12.4 Notes

- Density function. $\rho(x, y)$

$$M \approx \sum_{i=1}^{K} \sum_{j=1}^{L} \rho(x_{ij}, y_{ij}) \Delta A_{ij}$$

$$= \iint_D \rho(x, y)\, dA$$

charge density $\sigma(x, y)$

$$Q = \iint_D \sigma(x, y)\, dA$$

Moment of lamina about x and y axes

- $M_x = \iint_D y\rho(x, y)\, dA$; $M_y = \iint_D x\rho(x, y)\, dA$

- $(\bar{x}, \bar{y})$: center of mass where

- $\bar{x} = \frac{M_y}{M} = \frac{1}{M} \iint_D x\rho(x, y)\, dA$

- $\bar{y} = \frac{M_x}{M} = \frac{1}{M} \iint_D y\rho(x, y)\, dA$

- Moment of inertia about the origin:

$$I_o = \iint_D (x^2 + y^2)\rho(x, y)\, dA;$$

about x/y axes:

$$I_x = \iint_D y^2 \rho(x, y)\, dA; \quad I_y = \iint_D x^2 \rho(x, y)\, dA$$