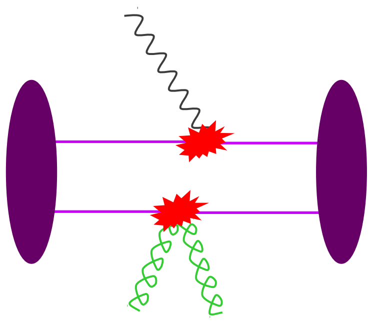
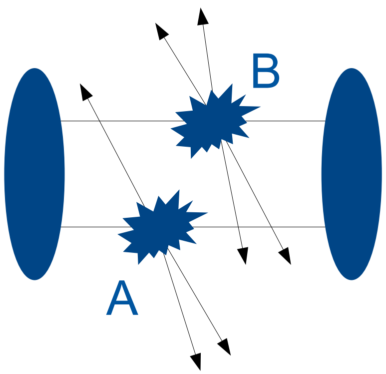
Theory of Double Parton Scattering



Jonathan Gaunt (CERN)

Based on JHEP 1706 (2017) 083 (JG, Diehl, Schoenwald)

The Sixth Annual Conference on Large Hadron Collider Physics, LHCP 2018, Bologna, Italy, 7th June 2018

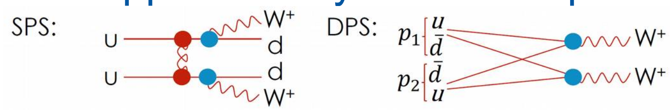


Double Parton Scattering (DPS) = when you have two separate hard interactions in a single proton-proton collision

In terms of total cross section for production of AB, DPS is power suppressed with respect to single parton scattering (SPS) mechanism: $\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \frac{\Lambda^2}{Q^2}$

Why then should we study DPS?

1. DPS can compete with SPS if SPS process is suppressed by small/multiple coupling constants (e.g. same sign WW).

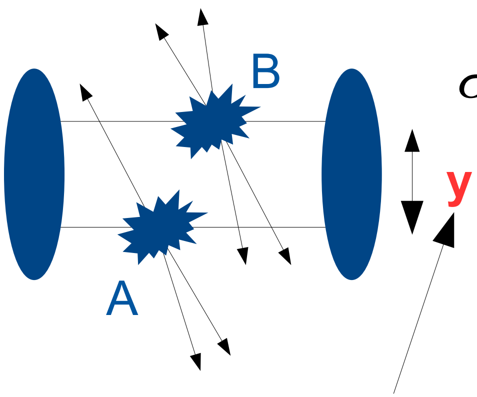


2. DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_{T,A}$, $\mathbf{q}_{T,B}$ – competitive with SPS in this region.
3. DPS becomes more important relative to SPS as the collider energy grows, and we probe smaller *x* values where there is a larger density of partons.
4. DPS reveals new information about the structure of the proton – in particular, correlations between partons in the proton.

Inclusive cross section for DPS

We know that in order to make a prediction for any process at the LHC, we need a factorisation formula (always hadrons/low energy QCD involved).

It's the same for double parton scattering. Postulated form for integrated double parton scattering cross section based on analysis of lowest order Feynman diagrams / parton model considerations:



$$\sigma_D^{(A,B)} = \overset{\text{Symmetry factor}}{\frac{m}{2}} \sum_{i,j,k,l} \int \overbrace{F_h^{ik}(x_1, x_2, \mathbf{y}; Q_A, Q_B)\, F_h^{jl}(x_1', x_2', \mathbf{y}; Q_A, Q_B)}^{\text{Collinear double parton distribution (DPD)}}$$

$$\times \underbrace{\hat{\sigma}_{ij}^{A}(x_1, x_1')\,\hat{\sigma}_{kl}^{B}(x_2, x_2')}_{\text{Parton level cross sections}} \mathrm{d}x_1\, \mathrm{d}x_1' \mathrm{d}x_2\, \mathrm{d}x_2' \mathrm{d}^2\mathbf{y}$$

N. Paver, D. Treleani, Nuovo Cim. A70 (1982) 215.
M. Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

y = separation in transverse space between the two partons

If one assumes

$$F_h^{ik}(x_1, x_2, \mathbf{y}; Q_A, Q_B) \simeq D_h^i(x_1; Q_A) D_h^k(x_2; Q_B) G(\mathbf{y})$$

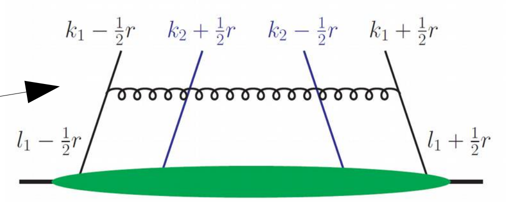
$$\sigma_D^{(A,B)} = \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

'DPS pocket formula'

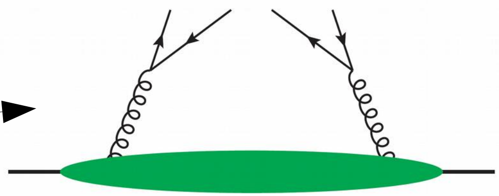
QCD evolution effects

Now we start trying to add in the effects of QCD evolution in DPS, going backwards from the hard interaction.

Some effects are similar to those encountered in SPS – i.e. (diagonal) emission from one of the parton legs. These can be treated in same way as for SPS.



However, there is a new effect possible here – when we go backwards from the hard interaction, we can discover that the two partons arose from the perturbative '1 → 2' splitting of a single parton.



