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Department of Physics

Investigating Fourier Transform and Filturing Of Signals

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This investigation examined the fourier transform of diffferent sources of signals, initially from the function generator. The signal function was filtured using the low pass frequency of the range of 0-100 Hz to filture the signal depending on the source that provided the signal. Ratios of peak magnitudes for fundamental frequencies and their harmonics were found to be 0.035 for the triangular wave and 0.034 for the square wave. The relationship between the coefficients of Fourier transform and the power spectrum was found to be directly proportional. The frequency of the oscillation for the uncovered solar panel was found to be 99.93 ± 0.05 Hz.

1 Introduction

Vibrating springs and heat problems gave rise to the formation of the Fourier analysis with which at first the solution to the wave equation was solved by using the travelling waves and the superposition of standing waves separately which was first solved by D'Alambert in 1747 and elaborated by Euler a year later, D.Bernoulli proposed that the solution might be given using Fourier series but Euler was not entirely convinced and Fourier later argued that the initial methods can be used intechangingly either way for all initial conditions and led others to completely prove the general function could be represented as a Fourier series.

Fourier series is regarded as being the way of representing non-trig periodic functions as an infinite sum of trigonometric functions.

1.1 Aims

The main aim of this investigation was to study the band pass filturing of different signals using the Fourier transform. Observing the effects filturing has on the signal, finding the peak voltages for different fundamental frequencies to verify the relationship that Fourier power spectrum value has with coefficients of the Fourier expansion, finding the frequency of the oscillation in the solar panel signal and investigating how the equaliser works.

2 Theory

2.1 Fourier Series

The fourier series aims at rewriting existing functions in terms of an infinite sum of trigonometric functions and is constructed as follows for an existing function of $f(x)$;

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\frac{2\pi kx}{T} + \sum_{k=1}^{\infty} b_k \sin\frac{2\pi kx}{T} \tag{1}$$

With coefficents a_k and b_k given by;

$$a_k = \frac{2}{T}\int_0^T f(x)\cos\frac{2\pi kx}{T}dx, \quad b_k = \frac{2}{T}\int_0^T f(x)\sin\frac{2\pi kx}{T}dx \tag{2}$$

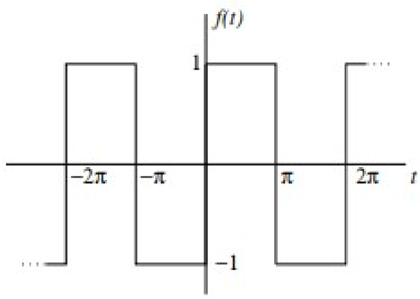
If the function of $f(x)$ were to be transfomed and the case of $f(x) = f(x+T)$ where T is the period had to be considered. The function can then be expanded in the following Fourier series:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega x} \tag{3}$$

Where ω and the coefficient of the Fourier function c_n are given by;

$$\omega = \frac{2\pi}{T}, \quad c_n = \frac{1}{T}\int_{\frac{T}{2}}^{\frac{T}{2}} f(x)e^{-in\omega x}dx$$

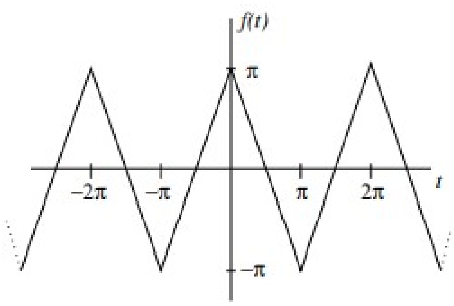
The fourier Analysis process converts a time series or any periodic waveform from its domain to a representation in the frequency domain, such time series can be presented in different forms some in square waves as the one shown below;



with its accompanying equation as follows;

$$f(x) = \frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + ...\right) \tag{4}$$

and also in this experiment we have examined the triangular wave which is also another time series and presented as follows;



which can be written as;

$$f(x) = \frac{8}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...\right) \tag{5}$$

2.2 Fourier transform

Fourier transform is a mathematical technique used to transform a function of time ($f(t) = f(x)$) which is not sinusoidal to a function of domain of frequency $F(\omega)$ and can be derived with accordance to the special case when the period $T \to \infty$ and is expressed in the following way;

$$F(\omega) = \frac{1}{\sqrt{2}\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

and it follows that ;

$$f(t) = \frac{1}{\sqrt{2}\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega$$

This derivation can possible occur with the condition that the function f(t) is intergrable and its discontinuities are finite in the given range.

