

## Formalized Axioms and Analysis

Formalizability Index: 0.36

Total Segments: 464

Formalizable Segments: 165

### Logic Reconstruction:

=== Formal Logic Reconstruction ===

(0) Sharpness-aware minimization (SAM) is a promising method to improve generalization.

Formal: SAM is a promising method to enhance generalization.

(1) The original proposal of SAM by Foret et al. signifies a foundational shift.

Formal: Foret et al.'s proposal of SAM marks a fundamental change.

(4) We study SAM for out-of-distribution (OOD) generalization.

Formal: SAM is studied for OOD generalization.

(5) The original SAM outperforms the Adam baseline by 4.76% in zero-shot out-of-distribution (OOD) generalization.

Formal: Original SAM surpasses Adam by 4.76% in zero-shot OOD generalization.

(6) An OOD (out-of-distribution) generalization bound can be provided in terms of sharpness for a part of the domain.

Formal: OOD generalization can be bounded by sharpness in a specific context.

(7) Gradual domain adaptation (GDA) is a form of out-of-distribution (OOD) generalization that utilizes intermediate domains.

Formal: GDA is OOD generalization using intermediate domains between source and target domain.

(8) The original SAM outperforms the baseline of Adam on each of the experimental datasets by 0.82% on average.

Formal: Original SAM exceeds Adam by 0.82% on average across datasets.

(8) The strongest SAM variants outperform Adam by 1.52% on average.

Formal: Top SAM variants outperform Adam by 1.52% on average.

(9) A generalization bound for SAM is provided in the GDA setting.

Formal: SAM has a generalization bound in GDA settings.

(10) Asymptotically, this generalization bound is no better than the one for self-training in the literature.

Formal: This asymptotic generalization bound matches self-training in GDA literature.

(11) There is a disconnection between the theoretical justification for SAM and its empirical performance.

Formal: Theoretical SAM justification and empirical performance are disconnected.

(12) Low sharpness alone does not account for all of SAM's generalization benefits.

Formal: SAM's generalization benefits aren't solely due to low sharpness.

(13) There are potential avenues for obtaining a tighter analysis for Stochastic Approximation Methods.

Formal: Potential exists for tighter SAM analysis in OOD context.

(14) Theoretical results provide a solid starting point for analyzing SAM in OOD settings.

Formal: Theoretical results are a basis for analyzing SAM in OOD.

(14) SAM can be applied to OOD settings to significantly improve accuracy.

Formal: SAM can enhance accuracy in OOD settings.

(14) Newer variants of SAM can be leveraged for further improvements in accuracy.

Formal: New SAM variants can further boost accuracy.

(15) Sharpness-Aware Minimization (SAM) is a promising new optimization algorithm.

Formal: SAM is an innovative optimization algorithm.

(16) A robust optimization procedure can lead to significant performance gains in the i.i.d. setting.

- Formal: Robust optimization can yield significant i.i.d. performance gains.
- (18) SAM remains understudied in the out-of-distribution (OOD) generalization setting.  
Formal: SAM is still understudied in OOD generalization.
- (19) A number of SAM variants have been proposed to improve the accuracy and efficiency of the original SAM.  
Formal: Several SAM variants aim to boost accuracy and efficiency.
- (22) The potential to enhance OOD (out-of-distribution) generalization exists.  
Formal: Enhancing OOD generalization potential is present.
- (24) There are eight SAM variants, including the original SAM.  
Formal: Eight SAM variants exist, original SAM included.
- (27) SAM can be used to improve zero-shot OOD generalization.  
Formal: SAM improves zero-shot OOD generalization.
- (27) The strongest SAM variants can be used for an even further improvement.  
Formal: Top SAM variants achieve further improvements.
- (29) A theoretical analysis of SAM under the distribution shift setting provides performance gains.  
Formal: Theoretical SAM analysis during distribution shift yields gains.
- (31) We extend the setting to gradual domain adaptation.  
Formal: The setting is extended to gradual domain adaptation.
- (32) SAM outperforms the Adam baseline by 0.82% on average.  
Formal: SAM surpasses Adam by 0.82% on average.
- (32) The strongest SAM variants achieve an even greater 1.52% average improvement over Adam.  
Formal: Top SAM variants exceed Adam by 1.52% on average.
- (32) SAM and the strongest SAM variants can be used for consistent performance gains in GDA.  
Formal: Use SAM and top variants for consistent GDA gains.
- (36) Our work is asymptotically the same as prior work in the GDA literature (Wang et al., 2022).  
Formal: Our work matches prior GDA literature asymptotically.
- (42) We further define the sharpness of  $\theta$  and the corresponding  $\rho$ -robust empirical loss.  
Formal: Sharpness of  $\theta$  and  $\rho$ -robust empirical loss are defined.
- (44) The  $\rho$ -robust risk of parameters  $\theta \in \Theta$  is the maximum loss obtained by perturbing  $\theta$  in the worst direction.  
Formal:  $\rho$ -robust risk is max loss from perturbing  $\theta$  with  $\|\delta\|_2 \leq \rho$ .
- (45) The  $\rho$ -sharpness of parameters  $\theta$  measures how much the loss increases when we perturb them.  
Formal:  $\rho$ -sharpness is loss increase from worst perturbation of  $\theta$  with  $\|\delta\|_2 \leq \rho$ .
- (46) Sharpness-aware minimization (SAM) proposes minimizing the  $\rho$ -robust empirical loss rather than the standard loss.  
Formal: SAM minimizes  $\rho$ -robust empirical loss instead of standard loss.
- (47) SAM's objective is to find a minimizer  $\theta^*$  of the form:  $\theta^* = \operatorname{argmin}_{\theta \in \Theta} \max_{\|\beta\|_2 \leq \rho} E(\theta + \beta)$ .  
Formal: SAM aims to find  $\theta^* = \operatorname{argmin}_{\theta \in \Theta} \max_{\|\beta\|_2 \leq \rho} E(\theta + \beta)$ .
- (49) The SAM gradient drops a second-order term arising from the chain rule.  
Formal: SAM gradient omits a chain rule second-order term.
- (51) Our results can be extended to any  $\ell_p$ -norm.  
Formal: Results extend to any  $\ell_p$ -norm.
- (53) This should not be confused with  $m$ -sharpness from Foret et al.  
Formal: Do not confuse with Foret et al.'s  $m$ -sharpness.
- (55) SAM (Sharpness-Aware Minimization) is a method for optimizing model parameters.  
Formal: SAM optimizes model parameters.
- (55) SAM uses variant-specific oracles for gradient computation, perturbation, and descent steps.  
Formal: SAM employs specific oracles for gradient, perturbation, and descent.
- (57) Adaptive SAM (ASAM) is a version of SAM that uses a scale-invariant version of the first-order approximation.

