Robustly constrained data-driven control

Albert Author[1] and Bernard D. Researcher[2]

***Abstract*— This is a very skeletal overview.**

I. Problem Setup

Consider a *single-input single-output* system $\mathbb{G}_P$ generating an output signal $y(t) \in \mathbb{R}$ corresponding to the input signal $u(t) \in \mathbb{R}$ for the time $t \in \mathbb{Z}$. Consider the data $\mathbb{D}_N = \{u(t), y(t); t \in 1, ..., N\}$ obtained by exciting the system.
A feedback controller is designed to control the system, using the VRFT methodology. For this, a reference model $\mathbb{M}_P$ is selected. The VRFT methodology designs a feedback controller $\mathbb{K}_P$, with the goal of making the closed-loop system $\mathbb{K}_P$-$\mathbb{G}_P$ behave similar to the reference model $\mathbb{M}_P$. To this end, the VRFT methodology utilizes the dataset $\mathbb{D}_N$. The desired closed-loop behavior is described by the LTI state-space model $\mathbb{M}_P$

$$\begin{aligned} x_M(t+1) &= A_M x_M(t) + B_M g(t) \\ y_d(t) &= C_M x_M(t) \end{aligned}$$

The VRFT methodology utilized to design the feedback controller $\mathbb{K}_P$ is now explained:

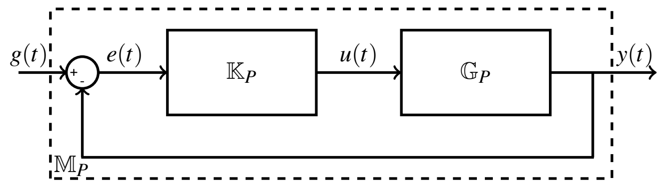
1) A virtual reference input $g(t)$ is calculated by setting $y_d(t) = y(t)$ obtained from the dataset $\mathbb{D}_N$, by the inverting the model $\mathbb{M}_P$. Let this mapping be defined by $g(t) = \mathbb{M}_P^{\dagger} y(t)$.
2) A feedback controller $\mathbb{K}_P$ described by $A_K(q^{-1})u(t) = B_K(q^{-1})(g(t) - y(t)$ is chosen, where

$$\begin{aligned} A_K(q^{-1}) &= 1 + \sum_{i=1}^{n_{a_K}} a_i^K q^{-i} \\ B_K(q^{-1}) &= \sum_{i=1}^{n_{b_K}} b_i^K q^{-i} \end{aligned}$$

   The parameters of the controller a_i^K and b_i^K are calculated by the VRFT methodology, such that the closed loop performance of $\mathbb{K}_P$-$\mathbb{G}_P$ matches open loop performance of the reference model $\mathbb{M}_P$.
3) This is done by solving the convex optimization problem

$$\min_{a_i^K, b_i^K} \quad \frac{1}{N} \sum_{t=1}^{N} \left| A_K(q^{-1})u(t) - B_K(q^{-1})(\mathbb{M}_P^{\dagger} y(t) - y(t)) \right|^2$$

   which minimizes the deviation between the control input calculated by the controller and $u(t)$ that is used to excite the system and obtain $y(t)$.
4) The synthesized controller $\mathbb{K}_P$ is placed before the plant, and the loop is closed. A reference step input is given to evaluate the controller performance.



The performance of this setup is improved by providing the reference signal $g(t)$ using an MPC controller in an outer loop. The objective of the MPC controller is to make the output signal $y(t)$ track the reference $r(t)$, while satisfying constraints on the plant output $y(t)$ and input $u(t)$. This is achieved by considering the closed-loop plant model $\mathbb{M}_P$ for the state propagation equation, and an augmented model with $[y(t), u(t)]$ as the output equation. This augmented state-space model is called $\mathbb{M}'_P$.

$$\begin{aligned} \zeta(t+1) &= A_\zeta \zeta(t) + B_\zeta g(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\zeta \zeta(t) + \begin{bmatrix} 0 \\ D_\zeta \end{bmatrix} g(t) \end{aligned}$$

The optimization problem solved by the MPC at each time step t for a horizon of N_P timesteps is shown below. The lower and upper bounds on $y(t)$ and $u(t)$ are $[y_{min}, y_{max}]$ and $[u_{min}, u_{max}]$ respectively.

$$\begin{aligned} \min_{\{g(t+k)\}_{k=1}^{N_P}} \quad & Q_y \sum_{k=1}^{N_P} (y(t+k|t) - r(t+k))^2 + Q_\varepsilon \varepsilon^2 \\ \text{subject to} \quad & \zeta(t+k+1) = A_\zeta \zeta(t+k) + B_\zeta g(t+k) \\ & \begin{bmatrix} y(t+k) \\ u(t+k) \end{bmatrix} = C_\zeta \zeta(t+k) + \begin{bmatrix} 0 \\ D_\zeta \end{bmatrix} g(t+k) \\ & y_{min} - V_y \varepsilon \leq y(t+k) \leq y_{max} + V_y \varepsilon \\ & u_{min} - V_u \varepsilon \leq u(t+k) \leq u_{max} + V_u \varepsilon \\ & \zeta(t|t) = \zeta(t) \end{aligned}$$

The quantities V_y and V_u in the MPC formulation are used to avoid infeasibility of the optimization problem over successive iterations, since the reference model $\mathbb{M}'_P$ might not accurately capture the dynamics of the unknown plant $\mathbb{G}_P$. This implies that constraint satisfaction is not guaranteed by the proposed formulation.

