Distributed motion planning for multiple vehicles transporting a flexible payload*

Sampath Kumar Mulagaleti[1], Ruben Van Parys[2] and Goele Pipeleers[2]

Abstract— **This paper deals with the development of control strategies for cooperatively transporting a flexible payload with multiple vehicles. To this end, an algorithm is developed which generates optimal trajectories for the vehicles to follow. The trajectories are parameterized using splines, and the optimization algorithm used to solve for the spline coefficients is based on an Alternating Direction Method of Multipliers (ADMM) formulation. The algorithm instructs that the optimization problem be solved repeatedly in a receding horizon fashion, making it fit into a distributed model predictive control (DMPC) framework. One ADMM iteration is performed per DMPC iteration, reducing the inter-agent communication rate. Numerical and experimental validation of the developed control scheme is performed and the results are presented.**

I. INTRODUCTION

Payload transportation tasks are ubiquitous in industrial environments. These payloads can vary greatly in weights, necessitating multiple transportation solutions of varying payload capacities be used within the same logistics structure. An alternative to this would be to use multiple vehicles to cooperatively tow a payload, with the number of vehicles chosen as per the towing requirements. To enable such a multi-agent system to work autonomously and transport the payload to a desired location, control strategies are required which perform coordinated motion of the vehicles.

Transportation and manipulation of objects with multiple (mobile) manipulators is a standard problem in robotics and control. Some of the early approaches to model and control such systems are presented in [1] [2] [3], and an optimal control approach in [4]. More powerful optimization-based control techniques can be formulated in a Model Predictive Control (MPC) framework, which explicitly incorporates the system constraints. Such techniques have been applied to perform trajectory tracking with multiple vehicles moving in a desired formation [5]. Optimization-based techniques can also be used to generate optimal state trajectories, which are sent to a low-level controller to track. When applied to robotic platforms, they fall under the umbrella of motion planning algorithms. Motion planning for multiple vehicles moving in a formation is done within an MPC framework in [6], utilizing a leader-follower strategy. Such a strategy is not robust against possible failure of the leader agent. This necessitates development of strategies in which each agent within the multi-agent system has equal role to play. This leads onto the domain of Distributed model predictive control (DMPC). Within the DMPC framework, the control problem is distributed over the agents. Since an optimization problem underlies the MPC framework, DMPC based techniques usually utilize distributed optimization methods to distribute the optimization problem over the agents [7]. Motion planning algorithms are designed within this framework in [8] and [9] for formation control of multiple agents.

A major difference between formation control problems and the payload transportation problem is that in the latter, the agents are dynamically coupled. This means that a change in state of one of the agents results in a change in state of the others. The coupled dynamics act as constraints on the centralized optimization problem, and decomposing the centralized problem for DMPC purposes would require decomposing the coupling dynamics constraint. Decoupling of dynamics is done using primal decomposition in [10], in which, linear systems are considered, and an optimal consensus problem is solved. In optimal consensus problems, multiple optimization variables are driven to the same value over iterations. In other words, consensus is achieved between different states of the system. Another approach based on dual decomposition, is discussed in [11]. In this, the variable splitting method is employed [8], with the introduction of copies of states of neighboring vehicles which affect local states of a host vehicle. More robust methods based on the ADMM are formulated in [12], which consider coupled objective functions in addition to coupled nonlinear dynamics. Another alternative is introduced in [13], where the ADMM-consensus problem introduced in [14] is combined with the idea of introducing copies of neighboring states from [11].

This work proposes a decentralized-consensus ADMM based DMPC scheme to solve the cooperative payload transportation problem. Decentralized-consensus is achieved by decoupling the 2nd ADMM step using a novel variable-copying scheme inspired from [9]. In order to reduce the communication and computation load, 1 ADMM iteration is performed per DMPC iteration, and the ADMM iterations are supported by inter-vehicle communication.

