

Date: 15/12/2022 | Time: 35 minutes

1. **[5 marks]** State the differences between parallel and perspective projection
2. Consider the following parameters for an orthographic ray-tracing:

Camera frame: $\mathbf{E} = [-1, 6, 15]^T$, $\mathbf{U} = [1, 0, 0]^T$, $\mathbf{V} = [0, 1, 0]^T$, $\mathbf{W} = [0, 0, 1]^T$

Image plane: $l = -10$, $r = 10$, $t = 15$, $b = -15$

Raster image resolution: 10×12

Sphere: $(x-5)^2 + (y-12)^2 + z^2 = 100$

A ray (with length = 25) is generated from the upper right corner pixel of the raster image.

- a) **[5 Marks]** Find the position of the ray origin on the image plane
- b) **[10 Marks]** Determine whether there will be any ray-sphere intersection

Solution:

1. Go through the book/slides
2. a) $[8, 19.75, 15]$
b) D: 77343.75, 2 intersection point exists

Date: 15/12/2022 | Time: 35 minutes

1. **[5 marks]** State the differences between raster and vector images
2. Consider the following parameters for an orthographic ray-tracing:

Camera frame: $\mathbf{E} = [3, 6, 15]^T$, $\mathbf{U} = [1, 0, 0]^T$, $\mathbf{V} = [0, 1, 0]^T$, $\mathbf{W} = [0, 0, 1]^T$

Image plane: $l = -15$, $r = 15$, $t = 12$, $b = -12$

Raster image resolution: 8×10

Sphere: $(x+5)^2 + y^2 + (z-5)^2 = 64$

A ray (with length = 25) is generated from the lower left corner pixel of the raster image.

- a) **[5 Marks]** Find the position of the ray origin on the image plane
- b) **[10 Marks]** Determine whether there will be any ray-sphere intersection

Solution:

1. Go through the book/slides
2. a) $[-10.125, -4.8, 15.]$
b) D: 36735.93749999994, 2 intersection point exists

1. Increase the size of a square OACB by 100 percent so that point B remains fixed; four vertices of the square are: O(2,2), A(2,10), C(10,10) and B(10,2). You must -
 - a. **[2 marks]** Mention the steps to perform the task.
 - b. **[10 marks]** Determine the composite transformation matrix.
 - c. **[3 marks]** Find the final vertices.

2. **[5 Marks]** Explain the problems associated with it if homogeneous coordinates were not used in matrix transformation.

Solution:

1. Steps:

- a) Translate(-10, -2)
- b) Scale(2, 2)
- c) Translate(10, 2)

<p>T:</p> $\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ <p>S:</p> $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ <p>TI:</p> $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	<p>M:</p> $\begin{bmatrix} 2 & 0 & -10 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ <p>Composite Matrix: $T^{-1} * R * S * P$</p> <p>P':</p> $\begin{bmatrix} -6 & -6 & 10 & 10 \\ 2 & 18 & 18 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$
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1. Reflect a rectangle OACB along a line $\sqrt{3}y = x$. Four vertices of the rectangle are: A(4, 0), B(8, 0), C(8, 2) and O(4,2). You must -
 - a. **[2 marks]** Mention the steps to perform the task.
 - b. **[10 marks]** Determine the composite transformation matrix.
 - c. **[3 marks]** Find the final vertices.
2. **[5 Marks]** State the properties of affine transformation.

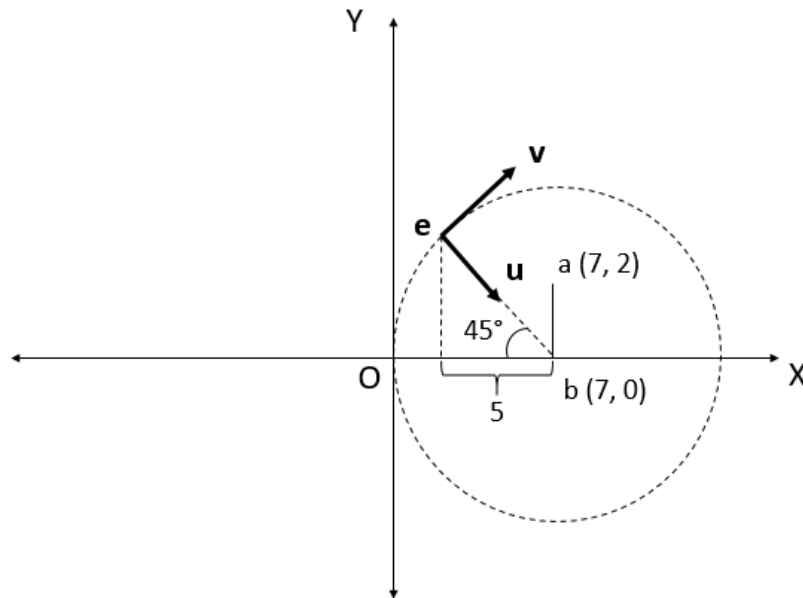
Solution:

