Robust data-driven predictive control

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***Abstract*— This is a very skeletal overview.**

I. Introduction

Design of control systems can broadly be classified into two categories: model-based and data-driven. Model-based control design techniques utilize an explicit model of the plant being controlled. This necessitates choosing a dynamical system parameterization that expresses the performance of the plant, and performing identification experiments to find the parameters. A good parameterization is one that sufficiently captures the dynamics, without being too complex for control design. Obtaining such a good model poses several challenges. To avoid this, one can resort to a data-driven controller design methodology.

Data-driven controller design methods avoid explicitly identifying the plant model. They synthesize a controller directly from I/O data obtained from the plant. A review of several such methods can be found in [1]. One such method, virtual reference feedback tuning (VRFT) introduced in [2], has been used to design a stabilizing feedback controller within a hierarchical controller framework in [3] for LTI/LPV systems. It employs an outer MPC controller, which utilizes the reference model selected for VRFT to generate a tracking signal. Performance bounds on the plant are translated into constraints on the optimization problem solved by the MPC controller. Since the reference model might not reasonably reflect the performance of the closed-loop plant, there is a possibility of constraint violation. To avoid this, one can use the techniques developed under the umbrella of robust MPC theory to guarantee constraint satisfaction in an MPC framework. A review of the techniques can be found in [4]. An alternative is to use a robust reference governor, which modifies the reference signal supplied to the plant in a way that constraints are satisfied. These are reviewed in [5]. Both the methodologies require an explicit model of the uncertainties present in the plant being controlled.

In this work, we propose a systematic methodology to develop a robust control framework, building on the hierarchical control design presented in [3]. A model of closed loop behavior of the plant is used for robust controller design. A parameterization of the closed loop behavior is chosen, and the parameters are identified using standard ARX estimation. Since the VRFT methodology shapes the closed loop performance similar to a user-selected reference model, identification of the closed loop model does not pose the same level of challenge as during model-based controller design. The model incorporates uncertainties as exogenous noise signals. The noises are assumed to lie within a bounded polyhedral set. The major contribution of this work is the formulation of an optimization problem which solves for these sets. Similar work was done for the estimation of parameter variability within ellipsoidal sets in [6]. To the best of authors' knowledge, no similar work has been done to calculate polyhedral noise sets. Following the identification of a model and corresponding noise sets, a robust reference governer is designed and appended to the control architecture presented in [6]. The option of robust reference governor is chosen for illustration, but the techniques presented can be easily extended to a robust MPC.

The paper is organized as follows. In Sec.II, the problem statement is formally presented. Background regarding the hierarchical data-driven control architecture and robust reference governor is presented in Sec.III. Sec.IV presents the techniques used to identify the closed-loop system model with bounds, which is the main contribution of this paper. The final Sec.V presents two simulation results, one verifying the techniques presented for bound identification, and one presenting the robust hierarchical control design framework.

II. Problem statement

Consider a *single-input single-output* system $\mathbb{G}_P$ generating an output signal $y(t) \in \mathbb{R}$ corresponding to the input signal $u(t) \in \mathbb{R}$ for the time $t \in \mathbb{Z}^+$. We aim to synthesize a controller that can make $y(t)$ accurately track any user defined reference signal, while robustly respecting the constraints

$$\begin{gathered} y_{min} \leq y(t) \leq y_{max} \\ u_{min} \leq u(t) \leq u_{max} \\ \forall t \in \mathbb{Z}^+ \end{gathered}$$

Following the data-driven controller synthesis methodology, we use the data $D_N = \{u(t), y(t); t \in 1, ..., N\}$ obtained by exciting the system to design the controller.

III. Background

A. Hierarchical approach

A feedback controller is designed to control the system, using the VRFT methodology. For this, a reference model $\mathbb{M}_P$ is selected, given by:

$$\begin{aligned} x_M(t+1) &= A_M x_M(t) + B_M g(t) \\ y_M(t) &= C_M x_M(t) \end{aligned}$$

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The VRFT methodology designs a feedback controller $\mathbb{K}_P$, with the goal of making the closed-loop system $\mathbb{K}_P$-$\mathbb{G}_P$ behave similar to the reference model $\mathbb{M}_P$. To this end, the VRFT methodology utilizes the dataset $\mathbb{D}_N$. The steps followed are as follows:

