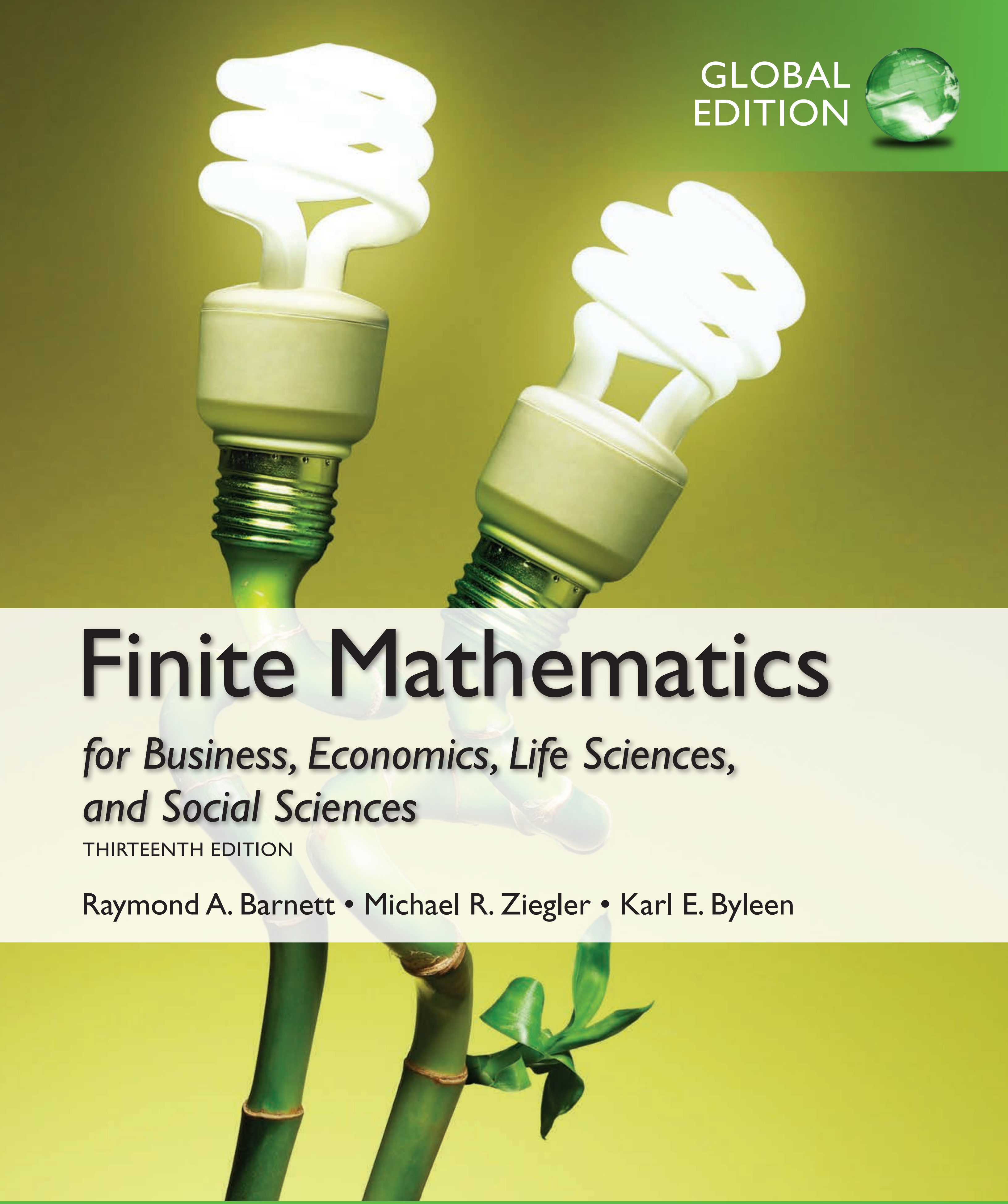

GLOBAL EDITION

Finite Mathematics

for Business, Economics, Life Sciences, and Social Sciences

THIRTEENTH EDITION

Raymond A. Barnett • Michael R. Ziegler • Karl E. Byleen

ALWAYS LEARNING

PEARSON

FINITE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

Thirteenth Edition
Global Edition

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CONTENTS

PREFACE

The thirteenth edition of *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a one-term course in finite mathematics for students who have had one to two years of high school algebra or the equivalent. The book's overall approach, refined by the authors' experience with large sections of college freshmen, addresses the challenges of teaching and learning when prerequisite knowledge varies greatly from student to student.

The authors had three main goals when writing this text:

- To write a text that students can easily comprehend
- To make connections between what students are learning and how they may apply that knowledge
- To give flexibility to instructors to tailor a course to the needs of their students.

