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Sixth Edition

Barnett Rich, PhD

Former Chairman, Department of Mathematics
Brooklyn Technical High School, New York City

Christopher Thomas, PhD

Professor and Chair, Department of Mathematics
Massachusetts College of Liberal Arts, North Adams, MA

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BARNETT RICH held a doctor of philosophy degree (PhD) from Columbia University and a doctor of jurisprudence (JD) from New York University. He began his professional career at Townsend Harris Hall High School of New York City and was one of the prominent organizers of the High School of Music and Art where he served as the Administrative Assistant. Later he taught at CUNY and Columbia University and held the post of chairman of mathematics at Brooklyn Technical High School for 14 years. Among his many achievements are the 6 degrees that he earned and the 23 books that he wrote, among them Schaum's Outlines of Elementary Algebra, Modern Elementary Algebra, and Review of Elementary Algebra.

CHRISTOPHER THOMAS has a BS from University of Massachusetts at Amherst and a PhD from Tufts University, both in mathematics. He first taught as a Peace Corps volunteer at the Mozano Senior Secondary School in Ghana. Since then he has taught at Tufts University, Texas A&M University, and the Massachusetts College of Liberal Arts. He has written Schaum's Outline of Math for the Liberal Arts as well as other books on calculus and trigonometry.

Preface to the First Edition

The central purpose of this book is to provide maximum help for the student and maximum service for the teacher.

Providing Help for the Student

This book has been designed to improve the learning of geometry far beyond that of the typical and traditional book in the subject. Students will find this text useful for these reasons:

(1) ***Learning Each Rule, Formula, and Principle***

Each rule, formula, and principle is stated in simple language, is made to stand out in distinctive type, is kept together with those related to it, and is clearly illustrated by examples.

(2) ***Learning Each Set of Solved Problems***

Each set of solved problems is used to clarify and apply the more important rules and principles. The character of each set is indicated by a title.

(3) ***Learning Each Set of Supplementary Problems***

Each set of supplementary problems provides further application of rules and principles. A guide number for each set refers the student to the set of related solved problems. There are more than 2000 additional related supplementary problems. Answers for the supplementary problems have been placed in the back of the book.

(4) ***Integrating the Learning of Plane Geometry***

The book integrates plane geometry with arithmetic, algebra, numerical trigonometry, analytic geometry, and simple logic. To carry out this integration:

(a) A separate chapter is devoted to analytic geometry.
(b) A separate chapter includes the complete proofs of the most important theorems together with the plan for each.
(c) A separate chapter fully explains 23 basic geometric constructions. Underlying geometric principles are provided for the constructions, as needed.
(d) Two separate chapters on methods of proof and improvement of reasoning present the simple and basic ideas of formal logic suitable for students at this stage.
(e) Throughout the book, algebra is emphasized as the major means of solving geometric problems through algebraic symbolism, algebraic equations, and algebraic proof.

(5) ***Learning Geometry Through Self-study***

The method of presentation in the book makes it ideal as a means of self-study. For able students, this book will enable then to accomplish the work of the standard course of study in much less time. For the less able, the presentation of numerous illustrations and solutions provides the help needed to remedy weaknesses and overcome difficulties, and in this way keep up with the class and at the same time gain a measure of confidence and security.

(6) *Extending Plane Geometry into Solid Geometry*

A separate chapter is devoted to the extension of two-dimensional plane geometry into three-dimensional solid geometry. It is especially important in this day and age that the student understand how the basic ideas of space are outgrowths of principles learned in plane geometry.

Providing Service for the Teacher

Teachers of geometry will find this text useful for these reasons:

(1) *Teaching Each Chapter*

Each chapter has a central unifying theme. Each chapter is divided into two to ten major subdivisions which support its central theme. In turn, these chapter subdivisions are arranged in graded sequence for greater teaching effectiveness.

(2) *Teaching Each Chapter Subdivision*

Each of the chapter subdivisions contains the problems and materials needed for a complete lesson developing the related principles.

(3) *Making Teaching More Effective Through Solved Problems*

Through proper use of the solved problems, students gain greater understanding of the way in which principles are applied in varied situations. By solving problems, mathematics is learned as it should be learned—by doing mathematics. To ensure effective learning, solutions should be reproduced on paper. Students should seek the why as well as the how of each step. Once students sees how a principle is applied to a solved problem, they are then ready to extend the principle to a related supplementary problem. Geometry is not learned through the reading of a textbook and the memorizing of a set of formulas. Until an adequate variety of suitable problems has been solved, a student will gain little more than a vague impression of plane geometry.

(4) *Making Teaching More Effective Through Problem Assignment*

The preparation of homework assignments and class assignments of problems is facilitated because the supplementary problems in this book are related to the sets of solved problems. Greatest attention should be given to the underlying principle and the major steps in the solution of the solved problems. After this, the student can reproduce the solved problems and then proceed to do those supplementary problems which are related to the solved ones.

Others Who Will Find This Text Advantageous

This book can be used profitably by others besides students and teachers. In this group we include: (1) the parents of geometry students who wish to help their children through the use of the book's self-study materials, or who may wish to refresh their own memory of geometry in order to properly help their children; (2) the supervisor who wishes to provide enrichment materials in geometry, or who seeks to improve teaching effectiveness in geometry; (3) the person who seeks to review geometry or to learn it through independent self-study.

BARNETT RICH
Brooklyn Technical High School
April, 1963

Introduction

Requirements

To fully appreciate this geometry book, you must have a basic understanding of algebra. If that is what you have really come to learn, then may I suggest you get a copy of Schaum's Outline of *College Algebra*. You will learn everything you need and more (things you don't need to know!)

If you have come to learn geometry, it begins at Chapter one.

As for algebra, you must understand that we can talk about numbers we do not know by assigning them variables like x, y, and A.

You must understand that variables can be combined when they are exactly the same, like $x + x = 2x$ and $3x^2 + 11x^2 = 14x^2$, but not when there is any difference, like $3x^2y - 9xy = 3x^2y - 9xy$.

You should understand the deep importance of the equals sign, which indicates that two things that appear different are actually exactly the same. If $3x = 15$, then this means that $3x$ is just another name for 15. If we do the same thing to both sides of an equation (add the same thing, divide both sides by something, take a square root, etc.), then the result will still be equal.

You must know how to solve an equation like $3x + 8 = 23$ by subtracting eight from both sides, $3x + 8 - 8 = 23 - 8 = 15$, and then dividing both sides by 3 to get $3x/3 = 15/3 = 5$. In this case, the variable was *constrained*; there was only one possible value and so x would have to be 5.

You must know how to add these sorts of things together, such as $(3x + 8) + (9 - x) = (3x - x) + (8 + 9) = 2x + 17$. You don't need to know that the ability to rearrange the parentheses is called *associativity* and the ability to change the order is called *commutativity*.

You must also know how to multiply them: $(3x + 8)\cdot(9 - x) = 27x - 3x^2 + 72 - 8x = -3x^2 + 19x + 72$

Actually, you might not even need to know that.

You must also be comfortable using more than one variable at a time, such as taking an equation in terms of y like $y = x^2 + 3$ and rearranging the equation to put it in terms of x like $y - 3 = x^2$. so $\sqrt{y - 3} = \sqrt{x^2}$ and thus $\sqrt{y - 3} = \pm x$, so $x = \pm\sqrt{y - 3}$.

