Project — Part 2

0. Project Rules and Advice:

- You may work alone or in a group of at most three.
- The project is due in class on Monday, April 18.
- There should be only one submission from each group. Make sure that the name of every member of the group appears on it.
- Your finished product should be typed (ideally, but not necessarily with LaTeX) or very neatly written.

1. Consider the linear program

$$(LP_0)\begin{cases} \text{Maximize } Z = cx, \\ \text{Subject to: } Ax \le b, \\ \qquad\qquad x \ge 0, \end{cases}$$

where $A = (a_{ij})$ is a 3×3 matrix,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}.$$

Think of LP_0 as a resource-allocation/profit-maximization problem for products P_1, P_2, P_3, made with resources R_1, R_2, R_3 by the manufacturing company Xco.

a. Use the simplex method to solve LP_0 with

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ 12 \\ 20 \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} 2.5 & 2 & 4 \end{bmatrix}.$$

Give an optimal solution x^* and the corresponding optimal tableau.

b. Write down the dual problem. Give the complementary optimal solution y^* to the dual.

c. Can Xco marginally increase its profit Z by purchasing more of resource R_1? If so, how much should they be willing to pay for one additional unit? What about R_2? Should Xco buy more? If so, at what unit price?

2. Suppose that Xco revises the model program (LP_0) to reflect changes in prices, supplies, etc. The revised problem is

$$(LP_1)\begin{cases} \text{Maximize } Z = \bar{c}x, \\ \text{Subject to: } \bar{A}x \le \bar{b}, \\ \qquad\qquad x \ge 0, \end{cases}$$

where

$$\bar{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} 10 \\ 12 \\ 20 \end{bmatrix} \quad \text{and} \quad \bar{c} = \begin{bmatrix} 3 & 2 & 4 \end{bmatrix}.$$

a. We'll solve (LP_1) by two different means. First solve the problem directly by the simplex method. Give the optimal solution $\bar{x}^*$ and the optimal tableau. How many iterations were required?

b. In the next two parts, we'll solve (LP_1) by sensitivity analysis. Give the matrices y^* and S^* from the optimal tableau for (LP_0). Use them to construct a revised tableau for (LP_1).

c. Is the revised tableau from part (b) in proper form? If so, reoptimize. If not, put it in proper form and then reoptimize. Give the optimal solution and optimal tableau for (LP_1).

d. Which solution method was faster? Direct application of the simplex method, or sensitivty analysis?

3. Suppose that Xco revises the model program (LP_0) *yet again.* The revised problem is

$$(LP_2)\begin{cases} \text{Maximize} & Z = cx, \\ \text{Subject to:} & Ax \leq \bar{b}, \\ & x \geq 0, \end{cases}$$

where A and c are as they were in (LP_0), and

$$\bar{b} = \begin{bmatrix} 14 \\ 12 \\ 20 \end{bmatrix}.$$

a. Solve (LP_2) by two different means. First treat the problem directly by the simplex method. You don't have to show your work. Just give the optimal solution $\bar{x}^*$ and the optimal tableau. How many iterations were required?

b. In the next two parts, you'll solve (LP_2) by sensitivity analysis. Give the matrices y^* and S^* from the optimal tableau for (LP_0). Use them to construct a revised tableau for (LP_2).

c. Is the revised tableau from part (b) in proper form? If not, put it in proper form.

d. Is the tableau (now in proper form) feasible? If not, use the dual simplex method to restore it to feasibility.

e. Is the (now feasible) tableau optimal? If not, use the simplex method to find an optimal solution. Give the optimal solution and optimal tableau for (LP_2).