2.3 Discrete Fourier transform

The DFT (Discrete Fourier Transform) is a famliy of methamatical techniques based on decomposing signals in time series to sinusoids, has both ita forward and inverse format and it also as the last one aims at converting time domain of a function of f(t) to a frequency domain.The data that needs to be transformed exsists on a grid of points at time interval of Δt and is converted or transformed onto a grid of frequency spacing $\Delta\omega$.This transformation is done in the following approach;

We first let $\Delta\omega = \frac{2\pi}{T}$, then we replace frequency with a finite set of N points and use the trapezoid rule ;

$$\begin{aligned} F(n\Delta\omega) &\approx \sum_{m=0}^{N-1} f(m\Delta t)e^{-in\Delta\omega m\Delta t} \\ &= \sum_{m=0}^{N-1} f(m\Delta t)e^{-i2\pi nm/N} \end{aligned}$$

T being the maximum time we have that $\Delta\omega = \frac{2\pi}{T}$ and therefore the inverse transform is then;

$$f(m\Delta t) = \frac{1}{N}\sum_{n=0}^{N-1} F(n\Delta\omega)e^{i2\pi mn/N}$$

With the grid spacing $\Delta t = \frac{T}{N}$, $t_n = n\Delta t$. We then let $\Delta\omega = \frac{2\pi}{T}$ and $\omega_n = n\Delta\omega$. Then as a result we have the followng;

$$\begin{aligned} F(\omega_n) &= \frac{1}{N}\sum_{m=0}^{N-1} f(t_m)e^{i\omega_n t_n} \\ &= \sum_{m=0}^{N-1} f(m\Delta t)e^{i2\pi nm/N} \end{aligned}$$

and the inverse transform is then;

$$f(t_m) = \sum_{n=0}^{N-1} F(\omega_n)e^{-2\pi mn/N}$$

2.4 Fast Fourier Trasform

FTT is the fast algorithm of computing a DFT or its inverse, this algorithm rapidly transforms the DFT by factorizing the existing matrix of the DFT. It is estimated to be about 1000 times faster than the DFT and can take N values of the order of 10^3 .

2.5 Power spectrum

Defined by;

$$P(\omega) = |F(\omega)|^2$$

The power spectrum determines how power is distributed into frequency on the signal and is regarded as the Fast-Fourier Transform (FTT) which is always applied to a finite domain whihc further causes power leaks.

2.6 Filters

In signal processing using filtures to filture out some regions for making the signal more defined we in this experiment used three simplest filtures namely the low pass filture, band pass filture and high pass filture with which in all these filtures frequencies are transmitted and filtured through according to the type of filture they belong in.

3 Experiment

3.1 Equipment

Function generator with a voltage of about 10 V was used to give the signal read through the myDAQ unit to the Fourier program. A breadboard was used to create a current flow from the function generator to the myDAQ unit by using wires connected to the AI 0_+ and AI 0_- ports of the myDAQ unit from the breadboard. Microphone was used to capture the signal from different types of instruments, voices and tunning forks. A Solar panel was used as the signal source and its signal was filtured on the Fourier transform program. Oscilloscope used for reading the frequency of the oscillation.