You should know about square roots, how $\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$. It is useful to keep in mind that there are many *irrational numbers*, like $\sqrt{2}$, which could never be written as a neat ratio or fraction, but only approximated with a number of decimals.

You shouldn't be scared when there are lots of variables, either, such as $F = \dfrac{gM_1M_2}{r^2}$; thus, $Fr^2 = gM_1M_2$ by cross-multiplication, so $r = \pm\sqrt{\dfrac{gM_1M_2}{F}}$.

Most important of all, you should know how to take a formula like $V = \frac{1}{3}\pi r^2 h$ and replace values and simplify. If $r = 5$ cm and $h = 8$ cm, then

$$V = \frac{1}{3}\pi(5\text{ cm})^2(8\text{ cm}) = \frac{200\pi}{3}\text{ cm}^3.$$

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Contents

CHAPTER 1

Lines, Angles, and Triangles

1.1 Historical Background of Geometry

The word *geometry* is derived from the Greek words *geos* (meaning *earth*) and *metron* (meaning *measure*). The ancient Egyptians, Chinese, Babylonians, Romans, and Greeks used geometry for surveying, navigation, astronomy, and other practical occupations.

The Greeks sought to systematize the geometric facts they knew by establishing logical reasons for them and relationships among them. The work of men such as Thales (600 B.C.), Pythagoras (540 B.C.), Plato (390 B.C.), and Aristotle (350 B.C.) in systematizing geometric facts and principles culminated in the geometry text *Elements*, written in approximately 325 B.C. by Euclid. This most remarkable text has been in use for over 2000 years.

1.2 Undefined Terms of Geometry: Point, Line, and Plane

1.2A Point, Line, and Plane are Undefined Terms

These undefined terms underlie the definitions of all geometric terms. They can be given meanings by way of descriptions. However, these descriptions, which follow, are not to be thought of as definitions.

1.2B Point

A *point* has position only. It has no length, width, or thickness.

A point is represented by a dot. Keep in mind, however, that the dot *represents* a point but *is not* a point, just as a dot on a map may represent a locality but is not the locality. A dot, unlike a point, has size.

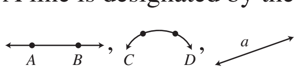
A point is designated by a capital letter next to the dot, thus point A is represented: $\cdot A$.

1.2C Line

A line has length but has no width or thickness.

A line may be represented by the path of a piece of chalk on the blackboard or by a stretched rubber band.

A line is designated by the capital letters of any two of its points or by a small letter, thus:

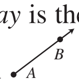


, or $\overleftrightarrow{AB}$.

A *line* may be straight, curved, or a combination of these. To understand how lines differ, think of a line as being generated by a moving point. A *straight line*, such as $\longleftrightarrow$, is generated by a point moving always in the same direction. A *curved line*, such as , is generated by a point moving in a continuously changing direction.

Two lines intersect in a point.

A straight line is unlimited in extent. It may be extended in either direction indefinitely.

A *ray* is the part of a straight line beginning at a given point and extending limitlessly in one direction: $\overrightarrow{AB}$ and 

designate rays.

In this book, the word *line* will mean "straight line" unless otherwise stated.

1.2D Surface

A *surface* has length and width but no thickness. It may be represented by a blackboard, a side of a box, or the outside of a sphere; remember, however, that these are representations of a surface but are not surfaces.

A plane surface (or *plane*) is a surface such that a straight line connecting any two of its points lies entirely in it. A plane is a flat surface.

Plane geometry is the geometry of plane figures—those that may be drawn on a plane. Unless otherwise stated, the word *figure* will mean "plane figure" in this book.

SOLVED PROBLEMS

1.1 Illustrating undefined terms

Point, line, and plane are undefined terms. State which of these terms is illustrated by (a) the top of a desk; (b) a projection screen; (c) a ruler's edge; (d) a stretched thread; (e) the tip of a pin.

Solutions

(a) surface; (b) surface; (c) line; (d) line; (e) point.

1.3 Line Segments

A straight line segment is the part of a straight line between two of its points, including the two points, called *endpoints*. It is designated by the capital letters of these points with a bar over them or by a small letter. Thus, $\overline{AB}$ or r represents the straight line segment $A \overset{r}{\text{——}} B$ between A and B.

The expression *straight line segment* may be shortened to *line segment* or to *segment*, if the meaning is clear. Thus, $\overline{AB}$ and *segment AB* both mean "the straight line segment AB."

1.3A Dividing a Line Segment into Parts

If a line segment is divided into parts:

1. The length of the whole line segment equals the sum of the lengths of its parts. Note that the length of $\overline{AB}$ is designated AB. A number written beside a line segment designates its length.
2. The length of the whole line segment is greater than the length of any part.

   Suppose $\overline{AB}$ is divided into three parts of lengths a, b, and c; thus $A \overset{a}{\text{——}} \bullet \overset{b}{\text{——}} \bullet \overset{c}{\text{————}} B$. Then $AB = a + b + c$. Also, AB is greater than a; this may be written as $AB > a$.

If a line segment is divided into two equal parts:

1. The point of division is the *midpoint* of the line segment.

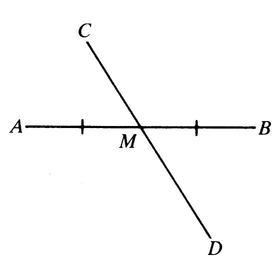


Fig. 1-1

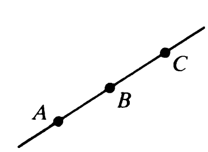


Fig. 1-2

2. A line that crosses at the midpoint is said to *bisect* the segment.

   Because $AM = MB$ in Fig. 1-1, M is the midpoint of $\overline{AB}$, and $\overline{CD}$ bisects $\overline{AB}$. Equal line segments may be shown by crossing them with the same number of strokes. Note that $\overline{AM}$ and $\overline{MB}$ are crossed with a single stroke.
3. If three points A, B, and C lie on a line, then we say they are *collinear*. If A, B, and C are collinear and $AB + BC = AC$, then B is between A and C (see Fig. 1-2).

1.3B Congruent Segments

Two line segments having the same length are said to be *congruent*. Thus, if $AB = CD$, then $\overline{AB}$ is congruent to $\overline{CD}$, written $\overline{AB} \cong \overline{CD}$.

SOLVED PROBLEMS

1.2 Naming line segments and points

See Fig. 1-3.

(a) Name each line segment shown.

(b) Name the line segments that intersect at A.

(c) What other line segment can be drawn using points A, B, C, and D?

(d) Name the point of intersection of $\overline{CD}$ and $\overline{AD}$.

(e) Name the point of intersection of $\overline{BC}$, $\overline{AC}$, and $\overline{CD}$.

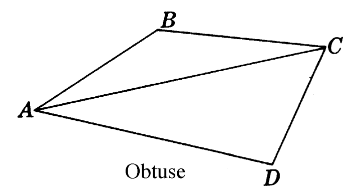


Fig. 1-3

Solutions

(a) $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AC}$, and $\overline{AD}$. These segments may also be named by interchanging the letters; thus, $\overline{BA}$, $\overline{CB}$, $\overline{DC}$, $\overline{CA}$, and $\overline{DA}$ are also correct.

(b) $\overline{AB}$, $\overline{AC}$, and $\overline{AD}$

(c) $\overline{BD}$

(d) D

(e) C

1.3 Finding lengths and points of line segments

See Fig. 1-4.

(a) State the lengths of $\overline{AB}$, $\overline{AC}$, and $\overline{AF}$.

