Date of Examination: 27/5/2021

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department: Computer Science and Engineering
Program: B.Sc. in Computer Science and Engineering
Semester Final Examination: Spring 2020
Year: 2nd Semester: 2nd
Course Number: CSE2201
Course Name: Numerical Methods

Time: 3 (Three) Hours **Full Marks: 60**

Use single answer script

Instructions:	i)	Answer script should be hand written and should be written in A4 white paper. You must submit the hard copy of this answer script to the Department when the university reopens.
	ii)	You must write the following information at the top page of each answer script: **Department:** **Program:** **Course no:** **Course Title:** **Examination:** **Semester (Session):** **Student ID:** **Signature and Date:**
	iii)	Write down Student ID, Course number and put your signature on top of every single page of the answer script.
	iv)	Write down page number at the bottom of every page of the answer script.
	v)	Upload the scan copy of your answer script in PDF format through provided **google form** at the respective course site (i.e., **google classroom)** using institutional email within the allocated time. Uploading clear and readable scan copy (uncorrupted) is your responsibility and you must cover all the pages of your answer script. However, for clear and readable scan copy of the answer script student should use only one side of a page for answering the questions.
	vi)	You must avoid **plagiarism,** maintain **academic integrity, and ethics**. You are not allowed to take any help from another individual and if taken so can result in stern disciplinary actions from the university authority.
	vii)	Marks allotted are indicated in the **right margin.**
	viii)	Necessary **charts/tables** are attached at the end of the question paper. You may use graph papers where necessary.
	ix)	Assume any reasonable data if needed.
	x)	Symbols and characters have their usual meaning.
	xi)	Before uploading, rename the PDF file as **CourseNo_StudentID.pdf** e.g., CSE2201_180204001.pdf

The answer script **(one single PDF file)** must be uploaded at designated location in the provided **Google Form link** available in the Google classroom.

There are 7 (Seven) Questions Answer any 5 (Five).

[Note: Your answer must contain detail steps of computation for each problem, marks will be deducted if not.]

Question 1. [Marks: 12]

a) Estimate the relative error of the final result in the evaluation of $w = \frac{x^2 y}{z}$. Given that $x = 1.2, y = 25.6 \; and \; z = 4.5 \; and \; \Delta x = \Delta y = \Delta z = 1$. **[3]**

b) Determine the approximate root of the equation $x^3 - 9x + 1 = 0$ by iterating 3 times using *Bisection Method* considering the root lies between 2 and 3. **[5]**

c) Discuss the situations with example where the fixed –point iteration process may not converge. **[4]**

Question 2. [Marks: 12]

a) If $X = 2.536$, find the absolute error and relative error when, **[3]**
   a) X is rounded and
   b) X is truncated to two decimal digits.

b) Solve the equation $e^{-x} - x = 0$ by using the Secant Method, starting with $x_0 = 0$ and $x_1 = 1$, accurate to 10^{-4}. **[5]**

c) Analyze the convergence rate of Newton-Raphson Method. **[4]**

Question 3. [Marks: 12]

a) Explain with an example the difference between the principal of Gauss-Seidel and Jacobi methods. **[3]**

b) Given the system of Linear Equations: **[5]**

$$x_1 + 2x_2 - 3x_3 = 4$$
$$2x_1 + 4x_2 - 6x_3 = 8$$
$$x_1 - 2x_2 + 5x_3 = 4$$

Solve by the Gauss Jordan Method. Show all steps of the computation.

c) Apply LU factorization method to decompose the following matrix A into Lower Triangular Matrix (L) and Upper Triangular Matrix (U). **[4]**

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

Question 4. [Marks: 12]

a) Prove the following: **[3]**

I. $(1 + \Delta)(1-\nabla) = 1$

II. $\Delta - \nabla = \Delta\nabla$

Here, Δ denotes forward difference operator and ∇ denotes backward operator.

b) Using Lagrange's interpolation formula find a polynomial which passes the points (0, -12), (1,0), (3,6), (4,12). **[5]**

c) Construct divide difference table up to 5th order for the following data table: **[4]**

X	0.1	0.3	0.5	0.7	0.9	1.1	1.3
f(x)	0.003	0.06A	0.148	0.248	0.370	0.518	0.697

[**Note: The symbol 'A' in the table above is the Last Digit (right most digit) of Your Student ID**]

Question 5. [Marks: 12]

a) Describe the trapezoidal method of computing Integrals. **[3]**

b) Evaluate the following integral with trapezoidal segments of n = 1, 2 4 and correct upto four decimal places using Romberg Integration: **[5]**

$$\int_0^1 \frac{dx}{1 + x^2}$$

Compare the results obtained from the method with the correct value 0.785398

c) Estimate the value of $y(0.02)$ using Euler's Method to solve the Ordinary Differential Equation $\frac{dy}{dx} = x^3 + y, y(0) = 1$ with step size of 0.01. **[4]**

Question 6. [Marks: 12]

a) Derive a three-point forward difference formula for estimating the first derivative of tabulated function. **[4]**

b) The following table gives the distance travelled by a bus at intervals of 2 minutes. **[4+4 =8]**

$t\ (s)$	0	2	4	6	8	10	12	14	16
$y(km)$	0	0.25	1	2.A	4	6.5	8.5	11	13

Estimate the Bus's:

i. Velocity ($\frac{dx}{dt}$) at $t = 5\ s$ and,

ii. Acceleration ($\frac{dv}{dt}$) at $t = 5\ s$.

[**Note: The symbol 'A' in the table above is the Last Digit (right most digit) of Your Student ID**]

Question 7. [Marks: 12]

a) Evaluate the normal equation for finding the parameters a and b to fit data to the function $y = a + bx$, using least square regression. **[4]**

b) The following data in the table represents the bacteria growth in liquid culture over of number of days: **[8]**

Day	0	4	8	12	16	20
Amount * 10^6	67.38	74.67	82.74	91.69	101.60	112.58

Find a 2nd order polynomial using least square regression to fit the above data. (Note: You are allowed to use Cramer rule to evaluate the coefficients)