4 Procedure

4.1 Reading Function Generator Signals

Long wires were connected to the AI ports 0_+ and 0_- of the myDAQ unit. The BNC cable was used to connect the function generator to the power supply terminals of the breadboard. A 'circuit-like' board on the breadboard which transfered the signal from the function generator to the myDAQ unit was created and the Fourier Transform program used to read off the signal and further filtured it under a low band pass frequency of 100 Hz. Peak voltage magnitudes of the filtured signal for both the fundamental frequency of 80 Hz and 300 Hz when the circuit was driven with a sinusoidal and square waves respectively were determined.

4.2 Reading Microphone signals

The microphone was connected using the myDAQ unit and the signal of sources read on the Fourier program. D and G type tuning forks were used as the sources of the signal, filtured and their curves recorded. Voice sang through the microphone, the signal recorded for analysis and filtured through the FT program.

4.3 Reading the signal from the Solar Panels

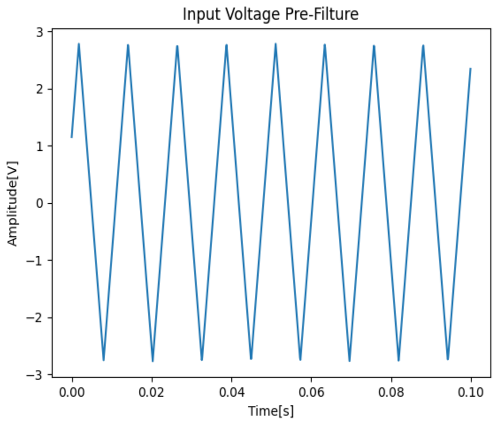
The solar panel was connected to the myDAQ unit using the 0_+ and 0_- ports . The signal when the solar panel was exposed to the light was filtured and recorded. The solar panel was covered with some sheet, its signal filtured and recorded for analysis.

4.4 Graphic Equalizer Signals for Different Sounds

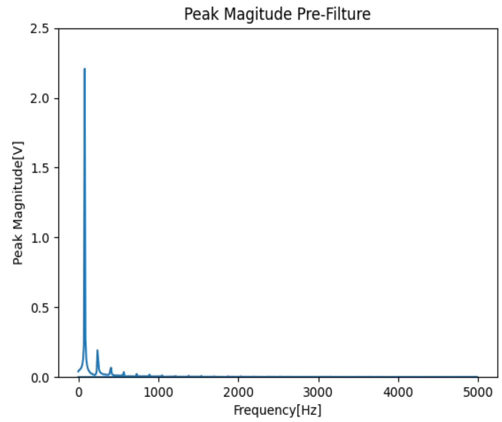
Using the audio input on the myDAQ unit, different audios of different pitches were played and sliders were alternated on the graphic equalizer, signal filtured and recorded for analysis .

5 Results

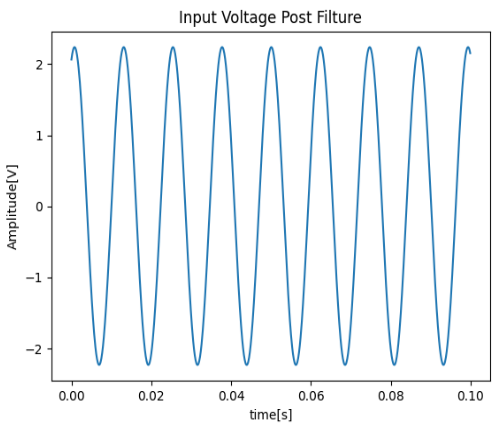
5.1 Signal from the Function Generator



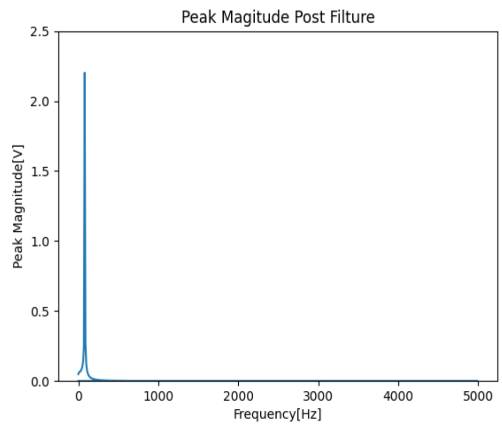
(a) Input from voltage the function generator