(b) Name two midpoints.

(c) Name two bisectors.

(d) Name all congruent segments.

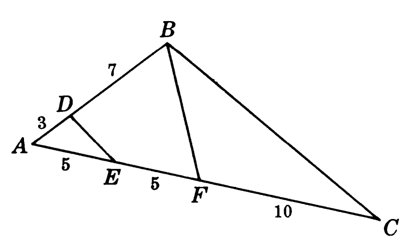


Fig. 1-4

Solutions

(a) $AB = 3 + 7 = 10$; $AC = 5 + 5 + 10 = 20$; $AF = 5 + 5 = 10$.

(b) E is midpoint of $\overline{AF}$; F is midpoint of $\overline{AC}$.

(c) $\overline{DE}$ is bisector of $\overline{AF}$; $\overline{BF}$ is bisector of $\overline{AC}$.

(d) $\overline{AB}$, $\overline{AF}$, and $\overline{FC}$ (all have length 10); $\overline{AE}$ and $\overline{EF}$ (both have length 5).

1.4 Circles

A *circle* is the set of all points in a plane that are the same distance from the *center*. The symbol for circle is $\odot$; for circles, Ⓢ. Thus, $\odot O$ stands for the circle whose center is O.

The *circumference* of a circle is the distance around the circle. It contains 360 *degrees* (360°).

A *radius* is a segment joining the center of a circle to a point on the circle (see Fig. 1-5). From the definition of a circle, it follows that the radii of a circle are congruent. Thus, $\overline{OA}$, $\overline{OB}$, and OC of Fig. 1-5 are radii of $\odot O$ and $\overline{OA} \cong \overline{OB} \cong \overline{OC}$.

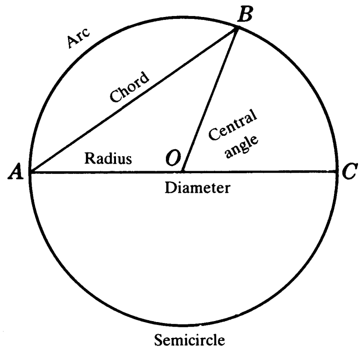


Fig. 1-5

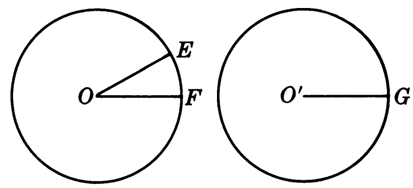


Fig. 1-6

A *chord* is a segment joining any two points on a circle. Thus, $\overline{AB}$ and $\overline{AC}$ are chords of $\odot O$.

A *diameter* is a chord through the center of the circle; it is the longest chord and is twice the length of a radius. $\overline{AC}$ is a diameter of $\odot O$.

An *arc* is a continuous part of a circle. The symbol for arc is $\frown$, so that $\overset{\frown}{AB}$ stands for arc AB. An arc of measure 1° is 1/360th of a circumference.

A *semicircle* is an arc measuring one-half of the circumference of a circle and thus contains 180°. A diameter divides a circle into two semicircles. For example, diameter $\overline{AC}$ cuts $\odot O$ of Fig. 1-5 into two semicircles.

A *central angle* is an angle formed by two radii. Thus, the angle between radii $\overline{OB}$ and $\overline{OC}$ is a central angle. A central angle measuring 1° cuts off an arc of 1°; thus, if the central angle between $\overline{OE}$ and $\overline{OF}$ in Fig. 1-6 is 1°, then $\overset{\frown}{EF}$ measures 1°.

Congruent circles are circles having congruent radii. Thus, if $\overline{OE} \cong \overline{O'G}$, then $\odot O \cong \odot O'$.

SOLVED PROBLEMS

1.4 Finding lines and arcs in a circle

In Fig. 1-7 find (a) OC and AB; (b) the number of degrees in $\overset{\frown}{AD}$; (c) the number of degrees in $\overset{\frown}{BC}$.

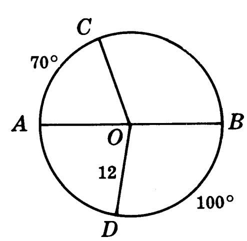


Fig. 1-7

Solutions

(a) Radius OC = radius OD = 12. Diameter AB = 24.

(b) Since semicircle ADB contains 180°, $\overset{\frown}{AD}$ contains 180° − 100° = 80°.

(c) Since semicircle ACB contains 180°, $\overset{\frown}{BC}$ contains 180° − 70° = 110°.

1.5 Angles

An *angle* is the figure formed by two rays with a common end point. The rays are the *sides* of the angle, while the end point is its *vertex*. The symbol for angle is ∠ or ∡; the plural is ∠s.

Thus, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the sides of the angle shown in Fig. 1-8(a), and A is its vertex.

1.5A Naming an Angle

An angle may be named in any of the following ways:

1. With the vertex letter, if there is only one angle having this vertex, as $\angle B$ in Fig. 1-8(b).
2. With a small letter or a number placed between the sides of the angle and near the vertex, as $\angle a$ or $\angle 1$ in Fig. 1-8(c).
3. With three capital letters, such that the vertex letter is between two others, one from each side of the angle. In Fig. 1-8(d), $\angle E$ may be named $\angle DEG$ or $\angle GED$.

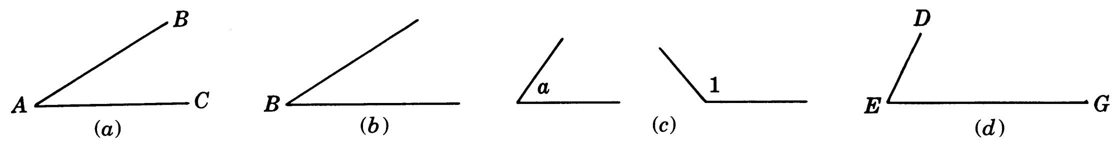


Fig. 1-8

1.5B Measuring the Size of an Angle

The size of an angle depends on the extent to which one side of the angle must be rotated, or turned about the vertex, until it meets the other side. We choose degrees to be the unit of measure for angles. The measure of an angle is the number of degrees it contains. We will write $m\angle A = 60°$ to denote that "angle A measures 60°."

The protractor in Fig. 1-9 shows that $\angle A$ measures of 60°. If $\overrightarrow{AC}$ were rotated about the vertex A until it met $\overrightarrow{AB}$, the amount of turn would be 60°.

In using a protractor, be sure that the vertex of the angle is at the center and that one side is along the $0°-180°$ diameter.

The size of an angle *does not* depend on the lengths of the sides of the angle.

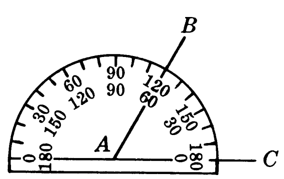


Fig. 1-9

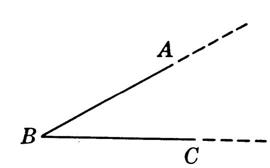


Fig. 1-10

The size of $\angle B$ in Fig. 1-10 would not be changed if its sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ were made larger or smaller.

No matter how large or small a clock is, the angle formed by its hands at 3 o'clock measures 90°, as shown in Figs. 1-11 and 1-12.

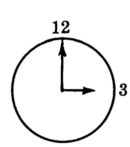


Fig. 1-11

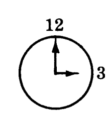


Fig. 1-12

Angles that measure less than 1° are usually represented as fractions or decimals. For example, one-thousandth of the way around a circle is either $\frac{360°}{1000}$ or 0.36°.

