Robust data-driven model predictive control

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***Abstract*— This paper presents a robust data-driven control approach for SISO systems subject to bound constraints on input and output. The approach builds on a previously presented hierarchical data-driven control architecture for constrained systems. Robustness is achieved by using a robust model predictive controller (RMPC) instead of a deterministic model predictive controller (MPC) within the architecture. The RMPC controller calculates a reference signal that robustly satisfies the plant constraints. To this end, it utilizes a model of the inner loop, the uncertainty of which is assumed to lie within a bounded disturbance set. A major contribution of this work is the development of a method to calculate this set from an identified ARX model. Numerical validations of the robust control architecture are performed, and the results are presented.**

I. INTRODUCTION

Design of control systems can broadly be classified into two categories: model-based and data-driven. Model-based control design techniques utilize an explicit model of the plant being controlled. This involves selecting a model that trades-off between complexity and accuracy. Model selection is followed by experiments to identify the parameters. These steps introduce several challenges. To avoid this, one can resort to a data-driven controller design methodology.

Data-driven controller design methods avoid explicitly identifying the plant model. They synthesize a controller directly from I/O data obtained from the plant. A review of several such methods can be found in [1]. One such method, virtual reference feedback tuning (VRFT) introduced in [2], has been used to design a stabilizing feedback controller within a hierarchical control architecture in [3] for LTI/LPV systems. The architecture employs an outer MPC controller, which utilizes the reference model selected for VRFT to generate a tracking signal. Performance bounds on the plant are translated into constraints on the optimization problem solved by the MPC controller. Since the reference model might not reasonably reflect the performance of the closed-loop plant, there is a possibility of constraint violation. To avoid this, one can use a robust MPC controller in the outer loop. A review of the concepts related to robust MPC controllers can be found in [4]. An alternative is to use a robust reference governor, which modifies the reference signal supplied to the plant in a way that constraints are satisfied. These are reviewed in [5]. Both the techniques need a model of the plant being controlled, with the model explicitly incorporating uncertainties.

In this work, we propose a robust data-driven control approach, building on the hierarchical control architecture presented in [3]. The robust controller uses a model of the inner closed loop, which consists of the plant and the VRFT-based feedback controller. This model is not the same as the reference model used for VRFT, but is separately identified using an ARX parameterization from closed loop experiments. The model incorporates uncertainties as exogenous disturbance signals, assumed to lie within a bounded polyhedral set. Since robust control methods explicitly incorporate uncertainty information in their formulation, identification of the polyhedral sets is necessary. In this work, a novel method to do so is presented. Such a method falls under the umbrella of identification for robust control. This field includes a large volume of literature on set-membership techniques for parameter estimation. A review of these can be found in [6]. A method to obtain ellipsoidal parameterizations of these sets during ARX estimation is discussed in [7]. To the best of authors' knowledge, no similar work has been done to calculate polyhedral sets of exogenous disturbance signals.

Following the identification of a model and corresponding disturbance sets, a robust reference governor is designed and appended to the control architecture presented in [3]. The option of robust reference governor is chosen for illustration, but the techniques presented can easily be extended to a robust MPC.

The paper is organized as follows. In Sec.II, the problem statement is formally presented. Background regarding the hierarchical data-driven control architecture and robust reference governor is presented in Sec.III. Sec.IV presents the techniques used to identify the closed-loop system model with disturbance bounds, which is the main contribution of this paper. The final Sec.V presents a numerical example implementing the robust hierarchical control design approach.

II. PROBLEM STATEMENT

Consider a *single-input single-output* system $\mathbb{G}_P$ generating an output signal $y(t) \in \mathbb{R}$ corresponding to the input signal $u(t) \in \mathbb{R}$ for the time $t \in \mathbb{Z}^+$. We aim to synthesize a controller that can make $y(t)$ accurately track any user defined reference signal, while robustly respecting the constraints:

$$\begin{aligned} y_{min} \leq y(t) \leq y_{max} \\ u_{min} \leq u(t) \leq u_{max} \\ \forall t \in \mathbb{Z}^+ \end{aligned} \tag{1}$$

Following the data-driven controller synthesis methodology, we use the data $D_N = \{u(t), y(t); t \in 1, ..., N\}$ obtained by

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exciting the system to design the controller.

