# Robustified data-driven model predictive control

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***Abstract*— This paper presents a robust data-driven control approach for linear systems subject to bound constraints on inputs and outputs. The approach builds on a previously presented hierarchical direct data-driven control architecture for constrained systems. Improved robustness is achieved by using a robust model predictive controller (RMPC) instead of a deterministic model predictive controller (MPC) in the outer loop of the architecture. The RMPC controller generates a reference signal for the inner loop under robust constraints based on a model of the inner loop and a bounded disturbance set. A major contribution of this work is the development of a method to calculate the disturbance set from an identified AutoRegressive model with eXogenous inputs (ARX). We validate numerically the robustified control architecture, showing significant improvements with respect to its deterministic counterpart.**

## I. INTRODUCTION

Design of control systems can be broadly classified into two categories: *model-based* and *model-free*. Model-based control design techniques use an explicit model of the plant being controlled, derived from physical principles or identified from data. This involves selecting a model that trades-off between complexity and accuracy. Model selection is followed by experiments to identify the parameters. These steps introduce several challenges, that one can avoid by resorting to a "direct" data-driven controller design methodology.

Direct data-driven controller design methods avoid explicitly identifying the plant model, so that we can label them as "model free". They synthesize a controller directly from I/O data obtained from the plant. A review of several such methods can be found in [1]. One such method, virtual reference feedback tuning (VRFT) introduced in [2], has been used to design a stabilizing feedback controller within a hierarchical control architecture in [3] for LTI/LPV systems. The architecture employs an outer MPC controller, which utilizes the reference model selected for VRFT to generate a tracking signal. Performance bounds on the plant are translated into constraints on the optimization problem solved by the MPC controller. Since the reference model might not reasonably reflect the performance of the closed-loop plant, there is the possibility of violating the constraints imposed on input/output variables.

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It is therefore desirable to use a robust MPC controller in the outer loop, which generates a tracking signal that robustly satisfies the system constraints. A review of the concepts related to robust MPC controllers can be found in [4]. An alternative is to use a robust reference governor, which modifies the reference signal supplied to the plant in a way that constraints are satisfied [5]. Both techniques not only need a model of the *closed-loop* system, but also a model of the associated uncertainties.

In this work, we propose a data-driven control approach with improved robustness properties. We build on the hierarchical control architecture presented in [3]. The robust controller uses a model with uncertainty of the inner closed-loop, which consists of the plant and the VRFT-based feedback controller. This model consists of the reference model used for VRFT appended with an ARX model describing the uncertainty. The latter is identified from closed-loop experiments.

The model incorporates uncertainties as exogenous disturbance signals, assumed to lie within an unknown bounded polyhedral set. Since robust control methods explicitly incorporate uncertainty information in their formulation, identification of the polyhedral uncertainty sets is necessary.

We propose a novel method to calculate the polyhedral sets of the exogenous disturbance signals. Such a method falls under the umbrella of identification for robust control. This field includes a large volume of literature on set-membership techniques for parameter estimation. A review of these can be found in [6]. A method to obtain ellipsoidal parameterizations of these sets during ARX estimation is discussed in [7]. To the best of the authors' knowledge, no similar work has been done to calculate polyhedral sets of input exogenous disturbance signals from output data.

Following the identification of a model and corresponding disturbance sets, an RMPC controller is then designed, which replaces the deterministic MPC controller in the outer loop of the hierarchical architecture presented in [3].

The paper is organized as follows. In Sect. II, the problem is formally stated. Background regarding the hierarchical data-driven control architecture and proposed robust model predictive controller is presented in Sect. III. Sect. IV presents the techniques used to identify the model of the closed-loop system with disturbance bounds, which is the main contribution of this paper. Finally, Sect. V presents a numerical example implementing the robust hierarchical control design approach.

## II. Problem statement

We consider, for simplicity, a *single-input single-output* (SISO) system $\mathbb{G}_P$ (unknown and not recovered from data) generating an output signal $y(t) \in \mathbb{R}$ corresponding to the input signal $u(t) \in \mathbb{R}$, $t \in \mathbb{Z}^+$. We aim at synthesizing a controller that can make $y(t)$ accurately track any user-defined reference signal, while satisfying the constraints

$$\begin{aligned} &y_{min} \leq y(t) \leq y_{max} \\ &u_{min} \leq u(t) \leq u_{max}, \quad t = 0, 1, \ldots \end{aligned} \tag{1}$$

Following the direct data-driven controller synthesis methodology, we use the dataset $D_N = \{u(t), y(t); t \in 1, \ldots, N\}$ obtained by exciting the system to design the controller.