(b) peak magnitude pre-filture

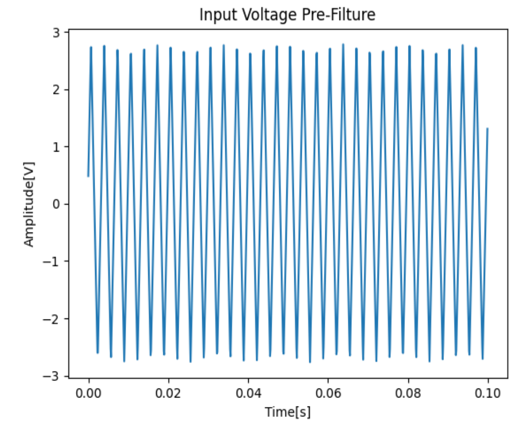


(c) post filtured voltage

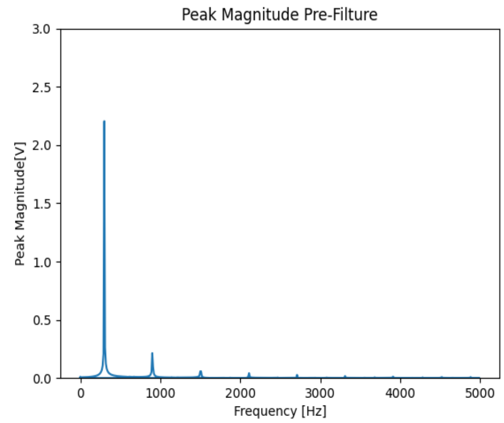


(d) post peak magnitude

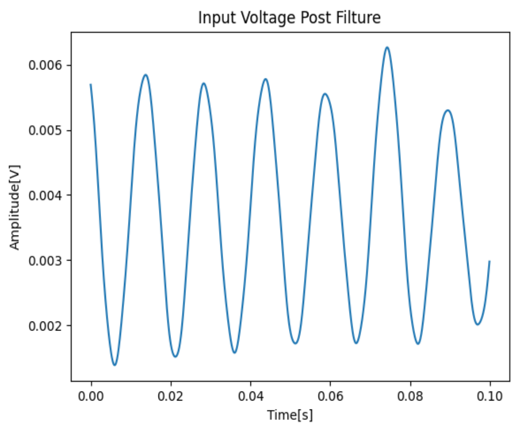
Figure 1: Signal from the function generator driven at 80 Hz on a triangular wave



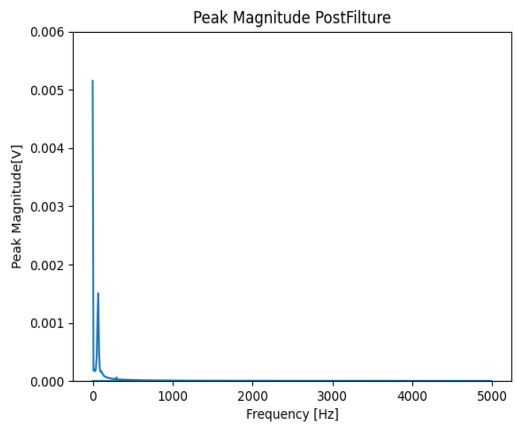
(a) Input from voltage the function generator



