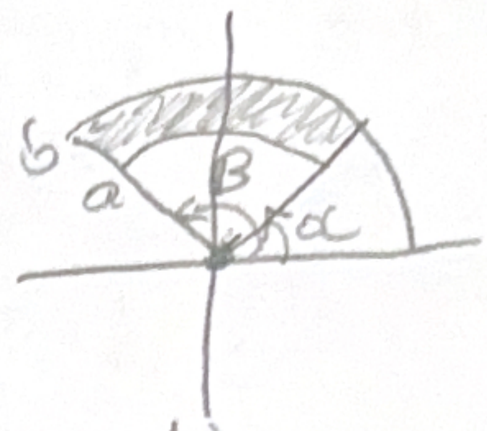
12.3 Notes

- Consider polar rectangle

$$R = \{(r,\theta) \mid \begin{matrix} a \le r \le b \\ \alpha \le \theta \le \beta \end{matrix}\}$$



Riemann sum of $f(x,y)$ in polar rectangle where

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f(r_i^* \cos\theta_j^*,\ r_i^* \sin\theta_j^*)\,\Delta A_{ij}$$

$$r_i^* = \frac{1}{2}(r_{i-1} + r_i),\quad \theta_j^* = \frac{1}{2}(\theta_{j-1} + \theta_j)$$

$$\Delta A_{ij} = r_i^* \Delta r_i \Delta\theta_j$$

$$f(r_i^* \cos\theta_j^*,\ r_i^* \sin\theta_j^*) = g(r_i, \theta_j)$$

riemann sum equates to

$$\int_\alpha^\beta \int_a^b g(r,\theta)\,dr\,d\theta$$

$$= \lim_{\substack{m\to\infty \\ n\to\infty}} \sum_{i=1}^{m}\sum_{j=1}^{n} g(r,\theta)\,\Delta A_{ij}$$

$$= \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta = \iint_R f(x,y)\,dA$$