- Formal: ASAM is a SAM variant using scale-invariant first-order approximation.
- (59) Adaptive SAM has one hyperparameter  $\rho$ .  
Formal: ASAM includes one hyperparameter  $\rho$ .
- (60) FisherSAM is a special case of ASAM with a specific normalization operator.  
Formal: FisherSAM is an ASAM variant with a distinct normalization operator.
- (62) K-SAM is a variant of SAM that only uses the top K data samples with the highest loss for gradient calculations.  
Formal: K-SAM uses top K loss samples for gradient calculations.
- (65) The idea behind Equation (12) is to remove the component of the descent gradient  $\nabla_{\theta} E(\theta) |_{\theta + \beta \nabla_{\theta} E(\theta)}$  paralleling the ascent gradient.  
Formal: Equation (12) eliminates descent gradient component paralleling the ascent gradient.
- (67) Using only the full-batch direction of the ascent-step gradient impairs performance.  
Formal: Reliance on full-batch ascent-step gradient direction hurts performance.
- (69) Computing the full-batch gradient  $\nabla_{\theta} E(\theta)$  is computationally prohibitive.  
Formal: Full-batch gradient calculation is computationally demanding.
- (70) FriendlySAM uses a specific perturbation method defined by an equation.  
Formal: FriendlySAM employs equation-defined perturbation.
- (72) NoSAM is a variant of SAM that only performs the SAM perturbation on normalization layers.  
Formal: NoSAM applies SAM perturbation solely on normalization layers.
- (74) The computational cost is compared across three specific metrics.  
Formal: Cost is compared using three distinct metrics.
- (77) EfficientSAM (ESAM) is a variant of SAM intended to make SAM more efficient.  
Formal: ESAM is a SAM variant for enhanced efficiency.
- (78) Stochastic weight perturbation is applied by performing the SAM perturbation on a fraction of the weights.  
Formal: SAM perturbation involves stochastic weight alteration.
- (79) Sharpness-sensitive data selection is performed by only computing gradients over data samples with highest loss.  
Formal: Select sharpness-sensitive data by targeting highest loss increasing samples.
- (83) A model  $\theta_S$  is trained on a source domain  $S \in \Delta(X \times Y)$  with a training set.  
Formal: Train model  $\theta_S$  on source domain  $S$  with training set.
- (86) The zero-shot OOD generalization error is given by  $ET(\theta_S)$ .  
Formal: Zero-shot OOD error is  $ET(\theta_S)$ .
- (92) The intermediate domains are evenly distributed between source and target.  
Formal: Intermediate domains are equally spaced between source and target.
- (95) The intermediate domains are also evenly distributed between source and target.  
Formal: Intermediate domains are evenly placed between source and target.
- (112) Variants such as LookSAM, F-SAM, and FisherSAM offer the strongest and most consistent improvements.  
Formal: LookSAM, F-SAM, FisherSAM show the best and consistent SAM improvements.
- (113) LookSAM and NoSAM perform the best among the variants with reduced computational cost.  
Formal: LookSAM and NoSAM excel among low computational cost variants.
- (113) LookSAM and NoSAM often outperform the original SAM in addition to being more efficient.  
Formal: LookSAM, NoSAM surpass original SAM and are more efficient.
- (117) FisherSAM takes the information geometry of the data into account, using an approximation of the Fisher matrix.  
Formal: FisherSAM uses data's information geometry to find max perturbation with Fisher matrix.
- (118) Under the cross-entropy loss used specifically in the experiments in Table 2, the Fisher information matrix equals the Hessian.  
Formal: In Table 2 experiments, Fisher matrix equals cross-entropy Hessian.
- (119) FisherSAM could be understood as a second-order approximation of the loss perturbation.  
Formal: FisherSAM is a second-order loss perturbation approximation.
- (120) Unlike FisherSAM, the connection between the FriendlySAM objective and OOD performance is not clear.

Formal: FriendlySAM's OOD link is less explicit than FisherSAM's.

(123) The modified perturbation makes FriendlySAM significantly more robust to the choice of the  $\rho$  hyperparameter.

Formal: FriendlySAM's modified perturbation enhances robustness to  $\rho$  selection.

(124) The optimal value of  $\rho$  depends intricately on the choice of dataset.

Formal: Optimal  $\rho$  value is dataset-dependent.

(124) FriendlySAM provides performance gains in experiments by penalizing sharpness adaptively and stably.

Formal: FriendlySAM aids performance by adaptively, stably penalizing sharpness.

(126) The Wasserstein distance is the smallest cost of moving mass between two distributions measured by the  $L_1$  norm.

Formal: Wasserstein distance is minimal cost of redistributing between two distributions.

(127) The loss functions considered in this paper are Lipschitz continuous.

Formal: Considered loss functions are Lipschitz continuous.

(127) For the loss function  $\ell$ , there exist constants  $\rho_1, \rho_2, \rho_3$  such that certain inequalities involving  $\ell$  and  $\ell^\sharp$  hold.

Formal: For  $\ell$ , constants  $\rho_1, \rho_2, \rho_3$  satisfy certain inequalities.

(130) This result holds for any choice of  $\mu, \nu$  on  $\mathcal{Y} \times \mathcal{X}$ .

Formal: Result applies to any  $\mu, \nu$  on  $\mathcal{Y} \times \mathcal{X}$ .

(131) Lemma 1 (Sharpness-Aware Error Difference Over Shifted Domains) posits a bound on the difference between the error of a model  $\theta$  on domain  $\mu$  and its error on domain  $\nu$ .

Formal: Lemma 1 bounds error difference between two distributions.

(132) Population error of a model  $\theta$  can be bounded in terms of its empirical sharpness.

Formal: Model  $\theta$ 's population error is bound by its empirical sharpness.

