Equivalence of NFAs and DFAs

- Every language that can be described by some NFA can also be described by some DFA.
- The DFA in practice has about as many states as the NFA, although it often has more transitions.
- However, in the worst case, the smallest DFA can have 2^n states while the smallest NFA for the same language has only *n* states.

Say, an NFA, $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$.

Our goal is the description of a DFA $D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L(D) = L(N)$

The components of D are as follows:

- The input *alphabets* of the two automata are same.
- The *start state* of *D* is the set containing only the start state of *N*.
- Q_D is the set of subsets (S) of Q_N; i.e., Q_D is the power set (the set of all the subsets of a set) of Q_N. Note that if Q_N has *n* states, then Q_D will have 2^n states. Often, not all these states are accessible from the start sate of Q_D and inaccessible states can be thrown away.
- F_D is the set of subsets (*S)* of Q_N such that $S \cap F_N \neq \emptyset$. That is, F_D is all sets of *N*'s states that include at least once accepting state of *N*.
- To compute $\delta_D(S, a)$ where $S \subseteq Q_N$ and input symbol *a* in Σ, we look at all the states *p* in S, see what states N goes to from *p* on input *a*, and take union of all those states.

$$\delta_D(S, a) = \cup\ \delta_N(p, a) \textit{ where } p \textit{ in } S.$$

Example: Design a nondeterministic finite automaton that accepts all and only the strings of 0's and 1's that end in 01 and find the equivalent DFA as well.

- Transition Diagram:
  [Book: Section 2.3.1- Fig 2.9]

❖ **Description of the equivalent DFA:**

- **Alphabet, Σ**: {0, 1}
- **Start State**: $\{q_0\}$
- **All Possible States**:
  $\emptyset$, $\{q_0\}$, $\{q_1\}$, $\{q_2\}$, $\{q_0, q_1\}$, $\{q_0, q_2\}$, $\{q_1, q_2\}$, $\{q_0, q_1, q_2\}$

- **All Possible Final States**:
  $\{q_2\}$, $\{q_0, q_2\}$, $\{q_1, q_2\}$, $\{q_0, q_1, q_2\}$

- **Transition Table**:

	0	**1**
∅	∅	∅
→ **$\{q_0\}$**	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_1\}$	∅	$\{q_2\}$
***$\{q_2\}$**	∅	∅
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
***$\{q_0, q_2\}$**	$\{q_0, q_1\}$	$\{q_0\}$
***$\{q_1, q_2\}$**	∅	$\{q_2\}$
***$\{q_0, q_1, q_2\}$**	$\{q_0, q_1\}$	$\{q_0, q_2\}$

The complete subset construction

- **Transition Diagram**:
  [Book: Section 2.3.5- Fig 2.14]

Book: Introduction to Automata Theory, Languages and Computation [3rd Ed.]