Distributed motion planning for multiple vehicles transporting a flexible payload*

Sampath Kumar Mulagaleti[1], Ruben Van Parys[2] and Goele Pipeleers[2]

Abstract— **This paper deals with the development of control strategies for cooperatively transporting a flexible payload with multiple vehicles. To this end, an algorithm is developed which generates optimal trajectories for the vehicles to follow. The trajectories are parameterized using splines, and the optimization algorithm used to solve for the spline coefficients is based on an Alternating Direction Method of Multipliers (ADMM) formulation. The algorithm instructs that the optimization problem be solved repeatedly in a receding horizon fashion, making it fit into a distributed model predictive control (DMPC) framework. One ADMM iteration is performed per DMPC iteration, reducing the inter-agent communication rate. Numerical and experimental validation of the developed control scheme is performed and the results are presented.**

I. INTRODUCTION

Payload transportation tasks are ubiquitous in industrial environments. These payloads can vary greatly in weights, necessitating multiple transportation solutions of varying payload capacities be used within the same logistics structure. An alternative to this would be to use multiple vehicles to cooperatively tow a payload, with the number of vehicles chosen as per the towing requirements. To enable such a multi-agent system to work autonomously and transport the payload to a desired location, cooperative control strategies are required. This work presents one such strategy from a motion-planning perspective. The algorithms developed within this strategy design optimal trajectories for the vehicles to follow to achieve the goal of payload transportation. These trajectories are fed to a low level controller for tracking. The generated trajectories respect various constraints presented by the dynamics of the system, individual vehicle kinematics, and avoid collisions between the vehicles. To account for dynamic disturbances, these trajectories are generated repeatedly with current initial conditions.

The trajectories result from solving an optimization problem. Since the payload transportation problem at hand is composed of multiple agents and there is no central agent with access to the states of all the transportation vehicles, the formulated optimization problem is solved in a distributed fashion. This means that each vehicle solves a local optimization problem to generate its own trajectory, and is aided by communication with other vehicles. The usual procedures for distributing an optimization problem over multiple agents are based on optimization decomposition techniques. One such technique, dual-decomposition, is used in [1] to generate trajectories for a flight formation problem in an offline manner. In order to make the motion planning-based control robust against plant-model mismatch, it is desirable to solve the motion planning problem repeatedly. This leads the problem into a distributed model predictive control (DMPC) framework.

The formation-control problem is solved in a DMPC framework in [2], in which the optimization problem is decomposed using the Alternating Direction Method of Multipliers (ADMM) algorithm. A major difference between formation control problems and the payload transportation problem is that in the latter, the agents are dynamically coupled. This means that a change in state of one of the agents results in a change in state of the others. These coupled dynamics act as constraints on the centralized optimization problem, and decomposing the centralized problem for DMPC purposes would require decomposing the coupling dynamics constraint. Decoupling of dynamics is done using primal decomposition in [3], in which, linear systems are considered, and an optimal consensus problem is solved. In optimal consensus problems, multiple optimization variables are driven to the same value over iterations. In other words, consensus is achieved between different states of the system. Another approach based on dual decomposition, is discussed in [4]. In this, the variable splitting method is employed [1], with the introduction of copies of states of neighboring vehicles which affect local states of a host vehicle. More robust methods based on the ADMM are formulated in [5], which consider coupled objective functions in addition to coupled nonlinear dynamics.

ADMM solves the optimization problem in a Gauss-Siedel fashion, with two Gauss-Siedel steps updating the primal variables and one updating the dual variables. Hence, each MPC iteration involves solving the three steps. The problem with using the techniques proposed in [5] is that the constraints are distributed over the first and second subproblems, leaving them without simple closed form solutions and hence making the algorithm slow. This makes practical implementation infeasible. This is overcome in [6], where the ADMM-consensus problem introduced in [7] is combined with the idea of introducing copies of neighboring states from [8]. The proposed formulation is the closest to what is used to solve the payload transport problem in this thesis, with a modification inspired by [2] to remove the

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[1]Albert Author is with Faculty of Electrical Engineering, Mathematics and Computer Science, University of Twente, 7500 AE Enschede, The Netherlands albert.author@papercept.net

[2]Bernard D. Researcheris with the Department of Electrical Engineering, Wright State University, Dayton, OH 45435, USA b.d.researcher@ieee.org

need to have a centralized controller to perform the 2nd ADMM step.

The rest of the paper is organized as follows:
Section II introduces the motion planning problem for cooperative payload transportation problem, and Section III describes how this problem can be solved in a online distributed manner. Results from simulations based on the proposed approach are presented in Section IV, and the results from an experimental validation in Section V.