(133) For any model  $\theta \in \Theta$  satisfying  $E(\theta) \leq E[\ell(\theta, \mu)] + \rho E[\ell^\sharp(\theta, \mu)]$  for some  $\rho > 0$ , with high probability  $1 - \delta$ ,  $E(\theta) \leq \hat{E}_\rho(\theta) + O((k \ln(\ell^\sharp(\theta, \mu)^2 / \rho^2) + \ln(n/\delta))/n)$ .

Formal: For  $\theta \in \Theta$ ,  $E(\theta) \leq E[\ell(\theta, \mu)] + \rho E[\ell^\sharp(\theta, \mu)]$  implies  $E(\theta) \leq \hat{E}_\rho(\theta) + O((k \ln(\ell^\sharp(\theta, \mu)^2 / \rho^2) + \ln(n/\delta))/n)$ .

(133) Using the error difference lemma from Lemma 1 and the PAC-Bayesian bound from Lemma 2, a bound on the error of a model  $\theta_\mu$  on domain  $\mu$  is derived.

Formal: Error difference lemma and PAC-Bayesian bound offer OOD generalization bound for SAM.

(134) This result upper bounds the error of a model  $\theta_\mu$  on domain  $\mu$  by the error of  $\theta_\nu$  on domain  $\nu$ , the error of  $\theta_\mu$  on domain  $\nu$ , and the error of  $\theta_\nu$  on domain  $\mu$ .

Formal: This result bounds  $\theta_\mu$ 's error on  $\mu$  using  $\theta_\nu$ 's error on  $\nu$ , PAC complexity,  $\theta_\mu$  sharpness,  $\theta_\nu$  error on  $\mu$ .

(135) Theorem 1 (Sharpness-Aware Domain Adaptation Error): For distributions  $\mu$  and  $\nu$  over  $\mathcal{X} \times \mathcal{Y}$  and a model  $\theta$ , the error of  $\theta_\mu$  on domain  $\mu$  is bounded by the error of  $\theta_\nu$  on domain  $\nu$  and the error of  $\theta_\nu$  on domain  $\mu$ .

Formal: Theorem 1: Error on  $\theta_\mu$  bound by  $\theta_\nu$  error and extra terms for distributions  $\mu, \nu$ .

(141) Each domain  $t \in [T]$  is a distribution  $\mu_t$  over  $\mathcal{X} \times \mathcal{Y}$ .

Formal: Each domain  $t \in [T]$  is distribution  $\mu_t$  on  $\mathcal{X} \times \mathcal{Y}$ .

(145) The distribution shift between successive pairs of gradually shifted distributions and the average distribution shift are mathematically defined.

Formal: Successive and average distribution shifts are mathematically defined.

(145) In gradual domain adaptation, a learner progressively accesses unlabeled examples from intermediate domains.

Formal: Gradual DA uses intermediate domains for minimizing target domain's error.

(146) Adding Gaussian perturbation around  $\theta_\mu$  increases the expected loss.

Formal: Gaussian perturbation near  $\theta_\mu$  raises expected loss.

(155) We choose the optimal number of intermediate domains  $T^\star$  for SAM from Figure 1.

Formal: Optimal intermediate domains  $T^\star$  for SAM selected from Figure 1.

(158) SAM can be applied to GDA to consistently achieve stronger performance.

Formal: SAM consistently boosts GDA performance.

(159) SAM is not too sensitive to the perturbation radius hyperparameter.

Formal: SAM's sensitivity to perturbation radius is minimal.

(160) SAM with  $\rho = 0.2$  leads to the strongest performance on Rotated MNIST, Portraits, and Color MNIST.

Formal: Using  $\rho = 0.2$  achieves top SAM performance on Rotated MNIST, Portraits, Color MNIST.

(160) SAM with  $\rho = 0.05$  leads to the strongest performance on Covertypes.

Formal: Using  $\rho = 0.05$  achieves top SAM performance on Covertypes.

(161) The strongest SAM variants lead to an average improvement of 1.42% compared to using AdamW.

- Formal: Top SAM variants outperform Adam by 1.42% on average.
- (163) The SAM variants with reduced computational cost tend to underperform Adam on GDA.  
Formal: Low-cost SAM variants underperform Adam in GDA.
- (164) FriendlySAM outperforms SAM by 0.40% on average across all datasets.  
Formal: FriendlySAM exceeds SAM by 0.40% on average over datasets.
- (164) FisherSAM slightly underperforms SAM by 0.01%.  
Formal: FisherSAM is 0.01% less efficient than SAM.
- (165) SAM can be used to consistently improve target domain error for GDA.  
Formal: SAM consistently reduces target domain error in GDA.
- (166) The segment claims that there is an extension of Theorem 1 to the setting of gradual domain adaptation.  
Formal: Theorem 1 is extended for GDA.
- (166) GDA is presented as a technique which improves target domain error by performing gradual self-training.  
Formal: GDA uses gradual self-training on unlabeled intermediates to improve target error.
- (173) The discrepancy measure captures the non-stationarity of the gradually shifting data.  
Formal: Discrepancy measure reflects data's non-stationarity.
- (174) The sequential Rademacher complexity generalizes the standard Rademacher complexity to the online learning setting.  
Formal: Sequential Rademacher complexity adapts standard complexity for online learning.
- (176) Each of the  $nT$  samples is viewed as the smallest element of the adaptation process.  
Formal:  $nT$  samples are smallest adaptation process elements.
- (178) Generalization bound can be stated for GDA performed using SAM.  
Formal: Generalization bound exists for SAM-used GDA.
- (179) The population risk of the gradually adapted model  $\theta_T$  can be bounded under certain conditions.  
Formal: Population risk for gradually adapted  $\theta_T$  is boundable.
- (182) By applying Corollary 2 of Kuznetsov & Mohri (2020a), a preliminary bound on the error in the target domain is obtained.  
Formal: Error bound for target domain  $ET(\theta_T)$  obtained using Kuznetsov & Mohri.
- (184) The bound we obtain in Theorem 2 is of a specific mathematical form.  
Formal: Theorem 2 offers mathematically specific bound.
- (189) Adding Gaussian perturbation to each solution  $\theta_t$  increases the expected loss.  
Formal: Each  $\theta_t$  with Gaussian perturbation raises expected loss.
- (189) Theorem 1 must relate the parameters of the successive domains separately.  
Formal: Theorem 1 requires separate parameter relationships.
- (190) The PAC Bayes bound introduces an additional weight norm term  $W_{\text{avg}}$ .  
Formal: PAC Bayes bound adds  $W_{\text{avg}}$  weight norm term.
- (191) The PAC Bayesian bound yields a different sample complexity term compared to the original Rademacher bound.  
Formal: PAC Bayesian bound differs from Rademacher in sample complexity.
- (197) The overall sample complexity is expected to remain the same between our Theorem 2 and the Rademacher bound.  
Formal: Our Theorem 2 and Wang et al. expect consistent sample complexity.
- (200) A tighter error difference between shifted domains would improve the analysis.  
Formal: Tighter shifted domain error difference enhances analysis.
- (201) A localized analysis could exploit implicit properties of SAM.  
Formal: Localized analysis can leverage SAM implicit properties.
- (203) There is a relationship between sharpness and generalization.  
Formal: Sharpness is related to generalization.
- (206) Sharpness is among the empirical measures most strongly correlated with generalization.  
Formal: Sharpness strongly correlates with generalization empirically.
- (208) Sharp minima can generalize under reparameterizations that cause flat minima to become arbitrary.

