CMP_SC 3050: Introduction to Greedy Algorithms

Rohit Chadha

September 25, 2014

Used for optimization problems (i.e. when we have to optimize some objective)

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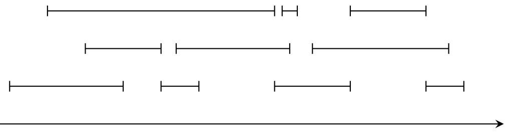
- When we have a choice to make, make the one that looks best right now.
- Make a locally optimal choice in hope of getting a globally optimal solution.
- Warning: Greedy algorithms dont always yield an optimal solution, but many times they do

Activity selection

Input: A set $\{a_1, a_2, \ldots, a_n\}$ of activities with start and finish times that require exclusive use of a common resource. $s(i)$ is the start time of activity i and $f(i)$ is the finish time of activity i.

Output: Schedule as many activities as possible



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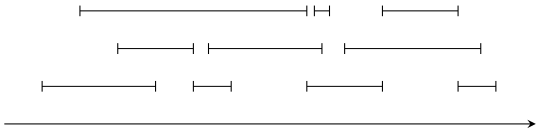

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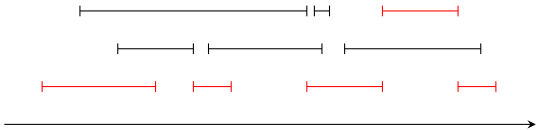

Greedy Template

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Initially R is the set of all activities
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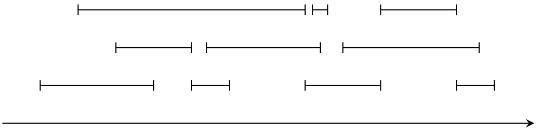
Main task: Decide the order in which to process activities in R

ES SP FC EF

Earliest Start Time

Process activities in the order of their starting times, beginning with those that start earliest.



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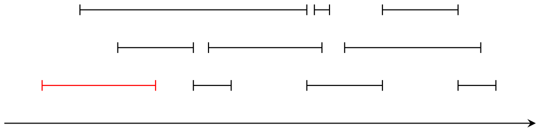

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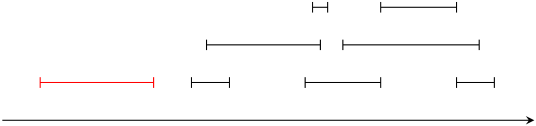

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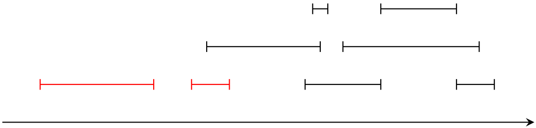

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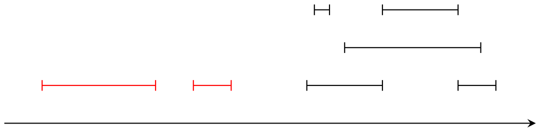

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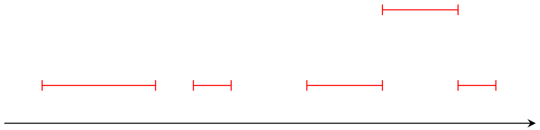

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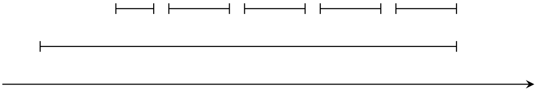

Figure : Counter example for earliest start time

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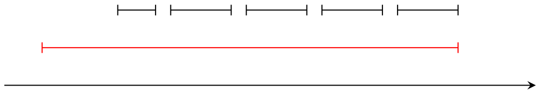

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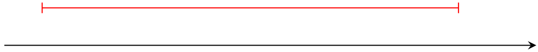

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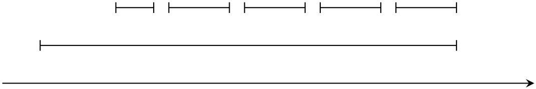

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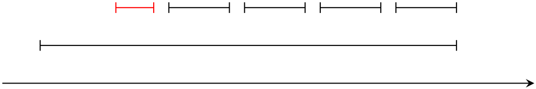

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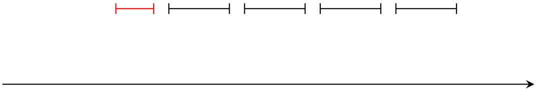