(b) peak magnitude pre-filture

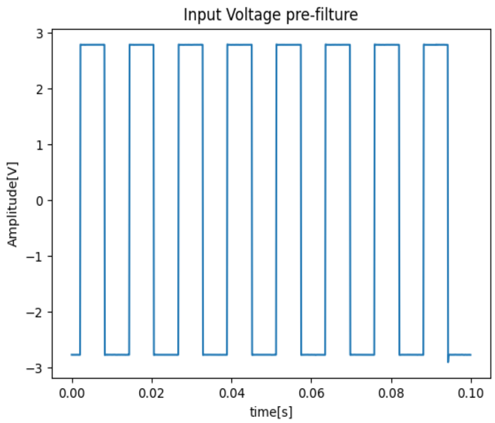


(c) post filtured voltage

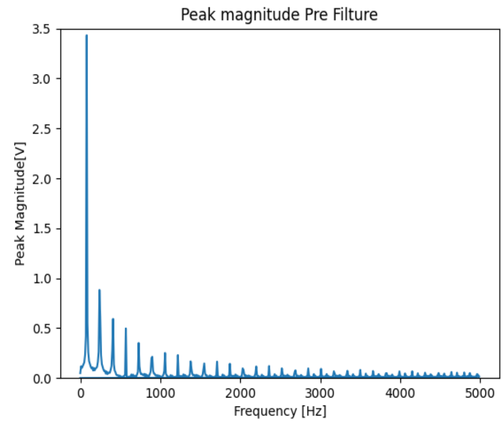


(d) post peak magnitude

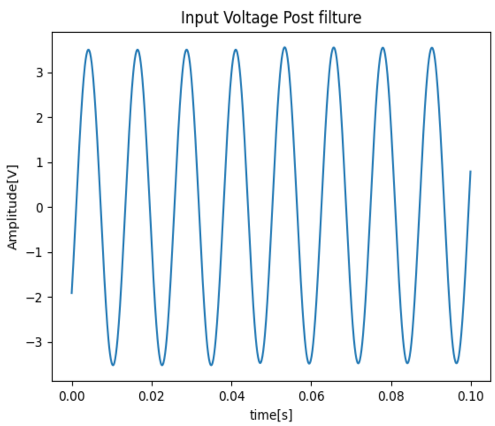
Figure 2: Signal from the function generator driven at 300 Hz on a triangular wave



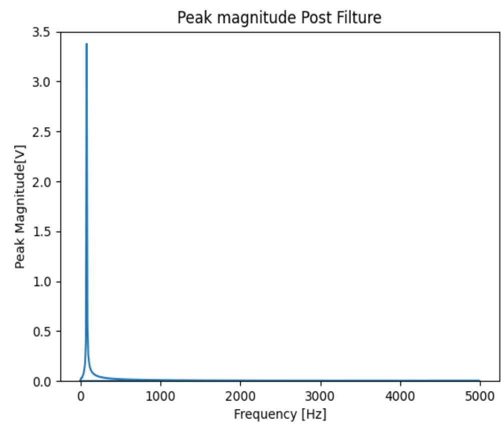
(a) Input from voltage the function generator