III. Background

A. Hierarchical approach

A feedback controller is designed to control the system, using the VRFT methodology. For this, a reference model $\mathbb{M}_P$ is selected, given by:

$$\begin{aligned} x_M(t+1) &= A_M x_M(t) + B_M g(t) \\ y_M(t) &= C_M x_M(t) \end{aligned}$$

The VRFT methodology designs a feedback controller $\mathbb{K}_P$, with the goal of making the closed-loop system $\mathbb{K}_P$-$\mathbb{G}_P$ behave similar to the reference model $\mathbb{M}_P$. It utilizes the dataset $\mathbb{D}_N$ for controller synthesis. The steps followed are summarized:

1) A virtual reference input $g(t)$ is calculated by setting $y_M(t) = y(t)$ obtained from the dataset $\mathbb{D}_N$, by inverting the model $\mathbb{M}_P$. Let this mapping be defined by $g(t) = \mathbb{M}_P^{\dagger} y(t)$.
2) A feedback controller $\mathbb{K}_P$ described by $A_K(q^{-1})u(t) = B_K(q^{-1})(g(t) - y(t))$ is chosen, where
$$\begin{aligned} A_K(q^{-1}) &= 1 + \sum_{i=1}^{n_{a_K}} a_i^K q^{-i} \\ B_K(q^{-1}) &= \sum_{i=1}^{n_{b_K}} b_i^K q^{-i} \end{aligned}$$
3) The parameters a_i^K and b_i^K of the controller are calculated such that the closed loop performance of $\mathbb{K}_P$-$\mathbb{G}_P$ matches open loop performance of the reference model $\mathbb{M}_P$. This is done by solving the convex optimization problem
$$\min_{a_i^K, b_i^K} \quad \frac{1}{N} \sum_{t=1}^{N} \left| A_K(q^{-1})u(t) - B_K(q^{-1})(\mathbb{M}_P^{\dagger} y(t) - y(t)) \right|^2 \tag{2}$$
which minimizes the deviation between the control input calculated by the controller and $u(t)$ that is used to excite the system and obtain $y(t)$.
4) The synthesized controller $\mathbb{K}_P$ is placed before the plant, and the loop is closed.

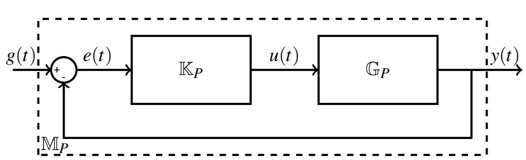

Fig. 1: Feedback controller designed using VRFT

To satisfy plant constraints, the reference signal $g(t)$ is generated using an MPC controller in an outer loop. The objective of the MPC controller is to make the output signal $y(t)$ track the reference $r(t)$, while satisfying constraints on the plant output $y(t)$ and input $u(t)$. This is achieved by considering the closed-loop plant model $\mathbb{M}_P$ for the state propagation equation, and an augmented model with $[y(t), u(t)]$ as the output equation. This augmented state-space model is called $\mathbb{M}'_P$.

$$\begin{aligned} \zeta(t+1) &= A_\zeta \zeta(t) + B_\zeta g(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\zeta \zeta(t) + D_\zeta g(t) \end{aligned}$$

The optimization problem solved by the MPC at each time step t for a horizon of N_P timesteps is shown in (3).

$$\begin{aligned} \min_{\{g(t+k)\}_{k=1}^{N_P}} \quad & Q_y \sum_{k=1}^{N_P} (y(t+k|t) - r(t+k))^2 + Q_\varepsilon \varepsilon^2 \\ \text{subject to} \quad & \zeta(t+k+1) = A_\zeta \zeta(t+k) + B_\zeta g(t+k) \\ & \begin{bmatrix} y(t+k) \\ u(t+k) \end{bmatrix} = C_\zeta \zeta(t+k) + \begin{bmatrix} 0 \\ D_\zeta \end{bmatrix} g(t+k) \\ & y_{min} - V_y \varepsilon \leq y(t+k) \leq y_{max} + V_y \varepsilon \\ & u_{min} - V_u \varepsilon \leq u(t+k) \leq u_{max} + V_u \varepsilon \\ & \zeta(t|t) = \zeta(t) \end{aligned} \tag{3}$$

The quantities V_y and V_u in the MPC formulation are used as soft constraints to avoid infeasibility of the optimization problem over successive iterations, since the reference model $\mathbb{M}'_P$ might not accurately capture the dynamics of the closed loop $\mathbb{K}_P - \mathbb{G}_P$. This implies that constraint satisfaction is not guaranteed by the proposed formulation.

