

# Robust data-driven predictive control

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**Abstract**—This paper presents a robust data-driven control approach for SISO systems, by building on a previously presented hierarchical control architecture for constrained systems. To this end, a robust reference governor is designed. The reference governor alters the input from the outer model predictive controller (MPC) of the architecture, in a way that the system constraints are robustly satisfied. The reference governor utilizes a model of the inner loop. Uncertainty in the model is modeled as process noise. A novel method to calculate the bounds on this noise from ARX identification is discussed. Numerical validations of this method, and of the robust control architecture are performed, and the results are presented.

## I. INTRODUCTION

Design of control systems can broadly be classified into two categories: model-based and data-driven. Model-based control design techniques utilize an explicit model of the plant being controlled. This involves selecting a model that trades-off between complexity and accuracy. Then, identification of the model parameters is performed. These steps introduce several challenges. To avoid this, one can resort to a data-driven controller design methodology.

Data-driven controller design methods avoid explicitly identifying the plant model. They synthesize a controller directly from I/O data obtained from the plant. A review of several such methods can be found in [1]. One such method, virtual reference feedback tuning (VRFT) introduced in [2], has been used to design a stabilizing feedback controller within a hierarchical controller framework in [3] for LTI/LPV systems. It employs an outer MPC controller, which utilizes the reference model selected for VRFT to generate a tracking signal. Performance bounds on the plant are translated into constraints on the optimization problem solved by the MPC controller. Since the reference model might not reasonably reflect the performance of the closed-loop plant, there is a possibility of constraint violation. To avoid this, one can use the techniques developed under the umbrella of robust MPC theory to guarantee constraint satisfaction in an MPC framework. A review of the techniques can be found in [4]. An alternative is to use a robust reference governor, which modifies the reference signal supplied to the plant in a way that constraints are satisfied. These are reviewed in [5]. Both the methodologies require an explicit model of the uncertainties present in the plant being controlled.

In this work, we propose a systematic methodology to develop a robust data-driven control framework, building on the hierarchical control architecture presented in [3]. A model of closed loop behavior of the plant is used for controller design, which is different from the reference model used for VRFT. The parameters of this model are estimated using standard ARX estimation. The model incorporates uncertainties as exogenous noise signals, assumed to lie within a bounded polyhedral set. Since robust control methods explicitly incorporate the knowledge of these noise signals in their formulation, identification of the polyhedral sets is necessary. In this work, a method to do so is presented. Identification for robust control includes a large volume of literature on set-membership techniques for parameter estimation. A review of these can be found in [6]. A method to obtain ellipsoidal parameterizations of these sets during ARX estimation is discussed in [7]. To the best of authors' knowledge, no similar work has been done to calculate polyhedral sets of exogenous noise signals.

Following the identification of a model and corresponding noise sets, a robust reference governor is designed and appended to the control architecture presented in [3]. The option of robust reference governor is chosen for illustration, but the techniques presented can be easily extended to a robust MPC.

The paper is organized as follows. In Sec.II, the problem statement is formally presented. Background regarding the hierarchical data-driven control architecture and robust reference governor is presented in Sec.III. Sec.IV presents the techniques used to identify the closed-loop system model with bounds, which is the main contribution of this paper. The final Sec.V presents two simulation results, one verifying the techniques presented for bound identification, and one presenting the robust hierarchical control design framework.

## II. PROBLEM STATEMENT

Consider a *single-input single-output* system  $\mathbb{G}_P$  generating an output signal  $y(t) \in \mathbb{R}$  corresponding to the input signal  $u(t) \in \mathbb{R}$  for the time  $t \in \mathbb{Z}^+$ . We aim to synthesize a controller that can make  $y(t)$  accurately track any user defined reference signal, while robustly respecting the constraints:

$$\begin{aligned} y_{min} &\leq y(t) \leq y_{max} \\ u_{min} &\leq u(t) \leq u_{max} \\ \forall t &\in \mathbb{Z}^+ \end{aligned}$$

Following the data-driven controller synthesis methodology, we use the data  $D_N = \{u(t), y(t); t \in 1, \dots, N\}$  obtained by exciting the system to design the controller.

