<u>Consider the polynomial:</u>
<u>$A_7x^7 + A_6x^6 + A_5x^5 + A_4x^4 + A_3x^3 + A_2x^2 + A_1x^1 + A_0 = 0$ the order of the polynomial is 7 so possible number of roots are 7.</u>

Determining All Possible Roots:
All the methods discussed to date is estimate only one root. If we want to find all the roots in the given interval, then one alternative is to map a function graph and then identify different independent intervals that keep the roots together. It is possible to use these intervals to find the different roots.
Another approach is to use an incremental search strategy that covers the entire root interval. It means that even after the first root has been identified, the quest for a root continues. The method is to start at one end of the interval, say at point a, and then search for a root at each incremental interval until the other end is checked, say point b. The following algorithm describes the steps for implementing an incremental search technique using the bisection method for locating all roots.

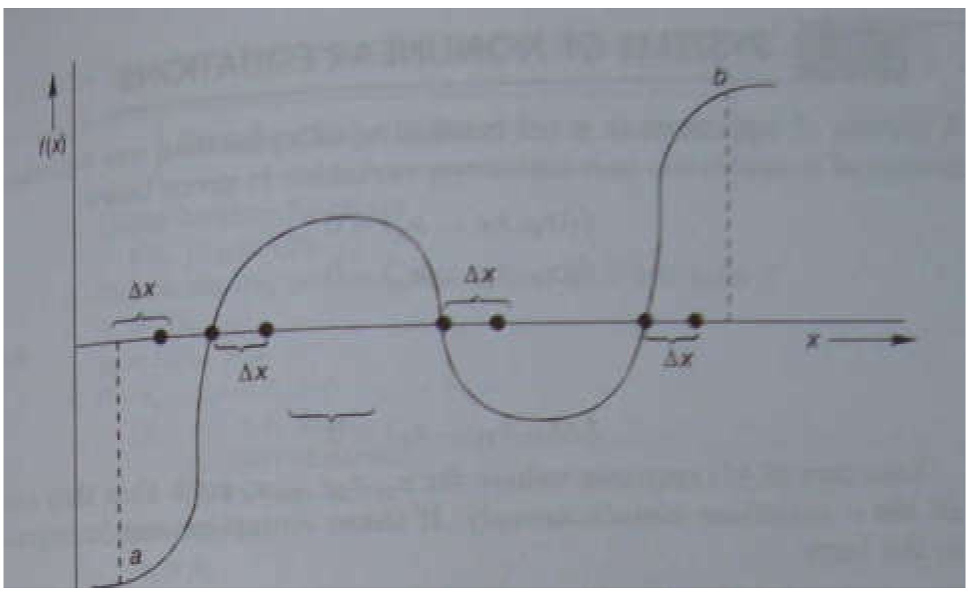

Figure 1: Incremental Search for All Possible Roots [1]

Algorithm
1. Choose lower limit a and upper limit b of the interval covering all the roots.
2. Decide the size of the increment interval Δx
3. set $x_1 = a$ and $x_2 = x_1 + \Delta x$
4. Compute $f_1 = f(x_1)$ and $f_2 = f(x_2)$
5. If $(f_1 * f_2) > 0$, then the interval does not bracket any root and go to step 9
6. Compute $x_0 = (x_1 + x_2)/2$ and $f_0 = f(x_0)$
7. If $(f_1 * f_2) < 0$, then set $x_2 = x_0$
   Else set $x_1 = x_0$ and $f_1 = f_0$
8. If $| (x_2 - x_1)/ x_2| < E$, then
   root = $(x_1+x_2) / 2$
   write the value of root
   go to step 9
   Else

go to step 6

9. If $x_2 < b$, then set $a = x_2$ and go to step 3
10. Stop.

- A major problem is to decide the increment size. A small size may mean more iterations and more execution time. If the size is large, then there is a possibility of missing the closely spaced roots.

Roots of Polynomials

The number of real roots can be obtained using Descarte's rule of sign. This rule states that

1. The number of positive real roots is equal (or less than by an even integer) to the number of sign changes in the co-efficient of the equation.
2. The number of negative real roots is equal (or less than by an even integer) to the number of sign changes in the co-efficient if x is replaced by –x.

