MTH224 3/8/23

$-F_X(a) = P(X \leq a) = \int_{-\infty}^{a} f_X(x)\,dx$

cumulative distribution function:

$-P(a < X \leq b) = F_X(b) - F_X(a)$

- pdf: $f(x)$, cdf: $F_X(x)$,

$F_X'(x) = f(x)$

example

a. pdf: $f_X(x) = \frac{3}{8}(4x - 2x^2)$

$x \in (0, 2)$

cdf: $\int_{-\infty}^{x} \frac{3}{8}(4y - 2y^2)\,dy = \int_{0}^{x} \frac{3}{8}(4y - 2y^2)\,dy$

$= \frac{3}{8}\left(4 \cdot \frac{y^2}{2}\Big|_0^x - 2\frac{y^3}{3}\Big|_0^x\right) = \frac{x^2}{4}(3 - x)$

verification:

$= \frac{3}{8}\left(\frac{4x^2}{2} - \frac{2x^3}{3}\right) = \frac{12x^2}{16} - \frac{6x^3}{24}$

$= \frac{3x^2}{4} - \frac{x^3}{4} = \frac{x^2}{4}(3 - x)$ ✓

- for continuous r.v., $E[X] = \sum_x x P_X(x)$

$-\mathrm{Var}(X) = E\left[(X - E[X])^2\right] = \int_{-\infty}^{\infty} (x - E[X])^2 \cdot f_X(x)\,dx$

$= E[X^2] - (E[X])^2$