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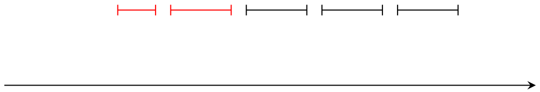

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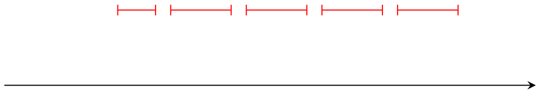

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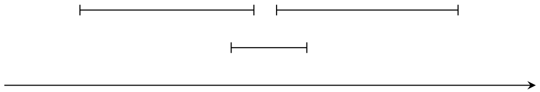

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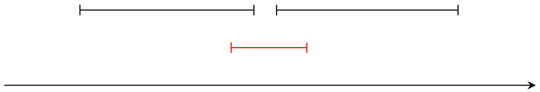

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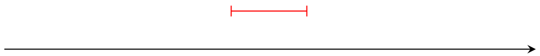

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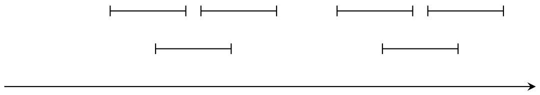

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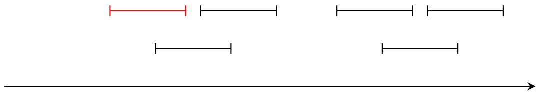

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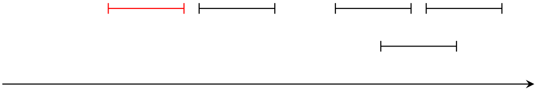

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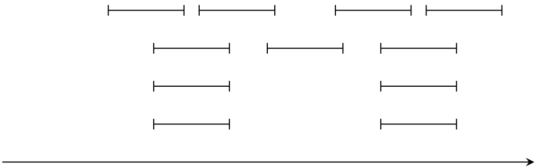

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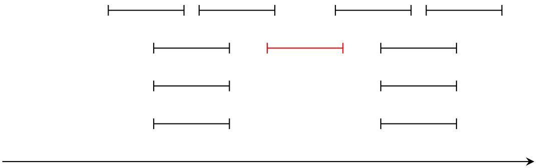

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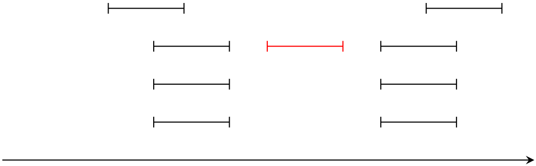

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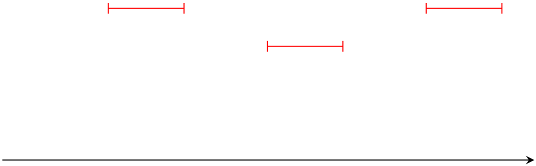

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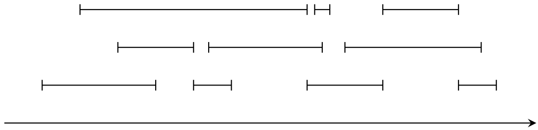

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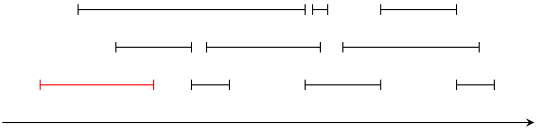

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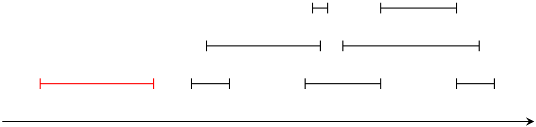

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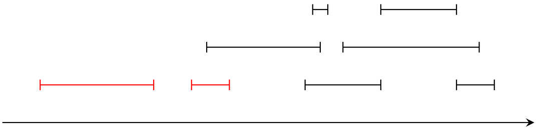

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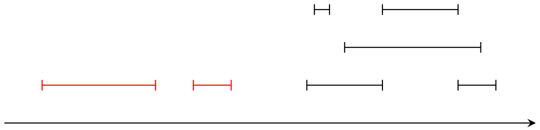

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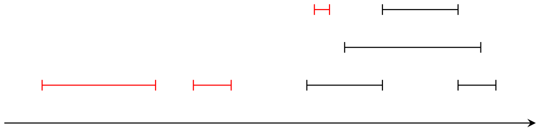

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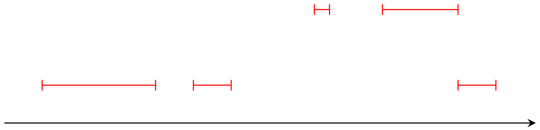

Optimal Greedy Algorithm

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Initially R is the set of all activities
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while R is not empty
    choose i ∈ R such that finishing time of i is least
    add i to A
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The greedy algorithm that picks activities in the order of their finishing times is optimal.