1) A virtual reference input $g(t)$ is calculated by setting $y_M(t) = y(t)$ obtained from the dataset $\mathbb{D}_N$, by the inverting the model $\mathbb{M}_P$. Let this mapping be defined by $g(t) = \mathbb{M}_P^{\dagger} y(t)$.
2) A feedback controller $\mathbb{K}_P$ described by $A_K(q^{-1})u(t) = B_K(q^{-1})(g(t) - y(t))$ is chosen, where

$$A_K(q^{-1}) = 1 + \sum_{i=1}^{n_{a_K}} a_i^K q^{-i}$$
$$B_K(q^{-1}) = \sum_{i=1}^{n_{b_K}} b_i^K q^{-i}$$

   The parameters of the controller a_i^K and b_i^K are calculated by the VRFT methodology, such that the closed loop performance of $\mathbb{K}_P$-$\mathbb{G}_P$ matches open loop performance of the reference model $\mathbb{M}_P$.
3) This is done by solving the convex optimization problem

$$\min_{a_i^K, b_i^K} \quad \frac{1}{N} \sum_{t=1}^{N} \left| A_K(q^{-1})u(t) - B_K(q^{-1})(\mathbb{M}_P^{\dagger} y(t) - y(t)) \right|^2 \tag{1}$$

   which minimizes the deviation between the control input calculated by the controller and $u(t)$ that is used to excite the system and obtain $y(t)$.
4) The synthesized controller $\mathbb{K}_P$ is placed before the plant, and the loop is closed. A reference step input is given to evaluate the controller performance.

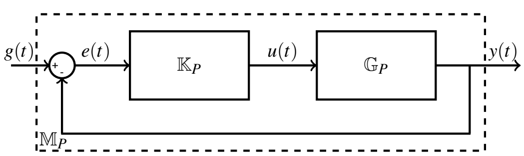

Fig. 1: Feedback controller designed using VRFT

The performance of this setup is improved by providing the reference signal $g(t)$ using an MPC controller in an outer loop. The objective of the MPC controller is to make the output signal $y(t)$ track the reference $r(t)$, while satisfying constraints on the plant output $y(t)$ and input $u(t)$. This is achieved by considering the closed-loop plant model $\mathbb{M}_P$ for the state propagation equation, and an augmented model with $[y(t), u(t)]$ as the output equation. This augmented state-space model is called $\mathbb{M}_P'$.

$$\begin{aligned} \zeta(t+1) &= A_\zeta \zeta(t) + B_\zeta g(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\zeta \zeta(t) + D_\zeta g(t) \end{aligned}$$

The optimization problem solved by the MPC at each time step t for a horizon of N_P timesteps is shown below. The lower and upper bounds on $y(t)$ and $u(t)$ are $[y_{min}, y_{max}]$ and $[u_{min}, u_{max}]$ respectively.

$$\begin{aligned} \min_{\{g(t+k)\}_{k=1}^{N_P}} \quad & Q_y \sum_{k=1}^{N_P} (y(t+k|t) - r(t+k))^2 + Q_\varepsilon \varepsilon^2 \\ \text{subject to} \quad & \zeta(t+k+1) = A_\zeta \zeta(t+k) + B_\zeta g(t+k) \\ & \begin{bmatrix} y(t+k) \\ u(t+k) \end{bmatrix} = C_\zeta \zeta(t+k) + \begin{bmatrix} 0 \\ D_\zeta \end{bmatrix} g(t+k) \\ & y_{min} - V_y \varepsilon \le y(t+k) \le y_{max} + V_y \varepsilon \\ & u_{min} - V_u \varepsilon \le u(t+k) \le u_{max} + V_u \varepsilon \\ & \zeta(t|t) = \zeta(t) \end{aligned} \tag{2}$$

The quantities V_y and V_u in the MPC formulation are used to avoid infeasibility of the optimization problem over successive iterations, since the reference model $\mathbb{M}_P'$ might not accurately capture the dynamics of the unknown plant $\mathbb{G}_P$. This implies that constraint satisfaction is not guaranteed by the proposed formulation.

B. Robust reference governor

The robust reference governer is a signal regulator which alters the command input such that the system robustly satisfies constraints. It does so by utilizing knowledge of disturbances acting on the system. A schematic of the control system is shown in Fig. 2.