This 'perturbative splitting' yields a contribution to the DPD of the following form:

Single PDF

$$F(x_1, x_2, y) \propto \alpha_s \frac{f(x_1 + x_2)}{x_1 + x_2} P\left(\frac{x_1}{x_1+x_2}\right) \frac{1}{y^2}$$

Dimensionful part

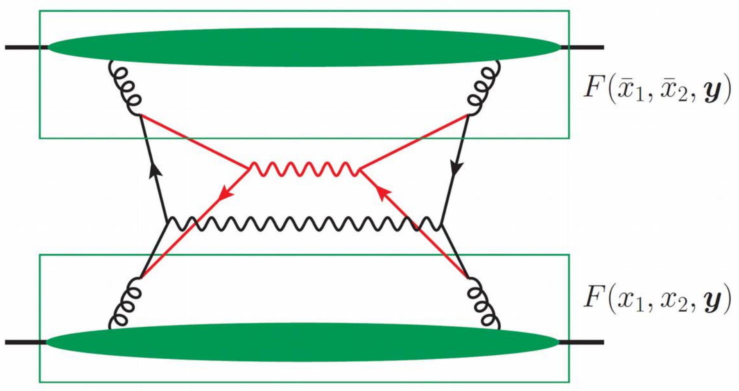
Perturbative splitting kernel

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Problems...

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2y}{y^4} = ?$$



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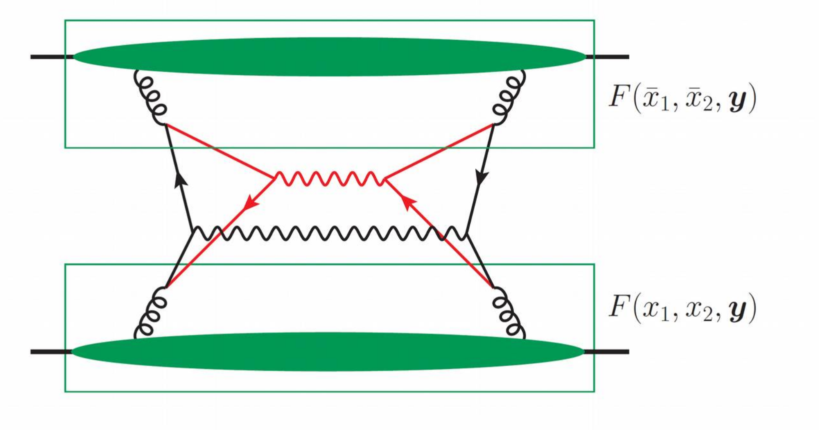
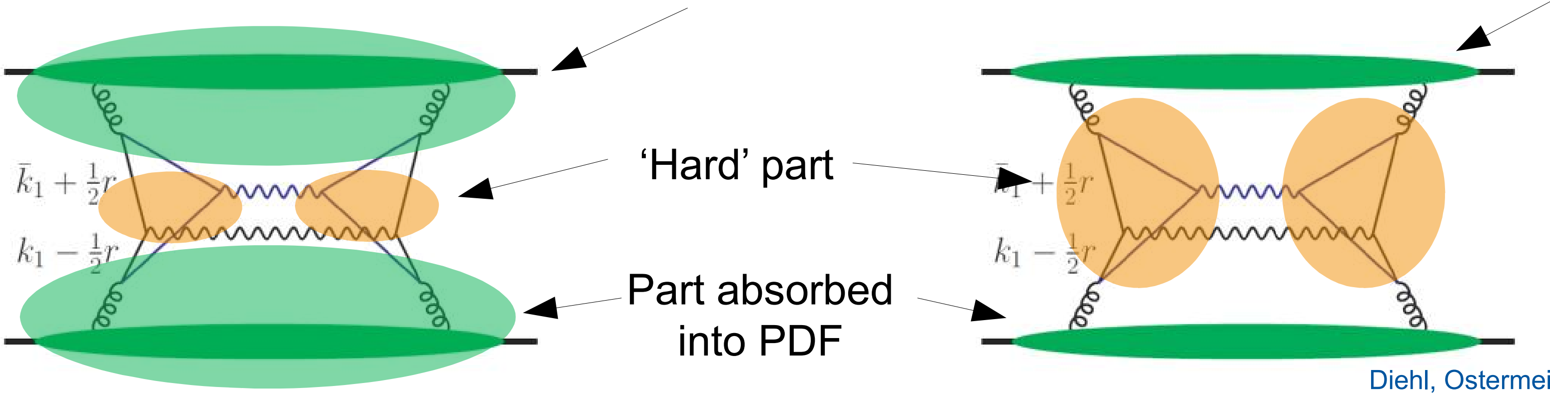


This is related to the fact that this graph can also be regarded as an SPS loop correction



$$\frac{\Lambda^2}{Q^4} \longrightarrow \frac{1}{Q^2}$$

Power divergence!

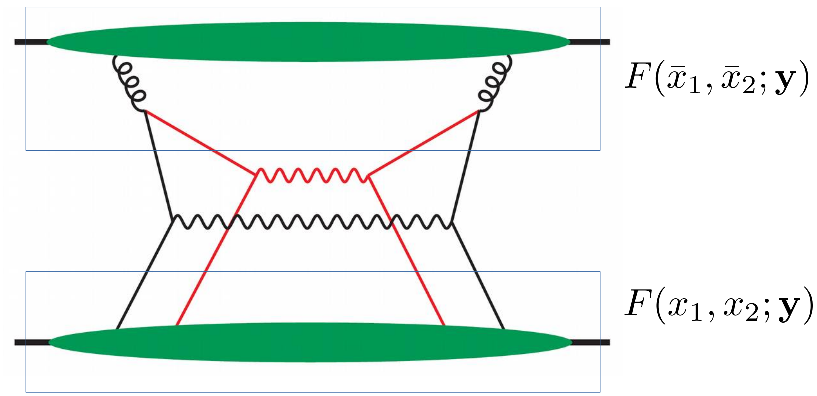
Diehl, Ostermeier and Schafer (JHEP 1203 (2012)) Manohar, Waalewijn Phys.Lett. 713 (2012) 196 JG and Stirling, JHEP 1106 048 (2011) Blok et al. Eur.Phys.J. C72 (2012) 1963 Ryskin, Snigirev, Phys.Rev.D83:114047,2011 Cacciari, Salam, Sapeta JHEP 1004 (2010) 065

Single perturbative splitting graphs

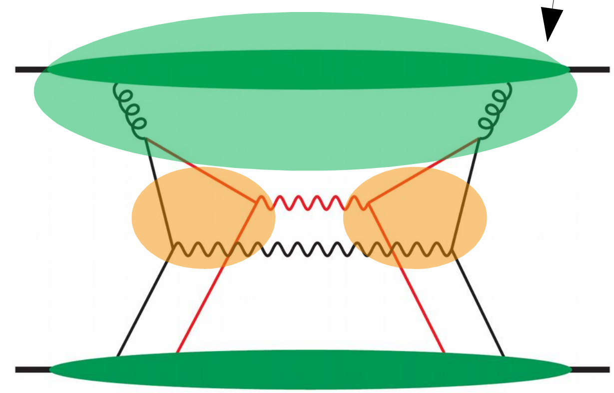
Also have graphs with perturbative 1→2 splitting in one proton only (2v1 graph).