In some fields, such as navigation and astronomy, small angles are measured in *minutes* and *seconds*. One degree is comprised of 60 minutes, written $1° = 60'$. A minute is 60 seconds, written $1' = 60''$. In this notation, one-thousandth of a circle is $21'36''$ because $\frac{21}{60} + \frac{36}{3600} = \frac{1296}{3600} = \frac{360}{1000}$.

1.5C Kinds of Angles

1. *Acute angle*: An acute angle is an angle whose measure is less than 90°.

   Thus, in Fig. 1-13 $a°$ is less than 90°; this is symbolized as $a° < 90°$.

2. *Right angle*: A right angle is an angle that measures 90°.

   Thus, in Fig. 1-14, $m(\text{rt. } \angle A) = 90°$. The square corner denotes a right angle.

3. *Obtuse angle*: An obtuse angle is an angle whose measure is more than 90° and less than 180°.

   Thus, in Fig. 1-15, 90° is less than $b°$ and $b°$ is less than 180°; this is denoted by $90° < b° < 180°$.

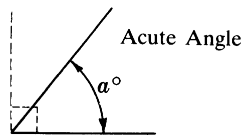


Fig. 1-13

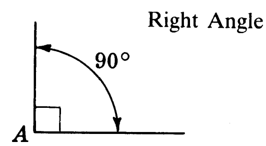


Fig. 1-14

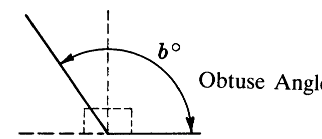


Fig. 1-15

4. *Straight angle*: A straight angle is an angle that measures 180°.

   Thus, in Fig. 1-16, $m(\text{st. } \angle B) = 180°$. Note that the sides of a straight angle lie in the same straight line. But do not confuse a straight angle with a straight line!

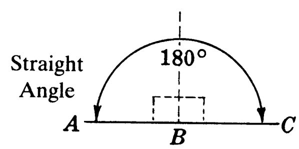


Fig. 1-16

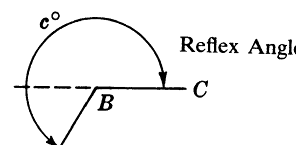


Fig. 1-17

5. *Reflex angle*: A reflex angle is an angle whose measure is more than 180° and less than 360°.

   Thus, in Fig. 1-17, 180° is less than $c°$ and $c°$ is less than 360°; this is symbolized as $180° < c° < 360°$.

1.5D Additional Angle Facts

1. *Congruent angles* are angles that have the same number of degrees. In other words, if $m\angle A = m\angle B$, then $\angle A \cong \angle B$.

   Thus, in Fig. 1-18, rt. $\angle A \cong$ rt. $\angle B$ since each measures 90°.

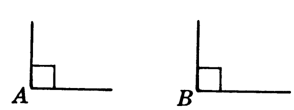


Fig. 1-18

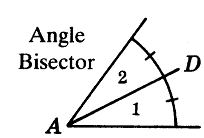


Fig. 1-19

2. A line that *bisects* an angle divides it into two congruent parts.

   Thus, in Fig. 1-19, if $\overline{AD}$ bisects $\angle A$, then $\angle 1 \cong \angle 2$. (Congruent angles may be shown by crossing their arcs with the same number of strokes. Here the arcs of $\angle$s 1 and 2 are crossed by a single stroke.)

3. *Perpendiculars* are lines or rays or segments that meet at right angles.

   The symbol for perpendicular is $\perp$; for perpendiculars, $\perp$s. In Fig. 1-20, $\overline{CD} \perp \overline{AB}$, so right angles 1 and 2 are formed.

4. A *perpendicular bisector* of a given segment is perpendicular to the segment and bisects it.

   In Fig. 1-21, $\overleftrightarrow{GH}$ is the bisector of $\overline{EF}$; thus, $\angle 1$ and $\angle 2$ are right angles and M is the midpoint of $\overline{EF}$.

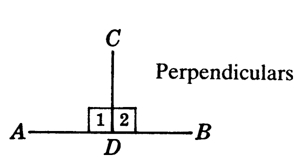


Fig. 1-20

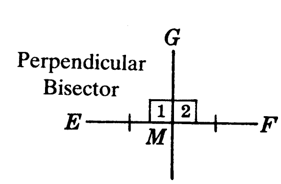


Fig. 1-21

SOLVED PROBLEMS

1.5 Naming an angle

Name the following angles in Fig. 1-22: (a) two obtuse angles; (b) a right angle; (c) a straight angle; (d) an acute angle at D; (e) an acute angle at B.

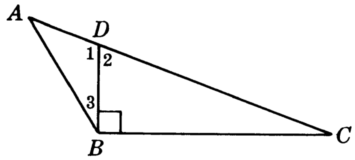


Fig. 1-22

Solutions

(a) $\angle ABC$ and $\angle ADB$ (or $\angle 1$). The angles may also be named by reversing the order of the letters: $\angle CBA$ and $\angle BDA$.

(b) $\angle DBC$

(c) $\angle ADC$

(d) $\angle 2$ or $\angle BDC$

(e) $\angle 3$ or $\angle ABD$

1.6 Adding and subtracting angles

In Fig. 1-23, find (a) $m\angle AOC$; (b) $m\angle BOE$; (c) the measure of obtuse $\angle AOE$.

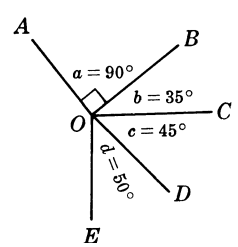


Fig. 1-23

Solutions

(a) $m\angle AOC = m\angle a + m\angle b = 90° + 35° = 125°$

(b) $m\angle BOE = m\angle b + m\angle c + m\angle d = 35° + 45° + 50° = 130°$

(c) $m\angle AOE = 360° - (m\angle a + m\angle b + m\angle c + m\angle d) = 360° - 220° = 140°$

1.7 Finding parts of angles

Find (a) $\frac{2}{5}$ of the measure of a rt. $\angle$; (b) $\frac{2}{3}$ of the measure of a st. $\angle$; (c) $\frac{1}{2}$ of 31°; (d) $\frac{1}{10}$ of 70°20′.

Solutions

(a) $\frac{2}{5}(90°) = 36°$

(b) $\frac{2}{3}(180°) = 120°$

(c) $\frac{1}{2}(31°) = 15\frac{1}{2}° = 15°30'$

(d) $\frac{1}{10}(70°20') = \frac{1}{10}(70°) + \frac{1}{10}(20') = 7°2'$

1.8 Finding rotations

In a half hour, what turn or rotation is made (a) by the minute hand, and (b) by the hour hand of a clock? What rotation is needed to turn (c) from north to southeast in a clockwise direction, and (d) from northwest to southwest in a counterclockwise direction (see Fig. 1-24)?

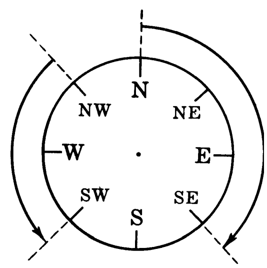


Fig. 1-24

Solutions

(a) In 1 hour, a minute hand completes a full circle of 360°. Hence, in a half hour it turns 180°.

(b) In 1 hour, an hour hand turns $\frac{1}{12}$ of 360° or 30°. Hence, in a half hour it turns 15°.