Many elements play a role in determining a book's effectiveness for students. Not only is it critical that the text be accurate and readable, but also, in order for a book to be effective, aspects such as the page design, the interactive nature of the presentation, and the ability to support and challenge all students have an incredible impact on how easily students comprehend the material. Here are some of the ways this text addresses the needs of students at all levels:

- Page layout is clean and free of potentially distracting elements.
- *Matched Problems* that accompany each of the completely worked examples help students gain solid knowledge of the basic topics and assess their own level of understanding before moving on.
- Review material (Appendix A and Chapters 1 and 2) can be used judiciously to help remedy gaps in prerequisite knowledge.
- A *Diagnostic Prerequisite Test* prior to Chapter 1 helps students assess their skills, while the *Basic Algebra Review* in Appendix A provides students with the content they need to remediate those skills.
- *Explore and Discuss* problems lead the discussion into new concepts or build upon a current topic. They help students of all levels gain better insight into the mathematical concepts through thought-provoking questions that are effective in both small and large classroom settings.
- Instructors are able to easily craft homework assignments that best meet the needs of their students by taking advantage of the variety of types and difficulty levels of the exercises. Exercise sets at the end of each section consist of a ***Skills Warm-up*** (four to eight problems that review prerequisite knowledge specific to that section) followed by problems divided into categories A, B, and C by level of difficulty, with level-C exercises being the most challenging.
- The MyMathLab course for this text is designed to help students help themselves and provide instructors with actionable information about their progress. The immediate feedback students receive when doing homework and practice in MyMathLab is invaluable, and the easily accessible e-book enhances student learning in a way that the printed page sometimes cannot.

Most important, all students get substantial experience in modeling and solving real-world problems through application examples and exercises chosen from business and economics, life sciences, and social sciences. Great care has been taken to write a book that is mathematically correct, with its emphasis on computational skills, ideas, and problem solving rather than mathematical theory.

Finally, the choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependencies chart on page 11). This text is one of three books in the authors' college mathematics series. The others are *Calculus for Business, Economics, Life Sciences, and Social Sciences*, and *College Mathematics for Business, Economics, Life Sciences, and Social Sciences*; the latter contains selected content from the other two books. *Additional Calculus Topics*, a supplement written to accompany the Barnett/Ziegler/Byleen series, can be used in conjunction with any of these books.

New to This Edition

Fundamental to a book's effectiveness is classroom use and feedback. Now in its thirteenth edition, *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of a substantial amount of both. Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. In this edition,

- The Diagnostic Prerequisite Test has been revised to identify the specific deficiencies in prerequisite knowledge that cause students the most difficulty with finite mathematics.
- Most exercise sets now begin with a ***Skills Warm-up***—four to eight problems that review prerequisite knowledge specific to that section in a just-in-time approach. References to review material are given for the benefit of students who struggle with the warm-up problems and need a refresher.
- Section 6.1 has been rewritten to better motivate and introduce the simplex method and associated terminology.
- Examples and exercises have been given up-to-date contexts and data.
- Exposition has been simplified and clarified throughout the book.
- An Annotated Instructor's Edition is now available, providing answers to exercises directly on the page (whenever possible). *Teaching Tips* provide less-experienced instructors with insight on common student pitfalls, suggestions for how to approach a topic, or reminders of which prerequisite skills students will need. Lastly, the difficulty level of exercises is indicated only in the AIE so as not to discourage students from attempting the most challenging "C" level exercises.
- *MyMathLab* for this text has been enhanced greatly in this revision. Most notably, a "Getting Ready for Chapter X" has been added to each chapter as an optional resource for instructors and students as a way to address the prerequisite skills that students need, and are often missing, for each chapter. Many more improvements have been made. See the detailed description on pages 14 and 15 for more information.

Trusted Features

Emphasis and Style

As was stated earlier, this text is written for student comprehension. To that end, the focus has been on making the book both mathematically correct and accessible to students. Most derivations and proofs are omitted, except where their inclusion adds significant insight into a particular concept as the emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. General concepts and results are typically presented only after particular cases have been discussed.

Design

One of the hallmark features of this text is the **clean, straightforward design** of its pages. Navigation is made simple with an obvious hierarchy of key topics and a judicious use of call-outs and pedagogical features. We made the decision to maintain a two-color design to help students stay focused on the mathematics and applications. Whether students start in the chapter opener or in the exercise sets, they can easily reference the content, examples, and *Conceptual Insights* they need to understand the topic at hand. Finally, a functional use of color improves the clarity of many illustrations, graphs, and explanations, and guides students through critical steps (see pages 77, 124, and 418).

Examples and Matched Problems

More than 300 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. The examples are annotated using blue text to the right of each step, and the problem-solving steps are clearly identified. **To give students extra help** in working through examples, dashed boxes are used to enclose steps that are usually performed mentally and rarely mentioned in other books (see Example 2 on page 20). Though some students may not need these additional steps, many will appreciate the fact that the authors do not assume too much in the way of prior knowledge.

EXAMPLE 9 Solving Exponential Equations Solve for x to four decimal places:

(A) $10^x = 2$ (B) $e^x = 3$ (C) $3^x = 4$

SOLUTION

(A) $10^x = 2$ Take common logarithms of both sides.