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### III. BACKGROUND

#### A. Hierarchical approach

A feedback controller is designed to control the system, using the VRFT methodology. For this, a reference model  $\mathbb{M}_P$  is selected, given by:

$$\begin{aligned} x_M(t+1) &= A_M x_M(t) + B_M g(t) \\ y_M(t) &= C_M x_M(t) \end{aligned}$$

The VRFT methodology designs a feedback controller  $\mathbb{K}_P$ , with the goal of making the closed-loop system  $\mathbb{K}_P - \mathbb{G}_P$  behave similar to the reference model  $\mathbb{M}_P$ . To this end, the VRFT methodology utilizes the dataset  $\mathbb{D}_N$ . The steps followed are as follows:

- 1) A virtual reference input  $g(t)$  is calculated by setting  $y_M(t) = y(t)$  obtained from the dataset  $\mathbb{D}_N$ , by inverting the model  $\mathbb{M}_P$ . Let this mapping be defined by  $g(t) = \mathbb{M}_P^\dagger y(t)$ .
- 2) A feedback controller  $\mathbb{K}_P$  described by  $A_K(q^{-1})u(t) = B_K(q^{-1})(g(t) - y(t))$  is chosen, where

$$\begin{aligned} A_K(q^{-1}) &= 1 + \sum_{i=1}^{n_{aK}} a_i^K q^{-i} \\ B_K(q^{-1}) &= \sum_{i=1}^{n_{bK}} b_i^K q^{-i} \end{aligned}$$

- 3) The parameters  $a_i^K$  and  $b_i^K$  of the controller are calculated such that the closed loop performance of  $\mathbb{K}_P - \mathbb{G}_P$  matches open loop performance of the reference model  $\mathbb{M}_P$ . This is done by solving the convex optimization problem

$$\min_{a_i^K, b_i^K} \frac{1}{N} \sum_{t=1}^N |A_K(q^{-1})u(t) - B_K(q^{-1})(\mathbb{M}_P^\dagger y(t) - y(t))|^2 \quad (1)$$

which minimizes the deviation between the control input calculated by the controller and  $u(t)$  that is used to excite the system and obtain  $y(t)$ .

- 4) The synthesized controller  $\mathbb{K}_P$  is placed before the plant, and the loop is closed.

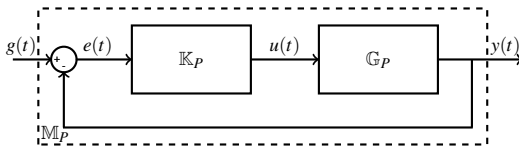


Fig. 1: Feedback controller designed using VRFT

To satisfy plant constraints, the reference signal  $g(t)$  is generated using an MPC controller in an outer loop. The objective of the MPC controller is to make the output signal  $y(t)$  track the reference  $r(t)$ , while satisfying constraints on the plant output  $y(t)$  and input  $u(t)$ . This is achieved by considering the closed-loop plant model  $\mathbb{M}_P$  for the state propagation equation, and an augmented model with  $[y(t), u(t)]$  as the output

equation. This augmented state-space model is called  $\mathbb{M}'_P$ .

$$\begin{aligned} \zeta(t+1) &= A_\zeta \zeta(t) + B_\zeta g(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\zeta \zeta(t) + D_\zeta g(t) \end{aligned}$$

The optimization problem solved by the MPC at each time step  $t$  for a horizon of  $N_P$  timesteps is shown in (2). The lower and upper bounds on  $y(t)$  and  $u(t)$  are  $[y_{min}, y_{max}]$  and  $[u_{min}, u_{max}]$  respectively.

$$\begin{aligned} \min_{\{g(t+k)\}_{k=1}^{N_P}} \quad & Q_y \sum_{k=1}^{N_P} (y(t+k|t) - r(t+k))^2 + Q_\varepsilon \varepsilon^2 \\ \text{subject to} \quad & \zeta(t+k+1) = A_\zeta \zeta(t+k) + B_\zeta g(t+k) \\ & \begin{bmatrix} y(t+k) \\ u(t+k) \end{bmatrix} = C_\zeta \zeta(t+k) + \begin{bmatrix} 0 \\ D_\zeta \end{bmatrix} g(t+k) \\ & y_{min} - V_y \varepsilon \leq y(t+k) \leq y_{max} + V_y \varepsilon \\ & u_{min} - V_u \varepsilon \leq u(t+k) \leq u_{max} + V_u \varepsilon \\ & \zeta(t|t) = \zeta(t) \end{aligned} \quad (2)$$

The quantities  $V_y$  and  $V_u$  in the MPC formulation are used to avoid infeasibility of the optimization problem over successive iterations, since the reference model  $\mathbb{M}'_P$  might not accurately capture the dynamics of the closed loop  $\mathbb{K}_P - \mathbb{G}_P$ . This implies that constraint satisfaction is not guaranteed by the proposed formulation.