(c) Add a turn of 90° from north to east and a turn of 45° from east to southeast to get $90° + 45° = 135°$.

(d) The turn from northwest to southwest is $\frac{1}{4}(360°) = 90°$.

1.9 Finding angles

Find the measure of the angle formed by the hands of the clock in Fig. 1-25, (a) at 8 o'clock; (b) at 4:30.

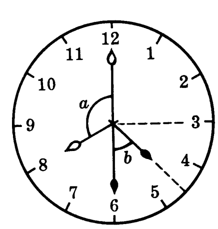


Fig. 1-25

Solutions

(a) At 8 o'clock, $m\angle a = \frac{1}{3}(360°) = 120°$.

(b) At 4:30, $m\angle b = \frac{1}{2}(90°) = 45°$.

1.10 Applying angle facts

In Fig. 1-26, (a) name two pairs of perpendicular segments; (b) find $m\angle a$ if $m\angle b = 42°$; (c) find $m\angle AEB$ and $m\angle CED$.

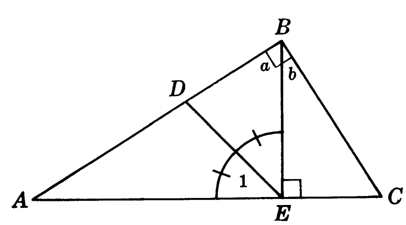


Fig. 1-26

Solutions

(a) Since $\angle ABC$ is a right angle, $\overline{AB} \perp \overline{BC}$. *Since* $\angle BEC$ is a right angle, $\overline{BE} \perp \overline{AC}$.

(b) $m\angle a = 90° - m\angle b = 90° - 42° = 48°$.

(c) $m\angle AEB = 180° - m\angle BEC = 180° - 90° = 90°$. $m\angle CED = 180° - m\angle 1 = 180° - 45° = 135°$.

1.6 Triangles

A *polygon* is a closed plane figure bounded by straight line segments as sides. Thus, Fig. 1-27 is a polygon of five sides, called a *pentagon*; it is named pentagon *ABCDE*, using its letters in order.

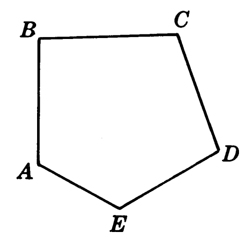


Fig. 1-27

A *quadrilateral* is a polygon having four sides.

A *triangle* is a polygon having three sides. A *vertex* of a triangle is a point at which two of the sides meet. (*Vertices* is the plural of vertex.) The symbol for triangle is $\triangle$; for triangles, $\triangle\!\!\!\triangle$.

A triangle may be named with its three letters in any order or with a Roman numeral placed inside of it. Thus, the triangle shown in Fig. 1-28 is $\triangle ABC$ *or* $\triangle$I; its sides are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$; its vertices are *A, B,* and *C*; its angles are $\angle A$, $\angle B$, and $\angle C$.

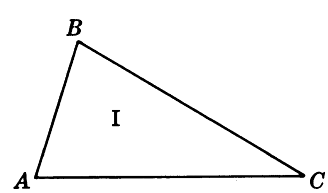


Fig. 1-28

1.6A Classifying Triangles

Triangles are classified according to the equality of the lengths of their sides or according to the kind of angles they have.

Triangles According to the Equality of the Lengths of their Sides (Fig. 1-29)

1. *Scalene triangle*: A scalene triangle is a triangle having no congruent sides.

   Thus in scalene triangle *ABC*, $a \neq b \neq c$. The small letter used for the length of each side agrees with the capital letter of the angle opposite it. Also, $\neq$ means "is not equal to."

2. *Isosceles triangle*: An isosceles triangle is a triangle having at least two congruent sides.

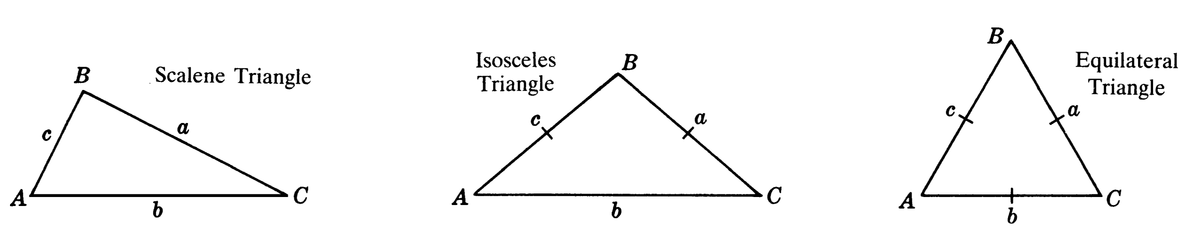


Fig. 1-29

Thus in isosceles triangle ABC, $a = c$. These equal sides are called the *legs* of the isosceles triangle; the remaining side is the *base, b*. The angles on either side of the base are the *base angles*; the angle opposite the base is the *vertex angle.*

3. *Equilateral triangle*: An equilateral triangle is a triangle having three congruent sides.

Thus in equilateral triangle ABC, $a = b = c$. Note that an equilateral triangle is also an isosceles triangle.

Triangles According to the Kind of Angles (Fig. 1-30)

1. *Right triangle*: A right triangle is a triangle having a right angle.

Thus in right triangle ABC, $\angle C$ is the right angle. Side c opposite the right angle is the *hypotenuse*. The perpendicular sides, a and b, are the *legs* or *arms* of the right triangle.

2. *Obtuse triangle*: An obtuse triangle is a triangle having an obtuse angle.

Thus in obtuse triangle DEF, $\angle D$ is the obtuse angle.

3. *Acute triangle*: An acute triangle is a triangle having three acute angles.

Thus in acute triangle HJK, $\angle H$, $\angle J$, and $\angle K$ are acute angles.

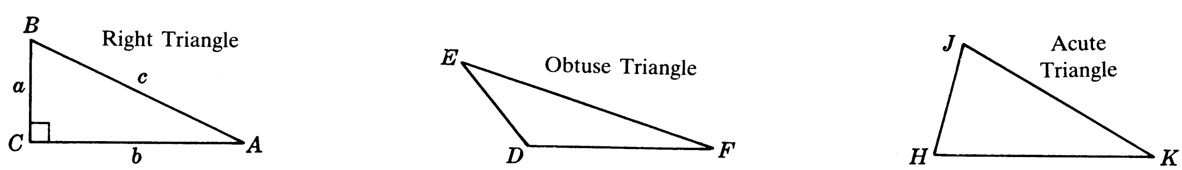


Fig. 1-30

1.6B Special Lines in a Triangle

1. *Angle bisector of a triangle*: An angle bisector of a triangle is a segment or ray that bisects an angle and extends to the opposite side.

Thus $\overline{BD}$, the angle bisector of $\angle B$ in Fig. 1-31, bisects $\angle B$, making $\angle 1 \cong \angle 2$.

2. *Median of a triangle*: A median of a triangle is a segment from a vertex to the midpoint of the opposite side.

Thus $\overline{BM}$, the median to $\overline{AC}$, in Fig. 1-32, bisects $\overline{AC}$, making $AM = MC$.