## III. Background

### A. Hierarchical approach

A feedback controller is designed to control the system, using the VRFT methodology. For this, a reference model $\mathbb{M}_P$ is selected, given by:

$$\begin{aligned} x_M(t+1) &= A_M x_M(t) + B_M g(t) \\ y_M(t) &= C_M x_M(t) \end{aligned}$$

For an approach to how to choose $\mathbb{M}_P$ optimally the reader is referred to the recent work [8]. Using the VRFT methodology, we design a linear feedback controller $\mathbb{K}_P$, with the goal of making the closed-loop system $\mathbb{K}_P$-$\mathbb{G}_P$ behave similar to the reference model $\mathbb{M}_P$. Note that the model $\mathbb{G}_P$ is unknown and not needed by the procedure, which is therefore a model-free one. The steps of the VRFT approach for synthesizing $\mathbb{K}_P$ are summarized below:

1) Use the dataset $D_N$, set $y_M(t) = y(t)$, calculate the virtual reference input $g(t)$ by by inverting the model $\mathbb{M}_P$, i.e, $g(t) = \mathbb{M}_P^{\dagger} y(t)$;
2) Parameterize the feedback controller $\mathbb{K}_P$ as $A_K(q^{-1})u(t) = B_K(q^{-1})(g(t) - y(t))$, where

$$\begin{aligned} A_K(q^{-1}) &= 1 + \sum_{i=1}^{n_{a_K}} a_i^K q^{-i} \\ B_K(q^{-1}) &= \sum_{i=1}^{n_{b_K}} b_i^K q^{-i}; \end{aligned}$$

3) Use system identification to estimate the coefficients $a_i^K$ and $b_i^K$, e.g., via least-squares:

$$\min_{a_i^K, b_i^K} \frac{1}{N} \sum_{t=1}^{N} \left( A_K(q^{-1})u(t) - B_K(q^{-1})(\mathbb{M}_P^{\dagger} y(t) - y(t)) \right)^2. \tag{2}$$

Problem (2) minimizes the deviation between the control input calculated by the controller and the signal $u(t)$ that was used to excite the system and obtain $y(t)$. To satisfy plant constraints, the reference signal $g(t)$ is generated using an MPC controller in an outer loop. The objective of the MPC controller is to make the output signal $y(t)$ track an arbitrary reference $r(t)$ while satisfying constraints on $y(t)$ and $u(t)$.

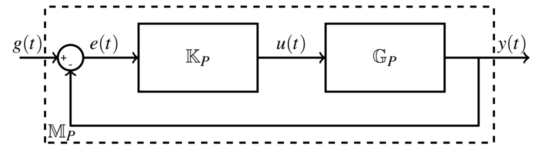


Fig. 1: Feedback controller designed using VRFT

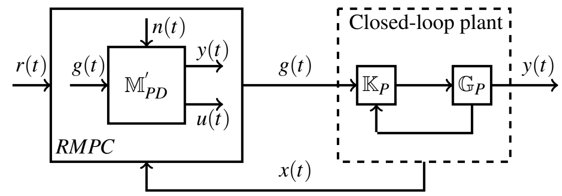


Fig. 2: Schematic of the robust control system

This is achieved by taking $\mathbb{M}_P$ and $\mathbb{K}_P$ as prediction model with output $[y(t), u(t)]$, that we denote as $\mathbb{M}'_P$:

$$\begin{aligned} \zeta(t+1) &= A_\zeta \zeta(t) + B_\zeta g(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\zeta \zeta(t) + D_\zeta g(t) \end{aligned}$$

At each time step $t$ MPC solves the following optimal control problem over a horizon of $N_P$ steps

$$\begin{aligned} \min_{\{g(t+k)\}_{k=1}^{N_P}} \quad & Q_y \sum_{k=1}^{N_P} (y(t+k|t) - r(t+k))^2 + Q_\varepsilon \varepsilon^2 \\ \text{subject to} \quad & \zeta(t+k+1) = A_\zeta \zeta(t+k) + B_\zeta g(t+k) \\ & \begin{bmatrix} y(t+k) \\ u(t+k) \end{bmatrix} = C_\zeta \zeta(t+k) + \begin{bmatrix} 0 \\ D_\zeta \end{bmatrix} g(t+k) \\ & y_{min} - V_y \varepsilon \leq y(t+k) \leq y_{max} + V_y \varepsilon \\ & u_{min} - V_u \varepsilon \leq u(t+k) \leq u_{max} + V_u \varepsilon \\ & \zeta(t|t) = \zeta(t) \end{aligned} \tag{3}$$

