11.7 Notes

- $f(x, y)$ has local minimum (maximum) at (a, b) if $f(x, y) \geq (\leq) f(a, b)$ when (x, y) is near (a, b).

- If f has local maximum/minimum and is differentiable at (a, b), then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

- Second derivative test:

$$f_x(a, b) = f_y(a, b) = 0.$$

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

$\{D > 0 \wedge f_{xx}(a, b) > 0\} \to f(a, b)$ local minimum

$\{D > 0 \wedge f_{xx}(a, b) < 0\} \to f(a, b)$ local max

$D < 0 \to$ neither; (a, b) is saddle point of f, and f crosses its tangent plane at (a, b)

$D = 0 \to$ no information