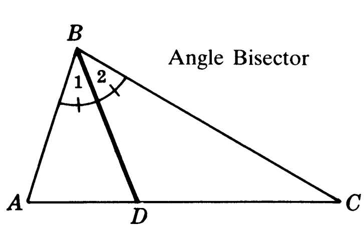


Fig. 1-31

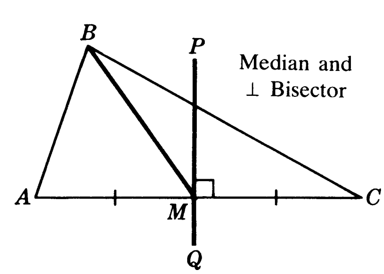


Fig. 1-32

3. *Perpendicular bisector of a side*: A perpendicular bisector of a side of a triangle is a line that bisects and is perpendicular to a side.

Thus $\overleftrightarrow{PQ}$, the perpendicular bisector of $\overline{AC}$ in Fig. 1-32, bisects $\overline{AC}$ and is perpendicular to it.

4. *Altitude to a side of a triangle*: An altitude of a triangle is a segment from a vertex perpendicular to the opposite side.

Thus $\overline{BD}$, the altitude to $\overline{AC}$ in Fig. 1-33, is perpendicular to $\overline{AC}$ and forms right angles 1 and 2. Each angle bisector, median, and altitude of a triangle extends from a vertex to the opposite side.

5. *Altitudes of obtuse triangle*: In an obtuse triangle, the altitude drawn to either side of the obtuse angle falls outside the triangle.

Thus in obtuse triangle ABC (shaded) in Fig. 1-34, altitudes $\overline{BD}$ and $\overline{CE}$ fall outside the triangle. In each case, a side of the obtuse angle must be extended.

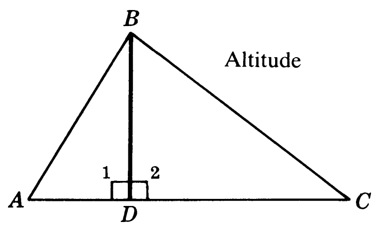


Fig. 1-33

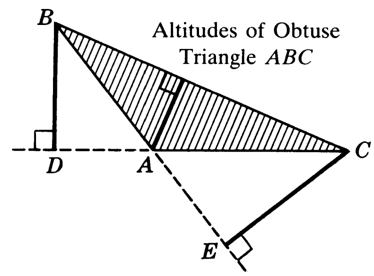


Fig. 1-34

SOLVED PROBLEMS

1.11 Naming a triangle and its parts

In Fig. 1-35, name (a) an obtuse triangle, and (b) two right triangles and the hypotenuse and legs of each. (c) In Fig. 1-36, name two isosceles triangles; also name the legs, base, and vertex angle of each.

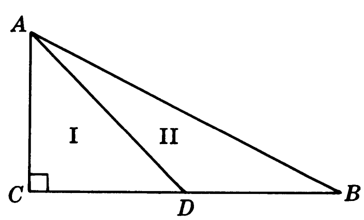


Fig. 1-35

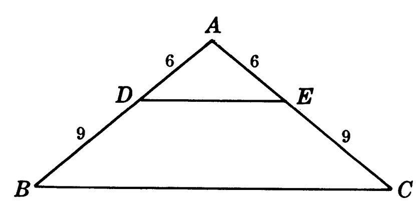


Fig. 1-36

Solutions

(a) Since $\angle ADB$ is an obtuse angle, $\angle ADB$ *or* $\triangle$II is obtuse.

(b) Since$\angle C$ is a right angle, $\triangle$I and $\triangle ABC$ are right triangles. In $\triangle$I, $\overline{AD}$ is the hypotenuse and $\overline{AC}$ and $\overline{CD}$ are the legs. In $\triangle ABC$, AB is the hypotenuse and $\overline{AC}$ and $\overline{BC}$ are the legs.

(c) Since $AD = AE$, $\triangle ADE$ is an isosceles triangle. In $\triangle ADE$, $\overline{AD}$ and $\overline{AE}$ are the legs, $\overline{DE}$ is the base, and $\angle A$ is the vertex angle.

Since $AB = AC$, $\triangle ABC$ is an isosceles triangle. In $\triangle ABC$, $\overline{AB}$ and $\overline{AC}$ are the legs, $\overline{BC}$ is the base, and $\angle A$ is the vertex angle.

1.12 Special lines in a triangle

Name the equal segments and congruent angles in Fig. 1-37, (a) if $\overline{AE}$ is the altitude to $\overline{BC}$; (b) if $\overline{CG}$ bisects $\angle ACB$; (c) if $\overline{KL}$ is the perpendicular bisector of $\overline{AD}$; (d) if $\overline{DF}$ is the median to $\overline{AC}$.

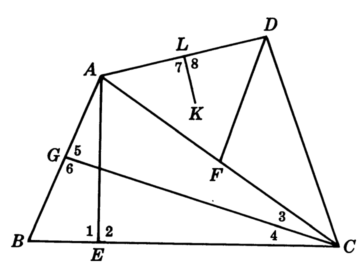


Fig. 1-37

Solutions

(a) Since $\overline{AE} \perp \overline{BC}$, $\angle 1 \cong \angle 2$.

(b) Since $\overline{CG}$ bisects $\angle ACB$, $\angle 3 \cong \angle 4$.

(c) Since $\overline{LK}$ is the $\perp$ bisector of $\overline{AD}$, $AL = LD$ and $\angle 7 \cong \angle 8$.

(d) Since $\overline{DF}$ is median to $\overline{AC}$, $AF = FC$.

1.7 Pairs of Angles

1.7A Kinds of Pairs of Angles

1. *Adjacent angles*: Adjacent angles are two angles that have the same vertex and a common side between them.

Thus, the entire angle of $c°$ in Fig. 1-38 has been cut into two adjacent angles of $a°$ and $b°$. These adjacent angles have the same vertex A, and a common side $\overrightarrow{AD}$ between them. Here, $a° + b° = c°$.

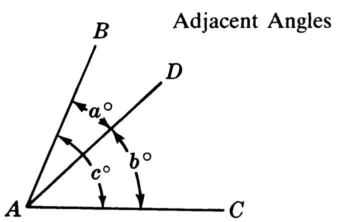


Fig. 1-38

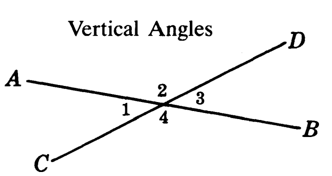


Fig. 1-39

2. *Vertical angles*: Vertical angles are two nonadjacent angles formed by two intersecting lines.

Thus, $\angle 1$ and $\angle 3$ in Fig. 1-39 are vertical angles formed by intersecting lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Also, $\angle 2$ and $\angle 4$ are another pair of vertical angles formed by the same lines.

3. *Complementary angles*: Complementary angles are two angles whose measures total 90°.

Thus, in Fig. 1-40(a) the angles of $a°$ and $b°$ are adjacent complementary angles. However, in (b) the complementary angles are nonadjacent. In each case, $a° + b° = 90°$. Either of two complementary angles is said to be the *complement* of the other.

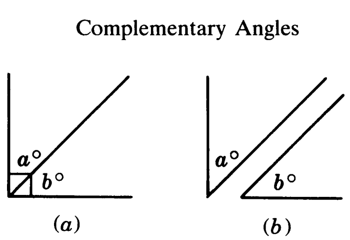


Fig. 1-40

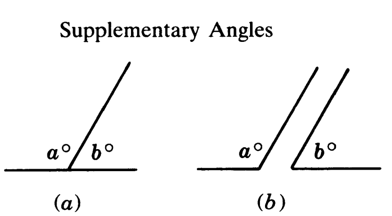


Fig. 1-41

4. *Supplementary angles*: Supplementary angles are two angles whose measures total 180°.