(b) peak magnitude pre-filture

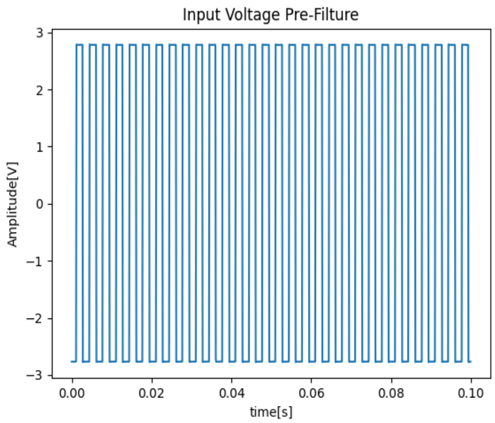


(c) post filtured voltage

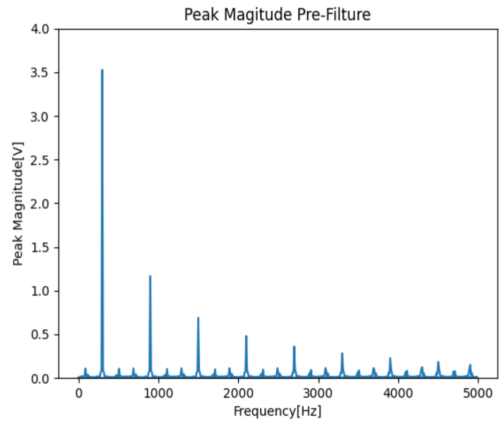


(d) post peak magnitude

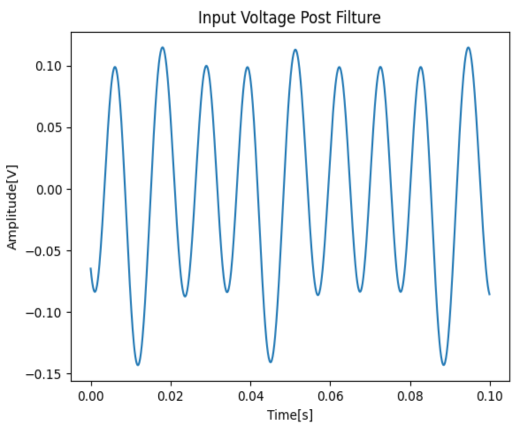
Figure 3: Signal from the function generator driven at 80 Hz on a square wave



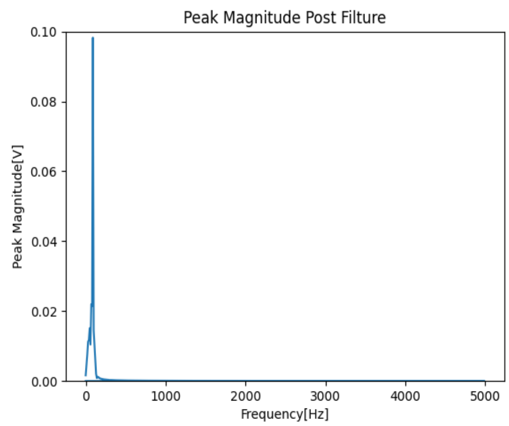
(a) Input from voltage the function generator



