Robustified RG, MIMO System

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Abstract

Your abstract.

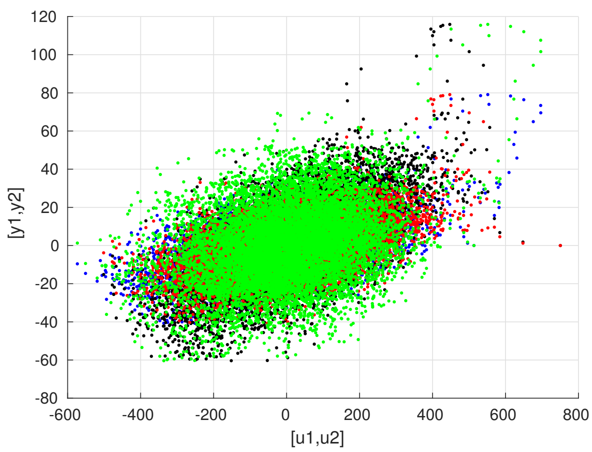
1 Introduction

Dealing with a mass-spring-damper MIMO system with 2 inputs (Forces) and 2 outputs(Positions). First, we perform experiments to do a MIMO-ARX identification of the system. Then, 2 separate dynamic models are extracted, one nominal and the other that generates the output error. This model that generates the output error is the disturbance model used to capture model uncertainty. Zero-measurement noise assumption is made.

2 Identification

Experiments are performed with the following parameters:

$$\begin{aligned} dt &= 0.1s & na &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\ N &= 10000 \\ u_1 &= 1000 * randn(1, N) & nb &= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \\ u_2 &= 1000 * randn(1, N) \\ \alpha &= 0.05 & nk &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{1}$$



Using the ARX identification toolbox from MATLAB, the following I/O model is identified

$$\begin{bmatrix} A_{11}(z^{-1}) & A_{12}(z^{-1}) \\ A_{21}(z^{-1}) & A_{22}(z^{-1}) \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} B_{11}(z^{-1}) & B_{12}(z^{-1}) \\ B_{21}(z^{-1}) & B_{22}(z^{-1}) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \tag{2}$$

Rewriting this as $\mathbf{A}y(t) = \mathbf{B}u(t) + \mathbf{C}w(t)$ and inverting $\mathbf{A}$, we get $y(t) = \mathbf{A}^{-1}\mathbf{B}u(t) + \mathbf{A}^{-1}\mathbf{C}w(t)$. This leads to two models with inputs $u(t)$ and $w(t)$, the sum of whose outputs captures all the dynamics of the system. It is made sure that the resultant systems are stable.

Converting the models $\Sigma_M : y_M(t) = \mathbf{A}^{-1}\mathbf{B}u(t)$ and $\Sigma_D : e(t) = \mathbf{A}^{-1}\mathbf{C}w(t)$ into state-space models and adding the outputs, we get the complete system as:

$$\begin{gathered}
\begin{bmatrix} x_M(k+1) \\ x_D(k+1) \end{bmatrix} = \begin{bmatrix} A_M & 0 \\ 0 & A_D \end{bmatrix} \begin{bmatrix} x_M(k) \\ x_D(k) \end{bmatrix} + \begin{bmatrix} B_M \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ B_D \end{bmatrix} w(k) \\
y(k) = \begin{bmatrix} C_M & C_D \end{bmatrix} \begin{bmatrix} x_M(k) \\ x_D(k) \end{bmatrix} + \begin{bmatrix} D_M \end{bmatrix} u(k) + \begin{bmatrix} D_D \end{bmatrix} w(k) \\
x_M \in \mathbb{R}^{n_M}, x_D \in \mathbb{R}^{n_D}
\end{gathered} \tag{3}$$

It is noted that since there is the assumption of zero-measurement noise, the control algorithm designed which used this models has to regulate the actual output $y(k)$, which includes the additive disturbance.