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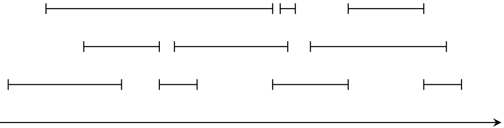

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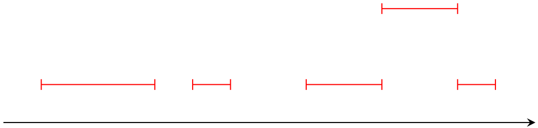

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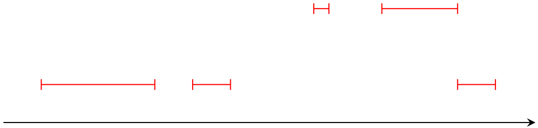

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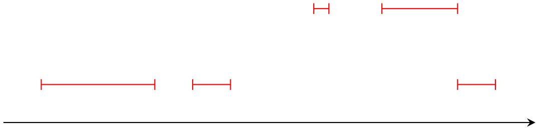

Instead we argue that O and A have the same number of elements

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- Now $s(j_{k+1}) \geq f(j_k)$, and so after removing all activities in conflict with $i_1, \ldots i_k$, j_{k+1} remains
- Greedy algorithm can still pick more activities!

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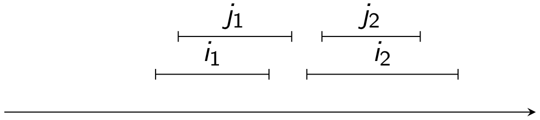
- Is this a problem?



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- Hence $f(i_2) \leq f(j_2)$
- We can continue the same argument for the 3rd, 4th, ... activities

Implementation and Running Time

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Initially R is the set of all activities
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- Now choosing least finishing time is $O(1)$
- Keep track of the finishing time of the last activity added to A (which can be implemented as a list). Then check if starting time of i later than that
- Thus, checking non-overlapping is $O(1)$

Implementation and Running Time

```
Initially R is the set of all activities
A is empty (* A will store all the activities that will be scheduled *)
while R is not empty
    choose i ∈ R such that finishing time of i is least
    if i does not overlap with activities in A
        add i to A
return the set A
```

- Pre-sort all activities based on finishing time. Takes $O(n \log n)$ time
- Now choosing least finishing time is $O(1)$
- Keep track of the finishing time of the last activity added to A (which can be implemented as a list). Then check if starting time of i later than that
- Thus, checking non-overlapping is $O(1)$
- Total time $O(n \log n + n) = O(n \log n)$

Extensions

- All activities need not be known at the beginning. Such *online* algorithms are a subject of research

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- All activities need not be known at the beginning. Such *online* algorithms are a subject of research
- Instead of maximizing the total number of activities, associate *value* with each activity that is scheduled. Try to schedule activities to maximize total value. Will be seen later in this course

Scheduling all Activities

Input A set of lectures, with start and end times

Output Find the minimum number of classrooms, needed to schedule all the lectures such two lectures do not occur at the same time in the same room.

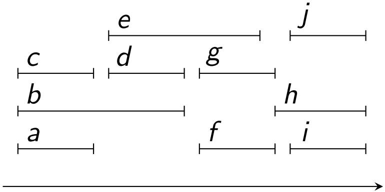

Figure : A schedule that uses 4 classrooms

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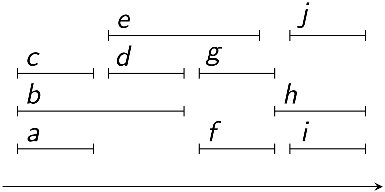

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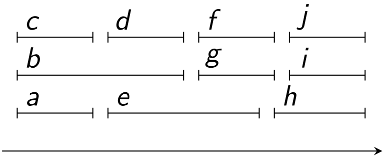

Figure : A schedule requiring 3 classrooms

Greedy Algorithm

```
Initially R is the set of all lectures
d = 0 (* number of classrooms *)
while R is not empty
    choose i ∈ R
    if i can be scheduled in some classroom k
        schedule lecture i in classroom k
    else
        allocate a new classroom d+1 and schedule lecture i in d+1
        d = d+1
```

What order should we process lectures in?