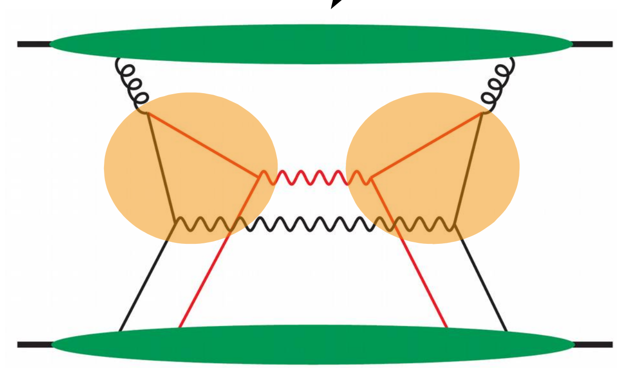
This has a log divergence:

$$\int d^2y/y^2 F_{\text{non-split}}(x_1, x_2; y)$$



Related to the fact that this graph can also be thought of as a twist 4 x twist 2 contribution to AB cross section





$$\frac{\Lambda^2}{Q^4} \longrightarrow \frac{\Lambda^2}{Q^4}$$

Logarithmic divergence

Blok et al., Eur.Phys.J. C72 (2012) 1963, Ryskin, Snigirev, Phys.Rev.D83:114047,2011, JG, JHEP 1301 (2013) 042

Desirable features of a solution to these issues

- Render DPS contribution finite, with no double counting between DPS and SPS.
- Retain concept of the DPD for an individual hadron, with a field theoretic definition. This allows us to investigate these functions using nonperturbative methods such as lattice calculations. C. Zimmerman, arXiv:1701.05479
- Should resum DGLAP logarithms in all types of diagram (1v1, 2v1, 2v2) where appropriate.
- Should permit a formulation at higher orders in perturbation theory (that is not too complicated in practice).
- Would like to re-use as much as possible existing SPS results (partonic cross sections, splitting functions).

Before JHEP 1706 (2017) 083 (JG, Diehl, Schoenwald), no solution satisfying all of these!

Our solution

JHEP 1706 (2017) 083 (JG, Diehl, Schoenwald)

[Focus for the moment only on the double perturbative splitting issue]

Insert a regulating function into DPS cross section formula:

$$\sigma_{\mathrm{DPS}} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$

Requirements: $\Phi(u) \to 0$ as $u \to 0$

$\Phi(u) \to 1$ for $u \gg 1$ e.g. $\Phi(u) = \theta(u - 1)$

In this way, we cut contributions with 1/y much bigger than the scale ν out of what we define to be DPS, and regulate the power divergence.

Note that the *F*s here contain both perturbative and nonperturbative splittings.

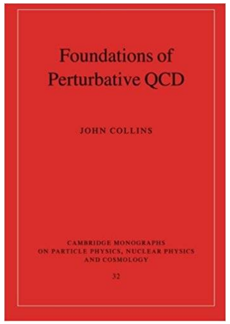
Our solution

Now we have introduced some double counting between SPS and DPS – we fix this by including a double counting subtraction:

$$\sigma_{\rm tot} = \sigma_{\rm DPS} + \sigma_{\rm SPS} - \sigma_{\rm sub}$$

The subtraction term is given by the DPS cross section with both DPDs replaced by fixed order splitting expression – i.e. combining the approximations used to compute double splitting piece in two approaches.

Subtraction term constructed along the lines of general subtraction formalism discussed in Collins pQCD book



Note: computation of subtraction term much easier than full SPS X sec

Straightforward extension of formalism to include twist 4 x twist 2 contribution and remove double counting with 2v1 DPS:

$$\sigma_{\rm tot} = \sigma_{\rm DPS} + \sigma_{\rm SPS} - \sigma_{\rm sub\ (1v1\ +\ 2v1)} + \sigma_{\rm tw4\times tw2}$$

Tw2 x tw 4 piece with hard part computed according to fixed order DPS expression

How the subtraction works

$$\sigma_{\rm tot} = \sigma_{\rm DPS} + \sigma_{\rm SPS} - \sigma_{\rm sub}$$

For small y (of order 1/Q) the dominant contribution to σ_{DPS} comes from the (fixed order) perturbative expression $\Longrightarrow \quad \sigma_{\rm DPS} \simeq \sigma_{\rm sub}$

& $\quad \sigma_{\rm tot} \simeq \sigma_{\rm SPS}$ (as desired)

(dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub})

For large y (much larger than 1/Q) the dominant contribution to σ_{SPS} is the region of the 'double splitting' loop where DPS approximations are valid

$\Longrightarrow \quad \sigma_{\rm SPS} \simeq \sigma_{\rm sub}$

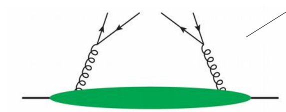
& $\quad \sigma_{\rm tot} \simeq \sigma_{\rm DPS}$ (as desired)

(similar considerations hold for 2v1 part of DPS and tw4xtw2 contribution)