The rest of the paper is organized as follows:
Section II introduces the motion planning problem for cooperative payload transportation problem, and Section III describes how this problem can be solved in a online

*This work was not supported by any organization

[1]Albert Author is with Faculty of Electrical Engineering, Mathematics and Computer Science, University of Twente, 7500 AE Enschede, The Netherlands albert.author@papercept.net

[2]Bernard D. Researcheris with the Department of Electrical Engineering, Wright State University, Dayton, OH 45435, USA b.d.researcher@ieee.org

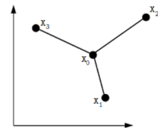

Fig. 1: Dynamic model decoupling

distributed manner. Results from simulations based on the proposed approach are presented in Section IV, and the results from an experimental validation in Section V.

II. PROBLEM FORMULATION

A. System model

The dynamic system under consideration is modeled as multiple holonomic vehicles attached through 1D spring-damper elements to a common payload. The payload and the holonomic vehicles are abstracted as point masses, and are constrained to move in a 2D plane. Dynamic spring-mass-damper models are used to describe the system in the optimization problem. Friction is ignored in the model. The free length of the flexible elements is assumed to be 0 m, resulting in the vehicles only applying a pulling force on the payload. A schematic of this model for 3 holonomic vehicles is shown in 1.

B. Optimal control problem

This section discusses the formulation of optimization problem with the payload and vehicle trajectories as the optimization variables. The solution of this problem are the position trajectories the vehicles and the payload should follow in order to transport the payload to a location closest to its goal $x_d \in \mathbb{R}^2$ within the time horizon T s over which the problem is solved. The payload trajectory is represented by the vector $x_0(t) \in \mathbb{R}^2$ and that of each vehicle i by $x_i(t) \in \mathbb{R}^2$. The formulation is shown in (1). The case of N=3 vehicles is dealt here. Extension to problems with more vehicles can be done by modifying the *intra-vehicle anti-collision* constraints.

$$
\begin{aligned}
\underset{\forall i: x_i(t), x_0(t)}{\text{minimize}} \quad & \int_0^T ||x_0(t) - x_d||_1 dt \\
\text{subject to} \quad & \underline{l} \preceq f_i(x_0(t), x_i(t)) \preceq \bar{l} \\
& g(x_0(t), x_1(t), x_2(t), x_3(t)) = 0 \\
& h_{ij}(x_0(t), x_i(t), x_j(t)) \leq 0 \\
& \underline{d} \leq d_i(x_i(t)) \leq \bar{d} \\
& x_0^{(k)}(0) = x_{0,0}^{(k)}, \quad x_i^{(k)}(0) = x_{0,i}^{(k)} \\
& x_0^{(k)}(T) = x_{T,0}^{(k)}, \quad x_i^{(k)}(T) = x_{T,i}^{(k)} \\
& k \in 0, 1, 2 \\
& \forall t \in [0, T], \quad \forall i, j \in \{1, 2, 3\}, i \neq j
\end{aligned} \tag{1}
$$

The function $f_i : \mathbb{R}^4 \to \mathbb{R}^3$ consists of individual vehicle i's dynamics, and flexible element i's length. The lower and upper bounds on these quantities are captured in vectors $\underline{l}$ and $\bar{l}$ respectively. The function $g : \mathbb{R}^8 \to \mathbb{R}$ represents the payload dynamics.

Intra-vehicle collisions are avoided using the third constraint, defined by the function $h_{ij} : \mathbb{R}^6 \to \mathbb{R}$. It limits the angles between the vehicles to lie at greater than 90° when subtended at the payload. It is reiterated that for N>3, this constraint should be reformulated. These functions are shown in (2). Note that the indication of time dependence of the variables is dropped for ease of notation.

