Sign and validate transactions by DSA (i.e. DSS, the signing equation defined in NIST FIPS 186-4): implement a DSA module which enables the user can sign the transactions and a miner can verify the signed transaction. (Note that since we set all inputs to zero, so  $m_i = Tx_i$ , i = 1, 2, 3.)

1) User 1: sign his transaction:  $Sig_{sk_1}(m_1) = (r_1, s_1)$  i.e, compute the signature over  $m_1, k_1, r_1 = \left(g^{k_1}mod(p)\right)mod(q), {k_1}^{-1}mod(q)$  and  $s = {k_1}^{-1}\left(h(m_1)\right) + xrmod(q)$ 

In accordance with the NIST FIPS 186-4 specification, the signature generation function was implemented using the following function provided in section 4.6 DSA Signature Generation:

```
# DSA signature function, p, q, g, k, sk are integers, Message are hex strings of even length.
def Sign(p, q, g, k, sk, Message):
  k_inv = inverse_mod(k, q)
  if k_inv is not None:
     r = pow(g, k, p) \% q
     N = len(bin(q)[2:])
     h_m = sha3_224_hex(Message)
     binary_h_m = str(bin(int(h_m, 16)))[2:].zfill(len(h_m[2:] * 4))
    outlen = len(binary_h_m[2:])
     z = binary_h_m[:min(N, outlen)]
    z_10 = int(z, 2)
    s = (k_inv * (z_10 + (sk * r))) % q
     if r != 0 and s != 0:
       return r,s
     raise Exception("either r or s were equal to 0 - generate a new value for k")
  raise Exception('k-1 was not found')
```

The set of input parameters p, q, g,  $k_1$ ,  $sk_1$ , and  $m_1$ , and output parameters  $r_1$  and  $s_1$  can be found in the Appendix.

2) A miner: verify  $Sig_{sk_1}(m_1)$ , i.e., compute the values of u, v, w, and verify whether v = r.

In accordance with the NIST FIPS 186-4 specification, the signature verification function was implemented using the following function provided in section 4.7 DSA Signature Verification and Validation:

```
# DSA verification function, p, q, g, k, pk are integers, Message are hex strings of even length.
def Verify(p, q, g, pk, Message, r, s):
  M_prime = Message
  r_prime = r
  s_prime = s
  y = pk
  N = len(bin(q)[2:])
  if not (0 < r_prime < q) or not (0 < s_prime < q):
    return False
  w = inverse_mod(s_prime, q)
  # print('w: ' + str(w))
  if w is not None:
    h_m = sha3_224_hex(M_prime)
    binary_h_m = str(bin(int(h_m, 16)))[2:].zfill(len(h_m[2:] * 4))
    outlen = len(binary_h_m[2:])
    z = binary_h_m[:min(N, outlen)]
    z_10 = int(z, 2)
    u1 = (z_10 * w) % q
    u2 = (r_prime * w) % q
     # print('u2: ' + str(u2))
    v1 = pow(g, u1, p)
    v2 = pow(y, u2, p)
     v = ((v1 * v2) \% p) \% q
     # print('v: ' + str(v))
    return True if v == r_prime else False
```

The set of input parameters p, q, g, pk1, m1, r, and s can be found in the Appendix. The output produced by the set of input parameters was True. The intermediate variables w, u1, u2, and v can be found in the Appendix.

3) User 2: sign his transaction  $m_2 = Tx_2$  using the key pair  $(sk_2, pk_2)$ , i.e. executing the same steps as user 1.

Using the function from part 1) The set of input parameters p, q, g, k2, sk2, and m2, and output parameters r2 and s2 can be found in the Appendix.