II. PROBLEM FORMULATION

A. System model

The dynamic system under consideration is modeled as multiple holonomic vehicles attached through 1D spring-damper elements to a common payload. The payload and the holonomic vehicles are abstracted as point masses, and are constrained to move in a 2D plane. Dynamic spring-mass-damper models are used to describe the system in the optimization problem. Friction is ignored from the model. The free length of the flexible elements is assumed to be 0m, resulting in the vehicles only applying a pulling force on the payload. A schematic of this model for 3 holonomic vehicles is shown in 1.

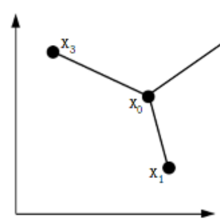

Fig. 1. Dynamic model decoupling

B. Optimal control problem

The payload position is represented by the vector $X_0(t)$ and each vehicle i's position by $X_i(t)$. The goal of the trajectory generation problem is to transport the payload to a location closest to the goal location respecting the various system constraints. The problem is solved for a time horizon of T s. The complete optimization problem required to generate the trajectories $X_0(t)$ and $X_i(t)$ is shown in (1).

$$\begin{aligned} \underset{\forall i: X_i(t), X_0(t)}{\text{minimize}} \quad & \int_0^T \big|\big|X_0(t) - X_d\big|\big|_1 dt \\ \text{subject to} \quad & f_i(X_0(t), X_i(t)) \leq 0 \\ & g(X_0(t), X_i(t), X_j(t)) = 0 \\ & \big\langle X_i(t) - X_0(t), X_j(t) - X_0(t)\big\rangle \leq 0 \\ & X_0(0) = x_0^i, X_i(0) = x_i^i \\ & \dot{X}_0(0) = u_0, \dot{X}_i(0) = u_i \\ & \forall t \in [0, T], \quad \forall i, j \in \{1, ., N\}, i \neq j \end{aligned} \tag{1}$$

The first constraint consists of individual vehicle i's dynamics, bounds on vehicle velocities and accelerations, and flexible element i's elongation limits. The second constraint represents the payload dynamics. These constraints are elaborated in (2).

$$f_i(X_0(t), X_i(t)) = \left| \begin{bmatrix} m_i & C_i & K_i \end{bmatrix} . \begin{bmatrix} \ddot{X}_i(t) \\ \dot{X}_i(t) - \dot{X}_0(t) \\ X_i(t) - X_0(t) \end{bmatrix} - \begin{bmatrix} \bar{F}_x \\ \bar{F}_y \end{bmatrix} \right|$$

$$\begin{gathered} g(X_0(t), X_i(t), X_j(t)) = m_0 \ddot{X}_0(t) + \\ \textstyle\sum_{i=0}^N (C_i(\dot{X}_i(t) - \dot{X}_0(t)) + K_i(X_i(t) - X_0(t))) \end{gathered} \tag{2}$$

Intra-vehicle collisions are avoided using the third constraint, which limits the angles between the vehicles to lie at greater than 90° when subtended at the payload. Initial conditions on the vehicle and payload position and velocity states are captured in the final two constraints.

III. SOLUTION STRATEGY

A. Spline parameterization

Problem (1) represents an infinite dimensional problem, since the optimization variables $X_0(t)$ and $X_i(t)$ are infinite dimensional, presenting an infinite number of constraints. In order to make the problem numerically tractable, a spline-based parameterization of the optimization variables is used. The continous optimization variables are hence represented as linear combinations of piecewise polynomial basis functions $b_k(t)$, as expressed in (3).

$$\begin{aligned} X_i(t) = \sum_{k=1}^{n} x_{i,k} b_k(t) \quad \forall i \in \{1, ., N\} \\ X_0(t) = \sum_{k=1}^{n} x_{0,k} b_k(t) \end{aligned} \tag{3}$$

The trajectories of each vehicle i are consequently represented by the coefficient set $\mathbf{X}_i = \{x_{i,k}\}_{k=1}^n$ and those of the payload by $\mathbf{X}_0 = \{x_{0,k}\}_{k=1}^n$. Substituting these parameterizations in (1) results in an optimization problem with finite number of variables. This is seen in (4).

$$\begin{aligned} \underset{\forall i: \mathbf{X}_i, \mathbf{X}_0}{\text{minimize}} \quad & J(\mathbf{X}_0) \\ \text{subject to} \quad & F_i(\mathbf{X}_0, \mathbf{X}_i) \leq 0 \\ & G(\mathbf{X}_0, \mathbf{X}_i, \mathbf{X}_j) = 0 \\ & H_{ij}(\mathbf{X}_0, \mathbf{X}_i, \mathbf{X}_j) \leq 0 \\ & \mathbf{X}_0(1) = x_0^i, \mathbf{X}_i(1) = x_i^i \\ & \textit{Make velocity things here??} \\ & \forall i, j \in \{1, ., N\}, i \neq j \end{aligned} \tag{4}$$

The objective in now a linear function in payload coefficients since the integration is performed over the basis functions, and the 1-norm results in an affine combination of the payload-spline coefficients. Substituting the parameterizations into the inequality constraints representing vehicle dynamics and intra-vehicle collision avoidance conditions results in functions which are in terms of the spline coefficients and time. These semi-infinite constraints (because of time dependence) can be converted to a finite number following the convex hull property [9] of splines. This property dictates that the constraints are conservatively satisfied if the inequality constraints are moved onto the coefficients of the combined function. These constraints are represented by F_i and H_{ij} respectively.