$$
\begin{aligned}
f_i(x_0, x_i) &= \begin{bmatrix} m_i \ddot{x}_i + c_i(\dot{x}_i - \dot{x}_0) + k_i(x_i - x_0) \\ (x_i - x_0)^T (x_i - x_0) \end{bmatrix} \\
g(x_0, x_1, x_2, x_3) &= m_0 \ddot{x}_0 + \sum_{i=1}^{3} \left[c_i(\dot{x}_i - \dot{x}_0) + k_i(x_i - x_0) \right] \\
h_{ij}(x_0(t), x_i(t), x_j(t)) &= \langle x_i(t) - x_0(t), x_j(t) - x_0(t) \rangle
\end{aligned} \tag{2}
$$

The rest of the constraints represent limits on vehicle velocities and accelerations, and initial and final conditions on the vehicle and payload position trajectories and higher derivatives.

III. SOLUTION STRATEGY

A. Spline parameterization

Problem (1) represents an infinite dimensional problem, since the optimization variables $x_0(t)$ and $x_i(t)$ are infinite dimensional, and enforcing the constraints at all instances results in an infinite number of constraints. In order to make the problem numerically tractable, a B-spline based parameterization of the optimization variables is used, along the lines of [15]. The continuous optimization variables are hence represented as linear combinations of piecewise polynomial basis functions $b_k(t)$, as expressed in (3).

$$
\begin{aligned}
x_i(t) &= \sum_{k=1}^{n} \mathsf{x}_{i,k} b_k(t) = \mathsf{x}_i^T b(t) \quad \forall i \in \{1, ., N\} \\
x_0(t) &= \sum_{k=1}^{n} \mathsf{x}_{0,k} b_k(t) = \mathsf{x}_0^T b(t)
\end{aligned} \tag{3}
$$

The trajectories of each vehicle i are consequently represented by the coefficient set $\mathsf{x}_i = \{\mathsf{x}_{i,k}\}_{k=1}^n$ and those of the payload by $\mathsf{x}_0 = \{\mathsf{x}_{0,k}\}_{k=1}^n$, for a chosen B-spline basis functions vector $b = [b_1, \ldots, b_n]^T$. Substituting these parameterizations in (1) results in an optimization problem with finite number of variables. A B-spline parameterization is chosen because of its convex hull property [15], which dictates that the resultant spline function lies within the convex hull of the coefficients. This is because at every time instant over which the spline is defined, the basis functions correspond to a non-negative partition of unity. Hence, the constraints on an infinite dimensional spline can be imposed on the coefficients of the corresponding B-spline basis functions, resulting in a finite number of constraints. Since the constraint functions f_i,g and h_{ij} are composed of polynomial combinations of the splines $x_i(t)$ and $x_0(t)$ and their derivatives, the resultant splines' coefficients are polynomial combinations of x_0 and x_i. Following the convex

Algorithm 1 Spline-based MPC

1: **Repeat** every ΔT: $k = 0, 1, \ldots$
2: Extract $x_i^k(t)$ from x_i^k for each Vehicle i.
3: Vehicle i starts following trajectory $x_i^k(t)$
4: Estimate $\hat{x}_i^k$ and $\hat{x}_0^k$ at time $(k+1)\Delta T$
5: Compute x_i^{k+1} and x_0^{k+1} by solving (4), using $\hat{x}_i^k$, $\hat{x}_0^k$ as initial conditions and hot start coefficients
6: **Until** target reached

hull property, these constraints are transformed to f_i,g and h_{ij} respectively. The resultant formulation is seen in (4).

$$\begin{aligned} \underset{\forall i:\mathsf{x}_i,\mathsf{x}_0}{\text{minimize}} \quad & \mathsf{J}(\mathsf{x}_0) \\ \text{subject to} \quad & \underline{l} \preceq \mathsf{f}_i(\mathsf{x}_0,\mathsf{x}_i) \preceq \overline{l} \\ & \mathsf{g}(\mathsf{x}_0,\mathsf{x}_1,\mathsf{x}_2,\mathsf{x}_3) = 0 \\ & \mathsf{h}_{ij}(\mathsf{x}_0,\mathsf{x}_i,\mathsf{x}_j) \le 0 \\ & \mathsf{x}_i \in \mathsf{X}_i \\ & \mathsf{x}_0 \in \mathsf{X}_0 \\ & \forall i,j \in \{1,2,3\}, i \neq j \end{aligned} \tag{4}$$