The quantities $V_y$ and $V_u$ in the MPC formulation are used as soft constraints to avoid infeasibility of the optimization problem over successive iterations, since the reference model $\mathbb{M}'_P$ might not accurately capture the dynamics of the closed loop $\mathbb{K}_P - \mathbb{G}_P$. This implies that constraint satisfaction is not guaranteed by the data-driven approach [3] recalled above.

### B. Robustified model predictive controller

**[AB: I changed the formulation completely, I suspect the previous one was not correct]**.

To improve constraint satisfaction, the deterministic MPC controller in the outer loop of the hierarchical control architecture is replaced by a robust MPC (RMPC) controller. A schematic of the control loop is shown in Figure 2.

In order to achieve robustness, the RMPC controller uses a different model $\mathbb{M}'_{PD}$ of the closed-loop system $\mathbb{K}_P$-$\mathbb{G}_P$

$$\begin{aligned} \gamma(t+1) &= A_\gamma \gamma(t) + B_\gamma^g g(t) + B_\gamma^n n(t) \\ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} &= C_\gamma \gamma(t) + D_\gamma^g g(t) + D_\gamma^n n(t) \end{aligned} \tag{4}$$

Model (4) has a deterministic input $g(t)$ and a disturbance input $n(t)$, which captures the effect of model uncertainties on system output. It is assumed that the disturbance input

$n(t)$ lies in an unknown bounded set $\mathcal{N}_\infty$. We will discuss in Sect. IV how to identify $\mathbb{M}'_{PD}$ and estimate $\mathcal{N}_\infty$.

By denoting as $y_\gamma(t) = [y(t)\; u(t)]^T$ the output of (4), the constraints in (1) can be rewritten as

$$Hy_\gamma(t) \leq h$$

Let $N_P$ be the prediction horizon of the RMPC controller. For simplicity we consider a control horizon of one step, so that the RMPC controller becomes a simpler robust reference governor (RRG). At each time instant $t$, given the current state $\hat{\gamma}(t) = \gamma(t)$, the RMPC controller/RRG solves the optimization problem

$$\begin{aligned} \min_{g(t)\in\mathbb{G}_\infty(\gamma(t))} \quad & \sum_{k=1}^{N_P} ([1\;0]\hat{y}_\gamma(t+k) - r(t+k))^2 \\ \text{subject to} \quad & \hat{\gamma}(t+k+1) = A_\gamma\hat{\gamma}(t+k) + B_\gamma^g g(t) \\ & \hat{y}_\gamma(t+k) = C_\gamma\hat{\gamma}(t+k) + D_\gamma^g g(t) \end{aligned} \tag{5}$$

where

$$\mathbb{G}_\infty(\gamma(t)) = \{g :\; Hy_\gamma(t+k) \leq h,\; \forall n(k) \in \mathcal{N}_\infty,\; \forall k \geq 0\} \tag{6}$$

is the set of feasible values of $g$ such that, if a constant input signal $g(t+k) = g$ is applied to model (4) from the current observed state $\gamma(t)$, the output constraints $Hy_\gamma(t+k) \leq h$ are satisfied for all $k \geq 0$, for all possible disturbance realizations $n(t+k) \in \mathcal{N}_\infty$.

Since

$$\begin{aligned} y_\gamma(t+k) = C_\gamma A_\gamma^k \gamma(t) + \left(C_\gamma \sum_{j=0}^{k-1} A_\gamma^j B_\gamma^g + D_\gamma^g\right) g + \\ C_\gamma \sum_{j=0}^{k-1} A_\gamma^j B_\gamma^n n(t+k-1-j) + D_\gamma^n n(t+k) \end{aligned}$$

the constraint $Hy_\gamma(t+k) \leq h$, $\forall n(t), \ldots, n(t+k) \in \mathcal{N}_\infty$, can be therefore rewritten as

$$\tilde{H}(k) g \leq h - HC_\gamma A_\gamma^k \gamma(t) - f^n(k) \tag{7}$$

where

$$\tilde{H}(k) = H\left(C_\gamma \sum_{j=0}^{k} A_\gamma^j B_\gamma^g + D_\gamma^g\right)$$

Each element $f_i^n(k)$ of the column vector $f^n(k)$ is calculated by solving the linear program

$$f_i^n(k) = \max_{\{n(t+j)\}_{j=0}^k \in \mathcal{N}_\infty} H_i\left(C_\gamma \sum_{j=0}^{k} A_\gamma^j B_\gamma^n n(k-1-j) + D_\gamma^n n(k)\right) \tag{8}$$

where the subscript $i$ in (8) indicates $i$th row.