$\log 10^x = \log 2$ Property 3

$x = \log 2$ Use a calculator.

$= 0.3010$ To four decimal places

(B) $e^x = 3$ Take natural logarithms of both sides.

$\ln e^x = \ln 3$ Property 3

$x = \ln 3$ Use a calculator.

$= 1.0986$ To four decimal places

(C) $3^x = 4$ Take either natural or common logarithms of both sides. (We choose common logarithms.)

$\log 3^x = \log 4$ Property 7

$x \log 3 = \log 4$ Solve for x.

$x = \dfrac{\log 4}{\log 3}$ Use a calculator.

$= 1.2619$ To four decimal places

Matched Problem 9 Solve for x to four decimal places:

(A) $10^x = 7$ (B) $e^x = 6$ (C) $4^x = 5$

Each example is followed by a similar *Matched Problem* for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Explore and Discuss

Most every section contains *Explore and Discuss* problems at appropriate places to encourage students to think about a relationship or process before a result is stated or to investigate additional consequences of a development in the text. This serves to foster critical thinking and communication skills. The Explore and Discuss material can be used for in-class discussions or out-of-class group activities and is effective in both small and large class settings.

Explore and Discuss 2 How many x intercepts can the graph of a quadratic function have? How many y intercepts? Explain your reasoning.

New to this edition, annotations in the instructor's edition provide tips for less-experienced instructors on how to engage students in these Explore and Discuss activities, expand on the topic, or simply guide student responses.

Exercise Sets

The book contains over 4,200 carefully selected and graded exercises. Many problems have multiple parts, significantly increasing the total number of exercises. Exercises are paired so that consecutive odd- and even-numbered exercises are of the same type and difficulty level. Each exercise set is designed to allow instructors to craft just the right assignment for students. Exercise sets are categorized as Skills Warm-up (review of prerequisite knowledge), and within the Annotated Instructor's Edition only, as A (routine easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) to make it easy for instructors to create assignments that are appropriate for their classes. The *writing exercises*, indicated by the icon ✎, provide students with an opportunity to express their understanding of the topic in writing. Answers to all odd-numbered problems are in the back of the book. Answers to application problems in linear programming include both the mathematical model and the numeric answer.

Applications

A major objective of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Index of Applications at the back of the book). Almost every exercise set contains application problems, including applications from business and economics, life sciences, and social sciences. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems inspired by professional journals and books. No specialized experience is required to solve any of the application problems.

Additional Pedagogical Features

The following features, while helpful to any student, are particularly helpful to students enrolled in a large classroom setting where access to the instructor is more challenging or just less frequent. These features provide much-needed guidance for students as they tackle difficult concepts.

- **Call-out boxes** highlight important definitions, results, and step-by-step processes (see pages 106, 112–113).
- **Caution statements** appear throughout the text where student errors often occur (see pages 154, 159, and 192).

CAUTION Note that in Example 11 we let $x = 0$ represent 1900. If we let $x = 0$ represent 1940, for example, we would obtain a different logarithmic regression equation, but the prediction for 2015 would be the same. We would *not* let $x = 0$ represent 1950 (the first year in Table 1) or any later year, because logarithmic functions are undefined at 0. ▲

- **Conceptual Insights**, appearing in nearly every section, often make explicit connections to previous knowledge, but sometimes encourage students to think beyond the particular skill they are working on and see a more enlightened view of the concepts at hand (see pages 75, 156, 232).

CONCEPTUAL INSIGHT

The notation (2.7) has two common mathematical interpretations: the ordered pair with first coordinate 2 and second coordinate 7, and the open interval consisting of all real numbers between 2 and 7. The choice of interpretation is usually determined by the context in which the notation is used. The notation $(2, -7)$ could be interpreted as an ordered pair but not as an interval. In interval notation, the left endpoint is always written first. So, $(-7, 2)$ is correct interval notation, but $(2, -7)$ is not.

- The newly revised **Diagnostic Prerequisite Test**, located at the front of the book, provides students with a tool to assess their prerequisite skills prior to taking the course. The **Basic Algebra Review**, in Appendix A, provides students with seven sections of content to help them remediate in specific areas of need. Answers to the Diagnostic Prerequisite Test are at the back of the book and reference specific sections in the Basic Algebra Review or Chapter 1 for students to use for remediation.

Graphing Calculator and Spreadsheet Technology

Although access to a graphing calculator or spreadsheets is not assumed, it is likely that many students will want to make use of this technology. To assist these students, optional graphing calculator and spreadsheet activities are included in appropriate places. These include brief discussions in the text, examples or portions of examples solved on a graphing calculator or spreadsheet, and exercises for the student to solve. For example, linear regression is introduced in Section 1.3, and regression techniques on a graphing calculator are used at appropriate points to illustrate mathematical modeling with real data. All the optional graphing calculator material is clearly identified with the icon and can be omitted without loss of continuity, if desired. Optional spreadsheet material is identified with the icon . Graphing calculator screens displayed in the text are actual output from the TI-84 Plus graphing calculator.

