Ex. Find the angles which the vector $\bar{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the co-ordinate axes.

Sol^n: Let α, β, γ be the angles which A makes with the positive x, y, z axes respectively.

Now $\bar{A} \cdot \hat{i} = |A| \cdot (1) \cos\alpha = \sqrt{9+36+4} \cdot \cos\alpha = 7 \cos\alpha$

Again $(3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot \hat{i} = 3$

Then $\cos\alpha = \dfrac{\bar{A} \cdot \hat{i}}{7} = \dfrac{3}{7} = 0.4286$

$\therefore \alpha = \cos^{-1}(0.4285) = 64.6°$ approximately.

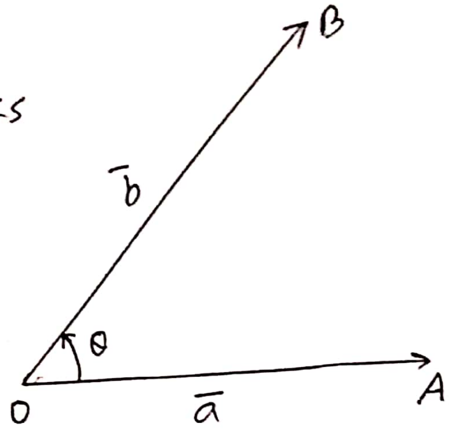
Similarly, $\cos\beta = -6/7$, $\beta = 149°$

and $\cos\gamma = 2/7$, $\gamma = 73.4°$.

Note: The cosines of α, β and γ are called the direction cosines of $\bar{A}$.

Scalar or Dot product of two vectors:

The scalar or dot product of two vectors $\bar{a}$ and $\bar{b}$ is defined to be $|\bar{a}||\bar{b}|\cos\theta$ (a scalar) where θ is the angle between $\bar{a}$ and $\bar{b}$. That is

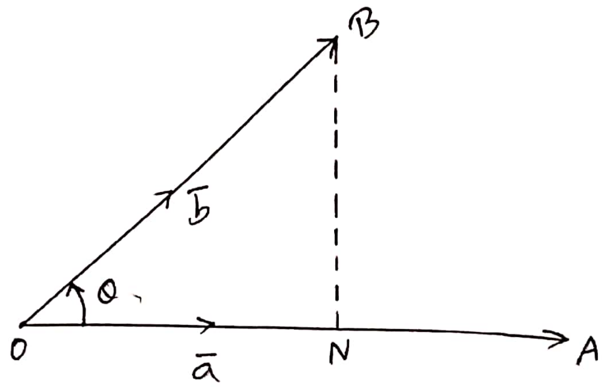


$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$$

Note: The scalar product is commutative $\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$

Geometrical interpretation:

The scalar product of two vectors is the product of one vector and the length of the projection of the other in the direction of the first.



Let $\overrightarrow{OA} = \bar{a}$ and $\overrightarrow{OB} = \bar{b}$

Then $\bar{a}\cdot\bar{b} = |\overrightarrow{OA}|\cdot|\overrightarrow{OB}|\cos\theta$

$$= OA\cdot OB\cdot\frac{ON}{OB}$$

$$= OA\cdot ON$$

$= (\text{Length of } \bar{a})(\text{Projection of } \bar{b} \text{ along } \bar{a})$

Ex. 1 Find the projection of the vector $\bar{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\bar{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

Soln: An unit vector in the direction of $\bar{B}$ is $\hat{b} = \frac{\bar{B}}{|\bar{B}|}$

$$= \frac{4\hat{i} - 4\hat{j} + 7\hat{k}}{\sqrt{(4)^2 + (-4)^2 + (7)^2}}$$

$$= \frac{4\hat{i} - 4\hat{j} + 7\hat{k}}{\sqrt{81}} = \frac{4\hat{i} - 4\hat{j} + 7\hat{k}}{9}$$

$$= \frac{4}{9}\hat{i} - \frac{4}{9}\hat{j} + \frac{7}{9}\hat{k}$$

$\therefore$ Projection of $\bar{A}$ on the vector $\bar{B} = \bar{A} \cdot \hat{b}$

$$= (\hat{i} - 2\hat{j} + \hat{k}) \cdot \left(\frac{4}{9}\hat{i} - \frac{4}{9}\hat{j} + \frac{7}{9}\hat{k}\right)$$

$$= (1)\left(\frac{4}{9}\right) + (-2)\left(-\frac{4}{9}\right) + (1)\left(\frac{7}{9}\right)$$

$$= \frac{19}{9}. \text{ (Ans)}$$

Note: $\hat{i} \cdot \hat{i} = |\hat{i}| \cdot |\hat{i}| \cos 0^\circ = 1 \cdot 1 \cdot 1 = 1$

$\hat{i} \cdot \hat{j} = |\hat{i}| \cdot |\hat{j}| \cos 90^\circ = 1 \cdot 1 \cdot 0 = 0$.

The cross or vector product:

The cross or vector product of $\bar{A}$ and $\bar{B}$ is a vector $\bar{C} = \bar{A} \times \bar{B}$. The magnitude of $\bar{A} \times \bar{B}$ is defined as the product of the magnitudes of $\bar{A}$ and $\bar{B}$ and the sine of the angle θ between them. The direction of the vector $\bar{C} = \bar{A} \times \bar{B}$ is perpendicular to the plane of $\bar{A}$ and $\bar{B}$ and such that $\bar{A}, \bar{B}$ and $\bar{C}$ form a right-handed system. In symbols, $\bar{A} \times \bar{B} = AB \sin\theta \, \hat{u}, \quad 0 \leq \theta \leq \pi$

where $\hat{u}$ is a unit vector indicating the direction of $\bar{A} \times \bar{B}$.

* If $\bar{A} = \bar{B}$, or if $\bar{A}$ is parallel to $\bar{B}$, then $\sin\theta = 0$ and we define $\bar{A} \times \bar{B} = \bar{0}$.

* $\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$ where $\bar{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k}$ and

$\bar{B} = B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}$

c

Example 1: Determine a unit vector perpendicular to the plane of $\bar{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\bar{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.

Soln: $\bar{A} \times \bar{B}$ is a vector perpendicular to the plane of $\bar{A}$ and $\bar{B}$.

$$\therefore \bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$$

$$= 15\hat{i} - 10\hat{j} + 30\hat{k}$$

A unit vector perpendicular to the plane of $\bar{A}$ and $\bar{B}$, that is parallel to $\bar{A} \times \bar{B}$ is $\frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|}$

$$= \frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{\sqrt{(15)^2 + (-10)^2 + (30)^2}}$$

$$= \frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{\sqrt{225 + 100 + 900}}$$

$$= \frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{35}$$

$$= \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Another unit vector opposite in direction is $-\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$. (Ans)