B. Robust model predictive controller

The deterministic MPC controller in the outer loop of the hierarchical control architecture is replaced by a robust MPC controller. A schematic of the control loop is shown in Fig.2.

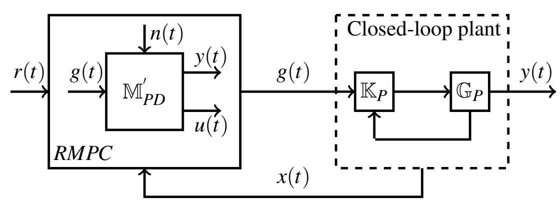

Fig. 2: Schematic of the robust control system

In order to achieve robustness, the robust MPC (RMPC) controller uses the model $\mathbb{M}'_{PD}$. $\mathbb{M}'_{PD}$ is a model of the closed loop performance of $\mathbb{K}_P$-$\mathbb{G}_P$, which is obtained by appending disturbance information over the model $\mathbb{M}'_P$. Identification of this model and corresponding bounds is discussed in Sec.IV. The state-space form of $\mathbb{M}'_{PD}$ is:

$$\begin{aligned} \gamma(t+1) &= A_\gamma \gamma(t) + B_\gamma^g g(t) + B_\gamma^n n(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\gamma \gamma(t) + D_\gamma^g g(t) + D_\gamma^n n(t) \end{aligned} \tag{4}$$

The model has a deterministic input $g(t)$ and a disturbance input $n(t)$, which captures the effect of model uncertainties on system output. It is assumed that the disturbance input lies in a bounded set $\mathscr{N}_\infty$. The columns of matrices B_γ and D_γ are separated for ease of notation. The output of (4) is denoted by $y_\gamma(t) = [y(t)\, u(t)]^T$. The constraints (1) on $y(t)$ and $u(t)$ are rewritten as:

$$Hy_\gamma(t) \leq h$$

At each time instant t, for a horizon of N_P time-steps, the RMPC controller solves the optimization problem:

$$\begin{aligned}
\min_{\{g(t+k)\}_{k=1}^{N_P}} \quad & \sum_{k=1}^{N_P} (y(t+k|t) - r(t+k))^2 \\
\text{subject to} \quad & \gamma(t+k+1) = A_\gamma \gamma(t+k) + B_\gamma^g g(t+k) \\
& y_\gamma(t+k) = C_\gamma \gamma(t+k) + D_\gamma^g g(t+k) \\
& Hy_\gamma(t) \leq h \\
& g(t+k) \in \mathbb{O}_\infty(\gamma(t)) \\
& \gamma(t|t) = \gamma(t)
\end{aligned} \tag{5}$$

where the set $\mathbb{O}_\infty(\gamma(t))$ is defined as:

$$\mathbb{O}_\infty(\gamma(t)) = \{g : \; Hy_\gamma(\tau) \leq h, \, \forall n(\tau) \in \mathcal{N}_\infty, \, \forall \tau \geq t\} \tag{6}$$

This is called maximal output admissible set. It is the set of feasible values for g for the current observed state $\gamma(t)$ such that if a constant input signal $g(\tau \geq t) = g$ is applied, the system output constraints $Hy_\gamma(\tau \geq t) \leq h$ remain satisfied for all τ, for all possible disturbance realizations $n(\tau) \in \mathcal{N}_\infty$, $\forall \tau \geq t$. At a time instant $\tau \geq t$, the system output $y_\gamma(\tau)$ for input signal $g(\tau \geq t) = g$ can be written as:

$$\begin{aligned}
y_\gamma(\tau) = C_\gamma A_\gamma^{\tau-t} \gamma(t) + \left(C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^g + D_\gamma^g \right) g + \\
C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^n n(\tau-k) + D_\gamma^n n(\tau)
\end{aligned}$$

For a particular future time instant τ, the set $\mathbb{O}_\tau(\gamma(t))$ is given by:

$$\begin{aligned}
\mathbb{O}_\tau(\gamma(t)) &= \{g : \; \tilde{H}(\tau) v \leq \tilde{h}(\tau)\} \\
\text{where} \quad \tilde{H}(\tau) &= H \left(C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^g + D_\gamma^g \right) \\
\tilde{h}(\tau) &= h - HC_\gamma A_\gamma^{\tau-t} \gamma(t) - f^n(\tau)
\end{aligned}$$

