Coin Change 2

Solution

You are given an integer array `coins` representing coins of different denominations and an integer `amount` representing a total amount of money.

Return *the number of combinations that make up that amount*. If that amount of money cannot be made up by any combination of the coins, return `0`.

You may assume that you have an infinite number of each kind of coin.

The answer is **guaranteed** to fit into a signed **32-bit** integer.

Example 1:

```
Input: amount = 5, coins = [1,2,5]
Output: 4
Explanation: there are four ways to make up the amount:
5=5
5=2+2+1
5=2+1+1+1
5=1+1+1+1+1
```

Example 2:

```
Input: amount = 3, coins = [2]
Output: 0
Explanation: the amount of 3 cannot be made up just with coins of 2.
```

Example 3:

```
Input: amount = 10, coins = [10]
Output: 1
```

Constraints:

- `1 <= coins.length <= 300`
- `1 <= coins[i] <= 5000`
- All the values of `coins` are **unique**.
- `0 <= amount <= 5000`

C++

```cpp
class Solution {
public:
    int change(int amount, vector<int>& coins) {
        
    }
};
```

Solution

Approach 1: Dynamic Programming

Template

This is a classical dynamic programming problem.

Here is a template one could use:

- Define the base cases for which the answer is obvious.
- Develop the strategy to compute more complex case from more simple one.
- Link the answer to base cases with this strategy.

Example

Let's pic up an example: amount = 11, available coins - 2 cent, 5 cent and 10 cent. Note, that coins are unlimited.

Base Cases: No Coins or Amount = 0

If the total amount of money is zero, there is only one combination: to take zero coins.

Another base case is no coins: zero combinations for `amount > 0` and one combination for `amount == 0`.

2 Cent Coins

Let's do one step further and consider the situation with one kind of available coins: 2 cent.

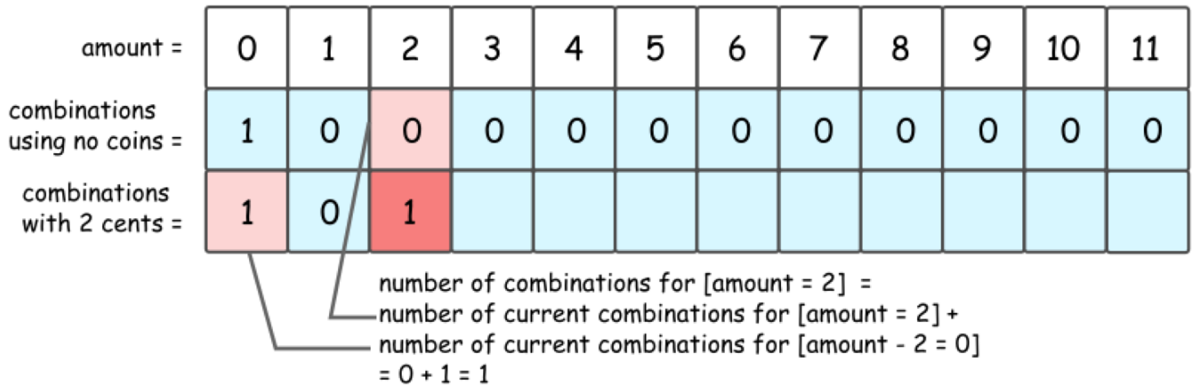
It's quite evident that there could be 1 or 0 combinations here, 1 combination for even amount and 0 combinations for the odd one.

> The same answer could be received in a recursive way, by computing the number of combinations for all amounts of money, from 0 to 11.