#### B. Robust reference governor

The robust reference governor is a signal regulator which alters the command input such that the system robustly satisfies constraints. It does so by utilizing knowledge of disturbances acting on the system. A schematic of the control system is shown in Fig.2.

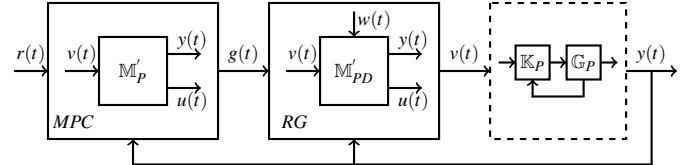


Fig. 2: Schematic of the robust control system

The MPC controller utilizes the model  $\mathbb{M}'_P$ , and the reference governor RG uses the model  $\mathbb{M}'_{PD}$ .  $\mathbb{M}'_{PD}$  is a model of the closed loop performance of  $\mathbb{K}_P - \mathbb{G}_P$ , which explicitly incorporates knowledge of the disturbances acting on  $\mathbb{M}'_P$ . Identification of this model and corresponding bounds is discussed in Sec.IV. The state space form of this model is written as:

$$\begin{aligned} \gamma(t+1) &= A_\gamma \gamma(t) + B_\gamma^v v(t) + B_\gamma^w w(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\gamma \gamma(t) + D_\gamma^v v(t) + D_\gamma^w w(t) \end{aligned}$$

The model has a deterministic input  $v(t)$  and a noise input  $w(t)$ , which captures the effect of model uncertainties on system output. The columns of matrices  $B_\gamma$  and  $D_\gamma$  are separated for ease of notation. The

outputs of the system are called  $y_\gamma(t) = [y(t) u(t)]^T$ . The constraints on these outputs are written as:

$$\left. \begin{array}{l} y_{\min} \leq y(t) \leq y_{\max} \\ u_{\min} \leq u(t) \leq u_{\max} \end{array} \right\} H y_\gamma(t) \leq h$$

At each time instant  $t$ , the reference governor solves the quadratic program:

$$\begin{aligned} \min_v \quad & \frac{1}{2} \|v - g(t)\|_2^2 \\ \text{subject to} \quad & (x, v) \in \mathbb{O}_\infty \end{aligned} \quad (3)$$

where the set  $\mathbb{O}_\infty$  is defined as:

$$\mathbb{O}_\infty = \{(x, v) : x = \gamma(t), H y_\gamma(\tau) \leq h \ \forall \tau \geq t\}$$

This is called a maximal output admissible set. It is set of feasible values for  $v$  for the current observed state  $x(t) = \gamma(t)$  such that if a constant input signal  $v(\tau \geq t) = v$  is applied, the system output constraints  $H y_\gamma(\tau \geq t) \leq h$  remain satisfied for all  $\tau$ . At a time instant  $\tau \geq t$ , the system output  $y_\gamma(\tau)$  for input signal  $v(\tau \geq t) = v$  can be written as:

$$\begin{aligned} y_\gamma(\tau) = & C_\gamma A_\gamma^{\tau-t} \gamma(t) + \left( C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^v + D_\gamma^v \right) v + \\ & C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^w w(\tau - k) + D_\gamma^w w(\tau) \end{aligned}$$

For a particular future time instant  $\tau$ , the set  $\mathbb{O}_\infty(\tau)$  is given by:

$$\mathbb{O}_\infty(\tau) = \{(x, v) : x = \gamma(t), \tilde{H}(\tau)v \leq \tilde{h}(\tau)\}$$

$$\begin{aligned} \text{where } \tilde{H}(\tau) = & H \left( C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^v + D_\gamma^v \right) \\ \tilde{h}(\tau) = & h - H C_\gamma A_\gamma^{\tau-t} \gamma(t) - f^w(\tau) \end{aligned}$$

Each element  $f_i^w(\tau)$  of the column vector  $f^w(\tau)$  is calculated by solving the linear program:

$$f_i^w(\tau) = \max_{\{w(k)\}_{k=1}^{\tau-t} \in \mathcal{W}_\infty} H \left( C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^w w(\tau - k) + D_\gamma^w w(\tau) \right)_i \quad (4)$$