3 State estimator

Following the identification, a state estimator is designed to estimate the states $x_D(k)$ of the disturbance model. This is because the signal $w(k)$ is unknown. An optimal estimator L_D is designed, which updates the states according to the following equations:

$$\begin{gathered}
\begin{bmatrix} \hat{x}_M(k+1) \\ \hat{x}_D(k+1) \end{bmatrix} = \begin{bmatrix} A_M & 0 \\ 0 & A_D \end{bmatrix} \begin{bmatrix} \hat{x}_M(k) \\ \hat{x}_D(k) \end{bmatrix} + \begin{bmatrix} B_M \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ L_D \end{bmatrix} (y(k) - \hat{y}(k)) \\
\hat{y}(k) = \begin{bmatrix} C_M & C_D \end{bmatrix} \begin{bmatrix} \hat{x}_M(k) \\ \hat{x}_D(k) \end{bmatrix} + \begin{bmatrix} D_M \end{bmatrix} u(k)
\end{gathered} \tag{4}$$

This results in error-dynamics between the actual and estimated system to evolve according to:

$$\begin{gathered}
\begin{bmatrix} e_M(k+1) \\ e_D(k+1) \end{bmatrix} = \begin{bmatrix} A_M & 0 \\ -L_D C_M & A_D - L_D C_D \end{bmatrix} \begin{bmatrix} e_M(k) \\ e_D(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_D - L_D D_D \end{bmatrix} w(k) \\
e_y(k) = \begin{bmatrix} C_M & C_D \end{bmatrix} \begin{bmatrix} e_M(k) \\ e_D(k) \end{bmatrix} + \begin{bmatrix} D_D \end{bmatrix} w(k)
\end{gathered} \tag{5}$$

In simplified notation, this is written as:

$$\begin{aligned}
e_x(k+1) &= A_L e_x(k) + B_L w(k) \\
e_y(k) &= C_L e_x(k) + D_L w(k)
\end{aligned} \tag{6}$$

4 Bound estimation

With the $u(k)$ and $y(k)$ signals used for identification, the estimator model (4) is simulated. The difference between the actual output and estimated output is captured as the signal $e_y(k)$, which is the output of the system (5). The input signal $w(k)$ to this system is unknown.
To design a robust controller, we intend to find the smallest set in which $w(k)$ lies. For this, we construct the set $\hat{E}_\infty$ defined as $\hat{E}_\infty = \{e_y : He_y \leq h\}$, which is a polyhedral box containing all the $e_y(k)$ vectors obtained from the identification and simulation. Using $\hat{E}_\infty$, we construct a set $\hat{W}_\infty$ which satisfies:

$$\begin{gathered}
\hat{W}_\infty = \{w : \hat{e}_y(k|e_x, w) \in \hat{E}_\infty \quad \forall e_x \in \mathbb{R}^{n_M+n_D}, k \in \mathbb{N}\} \\
\text{where} \quad \hat{e}_y(k|e_x, w) = C_L A_L^{k-1} e_x + C_L \sum_{t=1}^{k-1} A_L^{t-1} B_L w + D_L w
\end{gathered} \tag{7}$$

This set can be proven to be positive invariant, meaning that if $w(k) \in \hat{W}_\infty$ is applied to the system when $e_y(k) \in \hat{E}_\infty$, it will lead to $e_y(k+1) \in \hat{E}_\infty$.
To calculate $\hat{W}_\infty$, one can calculate $\hat{e}_y(k)$ for increasing values of k and an arbitrary initial condition

e_x, and constraint $\hat{e}_y(k)$ to lie within the estimated output-bound set $\hat{E}_\infty$. This is given by:

$$\text{For increasing values of } k: \quad \begin{aligned} H\hat{e}_y(k) &\leq h \\ H\big(C_L A_L^{k-1} e_x + C_L \sum_{t=1}^{k-1} A_L^{t-1} B_L w + D_L w\big) &\leq h \\ H\big(C_L \sum_{t=1}^{k-1} A_L^{t-1} B_L w + D_L\big) w &\leq h - C_L A_L^{k-1} e_x \\ G(k) w &\leq h - C_L A_L^{k-1} e_x \end{aligned} \tag{8}$$

For a stable estimator design, A_L is stable, and hence $C_L A_L^{k-1} e_x \to 0$ as $k \to 0$. Also, the finite termination property of $G(k)$ follows