(b) peak magnitude pre-filture



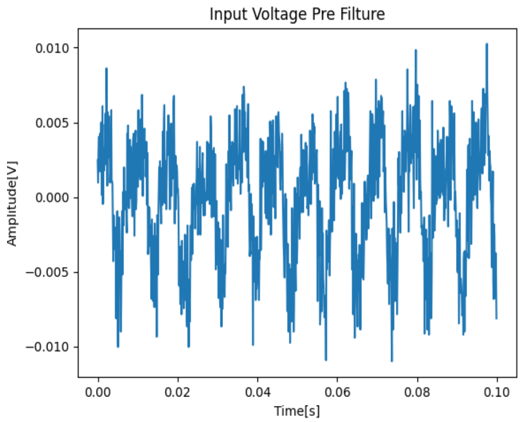
(c) post filtured voltage



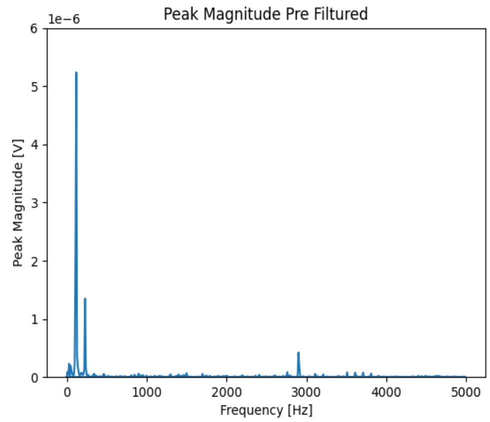
(d) post peak magnitude

Figure 4: Signal from the function generator driven at 300 Hz on a square wave

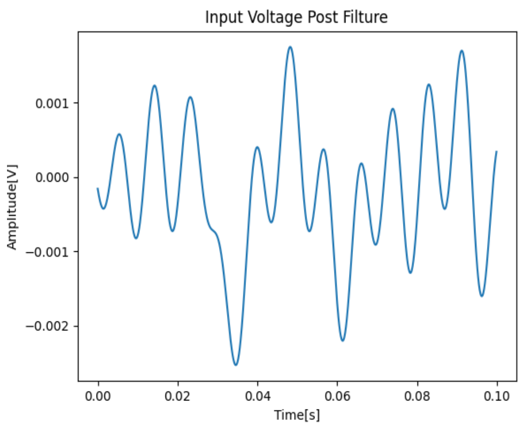
5.2 Signals from the Microphone



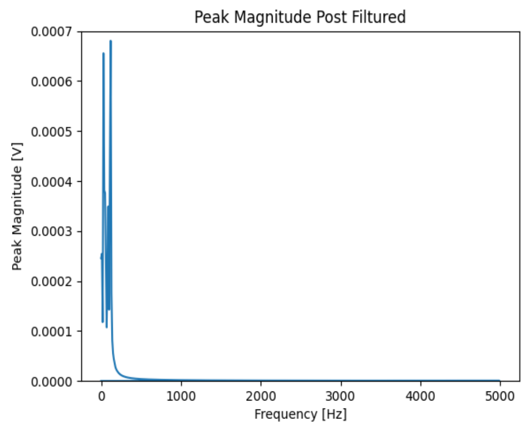
(a) Input from voltage the function generator