The MPC updating scheme proposed in [16] for optimization variables expressed in a B-spline basis can be used on a centralized processor, provided it has access to all x_i and x_0 variables. This scheme is reiterated in Algorithm 1. The vehicle and payload positions estimated at time $(k+1)\Delta T$ are incorporated into the feasible sets X_i and X_0 while solving the problem for x_i^{k+1} and x_0^{k+1}.

B. Distributed formulation

The purpose of this section is to split the optimization problem (4) into 3 separate problems, one for each vehicle. This means that each problem should contain optimization variables local to the vehicle it is assigned. The first step towards the distributed formulation is the introduction of copies of the payload variables x_0 on each vehicle. These variables are represented by x_{i0}. The substitution of these variables results in 3 copies of the payload dynamics constraint g, each one being $\mathsf{g}_i(\mathsf{x}_{i0},\mathsf{x}_i,\mathsf{x}_j) = 0$, where $\mathsf{x}_j = \{\mathsf{x}_1,\mathsf{x}_2,\mathsf{x}_3\} \ominus \mathsf{x}_i$.

The payload dynamics constraint is further decoupled by borrowing ideas from [11], which dictates that the variables $x_j(t)$ of vehicle j which affect the dynamics of vehicle i can be modeled as exogenous inputs $x_{ij}(t)$ acting on vehicle i. The new variables $x_{ij}(t)$ are hence local to vehicle i, and are parameterized by the spline coefficients x_{ij}. The payload dynamics constraint is thus further modified as $\mathsf{g}_i(\mathsf{x}_{i0},\mathsf{x}_i,\mathsf{x}_{ij}) = 0$. This substitution is supported by consensus constraints between x_j and x_{ij}. Following a modification of the consensus constraints presented in [9], consensus variables z_{ij} are introduced on each vehicle i, which act as a balance between x_j and x_{ij}. The modified optimization problem with local copy variables, consensus variables, and consensus constraints is shown in (5).

$$\begin{aligned} \underset{\forall i,j,i\neq j:\mathsf{x}_i,\mathsf{x}_{ij},\mathsf{z}_{ij},\mathsf{x}_{i0}}{\text{minimize}} \quad & \sum_{i=1}^{3} \mathsf{J}(\mathsf{x}_{i0}) \\ \text{subject to} \quad & \underline{l} \preceq \mathsf{f}_i(\mathsf{x}_{i0},\mathsf{x}_i) \preceq \overline{l} \\ & \mathsf{g}_i(\mathsf{x}_{i0},\mathsf{x}_i,\mathsf{x}_{ij}) = 0 \\ & \mathsf{h}_{ij}(\mathsf{x}_{i0},\mathsf{x}_i,\mathsf{x}_{ij}) \le 0 \\ & \mathsf{x}_{ij} = \mathsf{z}_{ij} \\ & \mathsf{x}_j = \mathsf{z}_{ij} \\ & \mathsf{x}_i \in \mathsf{X}_i \\ & \mathsf{x}_{i0} \in \mathsf{X}_0 \\ & \forall i,j \in \{1,2,3\}, i \neq j \end{aligned} \tag{5}$$

The optimization problem (5) can now be decoupled using the Alternating Direction Method of Multipliers (ADMM). The ADMM algorithm solves the dual of (5), after augumenting it with a quadratic term to improve convergence properties. To minimize duality gap, the fewest number of constraints are to be dualized according to [8]. Hence, only the consensus equality constraints are moved into the objective function to form the lagrangian. This appended objective function is seen in (6).