Subscript  $i$  in the above problem indicates that row  $i$  of the matrix is used in the linear program to calculate the corresponding  $w(k)$  sequence. According to the theory of maximal output admissible sets for linear systems as discussed in [8], the set  $\mathbb{O}_\infty = \mathbb{O}_\infty(\tau \rightarrow \infty)$  is reached at a finite value  $\tau_c$ . In the current setting, this happens when elements of  $f^w(\tau)$  converge to a constant value. Since the linear problem to be solved for  $f_i^w(\tau)$  does not depend on the state at current time instant  $t$ , it can be solved offline for increasing values of  $\tau$  after setting  $t = 1$ . When convergence is observed at time  $\tau_c$ , the value  $f^w(\tau_c)$  is stored. This results in problem (3) being simplified to

$$\begin{aligned} \min_v \quad & \frac{1}{2} \|v - g(t)\|_2^2 \\ \text{subject to} \quad & \tilde{H}(\tau_c)v \leq \tilde{h}(\tau_c) \end{aligned} \quad (5)$$

It must be noted that the value  $\tilde{h}(\tau_c)$  depends on the current observed state  $\gamma(t)$ . Thus, at each time instant, the reference governor reads the current state  $\gamma(t)$ , reference input  $g(t)$ , and calculates a new reference  $v$  which robustly satisfies output constraints on the system.

#### IV. DISTURBANCE SENSOR

This section discusses the development of the model  $\mathbb{M}'_{PD}$ , which is used by the robust reference governor to generate the signal  $v(t)$ . First, the overall model is presented. Then, a novel technique is introduced to quantify uncertainty bounds on the model obtained from ARX estimation.

##### A. Appended closed-loop model

In order to improve prediction of closed loop performance, a disturbance sensor  $\mathbb{D}$  is designed. The disturbance sensor is a dynamical system whose output is the discrepancy between output of the actual closed loop system  $\mathbb{K}_P\text{-}\mathbb{G}_P$  and reference closed loop system  $\mathbb{M}_P$ . The system consists of a deterministic part and a stochastic part, as seen in Fig. 3.

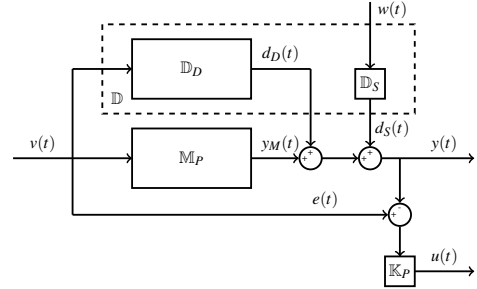


Fig. 3: Reference model appended with disturbance sensor.

This can be seen as a 2-input 2-output system  $\mathbb{M}'_{PD}$ , described as

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_P + \mathbb{D}_D & \mathbb{D}_S \\ \mathbb{K}_P(I - (\mathbb{M}_P + \mathbb{D}_D)) & -\mathbb{K}_P \mathbb{D}_S \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \quad (6)$$

Note that the dependence of the model on time shift operator  $q^{-1}$  is not shown for ease of notation. The signal  $w(t)$  is interpreted as an external noise signal. The state-space equivalent of this model is used in the robust reference governor that was previously discussed.

Since this model represents the closed loop behavior of the system, new closed loop measurements are required to estimate the parameters of the disturbance sensor. These are obtained by performing experiments with excitation signals  $\hat{v}(t)$  on the reference closed loop model  $\mathbb{M}_P$  and plant, and measuring the outputs  $\hat{y}_M(t)$  and  $\hat{y}(t)$  respectively. The signals are captured in the data set  $\hat{D}_N = \{\hat{v}(t), \hat{y}_M(t), \hat{y}(t); t \in 1, \dots, N\}$ . The disturbance sensor  $\mathbb{D}$  is parameterized as  $A_D(q^{-1})y_D(t) =$

$B_D(q^{-1})v(t) + w(t)$ , where

$$\begin{aligned} y_D(t) &= y(t) - y_M(t) = d_D(t) + d_S(t) \\ A_D(q^{-1}) &= 1 + \sum_{i=1}^{n_{aD}} a_i^D q^{-i} \\ B_D(q^{-1}) &= \sum_{i=1}^{n_{bD}} b_i^D q^{-i} \end{aligned}$$

Using standard ARX identification, the coefficients  $a_i^D$  and  $b_i^D$  are estimated by solving the optimization problem

$$\min_{a_i^D, b_i^D} \frac{1}{N} \sum_{t=1}^N |A_D(q^{-1})(\hat{y}(t) - \hat{y}_M(t)) - B_K(q^{-1})(\hat{v}(t))|^2 \quad (7)$$

