

COURSEWORK

IMPERIAL COLLEGE LONDON

DEPARTMENT OF COMPUTING

477 - Computational Optimisation

Author:

Jiahao Lin (CID: 00837321)

Date: November 6, 2016

1 Part 1

1.1 1

1) To prove is $\log \sum_{k=1}^{10} \exp(B_{jk})$ is convex, we define:

$$f(B_j) = \log \sum_{k=1}^{10} \exp(B_{jk}) \quad (1)$$

$$f(x) = \log \sum_{k=1}^{10} \exp(x_k) \quad (2)$$

First we need to compute the hessian of $f(x)$:

$$\nabla f(x) = \frac{1}{1^\top Z} \times z \quad (3)$$

$$\nabla^2 f(x) = \frac{1}{1^\top Z} \times \text{diag}(Z) - \frac{Z^\top Z}{1^\top Z^2} \quad (4)$$

$$\text{where } Z = \sum_{k=1}^{10} \exp(x_k) \quad (5)$$

To prove the hessian is positive semi-definite, we need to prove:

$$v^\top \nabla^2 f(x) v \geq 0 \quad (6)$$

$$v^\top \left(\frac{1}{1^\top Z} \times \text{diag}(Z) - \frac{Z^\top Z}{1^\top Z^2} \right) v \geq 0 \quad (7)$$

2)

3)

4)