$$\begin{aligned} \mathcal{L}_\rho &= \sum_{i=1}^{N} \Big(\mathsf{J}_i(\mathsf{x}_{i0}) + \sum_{j\neq i} \big(\lambda_{ij}^T(\mathsf{x}_{ij} - \mathsf{z}_{ij}) + \frac{\rho}{2}||\mathsf{x}_{ij} - \mathsf{z}_{ij}||_2^2\big) \\ &\qquad + \sum_{j\neq i} \big(\mu_{ij}^T(\mathsf{x}_j - \mathsf{z}_{ij}) + \frac{\rho}{2}||\mathsf{x}_j - \mathsf{z}_{ij}||_2^2\big)\Big) \\ &= \sum_{i=1}^{N} \mathcal{L}_{\rho,i}(\mathsf{x}_{i0},\mathsf{x}_{ij},\mathsf{z}_{ij},\lambda_{ij},\mathsf{x}_j,\mu_{ij}) \\ &= \sum_{i=1}^{N} \mathcal{L}_{\rho,i}(\mathsf{x}_{i0},\mathsf{x}_{ij},\mathsf{z}_{ij},\lambda_{ij},\mathsf{x}_i,\mathsf{z}_{ji},\mu_{ji}) \end{aligned} \tag{6}$$

The dual variables introduced in the Lagrangian corresponding to the equality constraints are λ_{ij} and μ_{ij}. The last equality in (6) is valid due to the bidirectionality of interaction amongst the vehicles: variables on vehicle i affecting the dynamics of vehicle j correspond to the variables on vehicle j affecting dynamics of vehicle i. Since the constraints are now completely decoupled, the feasible set on each vehicle i is represented by $\{\mathsf{x}_i,\mathsf{x}_{i0},\mathsf{x}_{ij}\} \in \Phi_i$. The dual function of (5) is shown in (7).

$$q(\lambda_{ij},\mu_{ij}) = \inf_{\substack{\{\mathsf{x}_i,\mathsf{x}_{i0},\mathsf{x}_{ij}\}\in\Phi_i \\ \forall i,j\in\{1,2,3\},i\neq j}} \mathcal{L}_\rho \tag{7}$$

The dual variables are found by maximizing the dual function q with respect to the lagrangian multipliers. The maximization is performed through gradient ascent in the ADMM algorithm. For this, the primal variables x_{ij}^k,x_j^k and z_{ij}^k from iteration k are updated in iteration $k+1$, and the

Algorithm 2 ADMM based Distributed MPC

1: Perform n ADMM iterations and get x_i^0, x_{ij}^0, x_{i0}^0, z_{ij}^0, λ_{ij}^0, μ_{ij}^0
2: **Repeat** every ΔT: $k = 0, 1, \ldots$
3: Extract $x_i^k(t)$, $x_{ij}^k(t)$ and $x_{i0}^k(t)$ from x_i^k, x_{ij}^k and x_{i0}^k on each Vehicle i.
4: Vehicle i starts following trajectory $x_i^k(t)$
5: Estimate $\hat{x}_i^k$ and $\hat{x}_{i0}^k$ at time $(k+1)\Delta T$, assuming perfect tracking of $x_{ij}^k(t)$
6: Update horizon and compute $\tilde{\mathsf{x}}_i^k$, $\tilde{\mathsf{x}}_{ij}^k$, $\tilde{\mathsf{x}}_{i0}^k$, $\tilde{\mathsf{z}}_{ij}^k$, $\tilde{\mathsf{z}}_{ji}^k$, $\tilde{\lambda}_{ij}^k$, $\tilde{\mu}_{ji}^k$
7: Compute $\{\mathsf{x}_i^{k+1}, \mathsf{x}_{ij}^{k+1}, \mathsf{x}_{i0}^{k+1}\}$, using $\hat{x}_i^k$ and $\hat{x}_{i0}^k$ as initial conditions:

$$\begin{pmatrix} \mathsf{x}_i^{k+1} \\ \mathsf{x}_{ij}^{k+1} \\ \mathsf{x}_{i0}^{k+1} \end{pmatrix} := \underset{\{\mathsf{x}_i, \mathsf{x}_{i0}, \mathsf{x}_{ij}\} \in \Phi_i}{\operatorname{argmin}} \mathcal{L}_{\rho,i}(\mathsf{x}_{i0}, \mathsf{x}_{ij}, \tilde{\mathsf{z}}_{ij}^k, \tilde{\lambda}_{ij}^k, \mathsf{x}_i, \tilde{\mathsf{z}}_{ji}^k, \tilde{\mu}_{ji}^k)$$