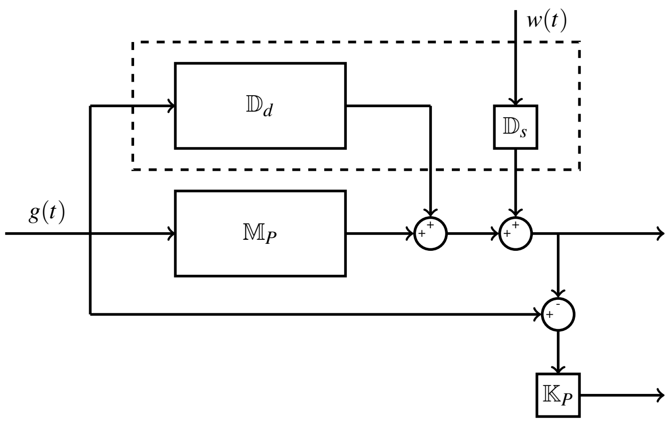
*This work was not supported by any organization

[1]Albert Author is with Faculty of Electrical Engineering, Mathematics and Computer Science, University of Twente, 7500 AE Enschede, The Netherlands `albert.author@papercept.net`

[2]Bernard D. Researcheris with the Department of Electrical Engineering, Wright State University, Dayton, OH 45435, USA `b.d.researcher@ieee.org`

II. Disturbance sensor

In order to improve prediction of closed loop performance, a disturbance sensor is designed. The disturbance sensor is a dynamical system whose output is the discrepancy between output of the actual closed loop system $\mathbb{K}_P$-$\mathbb{G}_P$ and reference closed loop system $\mathbb{M}_P$. The system consists of a deterministic part and a stochastic part, as seen in.



III. Contribution

References

[1] G. O. Young, Synthetic structure of industrial plastics (Book style with paper title and editor), in Plastics, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 1564.

[2] W.-K. Chen, Linear Networks and Systems (Book style). Belmont, CA: Wadsworth, 1993, pp. 123135.

[3] H. Poor, An Introduction to Signal Detection and Estimation. New York: Springer-Verlag, 1985, ch. 4.

[4] B. Smith, An approach to graphs of linear forms (Unpublished work style), unpublished.

[5] E. H. Miller, A note on reflector arrays (Periodical styleAccepted for publication), IEEE Trans. Antennas Propagat., to be publised.

[6] J. Wang, Fundamentals of erbium-doped fiber amplifiers arrays (Periodical styleSubmitted for publication), IEEE J. Quantum Electron., submitted for publication.

[7] C. J. Kaufman, Rocky Mountain Research Lab., Boulder, CO, private communication, May 1995.

[8] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, Electron spectroscopy studies on magneto-optical media and plastic substrate interfaces(Translation Journals style), IEEE Transl. J. Magn.Jpn., vol. 2, Aug. 1987, pp. 740741 [Dig. 9th Annu. Conf. Magnetics Japan, 1982, p. 301].

[9] M. Young, The Techincal Writers Handbook. Mill Valley, CA: University Science, 1989.

[10] J. U. Duncombe, Infrared navigationPart I: An assessment of feasibility (Periodical style), IEEE Trans. Electron Devices, vol. ED-11, pp. 3439, Jan. 1959.

[11] S. Chen, B. Mulgrew, and P. M. Grant, A clustering technique for digital communications channel equalization using radial basis function networks, IEEE Trans. Neural Networks, vol. 4, pp. 570578, July 1993.

[12] R. W. Lucky, Automatic equalization for digital communication, Bell Syst. Tech. J., vol. 44, no. 4, pp. 547588, Apr. 1965.

[13] S. P. Bingulac, On the compatibility of adaptive controllers (Published Conference Proceedings style), in Proc. 4th Annu. Allerton Conf. Circuits and Systems Theory, New York, 1994, pp. 816.

[14] G. R. Faulhaber, Design of service systems with priority reservation, in Conf. Rec. 1995 IEEE Int. Conf. Communications, pp. 38.

[15] W. D. Doyle, Magnetization reversal in films with biaxial anisotropy, in 1987 Proc. INTERMAG Conf., pp. 2.2-12.2-6.

[16] G. W. Juette and L. E. Zeffanella, Radio noise currents n short sections on bundle conductors (Presented Conference Paper style), presented at the IEEE Summer power Meeting, Dallas, TX, June 2227, 1990, Paper 90 SM 690-0 PWRS.

[17] J. G. Kreifeldt, An analysis of surface-detected EMG as an amplitude-modulated noise, presented at the 1989 Int. Conf. Medicine and Biological Engineering, Chicago, IL.

[18] J. Williams, Narrow-band analyzer (Thesis or Dissertation style), Ph.D. dissertation, Dept. Elect. Eng., Harvard Univ., Cambridge, MA, 1993.

[19] N. Kawasaki, Parametric study of thermal and chemical nonequilibrium nozzle flow, M.S. thesis, Dept. Electron. Eng., Osaka Univ., Osaka, Japan, 1993.

[20] J. P. Wilkinson, Nonlinear resonant circuit devices (Patent style), U.S. Patent 3 624 12, July 16, 1990.