Parton luminosities

Construct model of DPDs with ‘intrinsic’ and ‘splitting’ components:

$$F^{ij}(x_1, x_2, y, \mu) = F^{ij}_{\text{spl}}(x_1, x_2, y, \mu) + F^{ij}_{\text{int}}(x_1, x_2, y, \mu)$$



Perturbative splitting expression x large y suppression

Product of PDFs x smooth transverse profile

Study DPS luminosity (analogue of usual PDF luminosity for SPS)

$$\mathcal{L}_{a_1 a_2 b_1 b_2}(x_i, \bar{x}_i, \mu_i, \nu) = \int d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{a_1 a_2}(x_i, \boldsymbol{y}; \mu_i)\, F_{b_1 b_2}(\bar{x}_i, \boldsymbol{y}; \mu_i)$$

For cut-off function we use $\Phi(u) = \theta(u - b_0) \qquad b_0 = 2e^{-\gamma_E} = 1.1229...$

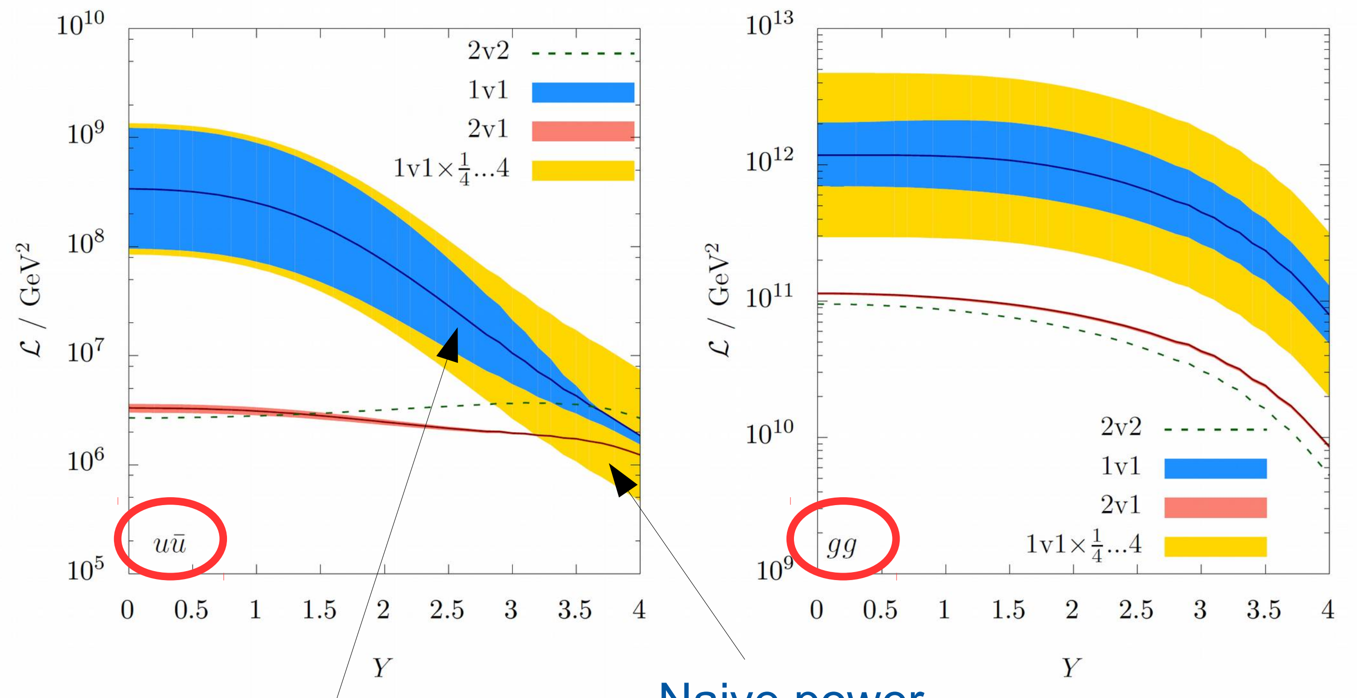
N.B. $\mathcal{L}$ / σ_{DPS} is not really ‘meaningful’ on its own. Can only measure $\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$

Large (small) dependence of $\mathcal{L}$ on ν indicates loop corrections to SPS (and subtraction term) more (less) important.

DPS luminosities

$Q_A = Q_B = 80$ GeV, $\sqrt{s} = 14$TeV

Here: plot luminosities against rapidity of one hard system (other kept central):



1v1 much larger than others, with large ν variation – although variation somewhat smaller in gg channel (larger evolution effect).

Actual ν variation

Naive power counting expection for ν variation $\propto \nu^2$

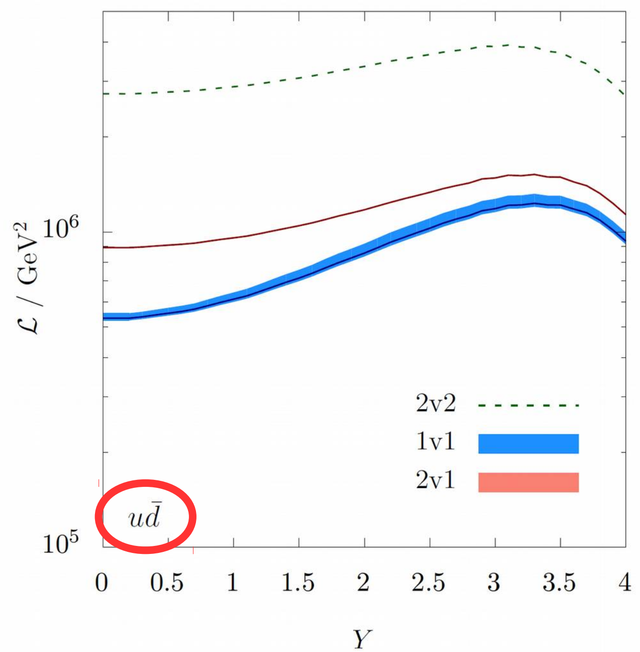
DPS luminosities

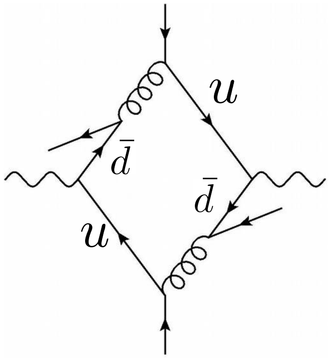
Some situations where ν variation is reduced – then don't need SPS and subtraction up to order containing double box.