Each element $f_i^n(\tau)$ of the column vector $f^n(\tau)$ is calculated by solving the linear program:

$$f_i^n(\tau) = \max_{\{n(k)\}_t^\tau \in \mathcal{N}_\infty} H \left(C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^n n(\tau-k) + D_\gamma^n n(\tau) \right)_i \tag{7}$$

The subscript i in (7) indicates that row i of the matrix is used in the linear program to calculate the corresponding $n(k)$ sequence. The maximal output admissible set is then given by:

$$\mathbb{O}_\infty(\gamma(t)) = \bigcup_{\tau=t}^{\infty} \mathbb{O}_\tau(\gamma(t)) \tag{8}$$

According to the theory of maximal output admissible sets for linear systems as discussed in [8], the sets $\mathbb{O}_\tau(\gamma(t))$ become redundant after a finite time τ_c. In the current setting, this happens when the elements of $f^n(\tau)$ converge to constant values. Since the linear program (7) does not depend on $\gamma(t)$, it can be solved offline for increasing values of τ after setting $t = 1$. The values of $f^n(\tau)$ are stored till convergence is observed at $\tau = \tau_c$. The corresponding parts $\tilde{H}(\tau)$ are also calculated and stored offline till $\tau = \tau_c$, after setting $t = 1$. The resulting set $\mathbb{O}_\infty(\gamma(t))$ is a set of linear inequality constraints on g, given by:

$$\begin{bmatrix} \tilde{H}(1) \\ \cdot \\ \cdot \\ \tilde{H}(\tau_c) \end{bmatrix} g \leq \begin{bmatrix} h \\ \cdot \\ \cdot \\ h \end{bmatrix} - \begin{bmatrix} HC_\gamma \\ \cdot \\ \cdot \\ HC_\gamma A_\gamma^{\tau_c - 1} \end{bmatrix} \gamma(t) - \begin{bmatrix} f^n(1) \\ \cdot \\ \cdot \\ f^n(\tau_c) \end{bmatrix} \tag{9}$$

Thus, at each time instant t, the RMPC controller reads the state $\gamma(t)$, builds the constraint set (9), and solves the optimization problem (5) to generate a reference trajectory $g(t)$ for the next N_P time-steps which robustly satisfies the constraints (1). Note that our framework can easily be extended to a multivariable system.

IV. Uncertain model estimation

This section discusses the development of the model $\mathbb{M}'_{PD}$, which is used by the RMPC controller to generate the signal $g(t)$. First, the overall model is presented. Then, a novel technique is introduced to quantify uncertainty bounds on the model obtained from ARX estimation.

A. Appended closed-loop model

In order to improve prediction of closed-loop performance, an output disturbance model $\mathbb{D}$ is designed. The disturbance model is a dynamical system whose output is the discrepancy between the actual output of the system in closed-loop with $\mathbb{K}_P$ and the output of the reference closed-loop system $\mathbb{M}_P$. The system consists of a deterministic part and a stochastic part, as seen in Fig. 3.

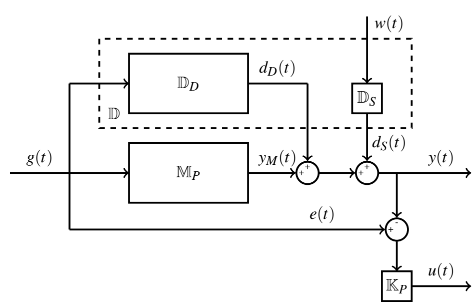

Fig. 3: Overall uncertain closed-loop model M'_{PD}.