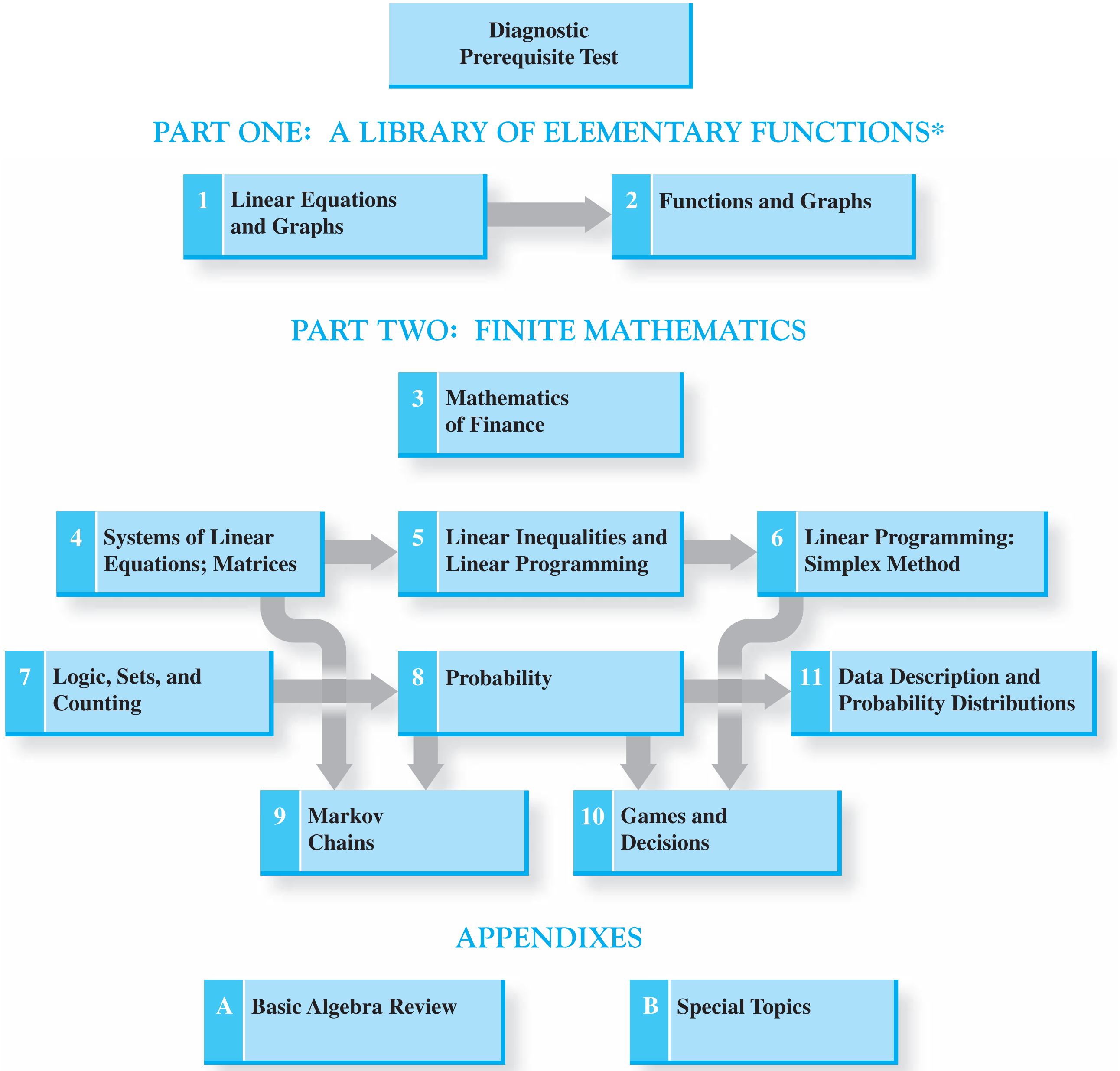
Chapter Reviews

Often it is during the preparation for a chapter exam that concepts gel for students, making the chapter review material particularly important. The chapter review sections in this text include a comprehensive summary of important terms, symbols, and concepts, keyed to completely worked examples, followed by a comprehensive set of Review Exercises. Answers to Review Exercises are included at the back of the book; *each answer contains a reference to the section in which that type of problem is discussed* so students can remediate any deficiencies in their skills on their own.

Content

The text begins with the development of a library of elementary functions in **Chapters 1 and 2**, including their properties and applications. Many students will be familiar with most, if not all, of the material in these introductory chapters. Depending on students'

Chapter Dependencies



*Selected topics from Part One may be referred to as needed in Part Two or reviewed systematically before starting Part Two.

preparation and the course syllabus, an instructor has several options for using the first two chapters, including the following:

(i) Skip Chapters 1 and 2 and refer to them only as necessary later in the course;

(ii) Cover Chapter 1 quickly in the first week of the course, emphasizing price–demand equations, price–supply equations, and linear regression, but skip Chapter 2;

(iii) Cover Chapters 1 and 2 systematically before moving on to other chapters.