1. Steps:

- a) Rotation(60)
- b) Reflect by y-axis
- c) Rootation(-60)

<p>R:</p> <pre>0.5 -0.87 0 0.87 0.5 0 0 0 1</pre> <p>Ref:</p> <pre>-1 0 0 0 1 0 0 0 1</pre> <p>RI:</p> <pre>0.5 0.87 0 -0.87 0.5 0 0 0 1</pre>	<p>M:</p> <pre>0.5 0.87 0.0 0.87 -0.5 0.0 0.0 0.0 1.0</pre> <p>Composite Matrix: R-1 * Ref * R * P</p> <p>P' :</p> <pre>2.0 4.0 5.73 3.73 3.46 6.93 5.93 2.46 1.0 1.0 1.0 1.0</pre>
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1. Here (in the figure), origin O and basis $\{x,y\}$ construct a 2D canonical coordinate system. Within this, line ab is our model (P_{xy}). Now, we want to view it from a new 2D camera with eye e and basis $\{u,v\}$; which is rotated by -45 degrees around b . Assume that, u is the viewing direction and b is the center of the circle.



- a. **[15 marks]** Determine the canonical-to-frame matrix
- b. **[5 marks]** Calculate and plot P_{uv} .

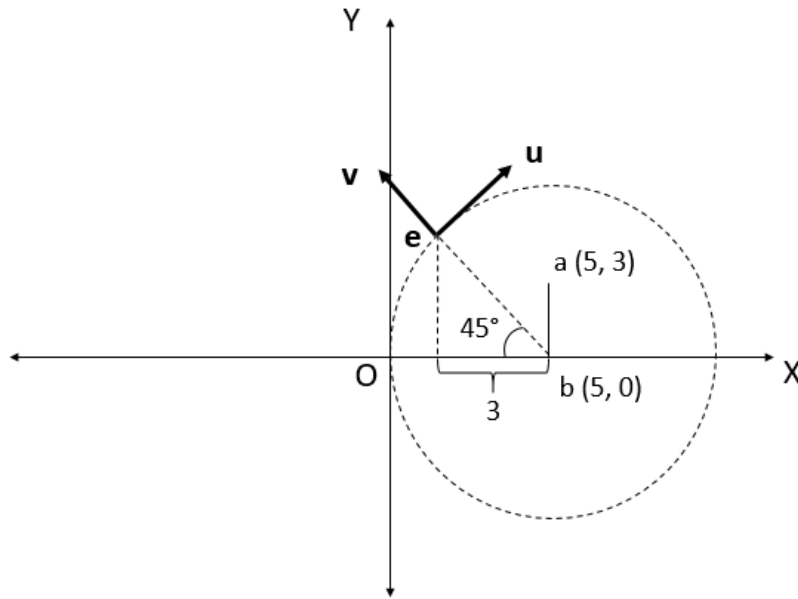
Solution:

- a) Canonical-to-frame matrix:

$$\begin{bmatrix} 0.71 & -0.71 & 2.06 \\ 0.71 & 0.71 & -4.9 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

- b) $P_u: (5.61, \quad 1.49, \quad 1.0)$
 $P_v: (7.03, \quad 0.07, \quad 1.0)$

1. Here (in the figure), origin O and basis $\{x,y\}$ construct a 2D canonical coordinate system. Within this, line ab is our model (P_{xy}). Now, we want to view it from a new 2D camera with eye e and basis $\{u,v\}$; which is rotated by $+45$ degrees around b . Assume that, u is the viewing direction and b is the center of the circle.