Examples:

(1) When the parton pairs in the relevant DPDs cannot be produced in a single leading-order splitting (e.g. $u\bar{d}$)

Relevant for same sign WW production!





Here splitting effects are not so pronounced – prominent correlations are related to number/momentum conservation effects.

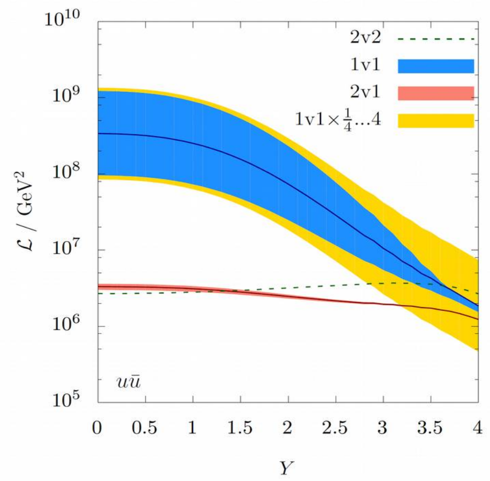
DPS luminosities

(2) When low x values in the DPDs are probed

E.g. Low mass Drell-Yan or heavy quark production at (HE-)LHC, with hard systems widely separated in rapidity

Here can have a significant contribution from pure splitting processes (where these can legitimately be thought of as DPS)

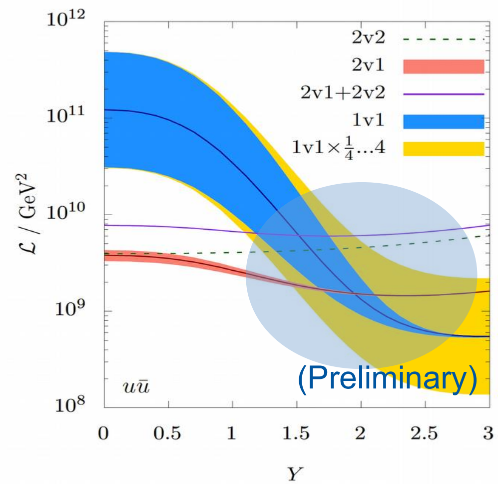
By combining studies of different types of processes, could probe size of splitting effects and compare with theory. More studies needed → WIP.



$\sqrt{s} = 14$ TeV $\rightarrow \sqrt{s} = 27$ TeV

$Q = 80\text{GeV} \rightarrow Q = 40\text{GeV}$

One particle at Y, another particle at -Y



Summary

- Power divergence in naive treatment of DPS including perturbative splittings (= 'leaking' of DPS into leading power SPS region).
- We have proposed a solution that retains the concept of a DPD for an individual hadron, and avoids double counting. Involves introduction of a regulator at the DPS cross section level, + a subtraction to remove double counting overlap between SPS and DPS.
- DPS luminosities: generically very large 1v1 with large uncertainty – have to compute SPS and subtraction up to two-loop.
- Certain scenarios where DPS is more prominent – e.g. processes with systems separated in rapidity, same sign WW. These are the most promising situations to make useful predictions and measurements of DPS.
- Framework has also been extended to measured transverse momentum

  Buffing, Diehl, Kasemets, JHEP 1801 (2018) 044.
- Interesting future direction: improve MC MPI models using insight from DPS theory studies.

CERN

Modelling the DPDs

Construct model of DPDs, with 'intrinsic' and 'splitting' components:

$$F^{ij}(x_1, x_2, y, \mu) = F^{ij}_{\text{spl}}(x_1, x_2, y, \mu) + F^{ij}_{\text{int}}(x_1, x_2, y, \mu)$$

Smooth transverse *y* profile, radius ~ R_p

'Usual' product of PDFs

Initialise at low scale μ_0 = 1 GeV

$$F_{a_1 a_2,\text{int}}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = \frac{1}{4\pi h_{a_1 a_2}} \exp\left[-\frac{y^2}{4h_{a_1 a_2}}\right] f_{a_1}(x_1; \mu_0)\, f_{a_2}(x_2; \mu_0)$$
$$\times (1 - x_1 - x_2)^2 (1 - x_1)^{-2} (1 - x_2)^{-2}$$

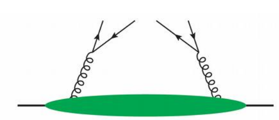
Factor to suppress DPD near phase space limit $x_1 + x_2 = 1$

Initialise at low scale μ_y ~ 1/y

Gaussian suppression at large y

Perturbative splitting expression

$$F_{a_1 a_2,\text{spl}}(x_1, x_2, \boldsymbol{y}; \mu_y, \mu_y) = \frac{1}{\pi y^2} \exp\left[-\frac{y^2}{4h_{a_1 a_2}}\right] \frac{f_{a_0}(x_1 + x_2; \mu_y)}{x_1 + x_2} \frac{\alpha_s(\mu_y)}{2\pi} P_{a_0 \to a_1 a_2}\left(\frac{x_1}{x_1 + x_2}\right)$$



Evolve both to scale *μ* using homogeneous double DGLAP $\frac{d}{d\log \mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$

DPS – numerical studies

With a model of the DPDs can begin to make predictions for DPS cross sections.

Note that depending on the circumstances, σ_{DPS} may not be very meaningful on its own.

Only measurable observable is actually σ_{tot} for production of AB: $\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$

σ_{DPS} contains a dependence on the unphysical parameter ν which, just on power counting grounds, should be very strong ($\propto \nu^2$). Subtraction term cancels ν dependence order-by-order. Need SPS and subtraction terms to two-loop order (or more) for meaningful prediction.