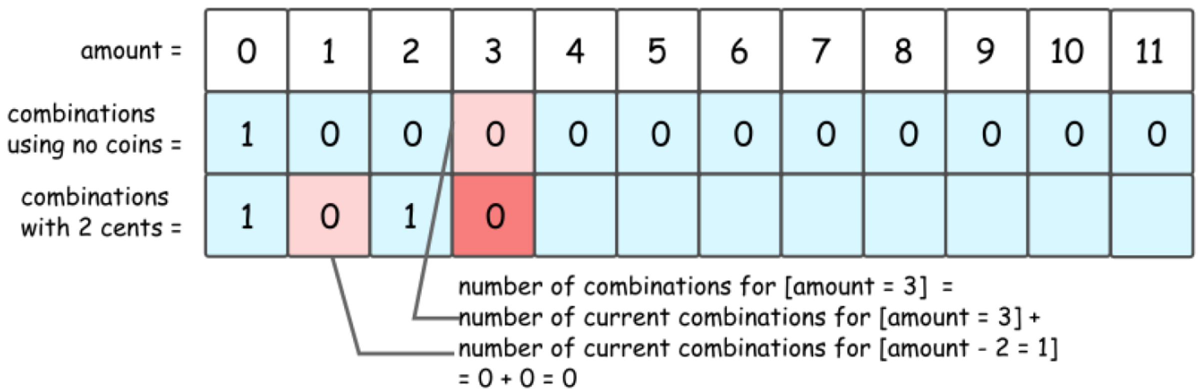
First, that's quite obvious that all amounts less than 2 are *not* impacted by the presence of 2 cent coins. Hence for `amount = 0` and for `amount = 1` one could reuse the results from the figure 2.

Starting from `amount = 2`, one could use 2 cent coins in the combinations. Since the amounts are considered gradually from 2 to 11, at each given moment one could be sure to add not more than one coin to the previously known combinations.

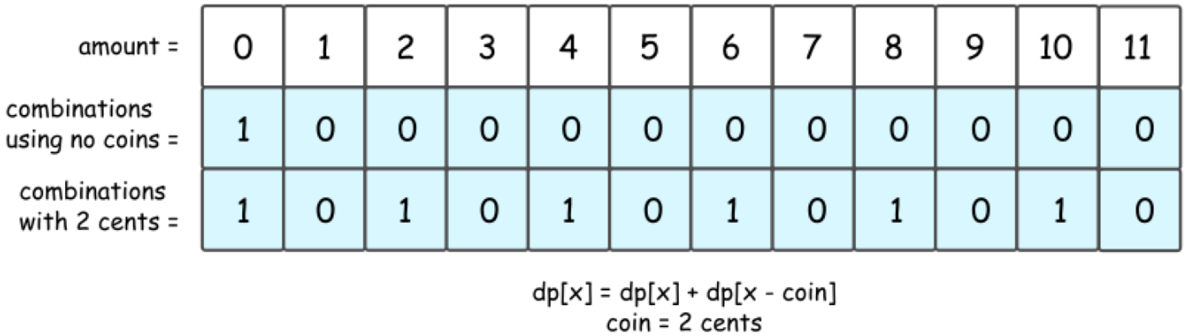
So let's pick up 2 cent coin, and use it to make up `amount = 2`. The number of combinations with this 2 cent coin is a number combinations for `amount = 0`, i.e. 1.



Now let's pick up 2 cent coin, and use it to make up `amount = 3`. The number of combinations with this 2 cent coin is a number combinations for `amount = 1`, i.e. 0.

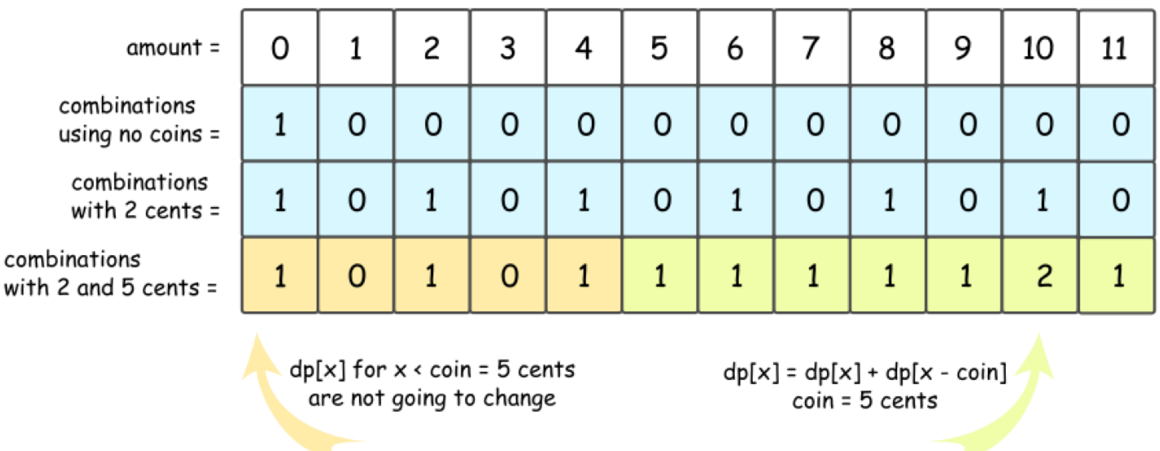


That leads to DP formula for number of combinations to make up the `amount = x`: `dp[x] = dp[x] + dp[x - coin]`, where `coin = 2 cents` is a value of coins we're currently adding.