8: Communication with agents j: :
send x_i^{k+1}, receive x_j^{k+1}
9: Compute z_{ij}^{k+1}:

$$\mathsf{z}_{ij}^{k+1} := \frac{1}{2}\left(\mathsf{x}_j^{k+1} + \mathsf{x}_{ij}^{k+1} + \frac{\tilde{\lambda}_{ij}^k + \tilde{\mu}_{ij}^k}{\rho}\right)$$

10: Compute λ_{ij}^{k+1} and μ_{ij}^{k+1}:

$$\lambda_{ij}^{k+1} := \tilde{\lambda}_{ij}^k + \rho(\mathsf{x}_{ij}^{k+1} - \mathsf{z}_{ij}^{k+1})$$
$$\mu_{ij}^{k+1} := \tilde{\mu}_{ij}^k + \rho(\mathsf{x}_j^{k+1} - \mathsf{z}_{ij}^{k+1})$$

11: Communication with agents j
send z_{ij}^{k+1} and μ_{ij}^{k+1}, receive z_{ji}^{k+1} and μ_{ji}^{k+1}
12: **Until** target reached

gradient of the dual function q is calculated in the directions of the lagrangian multipliers according to (8).

$$\begin{aligned} \nabla_{\lambda_{ij}} q &= \mathsf{x}_{ij}^{k+1} - \mathsf{z}_{ij}^{k+1} \\ \nabla_{\mu_{ij}} q &= \mathsf{x}_j^{k+1} - \mathsf{z}_{ij}^{k+1} \end{aligned} \tag{8}$$

An optimal step of length ρ in these directions is taken. The primal variables are updated in two steps in a Gauss-Siedel fashion, the first step updating the $\{\mathsf{x}_i, \mathsf{x}_{i0}, \mathsf{x}_{ij}\}$ variables and the second updating the z_{ij} variables.

Solving the optimization problem (4) till convergence will require several ADMM iterations. These iterations will replace (step 5) in Algorithm 1, and would be performed on individual vehicles in parallel. However, this would also result in a substantial computation and communication load for each MPC iteration. This is avoided by following the DMPC scheme proposed in [16], which performs 1 ADMM iteration per MPC iteration. The resultant is Algorithm 2, which includes the communication steps performed in one DMPC iteration.

The algorithm dictates that first, a few ADMM iterations be performed to find the first set of primal and dual variables. The primal x_i variables are then interpreted in terms of the trajectories $x_i(t)$ on each vehicle i, and the tracking of these trajectories begins. According to the spline-MPC scheme discussed in [16], $\hat{x}_i(\Delta T)$ and $\hat{x}_{i0}(\Delta T)$ are estimated locally on each vehicle, with perfectly tracked $x_{ij}(t)$ as the exogenous inputs into the local model. These estimates are passed as initial conditions into the MPC problem solving the trajectories from time ΔT s. This is followed by the usual 3 ADMM steps, with steps 7 and 9 for calculating the primal variables, and 10 for the dual variables. Since variables from the previous ADMM iteration are used in the current iteration, the spline coefficients are transformed to represent the spline in the current time frame, as discussed in [16]. Step 6 of the algorithm deals with this transformation. Step 9 is the direct least squares solution of the second ADMM step to update the z_{ij} variables, and the corresponding problem is shown in (9).