The disturbance sensor is then split into two parts, deterministic and stochastic. The deterministic part  $\mathbb{D}_D$  takes in the reference signal  $v(t)$  as an input, and the stochastic part  $\mathbb{D}_S$  takes in the noise  $w(t)$ . The I/O behavior of these parts are separately written as

$$\begin{aligned} d_D(t) &= \frac{B_D(q^{-1})}{A_D(q^{-1})} v(t) = \mathbb{D}_D v(t) \\ d_S(t) &= \frac{1}{A_D(q^{-1})} w(t) = \mathbb{D}_S w(t) \end{aligned}$$

The model has a noise input  $w(t)$ , which will be transformed to process and measurement noises on the state space model corresponding to (6). The following subsection discusses a technique to calculate upper and lower bounds  $w_{min}$  and  $w_{max}$  on the noise signal  $w(t)$ , which are explicitly provided to the robust reference governor for the calculation of  $\mathcal{O}_\infty$ .

### B. Noise input bounds calculation

A realization of the discrepancy  $d_S(t)$  between desired output  $y(t)$  and deterministic output  $y_M(t) + d_D(t)$  can be calculated from the data set  $\hat{D}_N$  as

$$\hat{d}_S(t) = \hat{y}(t) - \hat{y}_M(t) - \frac{B_D(q^{-1})}{A_D(q^{-1})} \hat{v}(t)$$

The maximum and minimum values of this discrepancy are labeled  $\hat{d}_{S,max}$  and  $\hat{d}_{S,min}$  respectively. If the ARX identification results in a stable deterministic disturbance sensor  $\mathbb{D}_D$ , the values  $\hat{d}_{S,max}$  and  $\hat{d}_{S,min}$  are finite. Further, if infinite closed loop data  $\hat{D}_\infty$  is collected for ARX estimation, the bounds on discrepancy are equal to the actual bounds  $d_{S,max}$  and  $d_{S,min}$ . The set of sequences  $d_S(t)$  satisfying these bounds are indicated as lying in a set  $\mathcal{D}_\infty$ , defined as

$$\mathcal{D}_\infty = \{d_S(t) : d_{S,min} \leq d_S(t) \leq d_{S,max} \quad \forall t \in (-\infty, \infty)\}$$

From these bounds, we attempt to calculate the set  $\mathcal{W}_\infty$  defined as

$$\mathcal{W}_\infty = \left\{ w(t) : \forall t \in (-\infty, \infty), \begin{array}{c} w_{min} \leq w(t) \leq w_{max} \\ \mathbb{D}_S w(t) \in \mathcal{D}_\infty \end{array} \right\}$$

This is the set of all  $w(t)$  sequences such that the output signal of  $\mathbb{D}_S$  corresponding to any  $w(t) \in \mathcal{W}_\infty$  lies within  $\mathcal{D}_\infty$ . It must be noted that the converse is not implied. That is, it does not mean that there cannot exist a noise sequence  $w(t)$  not belonging to  $\mathcal{W}_\infty$  but producing a corresponding output sequence belonging to  $\mathcal{D}_\infty$ .

$$\begin{aligned} \forall w(t) \in \mathcal{W}_\infty : \mathbb{D}_S w(t) &\in \mathcal{D}_\infty \\ \not\Rightarrow \nexists w(t) \notin \mathcal{W}_\infty : \mathbb{D}_S w(t) &\in \mathcal{D}_\infty \end{aligned}$$

This means that the bounds on residuals of the ARX identification (7) are not necessarily  $w_{min}$  and  $w_{max}$ , since the realization of the residuals need not necessarily belong to  $\mathcal{W}_\infty$ .