In practice the size of σ_{DPS} compared to its ν variation may be much larger than the naive expectation (due to e.g. evolution effects). In these cases $\sigma_{sub} << \sigma_{DPS}$ (σ_{sub} is roughly of the order of the ν variation of σ_{DPS}).

Actually these are the best circumstances to look at to measure DPS (since $\sigma_{sub} \sim \sigma_{DPS}$).

DPS – numerical studies

With this in mind we will look at just the DPS cross section. Actually we will plot the DPS luminosity, defined by:

$$\mathcal{L}_{a_1 a_2 b_1 b_2}(x_i, \bar{x}_i, \mu_i, \nu) = \int d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{a_1 a_2}(x_i, \boldsymbol{y}; \mu_i)\, F_{b_1 b_2}(\bar{x}_i, \boldsymbol{y}; \mu_i)$$

For cut-off function we use $\Phi(u) = \theta(u - b_0) \qquad b_0 = 2e^{-\gamma_E} = 1.1229\ldots$

Split luminosity into 2v2 ($F_{\text{int}} \otimes F_{\text{int}}$)

2v1 ($F_{\text{int}} \otimes F_{\text{spl}} + F_{\text{spl}} \otimes F_{\text{int}}$)

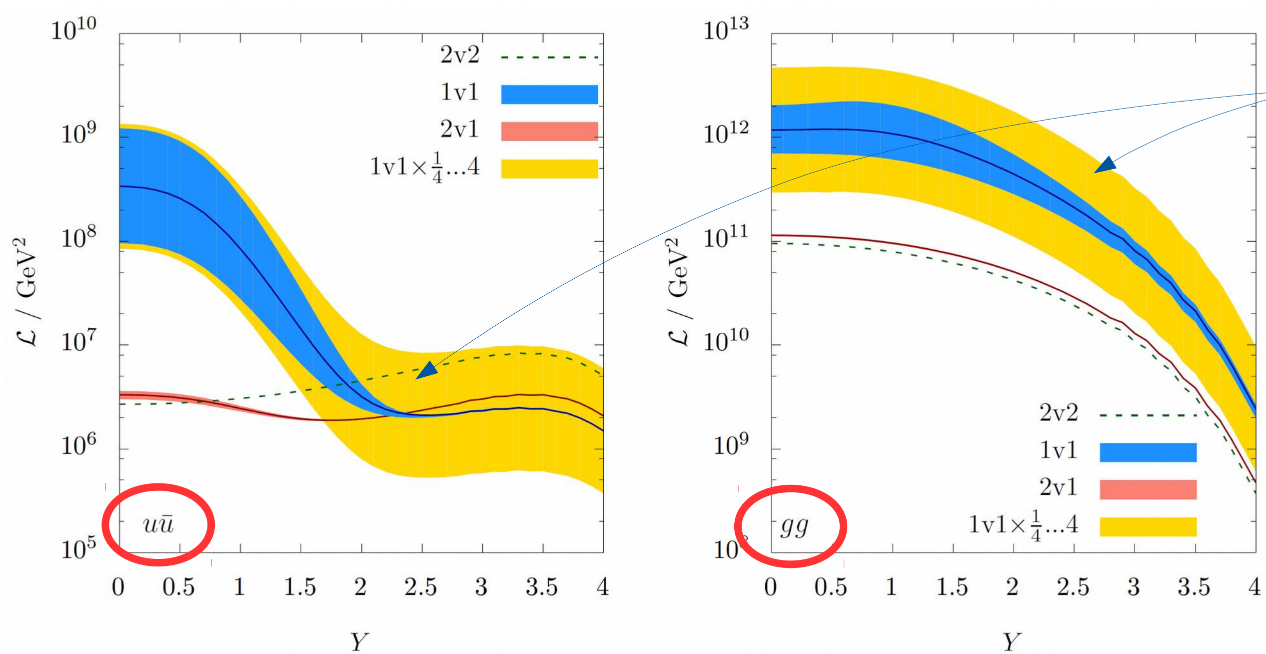
1v1 ($F_{\text{spl}} \otimes F_{\text{spl}}$)

Vary scale ν between $Q/2$ and $2Q$. If 1v1 term is large with very large ν variation, then we know immediately that we need SPS and subtraction terms up to high orders for accurate prediction of σ_{tot}, and DPS contribution not so prominent.

DPS luminosities

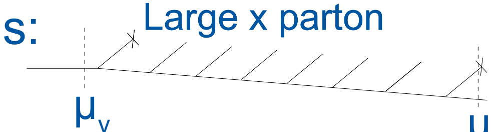
$Q_A = Q_B = 80$ GeV, $\sqrt{s} = 14$TeV

Now put one system at Y, other at -Y:



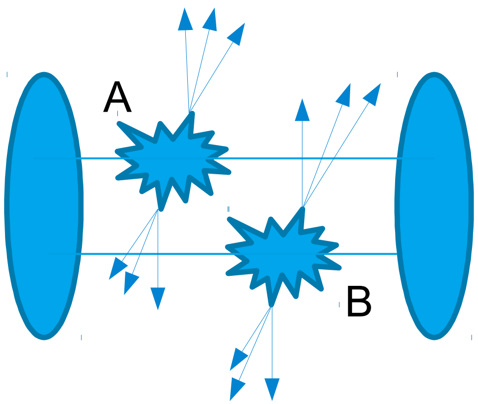
Reduction in ν variation (particularly drastic for $u\bar{u}$ channel)

Large Y → DPDs with one large x and one small x parton. Preferred scenario from the PoV of small x logarithms:



Favoured at large y (more evolution space $\mu_y \to \mu$)

Progress on formal side (factorisation proofs)