Thus, in Fig. 1-41(a) the angles of $a°$ and $b°$ are adjacent supplementary angles. However, in Fig. 1-41(b) the supplementary angles are nonadjacent. In each case, $a° + b° = 180°$. Either of two supplementary angles is said to be the *supplement* of the other.

1.7B Principles of Pairs of Angles

PRINCIPLE 1: *If an angle of $c°$ is cut into two adjacent angles of $a°$ and $b°$, then $a° + b° = c°$.*

Thus if $a° = 25°$ and $b° = 35°$ in Fig. 1-42, then $c° + 25° + 35° = 60°$.

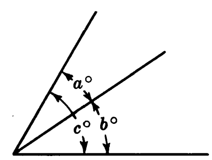


Fig. 1-42

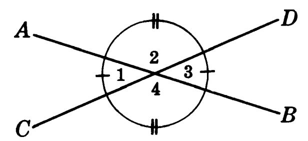


Fig. 1-43

PRINCIPLE 2: *Vertical angles are congruent.*

Thus if $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are straight lines in Fig. 1-43, then $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. Hence, if $m\angle 1 = 40°$, then $m\angle 3 = 40°$; in such a case, $m\angle 2 = m\angle 4 = 140°$.

PRINCIPLE 3: *If two complementary angles contain $a°$ and $b°$, then $a° + b° = 90°$.*

Thus if angles of $a°$ and $b°$ are complementary and $a° = 40°$, then $b° = 50°$ [Fig. 1-44(a) or (b)].

PRINCIPLE 4: *Adjacent angles are complementary if their exterior sides are perpendicular to each other.*

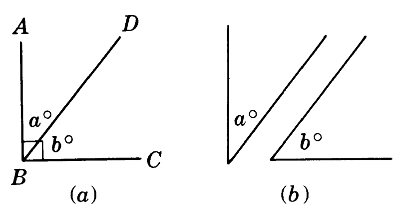


Fig. 1-44

Thus in Fig. 1-44(a), $a°$ and $b°$ are complementary since their exterior sides $\overline{AB}$ and $\overline{BC}$ are perpendicular to each other.

PRINCIPLE 5: *If two supplementary angles contain $a°$ and $b°$, then $a° + b° = 180°$.*

Thus if angles of $a°$ and $b°$ are supplementary and $a° = 140°$, then $b° = 40°$ [Fig. 1-45(a) or (b)].

PRINCIPLE 6: *Adjacent angles are supplementary if their exterior sides lie in the same straight line.*

Thus in Fig. 1-45(a) $a°$ and $b°$ are supplementary angles since their exterior sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ lie in the same straight line $\overrightarrow{AC}$.

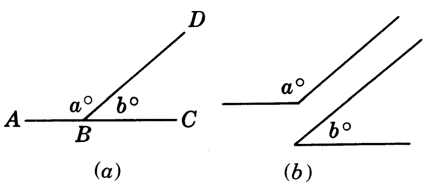


Fig. 1-45

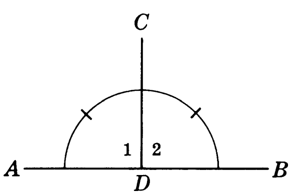


Fig. 1-46

PRINCIPLE 7: *If supplementary angles are congruent, each of them is a right angle. (Equal supplementary angles are right angles.)*

Thus if $\angle 1$ and $\angle 2$ in Fig. 1-46 are both congruent and supplementary, then each of them is a right angle.

SOLVED PROBLEMS

1.13 Naming pairs of angles

(a) In Fig. 1-47(a), name two pairs of supplementary angles.

(b) In Fig. 1-47(b), name two pairs of complementary angles.

(c) In Fig. 1-47(c), name two pairs of vertical angles.

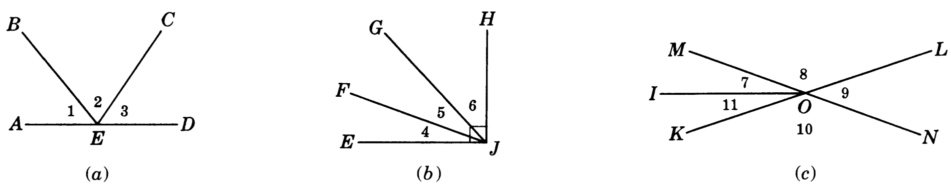


Fig. 1-47

Solutions

(a) Since their sum is 180°, the supplementary angles are (1) $\angle 1$ and $\angle BED$; (2) $\angle 3$ and $\angle AEC$.

(b) Since their sum is 90°, the complementary angles are (1) $\angle 4$ and $\angle FJH$; (2) $\angle 6$ and $\angle EJG$.

(c) Since $\overleftrightarrow{KL}$ and $\overleftrightarrow{MN}$ are intersecting lines, the vertical angles are (1) $\angle 8$ and $\angle 10$; (2) $\angle 9$ and $\angle MOK$.

1.14 Finding pairs of angles

Find two angles such that:

(a) The angles are supplementary and the larger is twice the smaller.

(b) The angles are complementary and the larger is 20° more than the smaller.

(c) The angles are adjacent and form an angle of 120°. The larger is 20° less than three times the smaller.

(d) The angles are vertical and complementary.

Solutions

In each solution, x is a number only. This number indicates the number of degrees contained in the angle. Hence, if $x = 60$, the angle measures 60°.

(a) Let $x = m$ (smaller angle) and $2x = m$ (larger angle), as in Fig. 1-48(a).
Principle 5: $x + 2x = 180$, so $3x = 180$; $x = 60$.
$2x = 120$. *Ans.* 60° and 120°

(b) Let $x = m$ (smaller angle) and $x + 20 = m$ (larger angle), as in Fig. 1-48(b).
Principle 3: $x + (x + 20) = 90$, or $2x + 20 = 90$; $x = 35$.
$x + 20 = 55$. *Ans.* 35° and 55°

(c) Let $x = m$ (smaller angle) and $3x - 20 = m$ (larger angle) as in Fig. 1-48(c).
Principle 1: $x + (3x - 20) = 120$, or $4x - 20 = 120$; $x = 35$.
$3x - 20 = 85$. *Ans.* 35° and 85°

(d) Let $x = m$ (each vertical angle), as in Fig. 1-48(d). They are congruent by Principle 2.
Principle 3: $x + x = 90°$, or $2x = 90$; $x = 45$. *Ans.* 45° each.

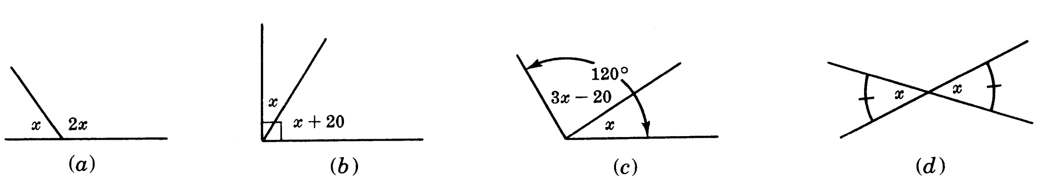


Fig. 1-48

1.15 Finding a pair of angles using two unknowns

For each of the following, be represented by a and b. Obtain two equations for each case, and then find the angles.