- Formal: Sharp minima can generalize despite reparameterizations causing flat minima sharpness.
- (209) A measure of sharpness is tied to the information geometry of the data and is invariant under reparameterization.
- Formal: Sharpness measure is data geometry-related and reparameterization-invariant.
- (210) Many works have explored algorithms that lead to flatter solutions.
- Formal: Research explores algorithms creating flatter solutions.
- (211) In the context of domain generalization (DG), sharpness affects generalization.
- Formal: In DG, sharpness impacts generalization.
- (212) A modified version of stochastic weight averaging leads to flatter minima with improved DG.
- Formal: Modified stochastic weight averaging results in flatter minima, improving DG.
- (213) Generalization bounds depend on the empirical robust loss in the source domain.
- Formal: Generalization bounds rely on source domain empirical robust loss.
- (215) A flatness-aware minimization algorithm for DG leads to improved performance.
- Formal: Flatness-aware minimization enhances DG performance.
- (217) Out-of-distribution (OOD) generalization bounds are presented based on sharpness.
- Formal: OOD generalization bounds are given in terms of sharpness.
- (219) Sharpness-Aware Minimization (SAM) was originally proposed in Foret et al.
- Formal: SAM was initially suggested by Foret et al.
- (220) A PAC Bayesian analysis provides a generalization bound in terms of the expected sharpness of the model.
- Formal: PAC Bayesian analysis offers a bound based on expected sharpness with isotropic Gaussian noise.
- (222) The practical implementation of SAM uses this first-order approximation.
- Formal: SAM's practical use employs first-order approximation.
- (224) The flatness of the final solution does not sufficiently capture the generalization benefit from SAM.
- Formal: Final solution flatness inadequately captures SAM's generalization benefit.
- (225) SAM leads to lower rank features with fewer active ReLU units.
- Formal: SAM reduces feature rank and ReLU unit activity.
- (225) SAM enhances feature quality by selecting more balanced features.
- Formal: SAM boosts feature quality by balancing selection.
- (225) SAM enhances robustness to label noise through implicitly regularizing the model Jacobian.
- Formal: SAM enhances label noise robustness by implicit Jacobian regularization.
- (225) SAM has an implicit denoising mechanism which prevents harmful overfitting in settings when SAM is used.
- Formal: SAM's implicit denoising deters harmful overfitting, unlike SGD.
- (226) Many variants of SAM have been proposed to improve the efficiency and accuracy of the original SAM.
- Formal: SAM variants aim to improve original SAM efficiency and accuracy.
- (229) Gradual self-training (GST) in GDA outperforms standard self-training without intermediate domains.
- Formal: GST in GDA overshadows standard self-training lacking intermediate domains.
- (230) These bounds have an exponential dependence on the number of intermediate domains  $T$ .
- Formal: Bounds exponentially depend on intermediate domains  $T$ .
- (234) The analysis can be generalized to any  $p$ -Lipschitz losses and Wasserstein distances of any order  $p \geq 1$ .
- Formal: Analysis generalizes to any  $p$ -Lipschitz losses and  $p \geq 1$  Wasserstein distances.
- (235) The existence of an optimal choice of intermediate domains  $T$  is suggested by the refined bounds.
- Formal: Refined bounds imply optimal intermediate domains  $T$  exist.
- (238) A new method of generating intermediate domains in an encoded feature space is proposed.
- Formal: New method proposed for creating intermediate domains in encoded feature space.
- (242) The main limitation of this work is the discrepancy between our theoretical analysis based on sharpness and prior work's asymptotic analysis.
- Formal: Major limitation is discrepancy in sharpness-based theoretical analysis and prior work's asymptotic analysis.
- (244) The analysis for SAM can be tightened.