The material in Part Two (Finite Mathematics) can be thought of as four units:

1. Mathematics of finance (Chapter 3)
2. Linear algebra, including matrices, linear systems, and linear programming (Chapters 4, 5, and 6)

3. Probability and statistics (Chapters 7, 8, and 11)
4. Applications of linear algebra and probability to Markov chains and game theory (Chapters 9 and 10)

The first three units are independent of each other, while the fourth unit is dependent on some of the earlier chapters (see chart on previous page).

- **Chapter 3** presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix B.1 addresses arithmetic and geometric sequences and can be covered in conjunction with this chapter, if desired.
- **Chapter 4** covers linear systems and matrices with an emphasis on using row operations and Gauss–Jordan elimination to solve systems and to find matrix inverses. This chapter also contains numerous applications of mathematical modeling using systems and matrices. To assist students in formulating solutions, all answers at the back of the book for application exercises in Sections 4.3, 4.5, and the chapter Review Exercises contain both the mathematical model and its solution. The row operations discussed in Sections 4.2 and 4.3 are required for the simplex method in Chapter 6. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 9.
- **Chapters 5 and 6** provide a broad and flexible coverage of linear programming. Chapter 5 covers two-variable graphing techniques. Instructors who wish to emphasize linear programming techniques can cover the basic simplex method in Sections 6.1 and 6.2 and then discuss either or both of the following: the dual method (Section 6.3) and the big M method (Section 6.4). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 6.2–6.4, and either omit the solution or use software to find the solution. To facilitate this approach, all answers at the back of the book for application exercises in Sections 6.2–6.4 and the chapter Review Exercises contain both the mathematical model and its solution. The simplex and dual solution methods are required for portions of Chapter 10.
- **Chapter 7** provides a foundation for probability with a treatment of logic, sets, and counting techniques.
- **Chapter 8** covers basic probability, including Bayes' formula and random variables.
- **Chapters 9 and 10** tie together concepts developed in earlier chapters and apply them to interesting topics. A study of Markov chains (Chapter 9) or game theory (Chapter 10) provides an excellent unifying conclusion to a finite mathematics course.
- **Chapter 11** deals with basic descriptive statistics and more advanced probability distributions, including the important normal distribution. Appendix B.3 contains a short discussion of the binomial theorem that can be used in conjunction with the development of the binomial distribution in Section 11.4.
- **Appendix A** contains a concise review of basic algebra that may be covered as part of the course or referenced as needed. As mentioned previously, **Appendix B** contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

Accuracy Check

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error free. If an error should be found, the authors would be grateful if notification were sent to Karl E. Byleen, 9322 W. Garden Court, Hales Corners, WI 53130; or by e-mail to kbyleen@wi.rr.com.

Student Supplements

Student's Solutions Manual

- By Garret J. Etgen, University of Houston
- This manual contains detailed, carefully worked-out solutions to all odd-numbered section exercises and all Chapter Review exercises. Each section begins with Things to Remember, a list of key material for review.
- ISBN-13: 978-0-321-94670-6

Graphing Calculator Manual for Applied Math

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- This manual contains detailed instructions for using the TI-83/TI-83 Plus/TI-84 Plus C calculators with this textbook. Instructions are organized by mathematical topics.
- Available in MyMathLab.

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- By Salvatore Sciandra,
  Niagara County Community College
- These worksheets for students contain unique examples to enforce what is taught in the lecture and/or material covered in the text. Instructor worksheets are also available and include answers.
- Available in MyMathLab.

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- The video lectures with optional captioning for this text make it easy and convenient for students to watch videos from a computer at home or on campus. The complete set is ideal for distance learning or supplemental instruction.
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Instructor Supplements

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- By Garret J. Etgen, University of Houston
- This manual contains detailed solutions to all even-numbered section problems.
- Available in MyMathLab or through http://www.pearsonglobaleditions.com/Barnett.

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- By Salvatore Sciandra,
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- Mini Lectures are provided for the teaching assistant, adjunct, part-time or even full-time instructor for lecture preparation by providing learning objectives, examples (and answers) not found in the text, and teaching notes.
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- These slides present key concepts and definitions from the text. They are available in MyMathLab or at http://www.pearsonglobaleditions.com/Barnett.

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- **Chapter-Level, Just-in-Time Remediation:** The MyMathLab course for these texts includes a short diagnostic, called Getting Ready, prior to each chapter to assess students' prerequisite knowledge. This diagnostic can then be tied to personalized homework so that each student receives a homework assignment specific to his or her prerequisite skill needs.
- **Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at the point of use.