Greedy Algorithm

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Initially R is the set of all lectures
d = 0 (* number of classrooms *)
while R is not empty
    choose i ∈ R such that start time of i is earliest
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```

What order should we process lectures in? According to start times (breaking ties arbitrarily)

Depth of Lectures

- For a set of lectures R, k are said to be in conflict if there is some time t such that there are k lectures going on at time t.

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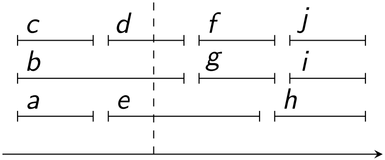

Depth and Number of Classrooms

For any set R of lectures, the number of classrooms required is at least the depth of R.

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All lectures that are in conflict must be scheduled in different rooms.

Number of Classrooms used by Greedy Algorithm

Let *dep* be the depth of the set of lectures R. The number of classrooms used by the greedy algorithm is at most *dep*.

- Suppose the greedy algorithm uses more that *dep* rooms. Let j be the first lecture that is scheduled in room $dep + 1$.

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- Since we process lectures according to start times, there are *dep* lectures that start (at or) before j and which are in conflict with j.
- Thus, at the starting time for j there are at least $dep + 1$ lectures going on, which contradicts the fact that the depth is *dep*.

Correctness

The greedy algorithm does not schedule two overlapping lectures in the same room.
The greedy algorithm is correct and uses the optimal number of classrooms.

Implementation and Running Time

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Initially R is the set of all activities
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Recall min-priority queues

- Like stacks, queues and lists, maintains a dynamic set
- Each element has a *key*. May have other data

Operations

$\textsc{Insert}(A, x)$:	inserts element x into priority queue A
$\textsc{Minimum}(A)$:	returns element of A with smallest key
$\textsc{Extract-Min}(A)$:	removes and returns element of A with smallest key
$\textsc{Decrease-Key}(A, x, k)$:	decreases value of element x's key to k

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Can be implemented using heaps. All operations except MINIMUM takes $O(\log n)$ time. MINIMUM takes $O(1)$ time

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- Pre-sort according to start times. Picking lecture with earliest start time can be done in $O(1)$ time.
- Keep track of the finish time of last lecture in each room.
- With priority queues, checking conflict takes $O(\log d)$ time.
- Total time $= O(n \log n + n \log dep) = O(n \log n)$

$0 - 1$ Knapsack Problem

Input: n items. Item i is worth \$ v_i and weighs w_i pounds

Output: Find a most valuable subset of items with total weight W. Have to either take an item or not take it–cannot take part of it.

Fractional Knapsack Problem

Input: n items. Item i is worth \$ v_i and weighs w_i pounds

Output: Find a most valuable subset of items with total weight W.
Can take part of an item.

Greedy algorithm and knapsack problem

Greedy algorithm and knapsack problem

Fractional knapsack has greedy solution!

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- Rank items by value/weight: $\frac{v_i}{w_i}$

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Greedy algorithm and knapsack problem

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- Rank items by value/weight: $\frac{v_i}{w_i}$
- Take items in decreasing order of value/weight
- Take all of the items with the greatest value/weight as possible, and possibly a fraction of the next item

Greedy algorithm and knapsack problem

Greedy algorithm does not work for $0 - 1$ knapsack problem!

Example:

Total capacity of knapsack, $W = 50$

i	1	2	3
v_i	60	100	120
w_i	10	20	30
$\frac{v_i}{w_i}$	6	5	4

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- Take items 1 and 2.
- Total value $= 160$ and *weight* $= 30$.

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- Take items 1 and 2.
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Have 20 pounds of capacity left over!

Optimal solution:

- Take items 2 and 3
- Total value $= 220$ and $weight = 50$

Greedy Analysis: Overview

- Greedy-choice property. Argue that after each step the solution of the greedy algorithm is at least as good as the solution of any other algorithm. Example, activity selection.

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- Structural property of solution. Observe some structural bound of every solution to the problem, and show that greedy algorithm achieves this bound. Example, scheduling all lectures.

This lecture was based on Sections 16.1 and 16.2 of the book
Your midterm is based on material upto now