$$\begin{aligned} \mathsf{z}_{ij}^{k+1} := \underset{\mathsf{z}_{ij}}{\operatorname{argmin}} \quad & (\tilde{\lambda}_{ij}^k)^T(\mathsf{x}_{ij}^{k+1} - \mathsf{z}_{ij}) + \frac{\rho}{2}\left|\left|\mathsf{x}_{ij}^{k+1} - \mathsf{z}_{ij}\right|\right|_2^2 \\ & + (\tilde{\mu}_{ij}^k)^T(\mathsf{x}_j^{k+1} - \mathsf{z}_{ij}) + \frac{\rho}{2}\left|\left|\mathsf{x}_j^{k+1} - \mathsf{z}_{ij}\right|\right|_2^2 \end{aligned} \tag{9}$$

From this least squares problem, it can be seen the consensus variables z_{ij} are pushed between the *expected* x_{ij} variables and the *actual* x_j variables, acting as a balance between them. In the first ADMM step (Step 7), the reverse occurs in that x_{ij} and x_j are pushed towards the previous *compromise* z_{ij} variables. Thus, over MPC (and hence, ADMM) iterations, a consensus is achieved between x_{ij} and x_j, resulting the problem dynamics being satisfied.

IV. Simulation results

The proposed MPC strategy was simulated using the *omg-tools* toolbox.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as 3.5-inch disk drive.
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.

- Do not mix complete spellings and abbreviations of units: Wb/m2 or webers per square meter, not webers/m2. Spell out units when they appear in text: . . . a few henries, not . . . a few H.
- Use a zero before decimal points: 0.25, not .25. Use cm3, not cc. (bullet list)

C. Equations

The equations are an exception to the prescribed specifications of this template. You will need to determine whether or not your equation should be typed using either the Times New Roman or the Symbol font (please no other font). To create multileveled equations, it may be necessary to treat the equation as a graphic and insert it into the text after your paper is styled. Number equations consecutively. Equation numbers, within parentheses, are to position flush right, as in (1), using a right tab stop. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in

$$\alpha + \beta = \chi \tag{1}$$

Note that the equation is centered using a center tab stop. Be sure that the symbols in your equation have been defined before or immediately following the equation. Use (1), not Eq. (1) or equation (1), except at the beginning of a sentence: Equation (1) is . . .

D. Some Common Mistakes

- The word data is plural, not singular.
- The subscript for the permeability of vacuum ?0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter o.
- In American English, commas, semi-/colons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an inset, not an insert. The word alternatively is preferred to the word alternately (unless you really mean something that alternates).
- Do not use the word essentially to mean approximately or effectively.
- In your paper title, if the words that uses can accurately replace the word using, capitalize the u; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones affect and effect, complement and compliment, discreet and discrete, principal and principle.
- Do not confuse imply and infer.
- The prefix non is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the et in the Latin abbreviation et al..
- The abbreviation i.e. means that is, and the abbreviation e.g. means for example.

V. USING THE TEMPLATE

Use this sample document as your LaTeX source file to create your document. Save this file as **root.tex**. You have to make sure to use the cls file that came with this distribution. If you use a different style file, you cannot expect to get required margins. Note also that when you are creating your out PDF file, the source file is only part of the equation. *Your TEX $\rightarrow$ PDF filter determines the output file size. Even if you make all the specifications to output a letter file in the source - if your filter is set to produce A4, you will only get A4 output.*

It is impossible to account for all possible situation, one would encounter using TEX. If you are using multiple TEX files you must make sure that the “MAIN“ source file is called root.tex - this is particularly important if your conference is using PaperPlaza’s built in TEX to PDF conversion tool.

A. Headings, etc

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced. Styles named Heading 1, Heading 2, Heading 3, and Heading 4 are prescribed.

B. Figures and Tables

Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation Fig. 1, even at the beginning of a sentence.