B. Distributed formulation

The objective of this section is to split the optimization problem (4) into N separate problems, one for each vehicle. This means that each problem should contain optimization variables local to the vehicle it is assigned. The first step towards the distributed formulation is the introduction of copies of the payload variables $\mathbf{X}_0$ on each vehicle. These variables are represented by $\mathbf{X}_{i0}$. The substitution of these variables results in N copies of the payload dynamics constraint G, each one being $G_i(\mathbf{X}_{i0}, \mathbf{X}_i, \mathbf{X}_j) = 0$.

The payload dynamics constraint is further decoupled by borrowing ideas from [8], which dictates that the states $\mathbf{X}_j$ of vehicle j which affect the dynamics of vehicle i can be modeled as exogenous inputs $\mathbf{X}_{ij}$ acting on vehicle i. The new variables $\mathbf{X}_{ij}$ are hence local to vehicle i. The constraint is thus modified as $G_i(\mathbf{X}_{i0}, \mathbf{X}_i, \mathbf{X}_{ij}) = 0$. This substitution is supported by consensus constraints between $\mathbf{X}_j$ and $\mathbf{X}_{ij}$. Following a modification of the consensus constraints presented in [2], consensus variables $\mathbf{Z}_{ij}$ are introduced which act as a balance between $\mathbf{X}_j$ and $\mathbf{X}_{ij}$. The corresponding consensus constraints are shown in (5).

$$\begin{array}{ll}
\underset{\forall i,j,i\neq j:\mathbf{X}_i,\mathbf{X}_{ij},\mathbf{Z}_{ij},\mathbf{X}_{i0}}{\text{minimize}} & \sum_{i=1}^{N} J_i(\mathbf{X}_{i0}) \\
\text{subject to} & F_i(\mathbf{X}_0, \mathbf{X}_i) \leq 0 \\
& G_i(\mathbf{X}_{i0}, \mathbf{X}_i, \mathbf{X}_{ij}) = 0 \\
& H_{ij}(\mathbf{X}_{i0}, \mathbf{X}_i, \mathbf{X}_{ij}) \leq 0 \\
& \mathbf{X}_j = \mathbf{Z}_{ij} \\
& \mathbf{X}_{ij} = \mathbf{Z}_{ij} \\
& \forall i, j \in \{1, ., N\}, i \neq j
\end{array} \tag{5}$$

The optimization problem (5) can now be decoupled using the Alternating Direction Method of Multipliers (ADMM), which is a popular technique used in DMPC formulations due to its superior convergence properties. According to this method, equality constraints are moved into the objective function forming a Lagrangian equivalent of the optimization problem, and the Lagrangian is augmented with a quadratic term to improve convergence properties. This appended objective function is seen in (6).

$$\begin{aligned}
\mathcal{L}=\sum_{i=1}^{N}\Big(J_i(\mathbf{X}_{i0})+\sum_{j\neq i}\big(\lambda_{iji}^T(\mathbf{X}_{ij}-\mathbf{Z}_{ij})+\frac{\rho}{2}||\mathbf{X}_{ij}-\mathbf{Z}_{ij}||_2^2+ \\
\lambda_{ijj}^T(\mathbf{X}_j-\mathbf{Z}_{ij})+\frac{\rho}{2}||\mathbf{X}_j-\mathbf{Z}_{ij}||_2^2\big)\Big)
\end{aligned} \tag{6}$$

The dual variables introduced in the Lagrangian corresponding to the equality constraints are λ_{iji} and λ_{ijj}. The dual problem of (5) is solved using dual gradient ascent. This method first solves for the primal variables by minimizing the Lagrangian (6) keeping the value of the dual variables fixed. At the primal variable solution, the gradient of the dual problem is calculated and the dual variables are updated by ascending in the gradient direction.