Since this model represents the closed-loop behavior of the system, new closed-loop measurements are required to estimate the parameters of the disturbance model $\mathbb{D}$. These are obtained by performing experiments with excitation signals $\hat{g}(t)$ on the reference closed-loop model $\mathbb{M}_P$ and the actual closed-loop plant under the control law $\mathbb{K}_P$, and measuring the outputs $\hat{y}_M(t)$ and $\hat{y}(t)$ respectively. The signals are captured in the data set $\hat{D}_N = \{\hat{g}(t), \hat{y}_M(t), \hat{y}(t); t \in 1, \ldots, N\}$. The disturbance model $\mathbb{D}$ is parameterized as the ARX model

$A_D(q^{-1})y_D(t) = B_D(q^{-1})v(t) + w(t)$, where

$$y_D(t) = y(t) - y_M(t) = d_D(t) + d_S(t)$$
$$A_D(q^{-1}) = 1 + \sum_{i=1}^{n_{a_D}} a_i^D q^{-i}$$
$$B_D(q^{-1}) = \sum_{i=1}^{n_{b_D}} b_i^D q^{-i}$$

Using standard ARX identification, the coefficients a_i^D and b_i^D are estimated by solving the standard linear regression problem

$$\min_{a_i^D, b_i^D} \quad \frac{1}{N}\sum_{t=1}^{N}\left(A_D(q^{-1})(\hat{y}(t) - \hat{y}_M(t)) - B_K(q^{-1})(\hat{v}(t))\right)^2 \tag{10}$$

The disturbance model is then split into two parts, deterministic and stochastic. The deterministic part $\mathbb{D}_D$ takes in the reference signal $g(t)$ as an input, and the stochastic part $\mathbb{D}_S$ is excited by the exogenous disturbance $w(t)$:

$$d_D(t) = \frac{B_D(q^{-1})}{A_D(q^{-1})}v(t) = \mathbb{D}_D g(t)$$
$$d_S(t) = \frac{1}{A_D(q^{-1})}w(t) = \mathbb{D}_S w(t)$$

The appended model shown is Fig.3 can be seen as the 2-input 2-output system $\mathbb{M}'_{PD}$.

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_P + \mathbb{D}_D & \mathbb{D}_S \\ \mathbb{K}_P(I - (\mathbb{M}_P + \mathbb{D}_D)) & -\mathbb{K}_P\mathbb{D}_S \end{bmatrix} \begin{bmatrix} g(t) \\ w(t) \end{bmatrix} \tag{11}$$

Alternatively, it can also be seen a system with $d_S(t)$ acting as measurement noise on the output $y(t)$. The model corresponding to this view-point is:

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_P + \mathbb{D}_D & I \\ \mathbb{K}_P(I - (\mathbb{M}_P + \mathbb{D}_D)) & -\mathbb{K}_P \end{bmatrix} \begin{bmatrix} g(t) \\ d_S(t) \end{bmatrix} \tag{12}$$

Note that the dependence of the model on time shift operator q^{-1} is not shown for ease of notation. Both $w(t)$ and $d_S(t)$ can be seen as exogenous disturbance signals, which cause model uncertainty. Hence, the state space equivalents of either of these models can be used in the RMPC controller, by replacing $n(t)$ in (4) with $w(t)$ if model (11) is used, and with $d_S(t)$ if (12) is used instead.

If the model described in (12) is considered for RMPC controller design, the bounds on $d_S(t)$ can be estimated directly from the prediction errors obtained during ARX identification performed in (10). These bounds are used to build the uncertainty set $\mathscr{D}_\infty$, the definition of which is formalized in the next subsection. Hence, the set $\mathscr{D}_\infty$ replaces $\mathscr{N}_\infty$ while solving the linear programs (7). In case of the model described in (11), the calculation of bounds on $w(t)$ is not as straightforward. The following subsection discusses a technique to calculate upper and lower bounds w_{min} and w_{max} on the disturbance signal $w(t)$, which build the set $\mathscr{W}_\infty$. Within the RMPC controller, $\mathscr{N}_\infty$ is then set equal to $\mathscr{W}_\infty$

B. Calculation of exogenous disturbance set

Samples $\hat{d}_S(t)$ between $\hat{y}(t)$ and the deterministic output $\hat{y}_M(t) + \hat{d}_D(t)$ can be simply calculated from the data set $\hat{D}_N$ as

$$\hat{d}_S(t) = \hat{y}(t) - \hat{y}_M(t) - \frac{B_D(q^{-1})}{A_D(q^{-1})}\hat{v}(t)$$