TABLE I: An Example of a Table

One	Two
Three	Four

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity Magnetization, or Magnetization, M, not just M. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write Magnetization (A/m) or Magnetization

We suggest that you use a text box to insert a graphic (which is ideally a 300 dpi TIFF or EPS file, with all fonts embedded) because, in an document, this method is somewhat more stable than directly inserting a picture.

Fig. 2: Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

A[m(1)], not just A/m. Do not label axes with a ratio of quantities and units. For example, write Temperature (K), not Temperature/K.

VI. CONCLUSIONS

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

REFERENCES

[1] D. Williams and O. Khatib, ”The virtual linkage: a model for internal forces in multi-grasp manipulation,” [1993] Proceedings IEEE International Conference on Robotics and Automation, Atlanta, GA, 1993, pp. 1025-1030 vol.1. doi: 10.1109/ROBOT.1993.292110

[2] O. Khatib. Force strategies for cooperative tasks in multiple mobile manipulation systems. Giralt G., Hirzinger G. (eds) Robotics Research. Springer, London, 1996.

[3] H. G. Tanner, K. J. Kyriakopoulos, and N. J. Krikelis. Modeling of multiple mobile manipulators handling a common deformable object. Journal of Robotic Systems, 15(11):599?623, 1998.

[4] R. Ritz and R. D?Andrea. Carrying a flexible payload with multiple flying vehicles. In 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems, pages 3465?3471, Nov 2013.

[5] S. Liu, D. Sun, and C. Zhu. Coordinated motion planning for multiple mobile robots along designed paths with formation requirement. IEEE/ASME Transactions on Mechatronics, 16(6):1021?1031, Dec 2011.

[6] M. Saska, V. Von ? asek, T. Krajn ??k, L. P?reu? cil, Coordination and navigation of heterogeneous mav?ugv formations localized by a hawk-eye-like approach under a model predictive control scheme, The International Journal of Robotics Research 33 (10) (2014) 1393?1412.

[7] S. Boyd, L. Xiao, and A. Mutapcic. Notes on decomposition methods, October 2003.

[8] R. L. Raffard, C. J. Tomlin, and S. P. Boyd. Distributed optimization for cooperative agents: application to formation flight. In 2004 43rd IEEE Conference on Decision and Control (CDC) (IEEE Cat. No.04CH37601), volume 3, pages 2453?2459 Vol.3, Dec 2004.

[9] R. V. Parys and G. Pipeleers. Online distributed motion planning for multi-vehicle systems. In 2016 European Control Conference (ECC), pages 1580?1585, June 2016.

[10] B. Johansson, A. Speranzon, M. Johansson, and K. H. Johansson. On decentralized negotiation of optimal consensus. Automatica, 44(4):1175 ? 1179, 2008.

[11] P. Giselsson and A. Rantzer. Distributed model predictive control with suboptimality and stability guarantees. In 49th IEEE Conference on Decision and Control. pp. 7272-7277.

[12] F. Farokhi, I. Shames, and K. H. Johansson. Distributed MPC Via Dual Decompo- sition and Alternative Direction Method of Multipliers, pages 115?131. Springer Netherlands, Dordrecht, 2014.

[13] T. H. Summers and J. Lygeros. Distributed model predictive consensus via the alternating direction method of multipliers. In 2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton), pages 79?84, Oct 2012.

[14] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein. Distributed optimization and statistical learning via the alternating direction method of multipliers. Found. Trends Mach. Learn., 3(1):1?122, Jan. 2011.

[15] W. Van Loock, G. Pipeleers, J. Swevers, B-spline parameterized optimal motion trajectories for robotic systems with guaranteed constraint satisfaction, Mechanical Sciences 6 (2) (2015) 163-171.

[16] Ruben Van Parys, Goele Pipeleers, Distributed MPC for multi-vehicle systems moving in formation, In Robotics and Autonomous Systems, Volume 97, 2017, Pages 144-152, ISSN 0921-8890, https://doi.org/10.1016/j.robot.2017.08.009.