(b) peak magnitude pre-filture



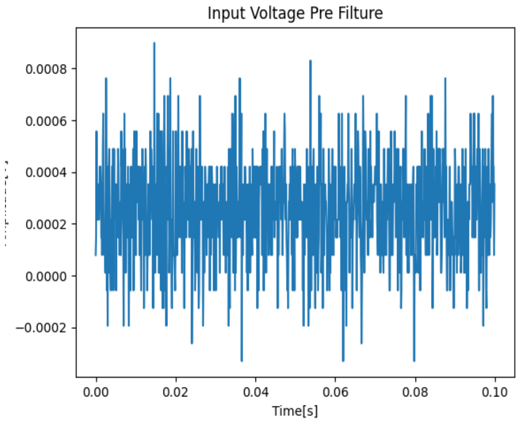
(c) post filtured voltage



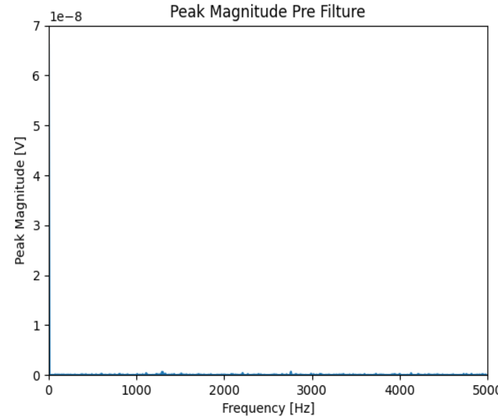
(d) post peak magnitude

Figure 5: Signal from the voices

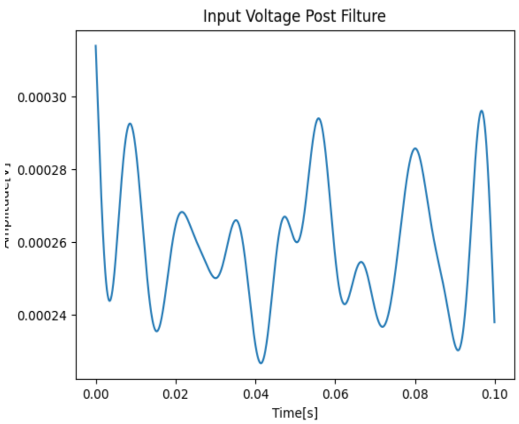
5.3 Signal from the Solar Panels



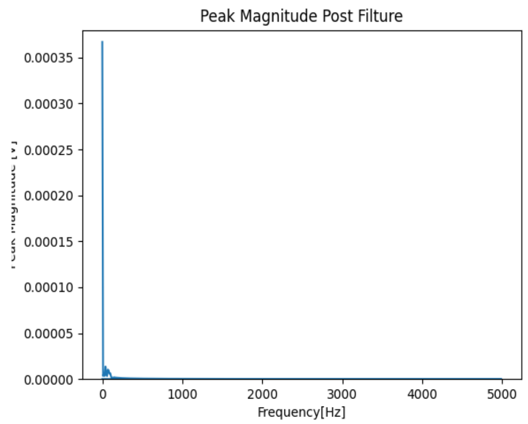
(a) Input from voltage the function generator



(b) peak magnitude pre-filture



(c) post filtured voltage



(d) post peak magnitude

Figure 6: Signal from the uncovered solar panel

6 Analysis

6.1 Function generator signal

All three waveforms(triangular, square and sinusoidal) were examined at frequencies of 80 Hz and 300 Hz. Then triangular and square waves were further examined at frequencies of 80 Hz and 300 Hz. When the signal was observed at 80 Hz as figure 1 shows , the following observations were made for a triangular wave;

Fundamental height(V)	Harmonics height(V)
1.75	0.062

For square waves at 80 Hz,

Fundamental height(V)	Harmonics height(V)
2.7	0.092

The square and triangular waves for 300 Hz results were;

Fundamental height(V)	Harmonics height(V)
1.75	0.062

For square waves,

Fundamental height(V)	Harmonics height(V)
2.54	0.076

The amplitude ratios were as follows;
For triangular waves;

$$r = \frac{Amp.Harmonic}{Amp.Fundamental} = \frac{0.062}{1.75} = 0.035$$

For Square waves;

$$r = \frac{Amp.Harmonic}{Amp.Fundamental} = \frac{0.092}{2.7} = 0.034$$

6.2 Microphone Signals

When the signal was not filtured using the Fourier transform as depicted in Figure 5 (a) , the signal was not well defined but the oscillations can be seen. As the signal undergoes low band pass frequency of 100 Hz, oscillations become well defined and other parameters can then be found with ease such as the period, frequency, etc. Peak magnitude after the Fourier transform becomes well defined as shown from Figure 5 (b) to (d).

6.3 Solar Panels

Voltage and current are variable since the solar panel is load dependent implying an oscillation in the amplitude of the signal read as seen on Figure 6 (a) and more clearly defined on the filtured signal in Figure 6 (c), the exposure time to the lights in the lab brought about these oscillations. The frequency of the oscillation using the oscilloscope was found to be 99.9276 Hz $\pm$ 0.05 Hz.

6.4 Graphic equalizer

Graphic equalizer gives the range of frequencies with which the signal from the sound source falls upon and gives the observer or listener an opportunity to alter these frequencies/ pitch of sound by using the virtual sliders provided on the graphic equalizer. When the sliders were altered in a way to put bass to zero, higher frequencies of high magnitude were recorded and otherwise. Tribble and bass were also made equivalent and it was observed that higher frequencies are still recorded, which implies tribble overpowers bass.

7 Conclusion

Using the signals captures for different signal sources, parameters such as the peak magnitudes for the fundamental and harmonic frequencies were found and recorded with their uncertainties. When a band pass of 100 Hz was used for filturing, the signal became a clear sinusoidal wave and parameters such as frequency were found. The ratios of the fundamental frequency to harmonic frequency for the peak magnitude were found to be 0.035 for the triangular wave and 0.034 for the square wave and it was concluded the relationship between power spectrum value and coefficients of the Fourier transform is proportional. The frequency of the oscillation was found to be 99.9276 ± 0.05 Hz.

8 References

(Pure and Applied Mathematics Volume 32) William F. Donoghue - Distributions and Fourier transforms-Academic Press (1969)

PHYLAB2 Experimental Physics Lab, Introduction to Fourier Analysis and Filters, PHY200W Experimental Lab(2022)