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- Whether you are just getting started with MyMathLab or you have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course.
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Diagnostic Prerequisite Test

Work all of the problems in this self-test without using a calculator. Then check your work by consulting the answers in the back of the book. Where weaknesses show up, use the reference that follows each answer to find the section in the text that provides the necessary review.

1. Replace each question mark with an appropriate expression that will illustrate the use of the indicated real number property:

(A) Commutative $(\cdot)$: $x(y + z) = ?$

(B) Associative $(+)$: $2 + (x + y) = ?$

(C) Distributive: $(2 + 3)x = ?$

Problems 2–6 refer to the following polynomials:

(A) $3x - 4$ (B) $x + 2$

(C) $2 - 3x^2$ (D) $x^3 + 8$

2. Add all four.

3. Subtract the sum of (A) and (C) from the sum of (B) and (D).

4. Multiply (C) and (D).

5. What is the degree of each polynomial?

6. What is the leading coefficient of each polynomial?

In Problems 7 and 8, perform the indicated operations and simplify.

7. $5x^2 - 3x[4 - 3(x - 2)]$

8. $(2x + y)(3x - 4y)$

In Problems 9 and 10, factor completely.

9. $x^2 + 7x + 10$ **10.** $x^3 - 2x^2 - 15x$

11. Write 0.35 as a fraction reduced to lowest terms.

12. Write $\frac{7}{8}$ in decimal form.

13. Write in scientific notation:

(A) 4,065,000,000,000 (B) 0.0073

14. Write in standard decimal form:

(A) 2.55×10^8 (B) 4.06×10^{-4}

15. Indicate true (T) or false (F):

(A) A natural number is a rational number.

(B) A number with a repeating decimal expansion is an irrational number.

16. Give an example of an integer that is not a natural number.

In Problems 17–24, simplify and write answers using positive exponents only. All variables represent positive real numbers.

17. $6(xy^3)^5$ **18.** $\dfrac{9u^8v^6}{3u^4v^8}$

19. $(2 \times 10^5)(3 \times 10^{-3})$ **20.** $(x^{-3}y^2)^{-2}$

21. $u^{5/3}u^{2/3}$ **22.** $(9a^4b^{-2})^{1/2}$

23. $\dfrac{5^0}{3^2} + \dfrac{3^{-2}}{2^{-2}}$ **24.** $(x^{1/2} + y^{1/2})^2$

In Problems 25–30, perform the indicated operation and write the answer as a simple fraction reduced to lowest terms. All variables represent positive real numbers.

25. $\dfrac{a}{b} + \dfrac{b}{a}$ **26.** $\dfrac{a}{bc} - \dfrac{c}{ab}$

27. $\dfrac{x^2}{y} \cdot \dfrac{y^6}{x^3}$ **28.** $\dfrac{x}{y^3} \div \dfrac{x^2}{y}$

29. $\dfrac{\dfrac{1}{7 + h} - \dfrac{1}{7}}{h}$ **30.** $\dfrac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$

31. Each statement illustrates the use of one of the following real number properties or definitions. Indicate which one.

Commutative $(+, \cdot)$	Associative $(+, \cdot)$	Distributive
Identity $(+, \cdot)$	Inverse $(+, \cdot)$	Subtraction
Division	Negatives	Zero

(A) $(-7) - (-5) = (-7) + [-(-5)]$

(B) $5u + (3v + 2) = (3v + 2) + 5u$

(C) $(5m - 2)(2m + 3) = (5m - 2)2m + (5m - 2)3$

(D) $9 \cdot (4y) = (9 \cdot 4)y$

(E) $\dfrac{u}{-(v - w)} = \dfrac{u}{w - v}$

(F) $(x - y) + 0 = (x - y)$

32. Round to the nearest integer:

(A) $\dfrac{17}{3}$ (B) $-\dfrac{5}{19}$

33. Multiplying a number x by 4 gives the same result as subtracting 4 from x. Express as an equation, and solve for x.

34. Find the slope of the line that contains the points $(3, -5)$ and $(-4, 10)$.

35. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the x axis.

36. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the y axis.

In Problems 37–40, solve for x.

37. $x^2 = 5x$

38. $3x^2 - 21 = 0$

39. $x^2 - x - 20 = 0$

40. $-6x^2 + 7x - 1 = 0$

PART 1

A LIBRARY OF ELEMENTARY FUNCTIONS

1 Linear Equations and Graphs

Introduction

We begin by discussing some algebraic methods for solving equations and inequalities. Next, we introduce coordinate systems that allow us to explore the relationship between algebra and geometry. Finally, we use this algebraic–geometric relationship to find equations that can be used to describe real-world data sets. For example, in Section 1.3 you will learn how to find the equation of a line that fits data on winning times in an Olympic swimming event (see Problems 27 and 28 on page 53). We also consider many applied problems that can be solved using the concepts discussed in this chapter.