Multiple Roots by Newton's Methods

A polynomial functions contains a multiple root at a point when the function is tangential to the x-axis at that point. For example, the equations: $x^3 - 7x^2 + 15x - 9 = 0$ has a double root at $x = 3$ (see figure 2).

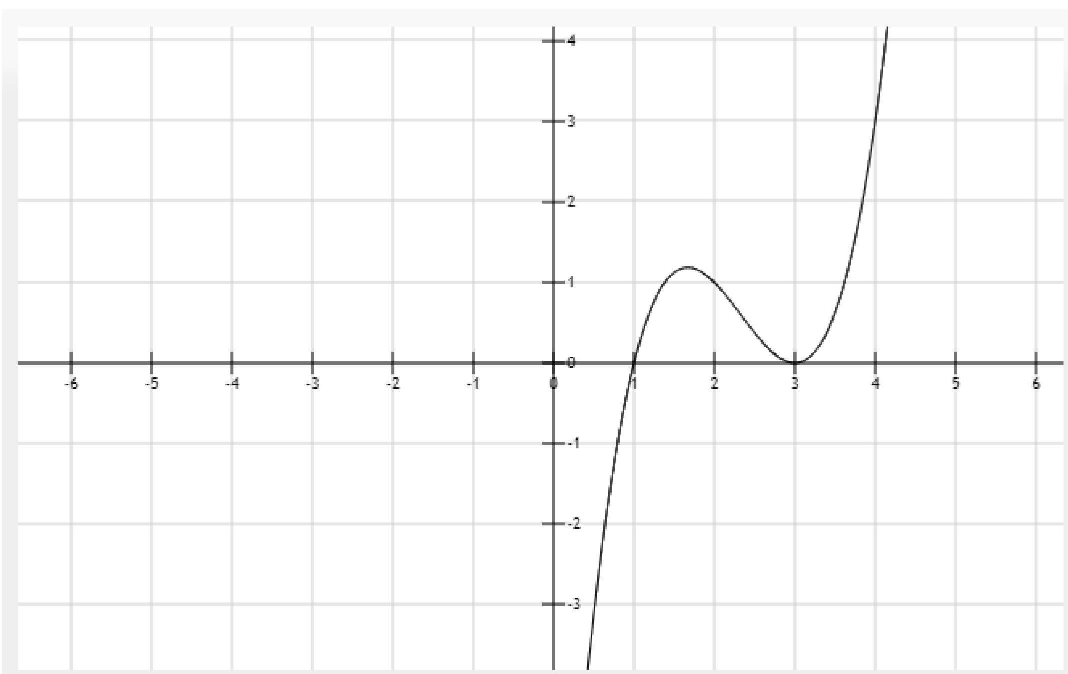

Figure 2

We can locate all real roots of a polynomial by repeatedly applying Newton-Raphson method and polynomial deflation to obtain polynomials of lower and lower degree. The deflation process is performed (n-1) times where n is the degree of the given polynomial. After (n-1) deflations, the quotient is the linear polynomial of type:

$a_1x + a_0 = 0$, and therefore the final root is given by, $x_r = a_0/ a_1$

Algorithm: Multiple Roots by Newton-Raphson Method

1. Obtain degree and co-efficient of polynomial (n and a_i).
2. Decide an initial estimate for the first root (x_0) and error criterion, E.
   Do while n >1
3. Find the root using Newton-Raphson algorithm
   $x_r = x_0 - f(x_0) / f'(x_0)$
4. Root (n) = x_r
5. Deflate the polynomial using synthetic division algorithm and make the factor polynomial a the new polynomial of order n-1.
6. Set $x_0 = x_r$ [Initial value of the new root]
   End of Do
7. Root (1) = $- a_0 / a_1$
8. Stop.

References:

1. **BalaGurushamy, E.** *Numerical Methods.* New Delhi : Tata McGraw-Hill, 2000.

2. **Steven C.Chapra, Raymon P. Cannale.** *Numerical Methods for Engineers.* New Delhi : Tata McGRAW-HILL, 2003. ISMN 0-07-047437-0.