To calculate the bounds  $w_{min}$  and  $w_{max}$ , first, the optimization problems shown in (8) are solved for increasing lengths of time horizon  $N$ .

$$\begin{aligned} \min_{X_j} \quad & \bar{w}_j(N) \\ \text{subject to} \quad & d_S(k) = - \sum_{i=1}^{n_{aD}} a_i^D d_S(k-i) + w(k), k = 1 : N \\ & d_{S,min} \leq d_S(k) \leq d_{S,max}, k = 1 : N-1 \\ & -\bar{w}_j(N) \leq w(k) \leq \bar{w}_j(N), k = 1 : N \\ & d_S(N) \in D_j \end{aligned}$$

where  $X_j = \left\{ \begin{array}{c} \{d_S(-n_{aD}+1), \dots, d_S(N)\}, \\ \{w(1), \dots, w(N)\}, \\ \bar{w}_j(N) \end{array} \right\}, j = \{1, 2\}$

$$\begin{aligned} D_1 &= \{d : d \leq d_{S,min}\} \\ D_2 &= \{d : d \geq d_{S,max}\} \end{aligned} \quad (8)$$

These equations solve for a sequence of inputs  $\{w(k), k = 1 : N\}$  such that the sequence of corresponding outputs at all instances except at time  $N$  lie within  $\mathcal{D}_\infty$ . This means that the chosen input sequence  $w(k)$  should drive  $d_S(N)$  out of  $\mathcal{D}_\infty$  into  $D_j$ . The initial conditions on  $d_S(k)$  are left free to be chosen by the problem, but are constrained to lie within  $\mathcal{D}_\infty$ . The solutions  $\bar{w}_j(N)$  are collected, and the set  $\mathcal{W}_\infty$  is calculated using (9). It is noted that the noise is assumed to be zero-mean. This can be rectified easily by letting the lower and upper bounds on  $w(k)$  in (8) be composed of different optimization variables.

$$\begin{aligned} w_{max} &= \max \left\{ \max_N \bar{w}_1(N), \max_N \bar{w}_2(N) \right\} \\ w_{min} &= \min \left\{ \min_N -\bar{w}_1(N), \min_N -\bar{w}_2(N) \right\} \end{aligned} \quad (9)$$

At lower values of  $N$ , there can exist an initial sequence  $\{d_S(-n_{aD}+1), \dots, d_S(1)\}$  that drives  $d_S(N)$  into  $D_j$  with very low control effort. At higher values of  $N$ , the effect of initial condition wears off, and a higher value of control effort is required to violate  $\mathcal{D}_\infty$ . Hence, solving (9) gives the values of  $w_{min}$  and  $w_{max}$ , that are the minimum and maximum values of the noise signal  $w(t)$  such that the steady state values of  $d_S(t)$  lie within  $\mathcal{D}_\infty$ .

We now have a closed-loop model with knowledge of noise bounds, and hence are ready to put it in a robust reference governor.

## V. CASE STUDIES

Two case studies are presented. The first one is to verify the formulation presented in (8) and (9). Then, numerical simulations are performed on a servo motor control problem, implementing the complete control scheme presented in Fig.2. Both the case studies are implemented using MATLAB R2017b, with optimization problems occasionally implemented with YALMIP [9].

### A. Noise input bound

Consider a MISO system with inputs  $\{u(t), w(t)\}$  and output  $x(t)$ , described by:

$$\begin{aligned} u(t+1) &= 0.995u(t) + 1 \\ x(t+1) &= 0.05x(t) + u(t) + w(t) \\ u(1) &= 0, \quad x(1) = 0 \end{aligned}$$

The input  $u(t)$  is deterministic and  $w(t)$  is noisy. An approximate ARX model of the system is calculated as  $A(q^{-1})\tilde{x}(t) = B(q^{-1})u(t) + w(t)$  after performing experiments on the system and collecting data. It is identified with  $A(q^{-1})$  and  $B(q^{-1})$  having one free parameter each. Following this, the methodology discussed in Sec.IV-B is used to calculate bounds on  $w(t)$ . First, the signal  $d_S(t) = x(t) - (B(q^{-1})/A(q^{-1}))u(t)$  is extracted from the experimental data, and  $\mathcal{D}_\infty$  is constructed. For increasing values of  $N$ , sequences  $\{w(k), k = 1 : N\}$  and corresponding  $\{d_S(k), k = 1 : N\}$  are calculated by solving the optimization problems in 4. The bounds  $-\bar{w}_j(N)$  and  $\bar{w}_j(N)$  on the input sequences are shown in Fig.4. The largest region where

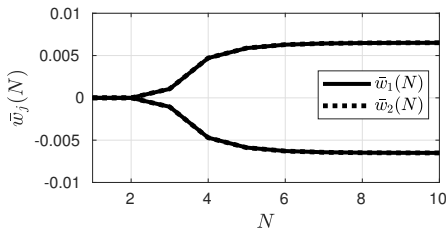


Fig. 4: Convergence of bounds to  $w_{min}$  and  $w_{max}$ , with zero mean noise assumption.