Since the linear program (8) does not depend on $\gamma(t)$, it can be solved for increasing values of $k$ offline. According to the theory maximal output admissible sets for linear systems [9] and assuming $A_\gamma$ is strictly Schur, constraint (7) becomes redundant after a finite time $k_c$. Hence, we get

$$\mathbb{G}_\infty(\gamma(t)) = \left\{ g :\; \begin{bmatrix} \tilde{H}(0) \\ \vdots \\ \tilde{H}(k_c) \end{bmatrix} g \leq \begin{bmatrix} h \\ \vdots \\ h \end{bmatrix} - \begin{bmatrix} HC_\gamma \\ \vdots \\ HC_\gamma A_\gamma^{k_c} \end{bmatrix} - \begin{bmatrix} f^n(0) \\ \vdots \\ f^n(k_c) \end{bmatrix} \right\} \tag{9}$$

Note that our framework can easily be extended to multivariable systems and to RMPC controllers with control horizon larger than one.

## IV. Estimation of uncertainty model

We discuss now how to get model $\mathbb{M}'_{PD}$ from data, which is used by the RMPC controller to generate the signal $g(t)$, as well as a novel technique to quantify the uncertainty set $\mathcal{N}_\infty$.

### A. Structure of uncertainty model

As depicted in Figure 3, we introduce an output disturbance model $\mathbb{D}$ modeling the discrepancy between the actual output of the system in closed-loop with $\mathbb{K}_P$ and the output of the reference closed-loop model $\mathbb{M}_P$.

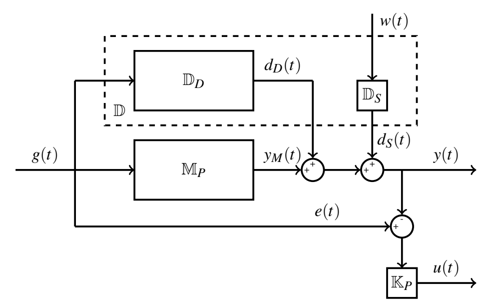


Fig. 3: Overall uncertain closed-loop model $M'_{PD}$.

Since this model represents the closed-loop behavior of the system, new closed-loop measurements are required to estimate the parameters of the disturbance model $\mathbb{D}$. These are obtained by performing experiments with excitation signals $\hat{g}(t)$ on the reference closed-loop model $\mathbb{M}_P$ and the actual closed-loop plant under the control law $\mathbb{K}_P$, measuring the output $\hat{y}(t)$. We also compute the nominal outputs $\hat{y}_M(t)$ for the same excitation. The signals are collected in the dataset $\hat{D}_N = \{\hat{g}(t), \hat{y}_M(t), \hat{y}(t); t \in 1, ..., N\}$.

By letting $y_D(t) = y(t) - y_M(t)$, the disturbance model $\mathbb{D}$ is parameterized as the ARX model $A_D(q^{-1})y_D(t) = B_D(q^{-1})g(t) + w(t)$, where

$$\begin{aligned} A_D(q^{-1}) &= 1 + \sum_{i=1}^{n_{a_D}} a_i^D q^{-i} \\ B_D(q^{-1}) &= \sum_{i=1}^{n_{b_D}} b_i^D q^{-i} \end{aligned}$$

Using standard ARX identification, the coefficients $a_i^D$ and $b_i^D$ are estimated by solving the standard linear regression problem

$$\min_{a_i^D, b_i^D} \; \frac{1}{N}\sum_{t=1}^{N} \left(A_D(q^{-1})(\hat{y}(t) - \hat{y}_M(t)) - B_K(q^{-1})(\hat{g}(t))\right)^2 \tag{10}$$

The disturbance model is then split into two parts. The deterministic part $\mathbb{D}_D$ takes in the reference signal $g(t)$ as an input, the uncertain part $\mathbb{D}_S$ is excited by the exogenous disturbance $w(t)$, which results in

$$d_D(t) = \frac{B_D(q^{-1})}{A_D(q^{-1})}v(t) = \mathbb{D}_D g(t)$$

$$d_S(t) = \frac{1}{A_D(q^{-1})}w(t) = \mathbb{D}_S w(t)$$

The overall model shown is Figure 3 can be seen as the 2-input 2-output system $\mathbb{M}'_{PD}$.