(a) The angles are adjacent, forming an angle of 88°. One is 36° more than the other.

(b) The angles are complementary. One is twice as large as the other.

(c) The angles are supplementary. One is 60° less than twice the other.

(d) The angles are supplementary. The difference of the angles is 24°.

Solutions

(a) $a + b = 88$
$a = b + 36$ *Ans.* 62° and 26°

(b) $a + b = 90$
$a = 2b$ *Ans.* 60° and 30°

(c) $a + b = 180$
$a = 2b - 60$ *Ans.* 100° and 80°

(d) $a + b = 180°$
$a - b = 24°$ *Ans.* 78° and 102°

SUPPLEMENTARY PROBLEMS

1.1. Point, line, and plane are undefined terms. Which of these is illustrated by (a) the tip of a sharpened pencil; (b) the shaving edge of a blade; (c) a sheet of paper; (d) a side of a box; (e) the crease of a folded paper; (f) the junction of two roads on a map? (1.1)

1.2. (a) Name the line segments that intersect at E in Fig. 1-49. (1.2)

(b) Name the line segments that intersect at D.

(c) What other line segments can be drawn using points A, B, C, D, E, and F?

(d) Name the point of intersection of $\overline{AC}$ and $\overline{BD}$.

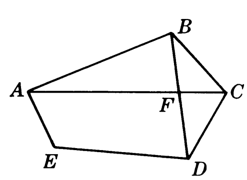


Fig. 1-49

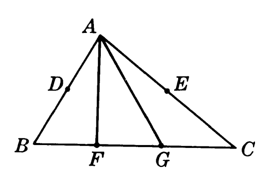


Fig. 1-50

1.3. (a) Find the length of $\overline{AB}$ in Fig. 1-50 if AD is 8 and D is the midpoint of $\overline{AB}$ (1.3)

(b) Find the length of $\overline{AE}$ if AC is 21 and E is the midpoint of $\overline{AC}$.

1.4. (a) Find OB in Fig. 1-51 if diameter $AD = 36$. (1.4)

(b) Find the number of degrees in $\widehat{AE}$ if E is the midpoint of semicircle $\widehat{AED}$. Find the number of degrees in (c) $\widehat{CD}$; (*d*) $\widehat{AC}$; (e) $\widehat{AEC}$.

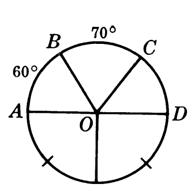


Fig. 1-51

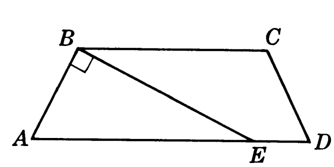


Fig. 1-52

1.5. Name the following angles in Fig. 1-52 (a) an acute angle at B; (b) an acute angle at E; (c) a right angle; (d) three obtuse angles; (e) a straight angle. (1.5)

1.6. (a) Find $m\angle ADC$ if $m\angle c = 45°$ and $m\angle d = 85°$ in Fig. 1-53. (1.6)

(b) Find $m\angle AEB$ if $m\angle e = 60°$.

(c) Find $m\angle EBD$ if $m\angle a = 15°$.

(d) Find $m\angle ABC$ if $m\angle b = 42°$.

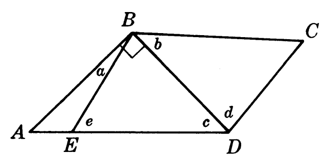


Fig. 1-53

1.7. Find (a) $\frac{5}{6}$ of a rt. $\angle$; (b) $\frac{2}{9}$ of a st. $\angle$; (c) $\frac{1}{3}$ of 31°; (d) $\frac{1}{5}$ of 45°55′. (1.7)

1.8. What turn or rotation is made (a) by an hour hand in 3 hours; (b) by the minute hand in $\frac{1}{3}$ of an hour? What rotation is needed to turn from (c) west to northeast in a clockwise direction; (d) east to south in a counterclockwise direction; (e) southwest to northeast in either direction? (1.8)

1.9. Find the angle formed by the hand of a clock (a) at 3 o'clock; (b) at 10 o'clock; (c) at 5:30 AM; (d) at 11:30 PM. (1.9)

1.10. In Fig. 1-54: (1.10)

(a) Name two pairs of perpendicular lines.

(b) Find $m\angle BCD$ if $m\angle 4$ is 39°.

If $m\angle 1 = 78°$, find (c) $m\angle BAD$; (d) $m\angle 2$; (e) $m\angle CAE$.

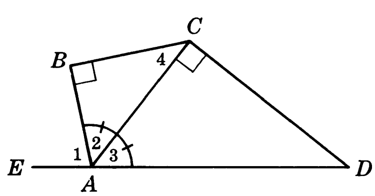


Fig. 1-54

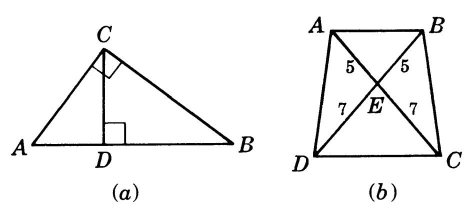


Fig. 1-55

1.11. (a) In Fig. 1-55(a), name three right triangles and the hypotenuse and legs of each. (1.11)

In Fig. 1-55(b), (b) name two obtuse triangles and (c) name two isosceles triangles, also naming the legs, base, and vertex angle of each.

1.12. In Fig. 1-56, name the congruent lines and angles (a) if $\overline{PR}$ is a $\perp$ bisector of $\overline{AB}$; (b) if $\overline{BF}$ bisects $\angle ABC$; (c) if $\overline{CG}$ is an altitude to $\overline{AD}$; (d) if $\overline{EM}$ is a median to $\overline{AD}$. (1.12)

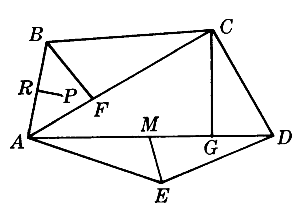


Fig. 1-56

1.13. In Fig. 1-57, state the relationship between: (1.13)

(a) $\angle 1$ and $\angle 4$ (d) $\angle 4$ and $\angle 5$

(b) $\angle 3$ and $\angle 4$ (e) $\angle 1$ and $\angle 3$

(c) $\angle 1$ and $\angle 2$ (f) $\angle AOD$ and $\angle 5$

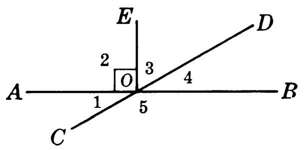


Fig. 1-57

1.14. Find two angles such that: (1.14)

(a) The angles are complementary and the measure of the smaller is 40° less than the measure of the larger.

(b) The angles are complementary and the measure of the larger is four times the measure of the smaller.

(c) The angles are supplementary and the measure of the smaller is one-half the measure of the larger.

(d) The angles are supplementary and the measure of the larger is 58° more than the measure of the smaller.

(e) The angles are supplementary and the measure of the larger is 20° less than three times the measure of the smaller.

(f) The angles are adjacent and form an angle measuring 140°. The measure of the smaller is 28° less than the measure of the larger.

(g) The angles are vertical and supplementary.

1.15. For each of the following, let the two angles be represented by a and b. Obtain two equations for each case, and then find the angles. (1.15)

(a) The angles are adjacent and form an angle measuring 75°. Their difference is 21°.

(b) The angles are complementary. One measures 10° less than three times the other.

(c) The angles are supplementary. One measures 20° more than four times the other.