$-\bar{w}_j(N) \leq w(k) \leq \bar{w}_j(N)$  is satisfied, is  $\mathcal{W}_\infty$ .

To verify the obtained bounds, the system identified through ARX identification is simulated with deterministic inputs  $u(t)$ , and a range of noise inputs  $w(t)$  sampled from  $\mathcal{W}_\infty$ . The simulation results are seen in Fig.5. It can be seen that output of the ARX model simulated with noise inputs  $w(t)$  covers the measurement range, and avoids conservatism. This is an indicator that the set  $\mathcal{W}_\infty$  is valid.

The simulation of ARX model with noise requires an

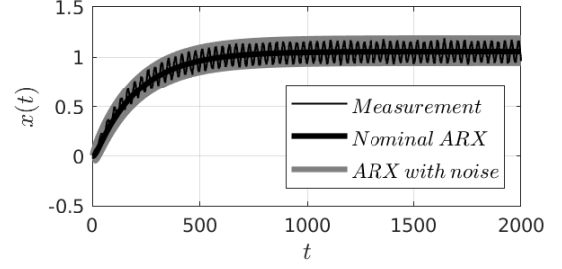


Fig. 5: Comparison of ARX model performance with constant  $w(t) = 0$  (nominal) and  $w(t) \in \mathcal{W}_\infty$ . The noise model gives bounded output  $x(t)$  without being conservative.

initial condition on the noise model  $\mathbb{D}_S$ . In the current example, it is set to 0. Since the actual initial condition of  $\mathbb{D}_S$  might not be 0, one might face problems with the model not covering the whole range of  $\mathcal{D}_\infty$  during the transient period. To correct this, a state estimator could be used which rectifies the effect of initial state discrepancy, thus covering the noise effects even during transients.

### B. Data driven MPC with robust reference governor

The plant consisting of a servo positioning system is controlled using the scheme shown in Fig.2. The plant dynamics are modeled with the following non-linear state space equations:

$$\begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \\ \dot{i}(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ \frac{-mgl}{J} \sin \theta(t) - \frac{b}{J} \omega(t) + \frac{K_m}{J} i(t) \\ \frac{-K_m}{L} \omega(t) - \frac{R}{L} i(t) + \frac{1}{L} u(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \omega(t) \\ i(t) \end{bmatrix}$$

Symbol	Parameter	Value
$R$	Motor resistance	$5\Omega$
$L$	Motor inductance	$5 \cdot 10^{-3} \text{H}$
$K_m$	Motor torque constant	$0.0847 \text{Nm/A}$
$J$	Complete disk inertia	$5 \cdot 10^{-5} \text{Nm}^2$
$b$	Friction coefficient	$3 \cdot 10^{-3} \text{Nms/rad}$
$m$	Additional mass	$3 \text{Kg}$
$l$	Mass offset	$2 \text{m}$

TABLE I: Physical parameters of servo motor system

The states of the system  $\theta(t), \omega(t)$  and  $i(t)$  are angle [rad] and rotational velocity [rad/s] of the servo motor, and armature current [A] respectively. The input  $u(t)$  is the voltage [V] applied across the motor, and output  $y(t)$  is the rotational angle. A VRFT methodology is used to design a stabilizing PD controller, which provides a voltage input  $u(t)$  to make the rotational

angle  $y(t)$  track a reference signal  $g(t)$ . To this end, experiments are conducted with a low-pass filtered white noise signal  $u(t)$  with a standard deviation of 10V. The output angle  $y(t)$  is recorded and the dataset  $\mathbb{D}_N$  is obtained. A slow reference closed loop model  $\mathbb{M}_P$  is chosen, given by:

$$\begin{aligned} x_M(t+1) &= 0.99x_M(t) + 0.01g(t) \\ y_M(t) &= x_M(t) \end{aligned}$$

The PD inner-loop controller  $\mathbb{K}_P$  is parameterized as:

$$u(t) = K_p e(t) + K_d \frac{e(t) - e(t-1)}{T_s}$$

Solving the optimization problem (1), the parameters  $K_p$  and  $K_d$  are calculated using the dataset  $D_N$ . The controller is placed in the inner loop within the hierarchical control scheme. After VRFT synthesis, an outer MPC is designed using the formulation in (2), to provide a reference signal  $g(t)$ . The output  $y(t)$  is constrained to lie between 0 rad and 4 rad, and the voltage input  $u(t)$  between  $-3.5$  V and  $3.5$  V. An MPC horizon of  $N_P = 20$  timesteps is chosen, and weights are  $Q_y = 1$  and  $Q_\varepsilon = 1$ .