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_P + \mathbb{D}_D & \mathbb{D}_S \\ \mathbb{K}_P(I - (\mathbb{M}_P + \mathbb{D}_D)) & -\mathbb{K}_P\mathbb{D}_S \end{bmatrix} \begin{bmatrix} g(t) \\ w(t) \end{bmatrix} \tag{11}$$

Alternatively, we can treat $d_S(t)$ as measurement noise on the output $y(t)$, leading to the following model

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_P + \mathbb{D}_D & I \\ \mathbb{K}_P(I - (\mathbb{M}_P + \mathbb{D}_D)) & -\mathbb{K}_P \end{bmatrix} \begin{bmatrix} g(t) \\ d_S(t) \end{bmatrix} \tag{12}$$

(the dependence of the model on time shift operator $q^{-1}$ is not shown for ease of notation). Both $w(t)$ and $d_S(t)$ can be seen as exogenous disturbance signals causing model uncertainty. Hence, the state-space equivalents of either of these models can be used in (5), by replacing $n(t)$ in (4) with $w(t)$ if model (11) is used, and with $d_S(t)$ if (12) is used instead.

If the model described in (12) is considered for RMPC design, the bounds on $d_S(t)$ can be estimated directly from the differences $\hat{y}(t) - \hat{y}_M(t) - d_D(t)$ after the ARX identification step performed in (10). These bounds are used to build the uncertainty set $\mathscr{D}_\infty$, whose definition will be formalized in the next section. Then, the set $\mathscr{D}_\infty$ replaces $\mathscr{N}_\infty$ while solving the linear programs (8).

In case of the model described in (11), the calculation of bounds on $w(t)$ is not as straightforward. The following section discusses a technique to calculate upper and lower bounds $w_{\min}$ and $w_{\max}$ on the disturbance signal $w(t)$, which define the set $\mathscr{W}_\infty$. Within the RMPC controller, $\mathscr{N}_\infty$ is then set equal to $\mathscr{W}_\infty$

### B. Calculation of exogenous disturbance set

Samples $\hat{d}_S(t)$ of the discrepancy $d_S(t)$ between $\hat{y}(t)$ and the deterministic output $\hat{y}_M(t) + \hat{d}_D(t)$ can be simply calculated from the data set $\hat{D}_N$ as

$$\hat{d}_S(t) = \hat{y}(t) - \hat{y}_M(t) - \frac{B_D(q^{-1})}{A_D(q^{-1})}\hat{g}(t)$$

The minimum and maximum values of $\hat{d}_S(t)$ recorded on the dataset are labeled $\hat{d}_{S,\min}$ and $\hat{d}_{S,\max}$, respectively. If the ARX identification results in a stable deterministic disturbance model $\mathbb{D}_D$, the values $\hat{d}_{S,\min}$ and $\hat{d}_{S,\max}$ are finite. Further, if infinite closed-loop data $\hat{D}_\infty$ were collected for ARX identification, the bounds on discrepancy would be equal to the true bounds $d_{S,\max}$ and $d_{S,\min}$ on $d_S(t)$. The set of sequences $\{d_S(t)\}$ satisfying these bounds are indicated as lying in a set $\mathscr{D}_\infty$, defined as

$$\mathscr{D}_\infty = \{\{d_S(t)\} : d_{S,\min} \le d_S(t) \le d_{S,\max}, \ \ \forall t \in (-\infty, \infty)\}$$