The maximum and minimum values of $\hat{d}_S(t)$ are labeled $\hat{d}_{S,max}$ and $\hat{d}_{S,min}$ respectively. If the ARX identification results in a stable deterministic disturbance model $\mathbb{D}_D$, the values $\hat{d}_{S,max}$ and $\hat{d}_{S,min}$ are finite. Further, if infinite closed-loop data $\hat{D}_\infty$ were collected for ARX identification, the bounds on discrepancy would be equal to the actual bounds $d_{S,max}$ and $d_{S,min}$ on $d_S(t)$. The set of sequences $\{d_S(t)\}$ satisfying these bounds are indicated as lying in a set $\mathscr{D}_\infty$, defined as

$$\mathscr{D}_\infty = \{\{d_S(t)\} : d_{S,min} \le d_S(t) \le d_{S,max} \ \ \forall t \in (-\infty, \infty)\}$$

From these bounds, we want to calculate the set $\mathscr{W}_\infty$ defined as

$$\mathscr{W}_\infty = \left\{ \{w(t)\} : \begin{matrix} w_{min} \le w(t) \le w_{max} \\ \mathbb{D}_S w(t) \in \mathscr{D}_\infty \end{matrix} \ \ \forall t \in (-\infty, \infty) \right\}$$

The set $\mathscr{W}_\infty$ contains all $\{w(t)\}$ sequences such that the corresponding steady-state output signal of $\mathbb{D}_S$ lies within $\mathscr{D}_\infty$. It must be noted that the converse is not implied. That is, it does not mean that there cannot exist a noise sequence $w(t)$ not belonging to $\mathscr{W}_\infty$ but producing a corresponding output sequence belonging to $\mathscr{D}_\infty$. This means that the bounds on residuals of the ARX identification (10), that can be obtained by inverting the (invertible) model $\mathbb{D}_S$, are not necessarily w_{min} and w_{max}.

To calculate the bounds w_{min} and w_{max}, first the optimization problems shown in (13) are solved for increasing lengths of time horizon N.

$$\begin{aligned} \min_{X_j} \quad & \bar{w}_j(N) \\ \text{subject to} \quad & d_S(k) = -\sum_{i=1}^{n_{a_D}} a_i^D d_S(k-i) + w(k), k = 1:N \\ & d_{S,min} \le d_S(k) \le d_{S,max}, \ \ k = 1:N-1 \\ & w(k) \le \bar{w}_j(N), \ \ k = 1:N \\ & d_S(N) \in D_j \end{aligned} \tag{13}$$

$$\text{where } X_j = \left\{ \begin{matrix} \{d_S(-n_{a_D}+1), \ldots, d_S(N)\}, \\ \{w(1), \ldots, w(N)\}, \\ \bar{w}_j(N) \end{matrix} \right\}, j = \{1, 2\}$$
$$D_1 = \{d : d \le d_{S,min}\}, \ \ D_2 = \{d : d \ge d_{S,max}\}$$

Problem (13) looks for a sequence of inputs $\{w(k), k = 1:N\}$ such that the sequence of corresponding outputs at all instances except at time N lie within $\mathscr{D}_\infty$. This means that the chosen input sequence $w(k)$ should drive $d_S(N)$ out of $\mathscr{D}_\infty$ into D_j. The initial conditions on $d_S(k)$ are left free to be chosen by the solver, but are constrained to lie within $\mathscr{D}_\infty$. The solutions $\bar{w}_j(N)$ are collected, and the set $\mathscr{W}_\infty$ is constructed with converged values of $\bar{w}_1(N)$ and $\bar{w}_2(N)$ corresponding to w_{min} and w_{max} respectively. At lower values of N, there can exist an initial sequence $\{d_S(-n_{a_D}+1),..,d_S(1)\}$ that drives $d_S(N)$ into D_j with very low input effort $w(k)$. At higher values of N, the effect of initial condition wears off, and a higher value of control effort is required to violate $\mathscr{D}_\infty$. Hence, solving (13) gives the values of w_{min} and w_{max}, that are the minimum and maximum values of the disturbance signal $w(t)$ such that the steady state values of $d_S(t)$ lie within $\mathscr{D}_\infty$.

We now have a closed-loop model with disturbance bound information, and hence are ready to use it in an RMPC controller.

V. Case study

Numerical simulations are performed on a servo motor control problem, implementing the complete control scheme presented in Fig 2. The sampling time is $T_s = 1ms$. RMPC controllers are implemented for closed-loop models corresponding to both (11) and (12). It is shown that using model (11) results is superior performance, and hence performing an additional step to calculate w_{min} and w_{max} is justified. Both the case studies are implemented using MATLAB R2017b, with optimization problems occasionally implemented using YALMIP [9].