1.1 Linear Equations and Inequalities

- Linear Equations
- Linear Inequalities
- Applications

The equation

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

and the inequality

$$\frac{x}{2} + 2(3x - 1) \ge 5$$

are both first degree in one variable. In general, a **first-degree**, or **linear**, **equation** in one variable is any equation that can be written in the form

$$\textbf{Standard form:} \quad ax + b = 0 \qquad a \ne 0 \tag{1}$$

If the equality symbol, $=$, in (1) is replaced by $<$, $>$, $\le$, or $\ge$, the resulting expression is called a **first-degree**, or **linear**, **inequality**.

A **solution** of an equation (or inequality) involving a single variable is a number that when substituted for the variable makes the equation (or inequality) true. The set of all solutions is called the **solution set**. When we say that we **solve an equation** (or inequality), we mean that we find its solution set.

Knowing what is meant by the solution set is one thing; finding it is another. We start by recalling the idea of equivalent equations and equivalent inequalities. If we perform an operation on an equation (or inequality) that produces another equation (or inequality) with the same solution set, then the two equations (or inequalities) are said to be **equivalent**. The basic idea in solving equations or inequalities is to perform operations that produce simpler equivalent equations or inequalities and to continue the process until we obtain an equation or inequality with an obvious solution.

Linear Equations

Linear equations are generally solved using the following equality properties.

THEOREM 1 Equality Properties

An equivalent equation will result if

1. The same quantity is added to or subtracted from each side of a given equation.
2. Each side of a given equation is multiplied by or divided by the same nonzero quantity.

EXAMPLE 1 Solving a Linear Equation Solve and check:

$$8x - 3(x - 4) = 3(x - 4) + 6$$

SOLUTION

$$\begin{aligned} 8x - 3(x - 4) &= 3(x - 4) + 6 && \text{Use the distributive property.} \\ 8x - 3x + 12 &= 3x - 12 + 6 && \text{Combine like terms.} \\ 5x + 12 &= 3x - 6 && \text{Subtract } 3x \text{ from both sides.} \\ 2x + 12 &= -6 && \text{Subtract 12 from both sides.} \\ 2x &= -18 && \text{Divide both sides by 2.} \\ x &= -9 \end{aligned}$$

CHECK

$$\begin{aligned} 8x - 3(x - 4) &= 3(x - 4) + 6 \\ 8(-9) - 3[(-9) - 4] &\stackrel{?}{=} 3[(-9) - 4] + 6 \\ -72 - 3(-13) &\stackrel{?}{=} 3(-13) + 6 \\ -33 &\stackrel{\checkmark}{=} -33 \end{aligned}$$

Matched Problem 1 Solve and check: $3x - 2(2x - 5) = 2(x + 3) - 8$

Explore and Discuss 1 According to equality property 2, multiplying both sides of an equation by a nonzero number always produces an equivalent equation. What is the smallest positive number that you could use to multiply both sides of the following equation to produce an equivalent equation without fractions?

$$\frac{x+1}{3} - \frac{x}{4} = \frac{1}{2}$$

EXAMPLE 2 Solving a Linear Equation Solve and check: $\dfrac{x+2}{2} - \dfrac{x}{3} = 5$

SOLUTION What operations can we perform on

$$\frac{x+2}{2} - \frac{x}{3} = 5$$

to eliminate the denominators? If we can find a number that is exactly divisible by each denominator, we can use the multiplication property of equality to clear the denominators. The LCD (least common denominator) of the fractions, 6, is exactly what we are looking for! Actually, any common denominator will do, but the LCD results in a simpler equivalent equation. So, we multiply both sides of the equation by 6:

$$\boxed{\begin{aligned} 6\left(\frac{x+2}{2} - \frac{x}{3}\right) &= 6 \cdot 5 \\ \overset{3}{\cancel{6}} \cdot \frac{(x+2)}{\underset{1}{\cancel{2}}} - \overset{2}{\cancel{6}} \cdot \frac{x}{\underset{1}{\cancel{3}}} &= 30 \end{aligned}}\,^{*}$$

$$\begin{aligned} 3(x+2) - 2x &= 30 && \text{Use the distributive property.} \\ 3x + 6 - 2x &= 30 && \text{Combine like terms.} \\ x + 6 &= 30 && \text{Subtract 6 from both sides.} \\ x &= 24 \end{aligned}$$

CHECK

$$\begin{aligned} \frac{x+2}{2} - \frac{x}{3} &= 5 \\ \frac{24+2}{2} - \frac{24}{3} &\overset{?}{=} 5 \\ 13 - 8 &\overset{?}{=} 5 \\ 5 &\overset{\checkmark}{=} 5 \end{aligned}$$

Matched Problem 2 Solve and check: $\dfrac{x+1}{3} - \dfrac{x}{4} = \dfrac{1}{2}$

In many applications of algebra, formulas or equations must be changed to alternative equivalent forms. The following example is typical.