According to the ADMM algorithm, the Lagrangian (6) is minimized in two Gauss-Siedel steps. The first step involves keeping the dual variables λ_{iji} and λ_{ijj} and *other* set of primal variables $\mathbf{Z}_{ij}$ constant from the previous ADMM iteration, and solving for the $\mathbf{X}$ variables. This step is easily split amongst individual vehicles and is solved for in parallel on each of the vehicles. The splitting is possible because of the bi-directionality of interaction, meaning that the Lagrangian terms corresponding to the constraint $\mathbf{X}_i = \mathbf{Z}_{ji}$ can be moved onto agent i provided that the values of $\mathbf{Z}_{ji}$ and λ_{jii} have been communicated by vehicle j to vehicle i in the previous ADMM iteration. This results in vehicle i solving an optimization problem with variables only local to it. The resultant problem solved by vehicle i is shown in (7).

Step 1 : Primal X variables

$$\begin{array}{ll}
\underset{\mathbf{x}_i,\,\mathbf{x}_{ij},\,\mathbf{x}_{i0}}{\text{minimize}} & J_i(\mathbf{X}_{i0})+\sum_{j\neq i}\big((\lambda_{iji}^T)^k(\mathbf{X}_{ij}-\mathbf{Z}_{ij}^k)+\frac{\rho}{2}||\mathbf{X}_{ij}-\mathbf{Z}_{ij}^k||_2^2+ \\
& \qquad (\lambda_{jii}^T)^k(\mathbf{X}_i-\mathbf{Z}_{ji}^k)+\frac{\rho}{2}||\mathbf{X}_i-\mathbf{Z}_{ji}^k||_2^2\big) \\
\text{subject to} & F_i(\mathbf{X}_0, \mathbf{X}_i) \leq 0 \\
& G_i(\mathbf{X}_{i0}, \mathbf{X}_i, \mathbf{X}_{ij}) = 0 \\
& H_{ij}(\mathbf{X}_{i0}, \mathbf{X}_i, \mathbf{X}_{ij}) \leq 0
\end{array} \tag{7}$$

The superscript k indicates that these variables are from the previous iteration. *Step 1* is followed by the communication of coefficients $\mathbf{X}_j$ belonging to vehicle j to vehicle i. Vehicle i uses these variables to calculate the $\mathbf{Z}_{ij}$ variables, which is the second Gauss-Siedel step to solve for the primal variables. This resultant problem solved by vehicle i is shown in (8).

Step 2 : Primal Z variables

$$\begin{array}{ll}
\underset{\mathbf{z}_{ij}}{\text{minimize}} & \sum_{j\neq i}((\lambda_{iji}^T)^k(\mathbf{X}_{ij}^{k+1}-\mathbf{Z}_{ij})+\frac{\rho}{2}||\mathbf{X}_{ij}^{k+1}-\mathbf{Z}_{ij}||_2^2+ \\
& \qquad (\lambda_{ijj}^T)^k(\mathbf{X}_j^{k+1}-\mathbf{Z}_{ij})+\frac{\rho}{2}||\mathbf{X}_j^{k+1}-\mathbf{Z}_{ij}||_2^2)
\end{array} \tag{8}$$

The 2nd step is solved as a Least Squares problem, utilizing the closed form solution. This step can be seen as a balancing step, finding a $\mathbf{Z}_{ij}$ trading off between the trajectory $\mathbf{X}_{ij}$ which is expected of vehicle j by vehicle i, and trajectory $\mathbf{X}_j$ which vehicle j can actually provide while respecting its constraints. *Step 2* is followed by the communication of $\mathbf{Z}_{ji}$ variables from vehicle j to vehicle i. These variables are used to update the $\mathbf{X}$ variables on vehicle i in step 1 of the next ADMM iteration.

In the final ADMM step, the dual variables λ_{iji} and λ_{ijj}

belonging to vehicle i are updated through gradient ascent. It can easily be seen from the Lagrangian formulation (6) that the gradients with respect to λ_{iji} and λ_{ijj} are $(\mathbf{X}_{ij} - \mathbf{Z}_{ij})$ and $(\mathbf{X}_j - \mathbf{Z}_{ij})$ respectively. Choosing the optimal step length of ρ according to [7], the dual variables are updated according to (9).

Step 3 : Dual variables

$$\lambda_{iji}^{k+1} = \lambda_{iji}^{k} + \rho(\mathbf{X}_{ij}^{k+1} - \mathbf{Z}_{ij}^{k+1}) \tag{9}$$

$$\lambda_{ijj}^{k+1} = \lambda_{ijj}^{k} + \rho(\mathbf{X}_{j}^{k+1} - \mathbf{Z}_{ij}^{k+1})$$

The updated dual variables λ_{jii} on vehicle j are communicated to vehicle i, which are used to update the $\mathbf{X}$ variables on vehicle i in step 1 of the next ADMM iteration. One sequence of these three ADMM steps are performing in one DMPC iteration, calculating a trajectory for the vehicles to follow for the next Ts. This is embedded in an online formulation according to the methodology described in [2].

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TABLE I

AN EXAMPLE OF A TABLE

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Three	Four

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Fig. 2. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

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APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

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