Define now the set

$$\mathscr{W}_\infty(w^*_{\min}, w^*_{\max}) = \left\{ \{w(t)\} : \begin{array}{c} w^*_{\min} \le w(t) \le w^*_{\max} \\ \mathbb{D}_S w(t) \in \mathscr{D}_\infty \\ \forall t \in (-\infty,\infty) \end{array} \right\} \tag{13}$$

of sequences $\{w(t)\}$ bounded by $w^*_{\min}$ and $w^*_{\max}$ such that the corresponding steady-state output signal of $\mathbb{D}_S$ lies within $\mathscr{D}_\infty$. Let $\mathscr{W}_\infty$ be the largest of the sets in (13), corresponding to the (unknown) bounds $w_{\min}$ and $w_{\max}$. The set $\mathscr{W}_\infty$ is the complete exogenous disturbance set of $w(t)$, which we want to find.
Let another set $\mathscr{W}^{**}_N$ be defined as

$$\mathscr{W}^{**}_N = \left\{ \{w(t)\} : w^N_{\min} \le w(t) \le w^N_{max} \ \ \forall t \in (-\infty, \infty) \right\} \tag{14}$$

where the bounds $w^N_{\min} = \bar{w}^*_1(N)$ and $w^N_{\max} = \bar{w}^*_2(N)$ on $\{w(t)\}$ are obtained by solving the optimization problems

$$\begin{aligned} \bar{w}^*_i(N) \quad = \quad & \min_{X_j} \quad \bar{w}_j \\ & \text{subject to} \quad d_S(k) = -\sum_{i=1}^{n_{a_D}} a^D_i d_S(k-i) + w(k), k = 1:N \\ & \qquad d_{S,\min} \le d_S(k) \le d_{S,\max}, \ \ k = 1:N-1 \\ & \qquad w(k) \le \bar{w}_j, \ \ k = 1:N \\ & \qquad d_S(N) \in D_j \end{aligned} \tag{15}$$

where $D_1 = \{d : d \le d_{S,\min}\}$, $D_2 = \{d : d \ge d_{S,\max}\}$, and

$$X_j = \left\{ \begin{array}{c} \{d_S(-n_{a_D}+1), \ldots, d_S(N)\}, \\ \{w(1), \ldots, w(N)\}, \\ \bar{w}_j \end{array} \right\}, j = \{1, 2\},$$

for increasing lengths of the time horizon $N$.

Problem (15) looks for a sequence of inputs $\{w(k), k = 1:N\}$ such that the sequence of corresponding outputs lie within $\mathscr{D}_\infty$ up to time $N-1$, while $d_S(N)$ gets out of $\mathscr{D}_\infty$ into $D_j$.

*Lemma 1:* Let $\bar{w}^*_i(N)$ be defined as in (15). Then the sequences $\{\bar{w}^*_1(N)\}^\infty_{N=1}$ and $\{\bar{w}^*_2(N)\}^\infty_{N=1}$ are, respectively, monotically decreasing and increasing and the following limits

$$\lim_{N \to +\infty} \bar{w}^*_1(N) = w^{**}_{\min}, \quad \lim_{N \to +\infty} \bar{w}^*_2(N) = w^{**}_{\max}$$

exist and are finite.
*Proof.* **[AB: TBC]** ■

Note that with low values of $N$ there can exist an initial sequence $\{d_S(-n_{a_D}+1), .., d_S(1)\}$ that drives $d_S(N)$ into $D_j$ with very low input effort $w(k)$. However, for larger $N$ the effect of initial condition wears off, and a higher value of input effort $w(t)$ is required to violate $\mathscr{D}_\infty$.

*Theorem 1:* Let $\mathscr{W}^{**}(N)$ be defined as in (14). Then

$$\mathscr{W}_\infty = \lim_{N \to +\infty} \mathscr{W}^{**}(N)$$

*Proof:* **[AB: To be redone, according to new notation and Lemma 1]**

Let $\mathscr{W}^*_\infty \subset \mathscr{W}^{**}_\infty$. This implies that for any sequence $\{d_S(t)\}^{N-1}_{-\infty} \in \mathscr{D}_\infty$ and $\{w(t)\}^N_{-\infty} \in \mathscr{W}^*_\infty$, we will have $d_S(N) \in \mathscr{D}_\infty$. Hence, the set $\mathscr{W}^{**}_\infty$ is the upper bound on the sets $\mathscr{W}^*_\infty$, which is equal to complete exogenous disturbance set $\mathscr{W}_\infty$.
■

We remark that in practice the set $\mathscr{D}_\infty$ is constructed only based on a finite dataset $\hat{D}_N$ of $N$ samples. Consequently, the computation of the set $\mathscr{W}_\infty$ is also based on a finite number of samples. Therefore, full robust constraint satisfaction based on the sets $\mathscr{D}_\infty$ or $\mathscr{W}_\infty$ is only achieved asymptotically for $N=\infty$ samples that capture all the possible values $d_S(t)$ can take.