- Formal: SAM analysis can be refined.
- (247) SAM contributes to out-of-distribution generalization.  
Formal: SAM aids OOD generalization.
- (248) The original SAM achieved a 4.76% average improvement over the Adam baseline.  
Formal: Original SAM improves Adam by 4.76% on average.
- (248) The strongest SAM variants achieved an 8.01% average improvement over the Adam baseline.  
Formal: Top SAM variants top Adam by 8.01% on average.
- (249) An OOD (Out-of-Distribution) generalization bound can be derived based on sharpness.  
Formal: OOD generalization bound is derivable from sharpness.
- (252) We provided an extension of our OOD generalization bound to get a generalization bound based on sharpness.  
Formal: We've extended our OOD bound to a sharpness-based generalization bound for GDA.
- (254) There is a discrepancy between theoretical and empirical results regarding SAM.  
Formal: Discrepancy exists between SAM's theoretical and empirical results.
- (255) Our theoretical results provide a starting point for doing this.  
Formal: Theoretical results offer a starting point.
- (255) Our empirical results suggest that SAM can be used empirically to achieve significant gains for OOD generalization.  
Formal: Empirical results indicate SAM's OOD generalization efficacy.
- (257) Sharpness-aware minimization leads to low-rank features.  
Formal: SAM results in low-rank features.
- (265) There is a cohesive theory that addresses how learning occurs across different domains.  
Formal: A cohesive theory explains multi-domain learning.
- (276) Domain generalization can be achieved by seeking flat minima.  
Formal: Seek flat minima for domain generalization.
- (279) Entropy-sgd biases gradient descent into wide valleys.  
Formal: Entropy-sgd directs gradient descent to wide valleys.
- (282) Sharpness-aware minimization generalizes better than SGD.  
Formal: SAM generalizes better than SGD.
- (285) Sharp minima can generalize for deep nets.  
Formal: Sharp minima generalize in deep networks.
- (293) Efficient sharpness-aware minimization leads to improved training of neural networks.  
Formal: Efficient SAM enhances neural network training.
- (296) Sharpness-aware minimization efficiently improves generalization.  
Formal: SAM efficiently boosts generalization.
- (305) Flat minima in the context of optimization and machine learning refer to regions in the parameter space that are invariant to perturbations.  
Formal: Flat minima in optimization offer invariant regions for better generalization.
- (311) Batch normalization accelerates deep network training.  
Formal: Batch normalization speeds up deep network training.
- (314) Averaging weights leads to wider optima and better generalization.  
Formal: Weight averaging provides wider optima for better generalization.
- (321) Large-batch training for deep learning leads to a generalization gap.  
Formal: Large-batch training induces a generalization gap.
- (321) Large-batch training results in sharp minima.  
Formal: Sharp minima arise from large-batch training.
- (329) Information geometry provides a framework for understanding optimization techniques like sharpness-aware minimization.  
Formal: Information geometry explains SAM optimization.
- (335) Self-training is an effective method for gradual domain adaptation.

- Formal: Self-training effectively facilitates gradual domain adaptation.
- (340) Discrepancy-based theory is useful for forecasting non-stationary time series.  
Formal: Discrepancy theory aids in non-stationary time series prediction.
- (346) Adaptive sharpness-aware minimization (Asam) is proposed for scale-invariant learning in deep  
Formal: ASAM supports scale-invariant deep neural network learning.
- (368) The natural gradient method provides new insights and perspectives.  
Formal: Natural gradient method offers fresh insights.
- (372) Normalization layers are all that sharpness-aware minimization needs.  
Formal: SAM requires only normalization layers.
- (378) Online learning can be understood through the framework of sequential complexities.  
Formal: Sequential complexities elucidate online learning.
- (384) Sharpness-aware minimization enhances feature quality via balanced learning.  
Formal: SAM improves feature quality through balanced learning.
- (388) Dropout is a simple way to prevent neural networks from overfitting.  
Formal: Dropout prevents neural network overfitting.
- (392) Gradual domain adaptation can be better understood through improved analysis.  
Formal: Enhanced analysis clarifies gradual domain adaptation.
- (396) Sharpness minimization algorithms do not only minimize sharpness to achieve better generalization.  
Formal: Sharpness algorithms use more than minimizing sharpness for generalization.
- (402) Averaging weights of multiple fine-tuned models improves accuracy without increasing inference time.  
Formal: Weight averaging of fine-tuned models enhances accuracy without extra inference time.
- (407) A theoretical framework for out-of-distribution generalization is necessary.  
Formal: OOD generalization requires a theoretical framework.
- (410) Flatness-aware minimization is a crucial method for improving domain generalization.  
Formal: Flatness-aware minimization crucially improves domain generalization.
- (414) Invariant representations are crucial for effective domain adaptation.  
Formal: Invariant representations are key to domain adaptation.
- (419) There are fundamental limits in invariant representation learning.  
Formal: Invariant representation learning has fundamental limits.
- (419) Tradeoffs exist in the process of learning invariant representations.  
Formal: Invariant representation learning involves tradeoffs.
- (422) Gradual domain adaptation via gradient flow is a viable method.  
Formal: GDA by gradient flow is viable.
- (425) Robust out-of-distribution generalization can be achieved through considerations of sharpness.  
Formal: Sharpness considerations secure robust OOD generalization.
- (428)  $|E_{\mu}(\theta_{\mu}) - E_{\nu}(\theta_{\nu})| \leq S_{\rho}(\theta_{\mu}) + O(\|\theta_{\mu} - \theta_{\nu}\| + W_{\rho}(\mu, \nu))$   
Formal: Error difference bound by sharpness and distribution distance.
- (430) Given distributions  $\mu, \nu$  over  $X \times Y$  and an error function  $E$  with loss satisfying Assumption 1 with  
Formal:  $E_{\mu}(\theta_{\mu})$  is error-bounded by  $E_{\nu}(\theta_{\nu})$  with high probability, based on Assumption 1.
- (430) A sharpness-aware generalization bound, along with Rademacher complexity and robust error d  
Formal: Sharpness-aware bound with Rademacher and robust error provides error expectation bound
- (431) If  $E(\theta) \leq E_{\theta} \sim N(0, \rho^2 I)[E(\theta + \theta)]$ , then with probability  $\geq 1 - \delta$ ,  $E(\theta) \leq \hat{E}_{\rho}(\theta) + O(\sqrt{(k \ln(\|\theta\|^2 / \rho^2))})$   
Formal: If  $E(\theta) \leq E_{\theta} \sim N(0, \rho^2 I)[E(\theta + \theta)]$ ,  $E(\theta)$  is bounded with high probability.
- (433) Theorem 2 is a key claim that is restated within the context of 'Total Sharpness-Aware Error Under  
Formal: Theorem 2 restates 'Total Sharpness-Aware Error Under GDA'.
- (434) The population risk of the gradually adapted model  $\theta_T$  can be bounded with high probability.



Formal: Population risk of gradually adapted  $\theta_T$  is bounded with high probability.

(438) The term  $ET(\theta_T)$  can be bounded by a sequence of steps applying Theorem 1.

Formal:  $ET(\theta_T)$  is bounded using Theorem 1 in steps.

(438)  $ET_{-1}(\theta_T)$  and other successive terms ( $ET_{-2}(\theta_T)$ ,  $ET_{-3}(\theta_T)$ , etc.) can be bound similarly using the

Formal: Terms like  $ET_{-1}(\theta_T)$  are similarly bound using derived formulas.

(439) The text provides a bound on the expected value  $ET(\theta_T)$  given initial conditions and average val

Formal: Bound on  $ET(\theta_T)$  uses initial conditions, average parameter values over  $T$ .

