COURSEWORK 2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING

477 - Computational Optimisation

Coursework 2

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1 Part 1

1.1 Q.1

a) To show that Δx_k is descent direction at x_k, we need to show that:

$$\nabla f(x_k)^\top \Delta x_k < 0 \tag{1}$$
$$\tag{2}$$

Since:

$$\nabla^2 f(x_k)\Delta x_k = -\nabla f(x_k) \tag{3}$$
$$\tag{4}$$

We can get:

$$\nabla f(x_k)^\top \Delta x_k = -\nabla f(x_k)^\top \frac{\nabla f(x_k)}{\nabla^2 f(x_k)} \tag{5}$$
$$= -\frac{\nabla f(x_k)^\top \nabla f(x_k)}{\nabla^2 f(x_k)} \tag{6}$$

We can see that this is less than zero since the numerator is positive definite and the same for denominator:

$$\nabla^2 f(x_k) \succeq m\boldsymbol{I} \tag{7}$$

b) We can set tolerane to e^{-08} and say that:

$$|f(x_{k+1}) - f(x_k)| < tor \tag{8}$$
$$||\nabla f(x_k)||_2 < tor \tag{9}$$
$$||x_{k+1} - x_k||_2 < tor \tag{10}$$

This is to check that the First Order Necessary Condition is satisfied.

c) Yes, since the function is strongly convex, but the condition is that the initial point x_0 has to be close enough to the optimal point.

d) First say that:

$$x_{k+1} = x_k + t_k \Delta x_k \tag{11}$$

Then

$$f(x_{k+1}) = f(x_k + t_k \Delta x_k) \tag{12}$$

Now use Taylor expansion to expand the above function into second order:

$$f(x_k + t_k \Delta x_k) \approx f(x_k) + t_k < \nabla f(x_k), \Delta x_k > + \frac{1}{2}\nabla^2 f(x_k)||t_k||_2^2||\Delta x_k||_2^2 \tag{13}$$

$$\mathbf{\Delta} x_k = -\frac{\nabla f(x_k)}{\nabla^2 f(x_k)} \tag{14}$$

$$< \nabla f(x_k), \mathbf{\Delta} x_k >= -\frac{\|\nabla f(x_k)\|_2^2}{\nabla^2 f(x_k)} \tag{15}$$

We need to show that:

$$f(x_k) + t_k < \nabla f(x_k), \mathbf{\Delta} x_k) > +\frac{1}{2}\nabla^2 f(x_k)\|t_k\|_2^2\|\mathbf{\Delta} x_k\|_2^2 \leq f(x_k) + \alpha t_k < \nabla f(x_k), \mathbf{\Delta} x_k) > \tag{16}$$

$$t_k < \nabla f(x_k), \mathbf{\Delta} x_k) > +\frac{1}{2}\nabla^2 f(x_k)\|t_k\|_2^2\|\mathbf{\Delta} x_k\|_2^2 \leq \alpha t_k < \nabla f(x_k), \mathbf{\Delta} x_k) > \tag{17}$$

$$\tag{18}$$

e)

2 Part 2

2.1 Q.2

a)

b)

c)