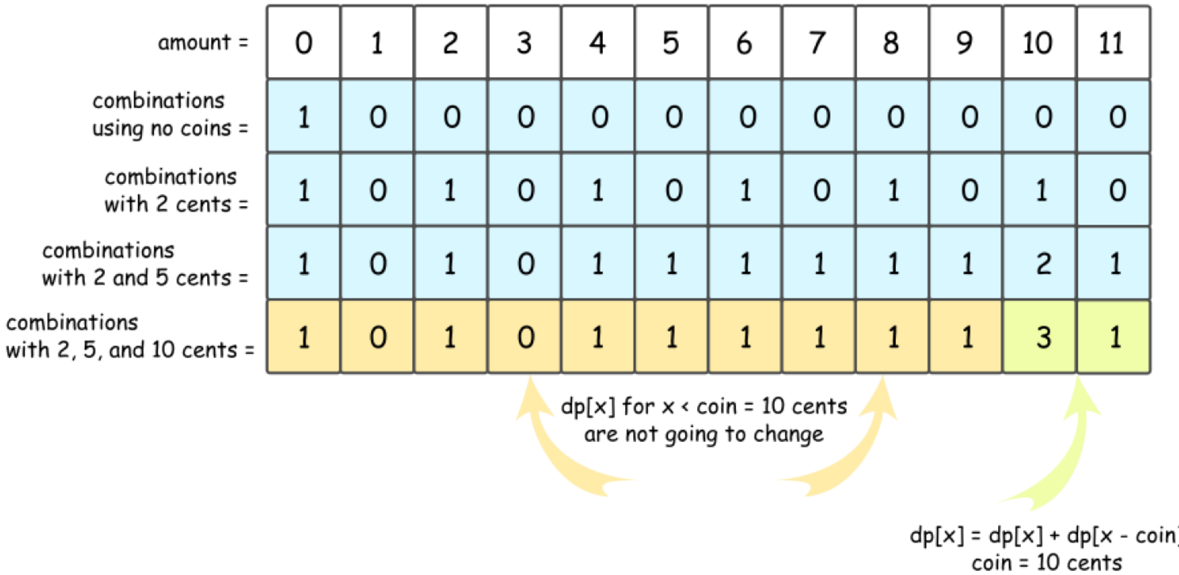
2 Cent Coins + 5 Cent Coins + 10 Cent Coins

Now let's add 5 cent coins. The formula is the same, but do not forget to add `dp[x]`, number of combinations with 2 cent coins.



The story is the same for 10 cent coins.

The story is the same for 10 cent coins.



Now the strategy is here:

- Add coins one-by-one, starting from base case "no coins".
- For each added coin, compute recursively the number of combinations for each amount of money from 0 to `amount`.

Algorithm

- Initiate number of combinations array with the base case "no coins": `dp[0] = 1`, and all the rest = 0.
- Loop over all coins:
  - For each coin, loop over all amounts from 0 to `amount`:
    - For each amount x, compute the number of combinations: `dp[x] += dp[x - coin]`.
- Return `dp[amount]`.

```
class Solution {
  public int change(int amount, int[] coins) {
    int[] dp = new int[amount + 1];
    dp[0] = 1;

    for (int coin : coins) {
      for (int x = coin; x < amount + 1; ++x) {
        dp[x] += dp[x - coin];
      }
    }
    return dp[amount];
  }
}
```

Complexity Analysis

- Time complexity: $\mathcal{O}(N \times \mathrm{amount})$, where N is a length of coins array.
- Space complexity: $\mathcal{O}(\mathrm{amount})$ to keep dp array.