(442)  $|E\mu(\theta) - E\nu(\theta)| \leq O(W_p(\mu, \nu))$

Formal: Error difference limited by Wasserstein distance.

(444) Proposition 1 (Discrepancy Bound - Lemma 2 of Wang et al.

Formal: Proposition 1 gives discrepancy bound per Wang et al.'s Lemma 2.

(446)  $\text{disc}(q_t) \leq O\left(\frac{1}{\sum_{k=0}^{t-1} q_k(t-k-1)} W_p(\mu_k, \mu_{k+1})\right)$

Formal:  $\text{disc}(q_t)$  is bounded by distribution shifts.

(446)  $\text{disc}(q_t) \leq O(t\Delta)$  when  $q_t = q$   $t := (1/t, \dots, 1/t)$

Formal:  $\text{disc}(q_t)$  follows  $O(t\Delta)$  under specific conditions.

(448) Definition 8 (Rademacher Complexity) introduces the concept of empirical Rademacher complex

Formal: Rademacher Complexity defines empirical complexity for models.

(450) The Rademacher Complexity of our model family is bounded for all distributions  $\mu \in \Delta(\mathcal{R}_d)$ .

Formal: Model's Rademacher Complexity is bounded for  $\mu \in \Delta(\mathcal{R}_d)$ .

(451) There exists some  $B > 0$  so that for any set of  $n$  samples drawn i.i.d.

Formal: Some  $B > 0$  exists for any i.i.d. sample set.

(452)  $R_\mu(\Theta) \leq B\sqrt{n}$  given that  $\mu$  is an element of  $\Delta(\mathcal{R}_d)$

Formal:  $R_\mu(\Theta) \leq B\sqrt{n}$  if  $\mu \in \Delta(\mathcal{R}_d)$ .

(453) Lemma 4 (Rademacher Complexity Generalization Bound): If Assumption 2 holds, then for any  $\theta$

Formal: Lemma 4: Under Assumption 2,  $\theta$ 's empirical vs. population error is boundable.

English Reconstruction:

=== English Reconstruction of the Argument ===

- Sharpness-aware minimization (SAM) is a promising method to improve generalization.
- The original proposal of SAM by Foret et al. signifies a foundational shift.
- We study SAM for out-of-distribution (OOD) generalization.
- The original SAM outperforms the Adam baseline by 4.76% in zero-shot out-of-distribution (OOD) ge
- An OOD (out-of-distribution) generalization bound can be provided in terms of sharpness for a partic
- Gradual domain adaptation (GDA) is a form of out-of-distribution (OOD) generalization that utilizes in
- The original SAM outperforms the baseline of Adam on each of the experimental datasets by 0.82%
- The strongest SAM variants outperform Adam by 1.52% on average.
- A generalization bound for SAM is provided in the GDA setting.
- Asymptotically, this generalization bound is no better than the one for self-training in the literature of
- There is a disconnection between the theoretical justification for SAM and its empirical performance.
- Low sharpness alone does not account for all of SAM's generalization benefits.
- There are potential avenues for obtaining a tighter analysis for Stochastic Approximation Methods (S
- Theoretical results provide a solid starting point for analyzing SAM in OOD settings.
- SAM can be applied to OOD settings to significantly improve accuracy.
- Newer variants of SAM can be leveraged for further improvements in accuracy.
- Sharpness-Aware Minimization (SAM) is a promising new optimization algorithm.
- A robust optimization procedure can lead to significant performance gains in the i.i.d.

- SAM remains understudied in the out-of-distribution (OOD) generalization setting.
- A number of SAM variants have been proposed to improve the accuracy and efficiency of the original SAM.
- The potential to enhance OOD (out-of-distribution) generalization exists.
- There are eight SAM variants, including the original SAM.
- SAM can be used to improve zero-shot OOD generalization.
- The strongest SAM variants can be used for an even further improvement.
- A theoretical analysis of SAM under the distribution shift setting provides performance gains.
- We extend the setting to gradual domain adaptation.
- SAM outperforms the Adam baseline by 0.82% on average.
- The strongest SAM variants achieve an even greater 1.52% average improvement over Adam.
- SAM and the strongest SAM variants can be used for consistent performance gains in GDA.
- Our work is asymptotically the same as prior work in the GDA literature (Wang et al., 2022).
- We further define the sharpness of  $\theta$  and the corresponding  $p$ -robust empirical loss.
- The  $p$ -robust risk of parameters  $\theta \in \Theta$  is the maximum loss obtained by perturbing  $\theta$  in the worst possible direction.
- The  $p$ -sharpness of parameters  $\theta$  measures how much the loss increases when we perturb them in the worst possible direction.
- Sharpness-aware minimization (SAM) proposes minimizing the  $p$ -robust empirical loss rather than the standard empirical loss.
- SAM's objective is to find a minimizer  $\theta^*$  of the form:  $\theta^* = \arg\min_{\theta \in \Theta} \max_{\|\beta\|_2 \leq p} E(\theta + \beta)$ .
- The SAM gradient drops a second-order term arising from the chain rule.
- Our results can be extended to any  $p$ -norm.
- This should not be confused with  $m$ -sharpness from Foret et al.
- SAM (Sharpness-Aware Minimization) is a method for optimizing model parameters.
- SAM uses variant-specific oracles for gradient computation, perturbation, and descent steps.
- Adaptive SAM (ASAM) is a version of SAM that uses a scale-invariant version of the first-order approximation.
- Adaptive SAM has one hyperparameter  $p$ .
- FisherSAM is a special case of ASAM with a specific normalization operator.
- K-SAM is a variant of SAM that only uses the top  $K$  data samples with the highest loss for gradient estimation.
- The idea behind Equation (12) is to remove the component of the descent gradient  $\nabla_{\theta} E(\theta) |_{\theta + \beta}$  lying in the direction of  $\beta$ .
- Using only the full-batch direction of the ascent-step gradient impairs performance.
- Computing the full-batch gradient  $\nabla_{\theta} E(\theta)$  is computationally prohibitive.
- FriendlySAM uses a specific perturbation method defined by an equation.
- NoSAM is a variant of SAM that only performs the SAM perturbation on normalization layers.
- The computational cost is compared across three specific metrics.
- EfficientSAM (ESAM) is a variant of SAM intended to make SAM more efficient.
- Stochastic weight perturbation is applied by performing the SAM perturbation on a fraction of the weights.
- Sharpness-sensitive data selection is performed by only computing gradients over data samples with high loss.
- A model  $\theta_S$  is trained on a source domain  $S \in \Delta(X \times Y)$  with a training set.
- The zero-shot OOD generalization error is given by  $ET(\theta_S)$ .
- The intermediate domains are evenly distributed between source and target.
- The intermediate domains are also evenly distributed between source and target.
- Variants such as LookSAM, F-SAM, and FisherSAM offer the strongest and most consistent improvements.
- LookSAM and NoSAM perform the best among the variants with reduced computational cost.
- LookSAM and NoSAM often outperform the original SAM in addition to being more efficient.
- FisherSAM takes the information geometry of the data into account, using an approximation of the Fisher information matrix.
- Under the cross-entropy loss used specifically in the experiments in Table 2, the Fisher information matrix is used.
- FisherSAM could be understood as a second-order approximation of the loss perturbation.
- Unlike FisherSAM, the connection between the FriendlySAM objective and OOD performance is not clear.