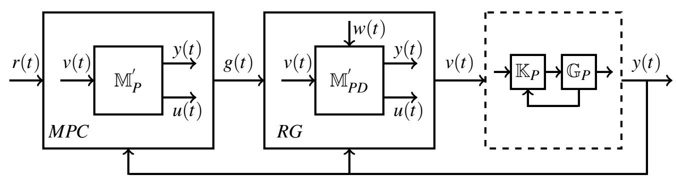

Fig. 2: Schematic of the control system

The *MPC* controller utilizes the model $\mathbb{M}_P'$, and the reference governer *RG* uses the model $\mathbb{M}_{PD}'$. $\mathbb{M}_{PD}'$ is a model of the closed loop performance of $\mathbb{K}_P$-$\mathbb{G}_P$. Identification of this model and corresponding bounds is discussed in Sec.IV. The state space form of this model is written as:

$$\begin{aligned} \gamma(t+1) &= A_\gamma \gamma(t) + B_\gamma^v v(t) + B_\gamma^w w(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\gamma \gamma(t) + D_\gamma^v v(t) + D_\gamma^w w(t) \end{aligned}$$

The model has a deterministic input $v(t)$ and a noise input $w(t)$, which captures the effect of measurement noise and un-modeled dynamics on the system output. The columns of matrices B_γ and D_γ are separated for ease of notation. The outputs of the system are called $y_\gamma(t) = [y(t)u(t)]^T$. The constraints on these outputs are written as:

$$\left. \begin{aligned} y_{min} \le y(t) \le y_{max} \\ u_{min} \le u(t) \le u_{max} \end{aligned} \right\} \quad H y_\gamma(t) \le h$$

At each time instant t, the reference governer solves the quadratic program:

$$\begin{aligned} \min_{v} \quad & \frac{1}{2}\|v-g(t)\|_2^2 \\ \text{subject to} \quad & (x(t),v) \in \mathbb{O}_\infty \end{aligned} \tag{3}$$

where the set $\mathbb{O}_\infty$ is defined as:

$$\mathbb{O}_\infty = \{(x(t),v) : x(t) = \gamma(t),\, Hy_\gamma(\tau) \le h \ \ \forall \tau \ge t\}$$

This is called an output-invariant set. It is set of feasible values for v for the current observed state $x(t) = \gamma(t)$ such that if a constant input signal $v(\tau \ge t) = v$ is applied, the system output constraints $Hy_\gamma(\tau \ge t) \le h$ remain satisfied for all τ. At a time instant $\tau \ge t$, the system output $y_\gamma(\tau)$ for input signal $v(\tau \ge t) = v$ can be written as:

$$\begin{aligned} y_\gamma(\tau) = C_\gamma A_\gamma^{\tau-t}\gamma(t) + \left(C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^v + D_\gamma^v\right) v + \\ C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^w w(\tau-k) + D_\gamma^w w(\tau) \end{aligned}$$

For a particular future time instant τ, the set $\mathbb{O}_\infty(\tau)$ is given by:

$$\begin{aligned} \mathbb{O}_\infty(\tau) &= \{(x(t),v) : x(t) = \gamma(t),\ \ \tilde{H}(\tau)v \le \tilde{h}(\tau)\} \\ \text{where} \quad \tilde{H}(\tau) &= H\left(C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^v + D_\gamma^v\right) \\ \tilde{h}(\tau) &= h - HC_\gamma A_\gamma^{\tau-t}\gamma(t) - f^w(\tau) \end{aligned}$$

Each element $f_i^w(\tau)$ of the column vector $f^w(\tau)$ is calculated by solving the linear program:

$$f_i^w(\tau) = \max_{\{w(k)\}_t^\tau \in \mathscr{W}_\infty} H\left(C_\gamma \sum_{k=1}^{\tau-t} A_\gamma^{k-1} B_\gamma^w w(\tau-k) + D_\gamma^w w(\tau)\right)_i \tag{4}$$