A. Robust data-driven MPC

The plant consisting of a servo positioning system is controlled using the scheme shown in Fig.2. The plant dynamics are modeled by the following non-linear state space equations:

$$\begin{bmatrix}\dot{\theta}(t)\\ \dot{\omega}(t)\\ \dot{i}(t)\end{bmatrix} = \begin{bmatrix}\omega(t)\\ \dfrac{-mgl}{J}sin\theta(t) - \dfrac{b}{J}\omega(t) + \dfrac{K_m}{J}i(t)\\ \dfrac{-K_m}{L}\omega(t) - \dfrac{R}{L}i(t) + \dfrac{1}{L}u(t)\end{bmatrix}$$

$$y(t) = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta(t)\\ \omega(t)\\ i(t)\end{bmatrix}$$

Symbol	Parameter	Value
R	Motor resistance	5Ω
L	Motor inductance	5.10^{-3}H
K_m	Motor torque constant	0.0847Nm/A
J	Complete disk inertia	5.10^{-5}Nm^2
b	Friction coefficient	3.10^{-3}Nms/rad
m	Additional mass	3Kg
l	Mass offset	2m

TABLE I: Physical parameters of servo motor system

The states of the system $\theta(t)$, $\omega(t)$, and $i(t)$ are angle $[rad]$ and rotational velocity $[rad/s]$ of the servo motor, and armature current $[A]$ respectively. The input $u(t)$ is the voltage $[V]$ applied across the motor, and the output $y(t)$ is the rotational angle. A VRFT methodology is used to design a stabilizing PD controller, which provides a voltage input $u(t)$ to make the rotational angle $y(t)$ track a reference signal $g(t)$. To this end, experiments are conducted with a low-pass filtered white noise signal $u(t)$ with a standard deviation of 10V. The output angle $y(t)$ is recorded and the dataset $\mathbb{D}_N$ is obtained. A slow reference closed loop model $\mathbb{M}_P$ is chosen, given by:

$$x_M(t+1) = 0.99x_M(t) + 0.01g(t)$$
$$y_M(t) = x_M(t)$$

The PD inner-loop controller $\mathbb{K}_P$ is parameterized as:

$$u(t) = K_p e(t) + K_d\frac{e(t)-e(t-1)}{T_s}$$

Solving the optimization problem (2), the parameters K_p and K_d are calculated using the dataset D_N. The controller is placed in the inner loop within the hierarchical control architecture. After VRFT synthesis, an outer MPC is designed using the formulation in (3), to provide a reference signal $g(t)$. The output $y(t)$ is constrained to lie between 0 rad and 4 rad, and the voltage input $u(t)$ between -3.5 V and 3.5 V. An MPC horizon of $N_P = 10$ timesteps is chosen, and weights are $Q_y = 1$ and $Q_\varepsilon = 1$.

Towards formulating the RMPC controller instead of the deterministic MPC controller, the disturbance model discussed in Sec IV is developed. First, the dataset $\hat{D}_N$ is built by performing closed loop experiments with input signal $\hat{g}(t)$ of standard deviation 10 rad. Then, a linear model for $\mathbb{D}$ parameterized by $n_{a_D} = 4$ and $n_{b_D} = 3$ is identified by solving (10). Two equivalent models of the appended system, corresponding to (11) and (12), are developed.

For the model corresponding to (11), the disturbance model is split into two parts, $\mathbb{D}_D$ and $\mathbb{D}_S$, with inputs $g(t)$ and $w(t)$ respectively. Bounds on the exogenous disturbance $w(t)$ for a horizon N are calculated by solving the linear problems (13). The evolution of these bounds with increasing values of horizon N is plotted in Fig 4. The values w_{min} and w_{max} are obtained at convergence of $\bar{w}_1(N)$ and $\bar{w}_2(N)$ respectively.

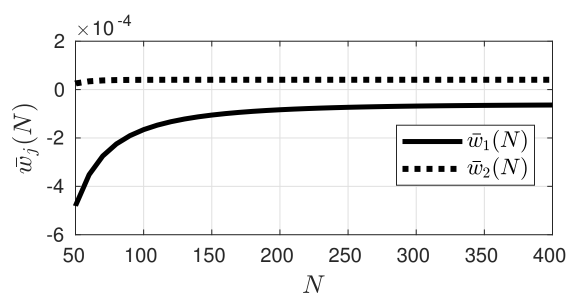

Fig. 4: Convergence of bounds on $w(t)$ acting as input to the disturbance model D_S.

