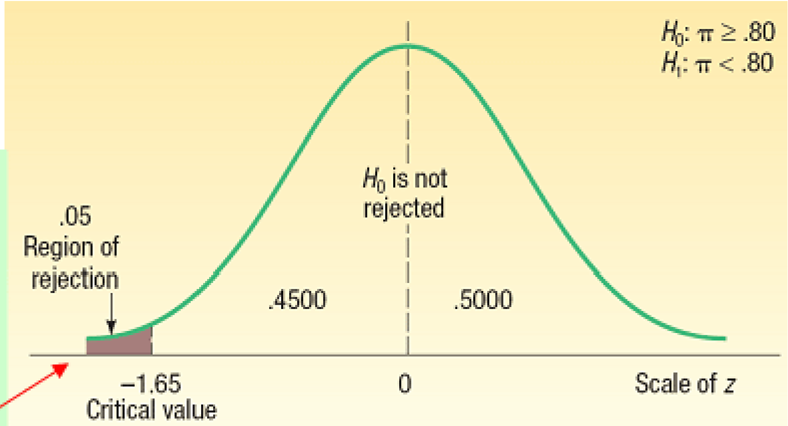
Reject H_0 if $Z < -Z_\alpha$

$$Z < -Z_\alpha$$

$$\frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} < -z_\alpha$$

$$\frac{\frac{1,550}{2,000}-.80}{\sqrt{\frac{.80(1-.80)}{2,000}}} < -1.65$$

$$-2.80 < -1.65$$



Step 5: Make a decision and interpret the result.

The computed value of z (2.80) is in the rejection region, so the null hypothesis is rejected at the .05 level. The difference of 2.5 percentage points between the sample percent (77.5 percent) and the hypothesized population percent (80) is statistically significant. The evidence at this point does not support the claim that the incumbent governor will return to the governor's mansion for another four years.

Type II Error

- Recall Type I Error, the level of significance, denoted by the Greek letter "α", is defined as the probability of rejecting the null hypothesis when it is actually true.

- Type II Error, denoted by the Greek letter "β",is defined as the probability of "accepting" the null hypothesis when it is actually false.

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Type II Error - Example

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: “Take a sample of 100 steel bars. At the .05 significance level if the sample mean strength falls between 9,922 psi and 10,078 psi, accept the lot. Otherwise the lot is to be rejected.”

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Type I and Type II Errors Illustrated

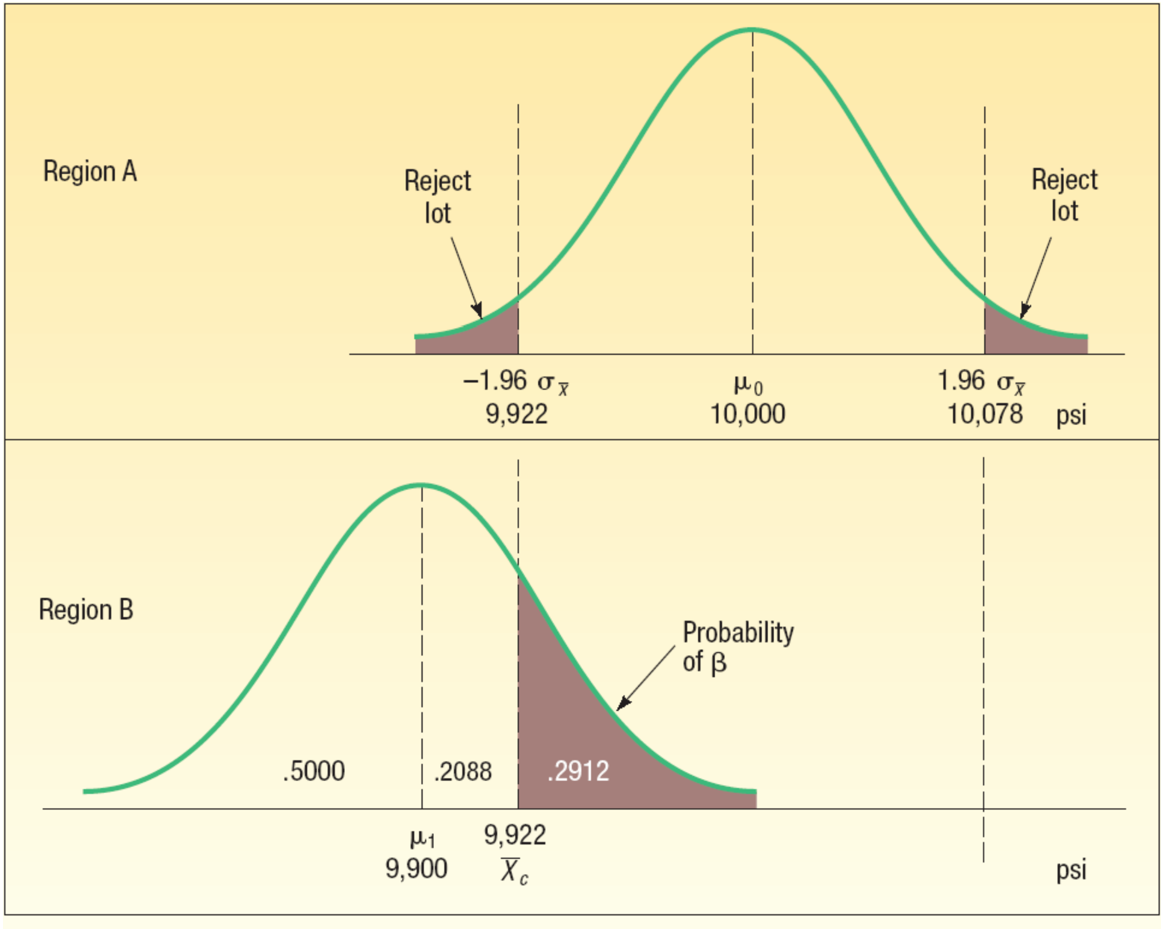

CHART 10–9 Charts Showing Type I and Type II Errors

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Type II Error Computed

The number of standard units (z value) between the mean of the incoming lot (9,900), designated by μ_1, and $\overline{X}_c$, representing the critical value for 9,922, is computed by:

TYPE II ERROR $$z = \frac{\overline{X}_c - \mu_1}{\sigma/\sqrt{n}} \qquad [10\text{–}4]$$

With $n = 100$ and $\sigma = 400$, the value of z is 0.55:

$$z = \frac{\overline{X}_c - \mu_1}{\sigma/\sqrt{n}} = \frac{9{,}922 - 9{,}900}{400/\sqrt{100}} = \frac{22}{40} = 0.55$$

The area under the curve between 9,900 and 9,922 (a z value of 0.55) is .2088. The area under the curve beyond 9,922 pounds is .5000 − .2088, or .2912; this is the probability of making a Type II error—that is, accepting an incoming lot of steel bars when the population mean is 9,900 psi.

Type II Errors For Varying Mean Levels

TABLE 10–4 Probabilities of a Type II Error for $\mu_0 = 10{,}000$ Pounds and Selected Alternative Means, .05 Level of Significance

Selected Alternative Mean (pounds)	Probability of Type II Error (β)	Probability of Not Making a Type II Error ($1 - \beta$)
9,820	.0054	.9946
9,880	.1469	.8531
9,900	.2912	.7088
9,940	.6736	.3264
9,980	.9265	.0735
10,000	—*	—
10,020	.9265	.0735
10,060	.6736	.3264
10,100	.2912	.7088
10,120	.1469	.8531
10,180	.0054	.9946

*It is not possible to make a Type II error when $\mu = \mu_0$.

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End of Chapter 10