Subscript i in the above problem indicates that row i of the matrix is used in the linear program to calculate the corresponding $w(k)$ sequence. According to the theory of output invariant sets for linear systems with polyhedral constraints on noise and outputs, the set $\mathbb{O}_\infty = \mathbb{O}_\infty(\tau \to \infty)$ is reached when the elements of $f^w(\tau)$ converge to a constant value. Since the linear problem to be solved for $f_i^w(\tau)$ does not depend on values at current time instant t, it can be solved offline for increasing values of τ. When convergence is observed at time τ_c, the value $f^w(\tau_c)$ is stored. This results in problem (3) being simplified to

$$\begin{aligned} \min_{v} \quad & \frac{1}{2}\|v-g(t)\|_2^2 \\ \text{subject to} \quad & \tilde{H}(\tau_c)v \le \tilde{h}(\tau_c) \end{aligned} \tag{5}$$

It must be noted that the value $\tilde{h}(\tau_c)$ depends on the current observed state $\gamma(t)$. Thus, at each time instant, the reference governer reads the current state $\gamma(t)$, reference input $g(t)$, and calculates a new reference $v(t)$ which satisfies output constraints on the system robustly.

IV. Disturbance sensor

This section discusses the major contribution of this work, which is the development of a methodology to calculate noise bounds on a system identified ARX techniques.

A. Appended closed-loop model

In order to improve prediction of closed loop performance, a disturbance sensor $\mathbb{D}$ is designed. The disturbance sensor is a dynamical system whose output is the discrepancy between output of the actual closed loop system $\mathbb{K}_P$-$\mathbb{G}_P$ and reference closed loop system $\mathbb{M}_P$. The system consists of a deterministic part and a stochastic part, as seen in Fig. 3.

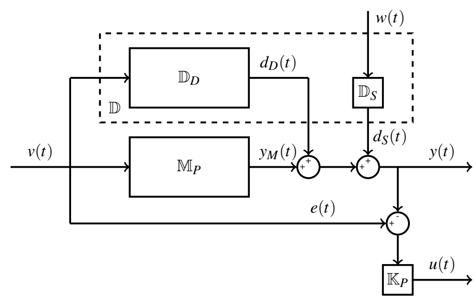

Fig. 3: Reference model appended with disturbance sensor

This can be seen as a 2-input 2-output system, whose transfer function is described as

$$\begin{bmatrix} Y(z) \\ U(z) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_P + \mathbb{D}_D & \mathbb{D}_S \\ \mathbb{K}_P(I - (\mathbb{M}_P + \mathbb{D}_D)) & -\mathbb{K}_P\mathbb{D}_S \end{bmatrix} \begin{bmatrix} V(z) \\ W(z) \end{bmatrix} \tag{6}$$

Note that the dependence of transfer functions on time shift operator z (or q^{-1}) is not denoted for ease of notation. The signal $w(t)$ is interpreted as an external noise signal. The state-space equivalent of this model is used in the robust reference governor that was previously discussed.

Since this model represents the closed loop behavior of the system, new closed loop measurements are required to estimate the parameters of the disturbance sensor. These are obtained by performing experiments with excitaion signals $\hat{v}(t)$ on the closed loop model and plant, and measuring the outputs $\hat{y}_M(t)$ and $\hat{y}(t)$ respectively. The output is captured as the data sequence $\hat{D}_N = \{\hat{v}(t), \hat{y}_M(t), \hat{y}(t); t \in 1,...,N\}$. The disturbance sensor $\mathbb{D}$ is parameterized as $A_D(q^{-1})y_D(t) = B_D(q^{-1})v(t) + w(t)$, where

$$\begin{aligned} y_D(t) = y(t) - y_M(t) &= d_D(t) + d_S(t) \\ A_D(q^{-1}) &= 1 + \sum_{i=1}^{n_{a_D}} a_i^D q^{-i} \\ B_D(q^{-1}) &= \sum_{i=1}^{n_{b_D}} b_i^D q^{-i} \end{aligned}$$