- [15 marks]** Determine the canonical-to-frame matrix
- [5 marks]** Calculate and plot P_{uv} .

Solution:

- Canonical-to-frame matrix:

0.71	0.71	-4.26
-0.71	0.71	-1.42
0.0	0.0	1.0

- $P_u: (1.42, -2.84, 1.0)$
 $P_v: (-0.71, -4.97, 1.0)$

1. Apply the midpoint line drawing algorithm to draw a line from (-1, -2) to (6, -5) and plot the points. Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=-x; swap(x, y); plot(-y, x)	(4) x=-x; plot(-x, y)
(5) x=-x; y=-y; plot(-x, -y)	(6) x=-x; y=-y; swap(x, y); plot(-y, -x)	(7) y=-y; swap(x, y); plot(y, -x)	(8) y=-y; plot(x, -y)

- a) **[15 marks]** Show the values of the decision variables and the points for each step (in a tabular format).
b) **[5 marks]** Plot the final points

Solution:

```

Octant:8
moves x y d
-----
-1 -2 -1
E 0 -2 5
NE 1 -3 -3
E 2 -3 3
NE 3 -4 -5
E 4 -4 1
NE 5 -5 -7
E 6 -5 -1
    
```

1. Apply the midpoint algorithm to draw a circle's portions of circumference centered at (-2,-1) on the 3rd, 4th, 5th and 6th octant with radius 6.

a) [15 marks] For each step, show the values of the decision variables and the points (in a tabular format).

b) [5 marks] Plot the final points.

Solution:

<p>X Y d</p> <p>0 6 -5</p> <p>1 6 -2</p> <p>2 6 3</p> <p>3 5 0</p> <p>4 4 1</p>	<p>octant 3</p> <p>x: [-2 -3 -4 -5 -6]</p> <p>y: [5 5 5 4 3]</p> <p>octant 4</p> <p>x: [-8 -8 -8 -7 -6]</p> <p>y: [-1 0 1 2 3]</p> <p>octant 5</p> <p>x: [-8 -8 -8 -7 -6]</p> <p>y: [-1 -2 -3 -4 -5]</p> <p>octant 6</p> <p>x: [-2 -3 -4 -5 -6]</p> <p>y: [-7 -7 -7 -6 -5]</p>
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1. Apply the midpoint line drawing algorithm to draw a line from (2, -3) to (-5, -6) and plot the points. Necessary adjustments of the original algorithm for different octants are provided below:

(1) plot(x, y)	(2) swap(x, y); plot(y, x)	(3) x=-x; swap(x, y); plot(-y, x)	(4) x=-x; plot(-x, y)
(5) x=-x; y=-y; plot(-x, -y)	(6) x=-x; y=-y; swap(x, y); plot(-y, -x)	(7) y=-y; swap(x, y); plot(y, -x)	(8) y=-y; plot(x, -y)

- a) **[15 marks]** Show the values of the decision variables and the points for each step (in a tabular format).
- b) **[5 marks]** Plot the final points

Solution:

```

Octant:5
moves x y d
-----
      2 -3 -1
E   1 -3  5
NE  -0 -4 -3
E   -1 -4  3
NE  -2 -5 -5
E   -3 -5  1
NE  -4 -6 -7
E   -5 -6 -1
    
```

1. Apply the midpoint algorithm to draw a circle's portions of circumference centered at (3,-2) on the 1st, 2nd, 7th and 8th octant with radius 7.
 - a) **[15 marks]** For each step, show the values of the decision variables and the points (in a tabular format).
 - b) **[5 marks]** Plot the final points.

Solution:

<p>X Y d</p> <p>0 7 -6</p> <p>1 7 -3</p> <p>2 7 2</p> <p>3 6 -3</p> <p>4 6 6</p> <p>5 5 7</p>	<p>octant 1</p> <p>x: [10 10 10 9 9 8]</p> <p>y: [-2 -1 0 1 2 3]</p> <p>octant 2</p> <p>x: [3 4 5 6 7 8]</p> <p>y: [5 5 5 4 4 3]</p> <p>octant 7</p> <p>x: [3 4 5 6 7 8]</p> <p>y: [-9 -9 -9 -8 -8 -7]</p> <p>octant 8</p> <p>x: [10 10 10 9 9 8]</p> <p>y: [-2 -3 -4 -5 -6 -7]</p>
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