## V. Case study

Numerical simulations are performed on a servo motor control problem, implementing the complete control scheme presented in Figure 2. The sampling time is $T_s = 1ms$. RMPC controllers are implemented for closed-loop models corresponding to both (11) and (12). It is shown that using model (11) results is superior performance, and hence performing an additional step to calculate $w_{\min}$ and $w_{\max}$ is justified. Both the case studies are implemented using MATLAB R2017b, with optimization problems occasionally implemented using YALMIP [10].

### A. Robust data-driven MPC

The plant consisting of a servo positioning system is controlled using the scheme shown in Figure 2. The plant dynamics are modeled by the following nonlinear state space equations:

$$\begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \\ \dot{i}(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ \frac{-mgl}{J} sin\theta(t) - \frac{b}{J}\omega(t) + \frac{K_m}{J} i(t) \\ \frac{-K_m}{L}\omega(t) - \frac{R}{L} i(t) + \frac{1}{L} u(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \omega(t) \\ i(t) \end{bmatrix}$$

The states of the system $\theta(t)$, $\omega(t)$, and $i(t)$ are angle

| Symbol | Parameter | Value |
|---|---|---|
| $R$ | Motor resistance | $5\Omega$ |
| $L$ | Motor inductance | $5.10^{-3}$H |
| $K_m$ | Motor torque constant | 0.0847Nm/A |
| $J$ | Complete disk inertia | $5.10^{-5}$Nm$^2$ |
| $b$ | Friction coefficient | $3.10^{-3}$Nms/rad |
| $m$ | Additional mass | 3Kg |
| $l$ | Mass offset | 2m |

TABLE I: Physical parameters of servo motor system

$[rad]$ and rotational velocity $[rad/s]$ of the servo motor, and armature current $[A]$ respectively. The input $u(t)$ is the voltage $[V]$ applied across the motor, and the output $y(t)$ is the rotational angle. A VRFT methodology is used to design a stabilizing PD controller, which provides a voltage input $u(t)$ to make the rotational angle $y(t)$ track a reference signal $g(t)$. To this end, experiments are conducted with a low-pass filtered white noise signal $u(t)$ with a standard deviation of 10V. The output angle $y(t)$ is recorded and the dataset $\mathbb{D}_N$ is obtained. A slow reference closed loop model $\mathbb{M}_P$ is chosen, given by:

$$\begin{aligned} x_M(t+1) &= 0.99x_M(t) + 0.01g(t) \\ y_M(t) &= x_M(t) \end{aligned}$$

The PD inner-loop controller $\mathbb{K}_P$ is parameterized as:

$$u(t) = K_p e(t) + K_d \frac{e(t) - e(t-1)}{T_s}$$

Solving the optimization problem (2), the parameters $K_p$ and $K_d$ are calculated using the dataset $D_N$. The controller is placed in the inner loop within the hierarchical control architecture. After VRFT synthesis, an outer MPC is designed using the formulation in (3), to provide a reference signal $g(t)$. The output $y(t)$ is constrained to lie between $-1$ rad and 1 rad, and the voltage input $u(t)$ between $-3.5$ V and 3.5 V. An MPC horizon of $N_P = 10$ timesteps is chosen, and weights are $Q_y = 1$ and $Q_\varepsilon = 1$.

Towards formulating the RMPC controller instead of the deterministic MPC controller, the disturbance model discussed in Sect. IV is developed. First, the dataset $\hat{D}_N$ is built by performing closed loop experiments with input signal $\hat{g}(t)$ of standard deviation 10 rad. Then, a linear model for $\mathbb{D}$ parameterized by $n_{a_D} = 4$ and $n_{b_D} = 3$ is identified by solving (10). Two equivalent models of the appended system, corresponding to (11) and (12), are developed.