For the model corresponding to (12), the disturbance model consists of $\mathbb{D}_D$ and a direct exogenous disturbance signal $d_S(t)$. The bounds $d_{S,min}$ and $d_{S,max}$ on $d_S(t)$ are set equal to the minimum and maximum values of prediction error

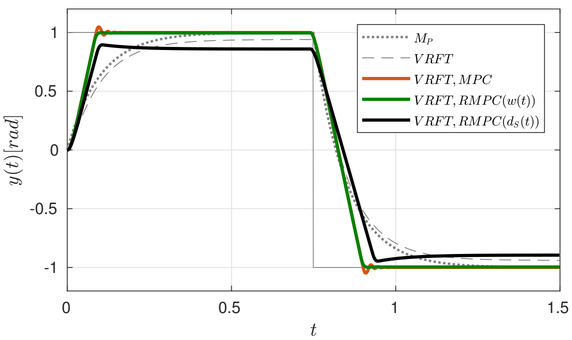

Fig. 5: The performance inner closed loop does not exactly match the reference model M_P. MPC improves the performance but results in small constraint violation. Constraint violation is robustly avoided by using a reference governor. Using (12) results in a conservative performance compared to (11).

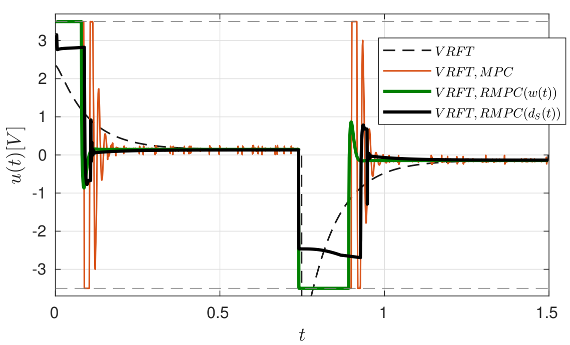

Fig. 6: Comparison of voltage input $u(t)$.

obtained during identification of the ARX model for $\mathbb{D}$ respectively.

These models and related disturbance bounds are used in the formulation of corresponding RMPC controllers. To this end, constraints on the reference signal $g(t)$ are constructed, as shown in (9). Solving the linear programs (7) till convergence to calculate maximal output admissible set $\mathbb{O}_\infty(\gamma(t))$ can result in very conservative constraints on $g(t)$. This is avoided by solving them only for the duration of RMPC horizon by setting $\tau_c = N_P T_s$.

The corresponding output admissible sets $\mathbb{O}_1(\gamma(t))$ till $\mathbb{O}_{N_P T_s}(\gamma(t))$ are calculated at each time-instant t by reading the state $\gamma(t)$, which is built with the assumption of no additional measurement noise. Following this, the optimization problem (5) is constructed and solved by the RMPC controller, generating a constraint respecting reference signal $g(t)$. Performance of the control system for all these cases is plotted in Fig 5 and Fig 6.

It can clearly be seen that utilizing an RMPC controller instead of a deterministic MPC controller in the outer loop results in robust constraint satisfaction. Also, using the closed-loop model corresponding to (12) as the plant model for RMPC controller design results in conservative performance when compared to using (11), for the same admissible set horizon $\tau_c = N_P T_s$. Since the calculation of bounds w_{min} and w_{max} (which are required to use (11) for RMPC synthesis) is performed offline, conservativeness of the control scheme can be reduced without any additional online operation. Note that increasing admissible set horizon to convergence results in conservative performance from either of the models, with (11) still outperforming (12).

VI. Conclusion

This paper builds on the hierarchical data-driven control of constrained systems, by introducing robustness with respect to constraint satisfaction. This is done by using a robust MPC controller. Uncertainty on the model utilized by the RMPC controller is modeled as disturbance input. The major contribution of this work is the formulation of a novel technique to compute bounds on this input from ARX identification. It is seen that robust constraint satisfaction is achieved without identifying an open-loop model of the nonlinear plant. Future work deals with extending the formulation to general Box-Jenkins models. Possible extensions to the control scheme includes modifications to incorporate LPV systems.

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