Using standard ARX identification, the coefficients

$$\begin{aligned} \max_{X_{min}} \quad & w_{min}^N \\ \text{subject to} \quad & d_S(k) = -\sum_{i=1}^{n_{a_D}} a_i^D d_S(k-i) + w(k), k = 1:N \\ & d_{S,min} \leq d_S(k) \leq d_{S,min}, k = 1:N-1 \\ & w_{min}^N \leq w(k), k = 1:N \\ & d_S(N) \leq d_{S,min} \end{aligned}$$	$$\begin{aligned} \min_{X_{max}} \quad & w_{max}^N \\ \text{subject to} \quad & d_S(k) = -\sum_{i=1}^{n_{a_D}} a_i^D d_S(k-i) + w(k), k = 1:N \\ & d_{S,min} \leq d_S(k) \leq d_{S,min}, k = 1:N-1 \\ & w(k) \leq w_{max}^N, k = 1:N \\ & d_S(N) \geq d_{S,max} \end{aligned}$$
$$\text{where } X_{min} = \left\{ \begin{matrix} \{d_S(-n_{a_D}+1), ..., d_S(N)\}, \\ \{w(1), ..., w(N)\}, \\ w_{min}^N \end{matrix} \right\}$$	$$\text{where } X_{max} = \left\{ \begin{matrix} \{d_S(-n_{a_D}+1), ..., d_S(N)\}, \\ \{w(1), ..., w(N)\}, \\ w_{max}^N \end{matrix} \right\}$$

TABLE I: Linear programs solved to calculate the set $\mathscr{W}_\infty$

a_i^D and b_i^D are estimated by solving the optimization problem

$$\min_{a_i^D, b_i^D} \quad \frac{1}{N} \sum_{t=1}^{N} \left| A_D(q^{-1})(\hat{y}(t) - \hat{y}_M(t)) - B_K(q^{-1})(\hat{v}(t)) \right|^2 \tag{7}$$

The disturbance sensor is then split into two parts, deterministic and stochastic. The deterministic part $\mathbb{D}_D$ takes in the reference signal $v(t)$ as an input, and the stochastic part $\mathbb{D}_S$ takes in the noise $w(t)$. The I/O behavior of these parts are separately written as

$$d_D(t) = \frac{B_D(q^{-1})}{A_D(q^{-1})} v(t) = \mathbb{D}_D v(t)$$

$$d_S(t) = \frac{1}{A_D(q^{-1})} w(t) = \mathbb{D}_S w(t)$$

The model has an additional noisy input $w(t)$, which will be transformed to process and measurement noises on the state space model corresponding to (6). The following subsection discusses a technique to calculate upper and lower bounds w_{min} and w_{max} on the noise signal $w(t)$, and which are explicitly provided to the robust reference governor.

B. Noise input bounds calculation

A realization of the discrepancy $d_S(t)$ between desired output $y(t)$ and deterministic output $y_M(t) + d_D(t)$ can be calculated from the data set $\hat{D}_N$ as

$$\hat{d}_S(t) = \hat{y}(t) - \hat{y}_M(t) - \frac{B_D(q^{-1})}{A_D(q^{-1})} \hat{v}(t)$$

The maximum and minimum values of this discrepancy are labeled $\hat{d}_{S,max}$ and $\hat{d}_{S,min}$ respectively. If the ARX identification results in a stable deterministic disturbance sensor $\mathbb{D}_D$, the values $\hat{d}_{S,max}$ and $\hat{d}_{S,min}$ are finite. Further, if infinite closed loop data $\hat{D}_\infty$ is collected for ARX estimation, the bounds on discrepancy are equal to the actual bounds $d_{S,max}$ and $d_{S,min}$. The set of sequences $d_S(t)$ satisfying these bounds are indicated as lying in a set $\mathscr{D}_\infty$, defined as

$$\mathscr{D}_\infty = \{d_S(t) : d_{S,min} \leq d_S(t) \leq d_{S,max} \ \ \forall t \in (-\infty, \infty)\}$$

From these bounds, we attempt to calculate the set $\mathscr{W}_\infty$ defined as

$$\mathscr{W}_\infty = \left\{ w(t) : \forall t \in (-\infty, \infty), \begin{matrix} w_{min} \leq w(t) \leq w_{max} \\ \mathbb{D}_S w(t) \in \mathscr{D}_\infty \end{matrix} \right\}$$

This is the set of all $w(t)$ sequences such that the output signal of $\mathbb{D}_S$ corresponding to any $w(t) \in \mathscr{W}_\infty$ lies within $\mathscr{D}_\infty$. It must be noted that the converse is not implied. That is, it does not mean that there cannot exist a noise sequence $w(t)$ not belonging to $\mathscr{W}_\infty$ but producing a corresponding output sequence belonging to $\mathscr{D}_\infty$.

$$\begin{aligned} & \forall w(t) \in \mathscr{W}_\infty : \mathbb{D}_S w(t) \in \mathscr{D}_\infty \\ & \qquad \not\Longrightarrow \nexists w(t) \notin \mathscr{W}_\infty : \mathbb{D}_S w(t) \in \mathscr{D}_\infty \end{aligned}$$