DPS power suppressed compared to SPS in terms of total cross section to produce AB:

$$\sigma_{DPS}/\sigma_{SPS} \sim \Lambda^2/Q^2$$

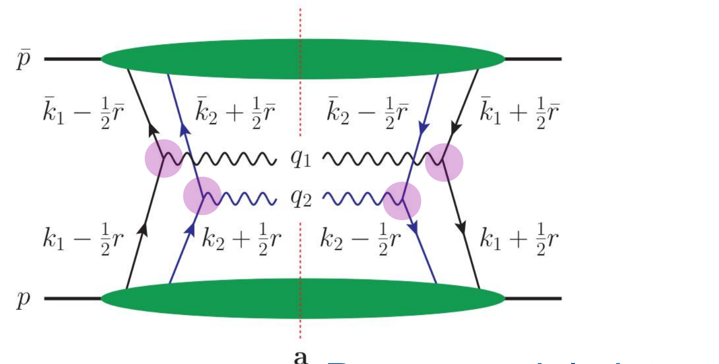
→ Experiments often have to use differential distributions to extract DPS signal

Key quantity from theory side: double differential cross section in p_T of A and B, for p_T << Q. For this quantity SPS and DPS are of the same power.

Does this quantity factorise? Desired end-goal:

$$\frac{d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i} = \frac{1}{S} \times \sum_{\substack{a_1,a_2=q,\Delta q,\delta q \\ \bar{a}_1,\bar{a}_2=\bar{q},\Delta\bar{q},\delta\bar{q}}} \left[\prod_{i=1}^{2} \hat{\sigma}_{i,a_i\bar{a}_i}(q_i^2) \int \frac{d^2\boldsymbol{z}_i}{(2\pi)^2}\, e^{-i\boldsymbol{z}_i\boldsymbol{q}_i}\right] \times \int d^2\boldsymbol{y}\, F_{a_1,a_2}(x_i,\boldsymbol{z}_i,\boldsymbol{y})\, F_{\bar{a}_1,\bar{a}_2}(\bar{x}_i,\boldsymbol{z}_i,\boldsymbol{y})$$

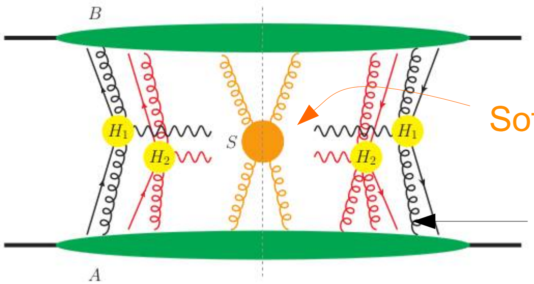
DTMD



Parton model picture

Obtaining this formula in QCD is not so simple!

Only consider colourless final states here – factorisation with colour in the final state is problematic

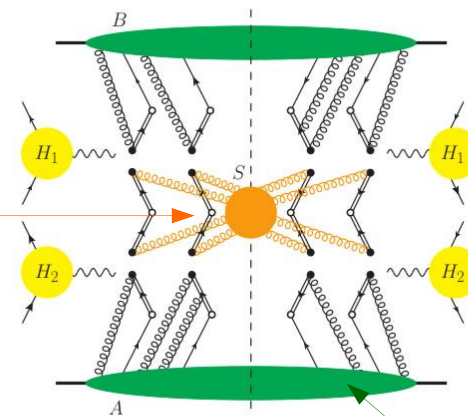


Soft and Glauber exchanges

Extra (unphysically polarised) gluon connections to hard

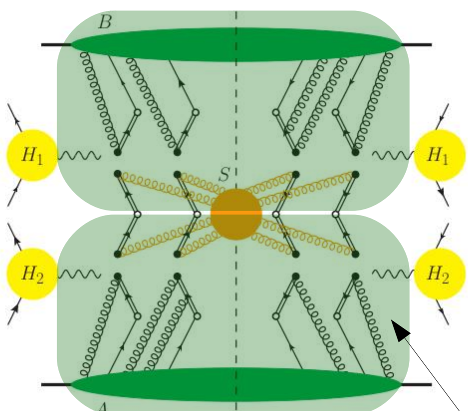
Initial picture

Cancel Glauber exchanges (Diehl, JG, Ostermeier, Ploessl, Schafer)
Use Ward identities to strip soft and collinear gluon attachments (Diehl, Ostermeier, Schafer)



Soft factor

Collinear factor



'Chop up soft factor and divide between DTMDs'

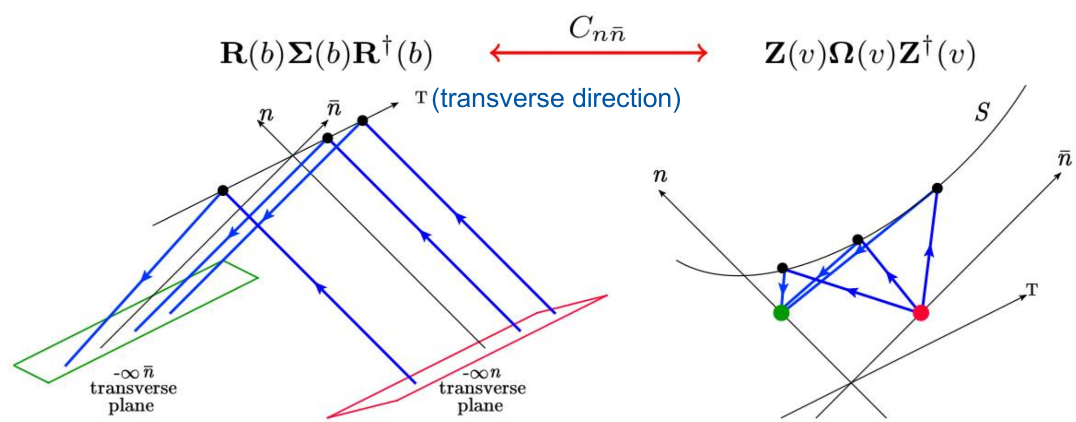
Both soft and collinear factors contain so-called rapidity divergences – have to be regulated using some appropriate regulator.

