Principle of Linear Impulse and Momentum

Momentum is the product of mass and velocity of an object:

$$p = mv$$

- p has the units $kg \cdot m/s$

Using kinematics, the equation of motion for a particle of mass m can be written as:

$$\sum F = ma = m\frac{dv}{dt}$$

$$\implies \sum \int_{t_1}^{t_2} F dt = m \int_{v_1}^{v_2} dv$$

$$\implies \sum \int_{t_1}^{t_2} F dt = mv_2 - mv_1$$

This last equation is reffered to as the principle of linear impulse and momentum.

Linear Impulse

The left (integral) side of the above equation is refferred to as the linear impulse.

$$I = \int_{t_1}^{t_2} F dt$$

- I has units of $N \cdot s$

If the force is expressed as a function of time, the impulse can be determined by direct evaluation of the integral. In particular, if the force is constant in both magnitude and direction, the resulting impulse becomes:

$$I = F_c \int_{t_1}^{t_2} dt = F_c(t_2 - t_1)$$

For problem solving, the principle of linear impulse and momentum equation is rewritten as:

$$mv_1 + \sum \int_{t_1}^{t_2} F dt = mv_2$$

Conservation of Linear Momentum

- If the principle of impulse and momentum is applied to a system of particles, then the collisions between the particles produce internal impulses that are equal, opposite, and collinear, and therefore cancel from the equation
- If an external impulse is small, that is, the force is small and the time is short, then the impulse can be classified as nonimpulsive and can be neglected. Consequently, momentum for the system of particles is conserved

$$\sum m_i(v_i)_1 = \sum m_i(v_i)_2$$

The conservation-of-momentum equation is useful for finding the final velocity of a particle when internal impulses are exerted between two particles and the initial velocities of the particles is known. If the internal impulse is to be determined, then one of the particles is isolated and the principle of impulse and momentum is applied to this particle.

Impact

- When two particles A and B have a direct impact, the internal impulse between them is equal, opposite, and collinear.
- The conservation of momentum for this system applies along the line of impact.

$$m_A(v_A)_1 + m_B(V_B)_1 = m_A(v_A)_2 + m_B(V_B)_2$$

Coefficient of Restitution

This experimentally determined coefficient depends upon the physical properties of the colliding particles. It can be expressed as the ratio of their relative velocity after collision to their relative velocity before collision. If the collision is elastic, no energy is lost and $e = 1$.

For a plastic collision $e = 0$.

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Oblique Impacts

Procedure for Analysis (Oblique Impact)

If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act *only in the x direction*, Fig. 15–15*b*. By resolving the velocity or momentum vectors into components along the x and y axes, Fig. 15–15*b*, it is then possible to write four independent scalar equations in order to determine $(v_{Ax})_2$, $(v_{Ay})_2$, $(v_{Bx})_2$, and $(v_{By})_2$.

- Momentum of the system is conserved *along the line of impact*, x axis, so that $\Sigma m(v_x)_1 = \Sigma m(v_x)_2$.
- The coefficient of restitution, $e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$, relates the relative-velocity *components* of the particles *along the line of impact* (x axis).
- If these two equations are solved simultaneously, we obtain $(v_{Ax})_2$ and $(v_{Bx})_2$.
- Momentum of particle A is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle A in this direction. As a result $m_A(v_{Ay})_1 = m_A(v_{Ay})_2$ or $(v_{Ay})_1 = (v_{Ay})_2$
- Momentum of particle B is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle B in this direction. Consequently $(v_{By})_1 = (v_{By})_2$.

Application of these four equations is illustrated in Example 15.11.