To calculate the bounds w_{min} and w_{max}, the optimization problems shown in **??** are solved for increasing lengths of time horizon N. These equations solve for a sequence of inputs $\{w(k), k = 1:N\}$ such that the sequence of corresponding outputs at all instances except at time N lie within $\mathscr{D}_\infty$. This means that the chosen input sequence $w(k)$ can drive the system output out of bounds in N time steps. The initial conditions on $d_S(k)$ are left free to be chosen by the problem, but are constrained to lie within $\mathscr{D}_\infty$. Thus, for a time horizon N, the input $\{w(k), k = 1:N\}$ is chosen such that, wherever the system starts inside the set $\mathscr{D}_\infty$, it will stay inside the set $\mathscr{D}_\infty$. This freedom to choose an initial condition requires the assumption that $\mathscr{D}_\infty$ is obtained from the infinite data set $\hat{D}_\infty$.

Any sequence $\{w(k) > w_{min}^N, k = 1:N\}$ will not drive $d_S(k \geq N)$ to the bound $d_{S,min}$. Similarly, $\{w(k) < w_{max}^N, k = 1:N\}$ will not drive $d_S(k \geq N)$ to the bound $d_{S,max}$. Hence, in order to explain the whole set $\mathscr{D}_\infty$, we need to find the lowest possible value of w_{min}^N and the highest possible value of w_{max}^N such that $d_S(k)$ is driven to its bounds in finite time k. This is achieved by increasing the value of N, which would require lower and higher values of $w(k)$ to make $d_S(N)$ reach $d_{S,min}$ and $d_{S,max}$ respectively. This leads to w_{min}^N and w_{max}^N asymptotically reaching their true values w_{min} and w_{max} as N increases, thus making the set $\mathscr{W}_\infty$ completely explain the set $\mathscr{D}_\infty$. It is noted that the presented method provides a methodology to quantify uncertainty in any general ARX identified model.

We now have a closed-loop model with knowledge of noise bounds, and hence are ready to put it in a robust reference governor.

V. Case studies

Two case studies are presented. The first one is to verify the formulation presented in (**??**). Then, numerical simulations are performed on a servo motor control problem, implementing the complete control scheme presented in Fig.2. Both the case studies are implemented using MATLAB R2017b, with optimization problems occasionally implemented with YALMIP [7].

A. Noise input bound

Consider a MISO system with inputs $\{u(t), w(t)\}$ and output $x(t)$, described by:

$$u(t+1) = 0.995u(t) + 1$$
$$x(t+1) = 0.05x(t) + u(t) + w(t)$$
$$u(1) = 0, \;\; x(1) = 0$$

The input $u(t)$ is deterministic and $w(t)$ is noisy. An approximate ARX model of the system is calculated as $A(q^{-1})\tilde{x}(t) = B(q^{-1})u(t) + w(t)$ after performing experiments on the system and collecting data. It is identified with $A(q^{-1})$ and $B(q^{-1})$ having 2 and 1 free parameters respectively. Following this, the methodology discussed in Sec.IV-B is used to calculate bounds on $w(t)$.

First, the signal $d_S(t) = x(t) - (B(q^{-1})/A(q^{-1}))u(t)$ is extracted from the experimental data, and $\mathscr{D}_\infty$ is constructed. For increasing values of N, sequences $\{w(k), k = 1:N\}$ and corresponding $\{d_S(k), k = 1:N\}$ are calculated by solving the optimization problems in 4. The bounds w^N_{min} and w^N_{max} on the input sequences are shown in Fig.4. The largest region where

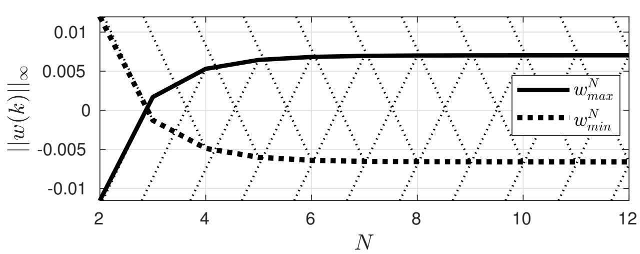

Fig. 4: Convergence of bounds to w_{min} and w_{max}

$w^N_{min} \leq w(k) \leq w^N_{max}$ is satisfied is $\mathscr{W}_\infty$. To verify the obtained bounds, the system identified through ARX identification is simulated with deterministic inputs $u(t)$, and a range of noise inputs $w(t)$ sampled from $\mathscr{W}_\infty$ and held constant throughout the simulation horizon. The simulation results are seen in Fig.5.