Towards developing the robust reference governor, a disturbance sensor discussed in Sec.IV is developed. First, the dataset  $\hat{D}_N$  is built by performing closed loop experiments with input signal  $\hat{v}(t)$  of standard deviation 10 rad. Then, a linear model for  $\mathbb{D}$  parameterized by  $n_{ad} = 4$  and  $n_{bd} = 3$  is identified by solving (7). This disturbance sensor is split into two parts,  $\mathbb{D}_D$  and  $\mathbb{D}_S$ , with inputs  $v(t)$  and noise  $w(t)$  respectively. Bounds on the noise  $w(t)$  are calculated by solving the linear problems (8) and (9). The evolution of these bounds with increasing values of horizon  $N$  is plotted in Fig.6.

The converged values  $w_{min}$  and  $w_{max}$  are passed

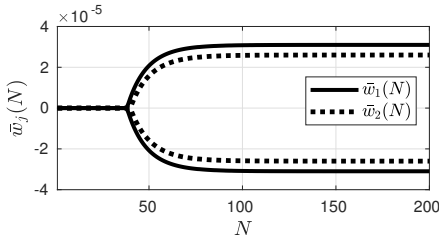


Fig. 6: Convergence of bounds on the noise  $w(t)$  acting on the disturbance sensor  $D_S$ .

into (4) and bounds on input  $v(t)$  are calculated. The quadratic program (5) is constructed, which is solved at each time step by the robust reference governor. The reference governor modifies the input  $g(t)$  to a constraint respecting  $v(t)$ . Steady-state Kalman filters are used to calculate the states  $\zeta(t)$  and  $\gamma(t)$  of the models  $\mathbb{M}'_P$  and  $\mathbb{M}'_{PD}$ , which are used by the MPC controller and robust reference governor respectively. Instead of using  $w(t)$  as process noise, one can treat

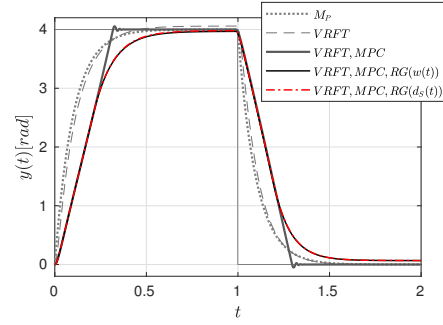


Fig. 7: The performance inner closed loop does not exactly match the reference model  $M_P$ . MPC improves the performance but results in small constraint violation. Constraint violation is robustly avoided by using a reference governor.

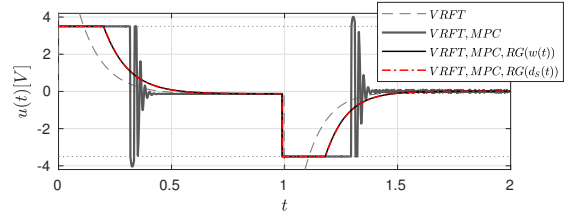


Fig. 8: Comparison of voltage input  $u(t)$ . Constraints are violated by the MPC controller. This is avoided by using a reference governor. The performance with the reference governor using either process noise model or measurement noise models is similar.

$d_S(t)$  as measurement noise after suitable transformations. The noise can be assumed to lie in the set  $\mathcal{D}_\infty$ . In this case, the calculation of  $\mathcal{O}_\infty$  for the robust reference governor becomes very simple, since there is no dependence of output on past input disturbances. This was implemented for the sake of comparison with the process noise case.

Performance of the control system for all these cases is plotted in Fig.7 and Fig.8. It can clearly be seen that with a loss of performance from MPC, the reference governor ensures robust constraint satisfaction. This loss in performance can be reduced by using a robust MPC controller instead of a reference governor. Since computational efficiency was not the focus of this work, the solving times are not noted here.

## VI. CONCLUSION

This paper builds on the hierarchical data-driven control of constrained systems, by introducing robustness. This is done by adding a robust reference governor. The major contribution of this work is the formulation of a novel technique to compute process noise bounds from ARX identification. Since the calculation of process noise bounds and maximal output admissible sets is performed offline, the control scheme can be

experimentally validated online in an efficient manner. Possible extensions to the work includes formulation of equivalent schemes for LPV and MIMO systems.

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