For the model corresponding to (11), the disturbance model is split into two parts, $\mathbb{D}_D$ and $\mathbb{D}_S$, with inputs $g(t)$ and $w(t)$ respectively. Bounds on the exogenous disturbance $w(t)$ for a horizon $N$ are calculated by solving the linear problems (15). The evolution of these bounds with increasing values of horizon $N$ is plotted in Figure 4. The values $w_{\min}$ and $w_{\max}$ are obtained at convergence of $\bar{w}_1(N)$ and $\bar{w}_2(N)$ respectively. For the model corresponding to (12), the

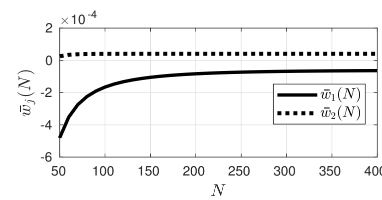


Fig. 4: Convergence of bounds on $w(t)$ acting as input to the disturbance model $D_S$.

disturbance model consists of $\mathbb{D}_D$ and a direct exogenous disturbance signal $d_S(t)$. The bounds $d_{S,\min}$ and $d_{S,\max}$ on $d_S(t)$ are set equal to the minimum and maximum values of prediction error obtained during identification of the ARX model for $\mathbb{D}$ respectively.

These models and related disturbance bounds are used in the formulation of corresponding RMPC controllers. To this end, constraints on the reference signal $g(t)$ are constructed, as shown in (9). Solving the linear programs (8) till convergence to calculate maximal output admissible set $\mathbb{G}_\infty(\gamma(t))$ can result in very conservative constraints on $g(t)$. This is avoided by solving them only for the duration of RMPC horizon by setting $\tau_c = N_P$. Note that the bounds used for the calculation

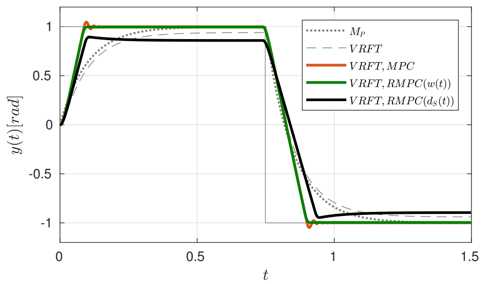


Fig. 5: The performance inner closed-loop does not exactly match the reference model $M_P$. MPC improves the performance but results in small constraint violation. Constraint violation is robustly avoided by using an RMPC controller instead of deterministic MPC. Using (12) results in a conservative performance compared to (11).

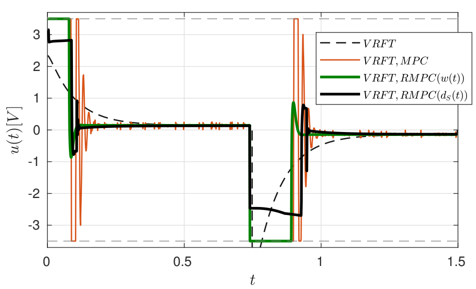


Fig. 6: Comparison of voltage input $u(t)$.

of constraint sets (9) are calculated with the assumption of infinite data $\hat{D}_\infty$, and hence actual robustness is achieved only asymptotically as $\lim_{N\to\infty} \hat{D}_N$.

The corresponding output admissible sets $\mathbb{O}_1(\gamma(t))$ till $\mathbb{O}_{N_P}(\gamma(t))$ are calculated at each time-instant $t$ by reading the state $\gamma(t)$, which is built with the assumption of no additional measurement noise. Following this, the optimization problem (5) is constructed and solved by the RMPC controller, generating a constraint satisfying reference signal $g(t)$. Performance of the control system for all these cases is plotted in Figure 5 and Figure 6. It can clearly be seen that utilizing an RMPC controller instead of a deterministic MPC controller in the outer loop results in robust constraint satisfaction. Also, using the closed-loop model corresponding to (12) as the plant model for RMPC controller design results in conservative performance when compared to using (11), for the same admissible set horizon $\tau_c = N_P$. Since the calculation of bounds $w_{\min}$ and $w_{\max}$ (which are required to use (11) for RMPC synthesis) is performed offline, conservativeness of the control scheme can be reduced without any additional online operation. Note that increasing admissible set horizon to convergence results in conservative performance from either of the models, with (11) still outperforming (12).

## VI. Conclusion

This paper builds on the hierarchical data-driven control of constrained systems, by introducing robustness with respect to constraint satisfaction. This is done by using a robust model predictive controller. Uncertainty on the model utilized by the RMPC controller is modeled as a disturbance input lying within an unknown bounded polyhedral set. In this work, a novel technique to compute this polyhedral set from ARX identification is presented. It is seen that utilizing the model and the disturbance set calculated using the presented technique results in robust constraint satisfaction, all without identifying an open-loop model of the non-linear plant. Future work deals with extending the formulation to general Box-Jenkins models. Possible extensions to the control scheme include modifications to incorporate LPV systems.

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