The simulation of ARX model with noise requires an initial condition on the noise model. However, the problem calculates $\mathscr{W}_\infty$ for all $\mathscr{D}_\infty$ without incorporating the knowledge of an initial condition (This is the main feature of the problem). In the current example, the ARX model with noise is simulated with a initial condition = 0. Since the actual initial condtion of the noise model might not be 0, one might face problems with the model not covering the whole range of $\mathscr{D}_\infty$ during the transient period. To correct this, a state estimator could be used which rectifies the effect of initial state discrepancy, thus covering the noise effects even during transients.

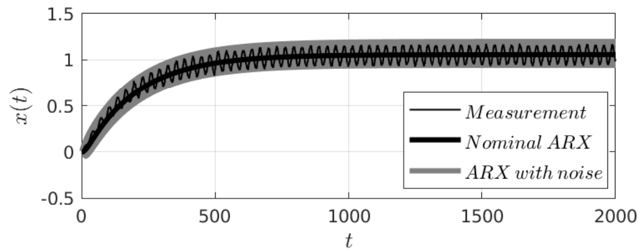

Fig. 5: Comparison of ARX model performance with constant $w(t) = 0$(nominal) and $w(t) \in \mathscr{W}_\infty$. The noise model gives bounded output $x(t)$ without being conservative.

B. Data driven MPC with robust reference governer

Simulations are performed on a model of a servo positioning system, controlled using the loop presented in Fig.2. The plant dynamics are modeled with the following non-linear state space equations:

$$\begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \\ \dot{i}(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ \dfrac{-mgl}{J} sin\theta(t) - \dfrac{b}{J}\omega(t) + \dfrac{K_m}{J} i(t) \\ \dfrac{-K_m}{L}\omega(t) - \dfrac{R}{L} i(t) + \dfrac{1}{L} u(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \omega(t) \\ i(t) \end{bmatrix}$$

Symbol	Parameter	Value
R	Motor resistance	5Ω
L	Motor inductance	5.10^{-3}H
K_m	Motor torque constant	0.0847Nm/A
J	Complete disk inertia	5.10^{-5}Nm^2
b	Friction coefficient	3.10^{-3}Nms/rad
m	Additional mass	3Kg
l	Mass offset	2m

TABLE II: Physical parameters of servo motor system

The states of the system $\theta(t)$,$\omega(t)$ and $i(t)$ are angle $[rad]$ and rotational velocity $[rad/s]$ of the servo motor, and armature current $[A]$ respectively. The input $u(t)$ is the voltage $[V]$ applied across the motor, and output $y(t)$ is the rotational angle. A VRFT methodology is used to design a stabilizing PD controller, which provides a voltage input $u(t)$ to make the rotational angle $y(t)$ track a reference signal $g(t)$. To this end, experiments are conducted with a low-pass filtered white noise signal $u(t)$ with a standard deviation of 10V. The output angle $y(t)$ is recorded and the dataset $\mathbb{D}_N$ is obtained. A slow reference closed loop model $\mathbb{M}_P$ is chosen, given by:

$$\begin{aligned} x_M(t+1) &= 0.99x_M(t) + 0.01g(t) \\ y_M(t) &= x_M(t) \end{aligned}$$

The PD inner-loop controller $\mathbb{K}_P$ is parameterized as:

$$u(t) = K_p e(t) + K_d \frac{e(t) - e(t-1)}{Ts}$$

Solving the optimization problem (1), the parameters K_p and K_d are calculated using the dataset D_N. The controller is placed in the inner loop within the hierarchical control scheme. Following VRFT synthesis, an outer MPC is designed using the formulation in (2), to provide a reference signal $g(t)$. The output $y(t)$ is constrained to lie between 0 rad and 4 rad, and the voltage input $u(t)$ between -3.5 V and 3.5 V. An MPC horizon of $N_P = 20$ timesteps is chosen, and weights are $Q_y = 1$ and $Q_\varepsilon = 1$. To improve prediction, a disturbance sensor discussed in Sec.IV is developed. First, the dataset $\hat{D}_N$ is built by performing closed loop experiments with input signal $\hat{v}(t)$ of standard deviation 10V. Then, a linear model for $\mathbb{D}$ parameterized by $n_{a_D} = 4$ and $n_{b_D} = 3$ is identified by solving (7). This disturbance sensor is split into two parts, $\mathbb{D}_D$ and $\mathbb{D}_S$, with inputs $v(t)$ and noise $w(t)$ respectively. Bounds on the noise $w(t)$ are calculated by solving the linear problems (**??**). The evolution of these bounds with increasing values of horizon N is plotted in Fig.6. The converged values w_{min} and

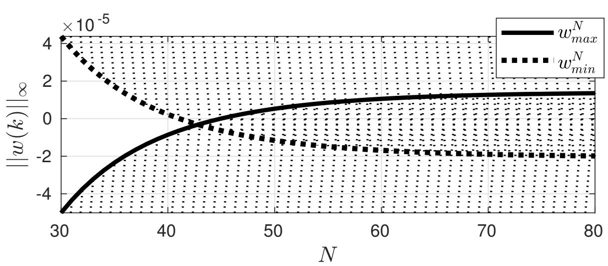

Fig. 6: Comparison of voltage input $u(t)$. Constraints are violated by the MPC controller.

w_{max} are passed into (4) and bounds on input $v(t)$ are calculated. A quadratic program (5) is constructed as the robust reference governor. The reference governor modifies the input $g(t)$ to a constraint respecting $v(t)$. Alternatively, one can also treat the set $\mathscr{D}_\infty$ as measurement noise, and use the bounds $d_{S,min}$ and d_{S_max} in the calculation of $\mathbb{O}_\infty$. Hence, simulations were also performed with output disturbance noise model. Performance of the control system for all these cases is plotted in Fig.7 and Fig.8.

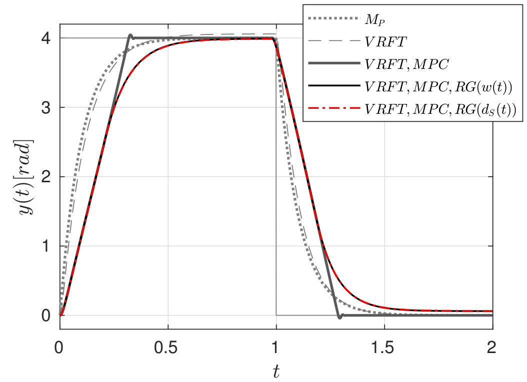

Fig. 7: The performance inner closed loop does not exactly match the reference model M_P. MPC improves the performance but results in small constraint violation. Constraint violation is robustly avoided by using a reference governor.

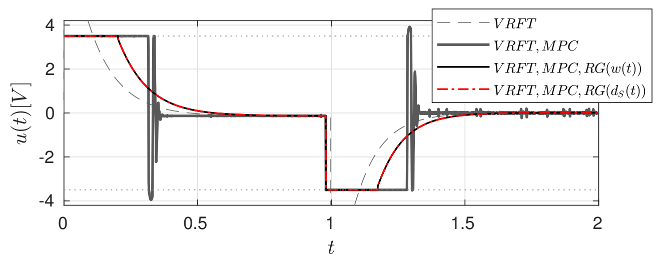

Fig. 8: Comparison of voltage input $u(t)$. Constraints are violated by the MPC controller. This is avoided by using a reference governor. The performance with the reference governer using either process noise model or measurement noise models is similar.

It can clearly be seen that with a slight loss of performance from MPC, the reference governor ensures constraint satisfaction. Since computational efficiency was not the focus of this work, the solving times are not noted here.

VI. Conclusion

This paper builds on the hierarchical data-driven control of constrained systems, by introducing robustness. This is done by adding a robust reference governor. The major contribution of this work is the formulation of a novel technique to compute process noise bounds from ARX identification. This formulation can prove to be useful, and is not limited to a control perspective. Possible extensions to the formulation are: a) Dealing with MIMO systems, b) Establishing the class of systems for which the formulation can be used, c) Efficient ways to deal with initial condition of the disturbance generating model $\mathbb{D}_S$. The performance of the control system in the case of using only measurement noise model instead of process noise $w(t)$ should be studied.

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