# Work and Energy

Work is the energy transferred to or from an object via the application of force along a displacement. In its simplest form, work ($U$) can be represented by the following formula:

$$U = Fs$$

The change in work of a force being excerted upon a particle can be described as the dot product between $F$ and $dr$, or rather:

$$dU = F \cos\theta ds$$

Where $F$ is the force being applied to the particle and $\theta$ is the angle between the force and the path. If the force isn't constant (variable force), we can rewrite this eqaution as such to find $U_{1-2}$:

$$U_{1-2} = \int_{s_1}^{s_2} F \cos\theta ds$$

## Work of Common Forces

### Weight

$$U_{1-2} = -W\Delta y$$

- $W$ is the gravitational force exerted on the object
- $\Delta y$ is the distance that the object has moved vertically

### Constant Force Moving Along a Straight Line

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

$$\implies U_{1-2} = F_c \cos(\Delta s)$$

- $F_c$ is the constant force being applied on the object
- $\theta$ is the angle between the path and the force
- $\Delta s$ is the change in position

### Spring Force

$$U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

- $k$ is the spring constant
- $s_1$, $s_2$ are the positions of the spring at the two points

## Principle of Work and Energy

By calculating the sum of all the work exerted on a particle, it is possible to determine the initial and final speeds of the particle:

$$\sum U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\implies T_1 + \sum U_{1-2} = T_2$$

**Notes:**

- Work is positive if the force and displacement are in the same direction, and negative if they are in opposing directions
- If work and displacement are perpandicular to each other, than no work is done (don't consider normal force when an object is sliding, for example)

# Power and Efficiency

Power is the amount of energy transfered or converted per unit of time. The power generated by a system that performs an amount of work within a given time interval can be expressed with the following equation:

$$P = \frac{dU}{dt}$$

$$\implies P = \vec{F} \cdot \vec{v}$$

- $P$ is the power of the system in Watts ($J/s$)
- $v$ represents the velocity of the particle
- $F$ is the force acting on the particle
- $U$ is the work done by the system
- like work, power follows the same sign convention

## Efficiency

(Mechanical) efficiency is defined as the ratio of the power produced to the power input.

$$\varepsilon = \frac{\text{power output}}{\text{power input}}$$

If the energy supplied to the system occurs during the same time interval that it is used, than the time cancels out and the efficiency can be re-written as:

$$\varepsilon = \frac{\text{energy output}}{\text{energy input}}$$

- the efficiency of a system is always less than 1 ($\varepsilon < 1$)

### Procedure for Analysis:

- First determine the external force $\vec{F}$ acting on the body which causes the motion. This force is usually developed by a machine or engine placed either within or external to the body.

- If the body is accelerating, it may be necessary to draw its free-body diagram and apply the equation of motion ($\sum \vec{F} = m\vec{a}$) to determine $F$.

- Once $\vec{F}$ and the velocity $\vec{v}$ of the particle where $\vec{F}$ is applied have been found, the power is determined by multiplying the force magnitude with the component of velocity acting in the direction of $\vec{F}$, (i.e., $P = \vec{F} \bullet \vec{v} = |F||v| \cos\theta$).

- In some problems the power may be found by calculating the work done by $\vec{F}$ per unit of time ($P_{avg} = \frac{\Delta U}{\Delta t}$)

# Conservative Forces and Energy

**Conservative Force** - a force where the work it exerts is indepentant of the path it takes, and only depends on the force's initial and final position along the path.

<u>Examples:</u>

- Weight of a particle
- Spring force

**Nonconservative Force** - a force where the work it exerts increases as the length of the path increases.

<u>Examples:</u>

- Friction

## Potential Energies

### Gravitational

$$V_g = Wy$$

- $W$ is the weight of the particle
- $y$ is the distance above a datum (positive) or distance below a datum (negative)

### Elastic

$$V_e = \frac{1}{2}ks^2$$

- $k$ is the spring constant

- $s$ is the distance from the equilibrium

Note: $V_e$ is always positive since the system always has potential to do work.

## Potential Function

If a particle is subjected to both gravtiational and elastic forces, the particles potential energy can be expressed as a potential function:

$$V = V_e + V_g$$

Work done by a conservative force moving the particel from one point to another is measured by the difference of this function:

$$U_{1-2} = V_1 - V_2$$
$$\implies$$

# Conservation of Energy

When a particle is acted upon by a system of both conservative and nonconservative forces, the portion of the work done by the conservative forces can be written in terms of the difference in their potential energies. As a result, the principle of work of energy can be written as:

$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{\text{noncons}} = T_2 + V_2$$

If only conservative forces are at work, the principle can be written as:

$$T_1 + V_1 = T_2 + V_2$$

- $T$ is kinetic energy
- $V$ is potential energy

$$\implies \Delta V = -U$$

## Procedure for Analysis

The conservation of energy equation can be used to solve problems involving velocity, displacement, and conservative force systems. It is generally easier to apply than the principle of work and energy because this equation requires specifying the particle's kinetic and potential energies at only two points along the path, rather than determining the work when the particle moves through a displacement. For application it is suggested that the following procedure be used.

#### Potential Energy

- Draw two diagrams showing the particle located at its initial and final points along the path.
- If the particle is subjected to a vertical displacement, establish the fixed horizontal datum from which to measure the particle's gravitational potential energy $V_g$.
- Data pertaining to the elevation y of the particle from the datum and the stretch or compression s of any connecting springs can be determined from the geometry associated with the two diagrams.

#### Conservation of Energy

- Apply the equation $T_1 + V_1 = T_2 + V_2$
- When determining the kinetic energy, remember that the particle's speed $v$ must be measured from an inertial reference frame