C. Inference Using Bayes’ Rule

i) From the definition of Conditional Probabilities and Commutativity of Conjunction we can derive the following Product rules:

$P(a \wedge b) = P(a \mid b) * P(b)$, where $P(b) > 0$, and
$P(a \wedge b) = P(b \mid a) * P(a)$, where $P(a) > 0$.

ii) From the product rules above we get the **Bayes’ rule/ law/ theorem**,

$P(b \mid a) = P(a \mid b) * P(b) / P(a)$, $P(a) > 0$ and $P(b) > 0$,

which underlies all modern AI systems for probabilistic inference.
[Probability of hypothesis given the evidence/ effect]

Example 1.

Say, the following facts are known.

‘toothache’ is caused by ‘cavity’ 60% of times : $P(\text{toothache} \mid \text{cavity}) = 0.6$
1 in 10 patients investigated has ‘cavity’ : $P(\text{cavity}) = 1/10 = 0.1$
1 in 4 patients investigated has ‘toothache’ : $P(\text{toothache}) = ¼ = 0.25$

We can now derive: $P(\text{cavity} \mid \text{toothache}) =$

Example 2.

Given,

$P(Water \mid Coal) = 0.4$,

$P(Coal) = 0.2$,

$P(Water) = 0.8$.

So, $P(Coal \mid Water) =$

D. Inference using Joint Probabilities of Consistent Models of the Environment

Recall the Monster that Smells:

? 1,3			
OK S 1,2	? 2,2		
OK 1,1	OK S 2,1	? 3,1	

- ✓ 3 reachable squares[fringe/ frontier]: [1,3], [2,2], [3,1]; 3 random variables: $M_{1,3}$, $M_{2,2}$, $M_{3,1}$;
- ✓ Pure logical inference can help no more;
- ✓ Probabilistic agent can do much better after computing $P(M_{1,3})$, $P(M_{2,2})$, $P(M_{3,1})$;
- ✓ We assume that OK means 'No Monster', and independent probability of a Monster at any unknown cell is 0.2.

Consistent models of the Environment for fringe variables $M_{1,3}$, $M_{2,2}$, $M_{3,1}$:

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i) $P(M_{1,3} \mid \text{evidence}) = \langle P(m_{1,3} \mid \text{evidence}), P(\neg m_{1,3} \mid \text{evidence}) \rangle$

ii) $P(m_{1,3} \mid \text{evidence}) =$

$$\frac{\text{Sum of the joint probabilities of models where } m_{1,3} \text{ holds}}{\text{Sum of the joint probabilities of all consistent models}} \approx 0.31$$

iii) $P(\neg m_{1,3} \mid \text{evidence}) \approx 0.69$

iv) $P(M_{1,3} \mid \text{evidence}) \approx \langle 0.31, 0.69 \rangle = \langle 31\%, 69\% \rangle$

$P(M_{2,2} \mid \text{evidence}) \approx \langle 0.86, 0.14 \rangle = \langle 86\%, 14\% \rangle$ [Self study]

$P(M_{3,1} \mid \text{evidence}) \approx \langle 0.31, 0.69 \rangle = \langle 31\%, 69\% \rangle$ [Self study]

So, which cell is safer to move to? M (1,3) AND M (3,1)