- The modified perturbation makes FriendlySAM significantly more robust to the choice of the  $\rho$  hyperparameter.
- The optimal value of  $\rho$  depends intricately on the choice of dataset.
- FriendlySAM provides performance gains in experiments by penalizing sharpness adaptively and stabilizing the training process.
- The Wasserstein distance is the smallest cost of moving mass between two distributions measured by the  $L_1$  norm.
- The loss functions considered in this paper are Lipschitz continuous.
- For the loss function  $\ell$ , there exist constants  $\rho_1, \rho_2, \rho_3$  such that certain inequalities involving  $\ell$  hold.
- This result holds for any choice of  $\mu, \nu$  on  $\mathcal{Y} \times \mathcal{X}$ .
- Lemma 1 (Sharpness-Aware Error Difference Over Shifted Domains) posits a bound on the difference between the population error and the empirical sharpness.
- Population error of a model  $\theta$  can be bounded in terms of its empirical sharpness.
- For any model  $\theta \in \Theta$  satisfying  $E(\theta) \leq E[\ell(\theta + \epsilon)]$  for some  $\epsilon > 0$ , with high probability (w.p.  $1 - \delta$ ),
- Using the error difference lemma from Lemma 1 and the PAC-Bayesian bound from Lemma 2, an OGD algorithm can be designed.
- This result upper bounds the error of a model  $\theta_\mu$  on domain  $\mu$  by the error of  $\theta_\nu$  on domain  $\nu$ , the same as the error difference lemma.
- Theorem 1 (Sharpness-Aware Domain Adaptation Error): For distributions  $\mu$  and  $\nu$  over  $\mathcal{X} \times \mathcal{Y}$  and an OGD algorithm,
- Each domain  $t \in [T]$  is a distribution  $\mu_t$  over  $\mathcal{X} \times \mathcal{Y}$ .
- The distribution shift between successive pairs of gradually shifted distributions and the average distribution is bounded.
- In gradual domain adaptation, a learner progressively accesses unlabeled examples from intermediate domains.
- Adding Gaussian perturbation around  $\theta_\mu$  increases the expected loss.
- We choose the optimal number of intermediate domains  $T$  for SAM from Figure 1.
- SAM can be applied to GDA to consistently achieve stronger performance.
- SAM is not too sensitive to the perturbation radius hyperparameter.
- SAM with  $\rho = 0.2$  leads to the strongest performance on Rotated MNIST, Portraits, and Color MNIST.
- SAM with  $\rho = 0.05$  leads to the strongest performance on Covertypes.
- The strongest SAM variants lead to an average improvement of 1.42% compared to using Adam.
- The SAM variants with reduced computational cost tend to underperform Adam on GDA.
- FriendlySAM outperforms SAM by 0.40% on average across all datasets.
- FisherSAM slightly underperforms SAM by 0.01%.
- SAM can be used to consistently improve target domain error for GDA.
- The segment claims that there is an extension of Theorem 1 to the setting of gradual domain adaptation.
- GDA is presented as a technique which improves target domain error by performing gradual self-training.
- The discrepancy measure captures the non-stationarity of the gradually shifting data.
- The sequential Rademacher complexity generalizes the standard Rademacher complexity to the online setting.
- Each of the  $nT$  samples is viewed as the smallest element of the adaptation process.
- Generalization bound can be stated for GDA performed using SAM.
- The population risk of the gradually adapted model  $\theta_T$  can be bounded under certain conditions.
- By applying Corollary 2 of Kuznetsov & Mohri (2020a), a preliminary bound on the error in the target domain is obtained.
- The bound we obtain in Theorem 2 is of a specific mathematical form.
- Adding Gaussian perturbation to each solution  $\theta_t$  increases the expected loss.
- Theorem 1 must relate the parameters of the successive domains separately.
- The PAC Bayes bound introduces an additional weight norm term  $W_{\text{avg}}$ .
- The PAC Bayesian bound yields a different sample complexity term compared to the original analysis.
- The overall sample complexity is expected to remain the same between our Theorem 2 and the main theorem.
- A tighter error difference between shifted domains would improve the analysis.
- A localized analysis could exploit implicit properties of SAM.
- There is a relationship between sharpness and generalization.
- Sharpness is among the empirical measures most strongly correlated with generalization.
- Sharp minima can generalize under reparameterizations that cause flat minima to become arbitrarily sharp.

