CO–495: Advanced Statistical Machine Learning & Pattern Recognition

Coursework #2: Hidden Markov Models

Exercise I

A function that is used to generate random samples from a Hidden Markov Model (HMM) is provided. The function generates samples from an HMM with discrete valued observations (in that case, `E` is the emission probability matrix), as well as from an HMM with continuous valued observations (in that case, 1D Gaussians, thus `E.mu` is a vector of means and `E.sigma2` is a vector of variances).

i) Given the observations generated from the HMM, programme functions that perform the EM algorithm to estimate the parameters. You should provide different functions that perform smoothing and filtering.

(50 marks)

ii) Given the parameters and a string of observations, programme a function that performs Viterbi decoding.

(20 marks)

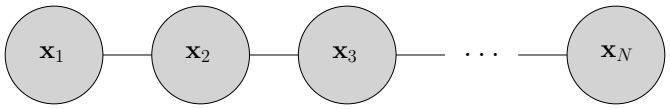

Figure 1: Markov Chain

Exercise II

i) Assume the stochastic automaton given in Fig. 2, which represents a Markov Chain for the word "kid", consisting of phonemes `k`,`I`,`d`, as well as a starting and an ending state `start` and `end`, respectively. Assume that N strings of length T are given. Apply the Maximum Likelihood algorithm to estimate $\pi_1, \ldots, \pi_5$, as well as the transition probabilities a_{ij}.

(15 marks)

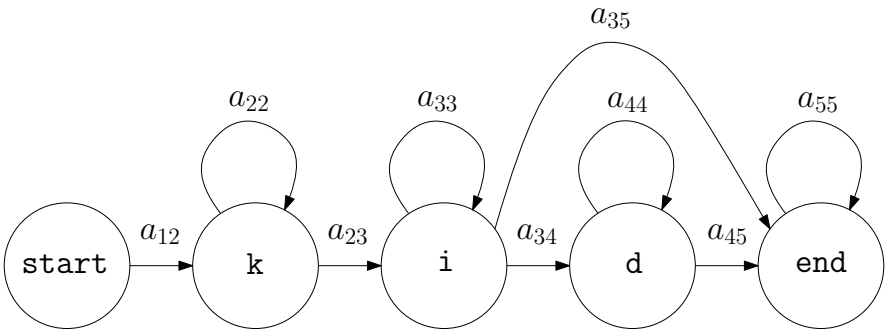

Figure 2: Stochastic automaton for the word "kid"

ii) Assume the Markov Chain of Fig. 1 and that the chain represents the hidden structure of an HMM with emission probabilities $p(\mathbf{x}|\mathbf{z})$ with discrete observations taking 5 values. Given a set of N sequences of length T of observations, devise the EM algorithm to find the parameters.

(15 marks)