Finite DTMDs

Proof that soft factor can be divided up between the DTMDs was given last year by Vladimirov (under “exponential” rapidity regulator of Li, Neill, Zhu, 1604.00392) [arXiv:1707.07606]

Proof starts with a conformal theory, and uses a conformal transform to map rapidity divergences to UV divergences:

Conformal transformation



Rapidity divergences here

UV divergences here

Proof extended order-by-order to QCD (inductive proof)

This shows divergent parts of soft can be appropriately divided up between DTMDs, leaving finite DTMDs and finite soft. Finite soft can also be divided up between DTMDs:

$$F_{\{f\}\leftarrow h_1}(\{x\},\{b\},\zeta,\nu^2) \to \mathbf{S} \times F_{\{f\}\leftarrow h_1}(\{x\},\{b\},\zeta,\nu^2).$$

$$\mathbf{\Sigma}_0^{-1}(\{b\},\mu,\nu^2) \to (\mathbf{S}^{-1})^T \mathbf{\Sigma}_0^{-1}(\{b\},\mu,\nu^2)\mathbf{S}^{-1}.$$

Pick $\mathbf{S}(b,\mu,\nu^2)\mathbf{\Sigma}_0(\{b\},\mu,\nu^2)\mathbf{S}^T(b,\mu,\nu^2) = \mathbf{I}.$

Buffing, Diehl, Kasemets, [arXiv:1708.03528]

This paper also contains various results on rapidity (and scale) evolution of DTMDs (and DPDFs)

Extension to measured transverse momenta

So far just discussed DPS at the total cross section level.

However, since DPS preferentially populates the small $\mathbf{q}_A$, $\mathbf{q}_B$ region, the transverse-momentum-differential cross section for the production of AB for small $\mathbf{q}_A$, $\mathbf{q}_B$ is also of significant interest. Need to adapt SPS TMD formalism to double scattering case.

The scheme can be readily adapted to solve double counting issues in this case.

TMD DPS cross section involves the so called DTMDs: ${}^{R}F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; \mu_1, \mu_2, \zeta)$

Complex objects with many arguments – difficult to model.

Significant simplification when $\Lambda << q_A, q_B << Q_A, Q_B$ – then cross section can be expressed in terms of same nonperturbative objects as total cross section (including DPDs) and perturbative kernels (most known at least to order α_s)

Buffing, Diehl, Kasemets, JHEP 1801 (2018) 044.

DTMDs at perturbative transverse momenta

DTMDs are complex objects with many arguments – difficult to model!:

Colour representation

Rapidity parameter

$$
{}^{R}F(\underbrace{x_1, x_2}_{\text{Mtm fractions of two partons}}, \underbrace{\boldsymbol{z}_1, \boldsymbol{z}_2}_{\text{Conjugate to average transverse mta of two partons}}, \boldsymbol{y}; \overbrace{\mu_1, \mu_2}^{\text{Two scales}}, \zeta)
$$

Average transverse separation of two partons

For $\Lambda << q_T << Q$ DTMDs can be expressed as convolutions of simpler collinear objects and perturbative kernels. Two regimes:

Buffing, Diehl, Kasemets, [arXiv:1708.03528]

$\boxed{\text{Large y}}$ $|\boldsymbol{y}| \sim 1/\Lambda$

$$
{}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \sum_{b_1 b_2} {}^{R}C_{a_1 b_1}(x_1, \boldsymbol{z}_1, \mu_1, x_1\zeta/x_2)
$$

$$
\underset{x_1}{\otimes} {}^{R}C_{a_2 b_2}(x_2, \boldsymbol{z}_2, \mu_2, x_2\zeta/x_1) \underset{x_2}{\otimes} {}^{R}F_{b_1 b_2}(x_i, \boldsymbol{y}; \mu_i, \zeta)
$$

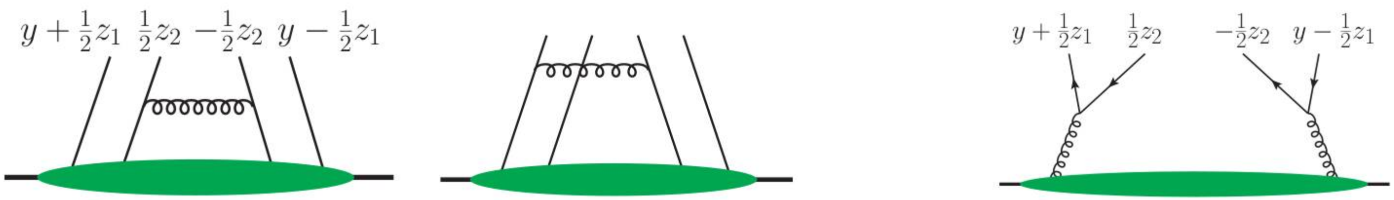
DPDFs

Analogous matching as for single parton TMDs (and kernels same for R=1)

Buffing, Diehl, Kasemets, [arXiv:1708.03528]

Small y $\quad |\boldsymbol{y}| \sim 1/q_T \sim |\boldsymbol{z}_i|$

$$F(x_i, \boldsymbol{z}_i, \boldsymbol{y}) = F_{\text{int}} + F_{\text{spl}}$$ + twist-three contribution, negligible for low x



$$F_{\text{int}} = G + C_{\text{tw4}}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) \otimes G \sim \Lambda^2$$

$G =$ twist 4 distribution

$C_{\text{tw4}} \propto \alpha_s$ (unknown)

$$F_{\text{spl}} \sim \frac{\boldsymbol{y}_+}{\boldsymbol{y}_+^2} \frac{\boldsymbol{y}_-}{\boldsymbol{y}_-^2} P_{\text{spl}} \cdot f(x_1 + x_2) \sim q_T^2$$

$f =$ PDF, $\quad \boldsymbol{y}_\pm = \boldsymbol{y} \pm \frac{1}{2}(\boldsymbol{z}_1 + \boldsymbol{z}_2)$

$P_{\text{spl}} \propto \alpha_s$ (known)

Large and small y expressions need to be appropriately combined, with a subtraction implemented to remove double counting (all worked out in arXiv:1708.03528).