- A measure of sharpness is tied to the information geometry of the data and is invariant under reparameterization.
- Many works have explored algorithms that lead to flatter solutions.
- In the context of domain generalization (DG), sharpness affects generalization.
- A modified version of stochastic weight averaging leads to flatter minima with improved DG.
- Generalization bounds depend on the empirical robust loss in the source domain.
- A flatness-aware minimization algorithm for DG leads to improved performance.
- Out-of-distribution (OOD) generalization bounds are presented based on sharpness.
- Sharpness-Aware Minimization (SAM) was originally proposed in Foret et al.
- A PAC Bayesian analysis provides a generalization bound in terms of the expected sharpness over a parameter space.
- The practical implementation of SAM uses this first-order approximation.
- The flatness of the final solution does not sufficiently capture the generalization benefit from SAM alone.
- SAM leads to lower rank features with fewer active ReLU units.
- SAM enhances feature quality by selecting more balanced features.
- SAM enhances robustness to label noise through implicitly regularizing the model Jacobian.
- SAM has an implicit denoising mechanism which prevents harmful overfitting in settings when SGD is used.
- Many variants of SAM have been proposed to improve the efficiency and accuracy of the original SAM.
- Gradual self-training (GST) in GDA outperforms standard self-training without intermediate domains.
- These bounds have an exponential dependence on the number of intermediate domains  $T$ .
- The analysis can be generalized to any  $p$ -Lipschitz losses and Wasserstein distances of any order  $p \geq 1$ .
- The existence of an optimal choice of intermediate domains  $T$  is suggested by the refined bounds.
- A new method of generating intermediate domains in an encoded feature space is proposed.
- The main limitation of this work is the discrepancy between our theoretical analysis based on sharpness and empirical results.
- The analysis for SAM can be tightened.
- SAM contributes to out-of-distribution generalization.
- The original SAM achieved a 4.76% average improvement over the Adam baseline.
- The strongest SAM variants achieved an 8.01% average improvement over the Adam baseline.
- An OOD (Out-of-Distribution) generalization bound can be derived based on sharpness.
- We provided an extension of our OOD generalization bound to get a generalization bound based on sharpness.
- There is a discrepancy between theoretical and empirical results regarding SAM.
- Our theoretical results provide a starting point for doing this.
- Our empirical results suggest that SAM can be used empirically to achieve significant gains for OOD generalization.
- Sharpness-aware minimization leads to low-rank features.
- There is a cohesive theory that addresses how learning occurs across different domains.
- Domain generalization can be achieved by seeking flat minima.
- Entropy-sgd biases gradient descent into wide valleys.
- Sharpness-aware minimization generalizes better than SGD.
- Sharp minima can generalize for deep nets.
- Efficient sharpness-aware minimization leads to improved training of neural networks.
- Sharpness-aware minimization efficiently improves generalization.
- Flat minima in the context of optimization and machine learning refer to regions in the parameter space.
- Batch normalization accelerates deep network training.
- Averaging weights leads to wider optima and better generalization.
- Large-batch training for deep learning leads to a generalization gap.
- Large-batch training results in sharp minima.
- Information geometry provides a framework for understanding optimization techniques like sharpness-aware minimization.
- Self-training is an effective method for gradual domain adaptation.

- Discrepancy-based theory is useful for forecasting non-stationary time series.
- Adaptive sharpness-aware minimization (Asam) is proposed for scale-invariant learning in deep neural networks.
- The natural gradient method provides new insights and perspectives.
- Normalization layers are all that sharpness-aware minimization needs.
- Online learning can be understood through the framework of sequential complexities.
- Sharpness-aware minimization enhances feature quality via balanced learning.
- Dropout is a simple way to prevent neural networks from overfitting.
- Gradual domain adaptation can be better understood through improved analysis.
- Sharpness minimization algorithms do not only minimize sharpness to achieve better generalization.
- Averaging weights of multiple fine-tuned models improves accuracy without increasing inference time.
- A theoretical framework for out-of-distribution generalization is necessary.
- Flatness-aware minimization is a crucial method for improving domain generalization.
- Invariant representations are crucial for effective domain adaptation.
- There are fundamental limits in invariant representation learning.
- Tradeoffs exist in the process of learning invariant representations.
- Gradual domain adaptation via gradient flow is a viable method.
- Robust out-of-distribution generalization can be achieved through considerations of sharpness.
- $|E_{\mu}(\theta_{\mu}) - E_{\nu}(\theta_{\nu})| \leq S_{\mu}(\theta_{\mu}) + O(\|\theta_{\mu} - \theta_{\nu}\| + W_{\mu}(\mu, \nu))$
- Given distributions  $\mu, \nu$  over  $X \times Y$  and an error function  $E$  with loss satisfying Assumption 1 with some  $\rho$ .
- A sharpness-aware generalization bound, along with Rademacher complexity and robust error difference.
- If  $E(\theta) \leq E_{\mu}(\theta) + \rho \|\theta - \theta_{\mu}\|$ , then with probability  $\geq 1 - \delta$ ,  $E(\theta) \leq \hat{E}_{\mu}(\theta) + O(\sqrt{k \ln(\|\theta - \theta_{\mu}\|^2 / \rho^2)} + \ln(1/\delta))$ .
- Theorem 2 is a key claim that is restated within the context of 'Total Sharpness-Aware Error Under Gradient Descent'.
- The population risk of the gradually adapted model  $\theta_T$  can be bounded with high probability.
- The term  $E_T(\theta_T)$  can be bounded by a sequence of steps applying Theorem 1.
- $E_{T-1}(\theta_T)$  and other successive terms ( $E_{T-2}(\theta_T)$ ,  $E_{T-3}(\theta_T)$ , etc.) can be bound similarly using the definition of sharpness-aware error.
- The text provides a bound on the expected value  $E_T(\theta_T)$  given initial conditions and average values of  $E_{T-1}(\theta_T)$ .
- $|E_{\mu}(\theta) - E_{\nu}(\theta)| \leq O(W_{\mu}(\mu, \nu))$
- Proposition 1 (Discrepancy Bound - Lemma 2 of Wang et al.
- $\text{disc}(q_t) \leq O(\sum_{k=0}^{t-1} q_k(t-k-1) W_{\mu}(\mu_k, \mu_{k+1}))$
- $\text{disc}(q_t) \leq O(t\Delta)$  when  $q_t = q_{\frac{1}{t}}$   $t := (1/t, \dots, 1/t)$
- Definition 8 (Rademacher Complexity) introduces the concept of empirical Rademacher complexity for a set of functions.
- The Rademacher Complexity of our model family is bounded for all distributions  $\mu \in \Delta(\mathcal{R}_d)$ .
- There exists some  $B > 0$  so that for any set of  $n$  samples drawn i.i.d.
- $R_{\mu}(\Theta) \leq B\sqrt{n}$  given that  $\mu$  is an element of  $\Delta(\mathcal{R}_d)$
- Lemma 4 (Rademacher Complexity Generalization Bound): If Assumption 2 holds, then for any  $\theta \in \Theta$ ,