EXAMPLE 3 Solving a Formula for a Particular Variable If you deposit a principal P in an account that earns simple interest at an annual rate r, then the amount A in the account after t years is given by $A = P + Prt$. Solve for

(A) r in terms of A, P, and t

(B) P in terms of A, r, and t

*Dashed boxes are used throughout the book to denote steps that are usually performed mentally.

SOLUTION (A)

$$A = P + Prt \quad \text{Reverse equation.}$$
$$P + Prt = A \quad \text{Subtract } P \text{ from both sides.}$$
$$Prt = A - P \quad \text{Divide both members by } Pt.$$
$$r = \frac{A - P}{Pt}$$

(B)

$$A = P + Prt \quad \text{Reverse equation.}$$
$$P + Prt = A \quad \text{Factor out } P \text{ (note the use of the distributive property).}$$
$$P(1 + rt) = A \quad \text{Divide by } (1 + rt).$$
$$P = \frac{A}{1 + rt}$$

Matched Problem 3 If a cardboard box has length L, width W, and height H, then its surface area is given by the formula $S = 2LW + 2LH + 2WH$. Solve the formula for

(A) L in terms of S, W, and H (B) H in terms of S, L, and W

Linear Inequalities

Before we start solving linear inequalities, let us recall what we mean by $<$ (less than) and $>$ (greater than). If a and b are real numbers, we write

$$\boldsymbol{a < b} \quad a \text{ is less than } b$$

if there exists a positive number p such that $a + p = b$. Certainly, we would expect that if a positive number was added to any real number, the sum would be larger than the original. That is essentially what the definition states. If $a < b$, we may also write

$$\boldsymbol{b > a} \quad b \text{ is greater than } a.$$

EXAMPLE 4 Inequalities

(A)	$3 < 5$	Since $3 + 2 = 5$
(B)	$-6 < -2$	Since $-6 + 4 = -2$
(C)	$0 > -10$	Since $-10 < 0$ (because $-10 + 10 = 0$)

Matched Problem 4 Replace each question mark with either $<$ or $>$.

(A) $2 \; ? \; 8$ (B) $-20 \; ? \; 0$ (C) $-3 \; ? \; -30$

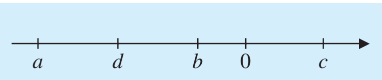


Figure 1 $a < b, c > d$

The inequality symbols have a very clear geometric interpretation on the real number line. If $a < b$, then a is to the left of b on the number line; if $c > d$, then c is to the right of d on the number line (Fig. 1). Check this geometric property with the inequalities in Example 4.

Explore and Discuss 2 Replace **?** with $<$ or $>$ in each of the following:

(A) $-1 \; ? \; 3$ and $2(-1) \; ? \; 2(3)$

(B) $-1 \; ? \; 3$ and $-2(-1) \; ? \; -2(3)$

(C) $12 \; ? \; -8$ and $\frac{12}{4} \; ? \; \frac{-8}{4}$

(D) $12 \; ? \; -8$ and $\frac{12}{-4} \; ? \; \frac{-8}{-4}$

Based on these examples, describe the effect of multiplying both sides of an inequality by a number.

The procedures used to solve linear inequalities in one variable are almost the same as those used to solve linear equations in one variable, but with one important exception, as noted in item 3 of Theorem 2.

THEOREM 2 Inequality Properties

An equivalent inequality will result, and the **sense or direction will remain the same** if each side of the original inequality

1. has the same real number added to or subtracted from it.
2. is multiplied or divided by the same *positive* number.

An equivalent inequality will result, and the **sense or direction will reverse** if each side of the original inequality

3. is multiplied or divided by the same *negative* number.

Note: Multiplication by 0 and division by 0 are not permitted.

Therefore, we can perform essentially the same operations on inequalities that we perform on equations, with the exception that **the sense of the inequality reverses if we multiply or divide both sides by a negative number**. Otherwise, the sense of the inequality does not change. For example, if we start with the true statement

$$-3 > -7$$

and multiply both sides by 2, we obtain

$$-6 > -14$$

and the sense of the inequality stays the same. But if we multiply both sides of $-3 > -7$ by -2, the left side becomes 6 and the right side becomes 14, so we must write

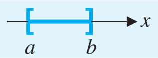
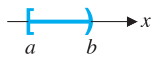
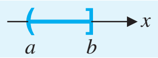
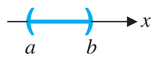
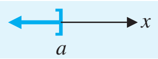
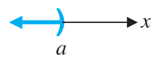
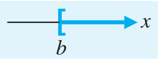
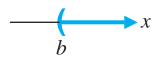
$$6 < 14$$

to have a true statement. The sense of the inequality reverses.

If $a < b$, the **double inequality** $a < x < b$ means that $\boldsymbol{a < x}$ **and** $\boldsymbol{x < b}$; that is, x is between a and b. **Interval notation** is also used to describe sets defined by inequalities, as shown in Table 1.

The numbers a and b in Table 1 are called the **endpoints** of the interval. An interval is **closed** if it contains all its endpoints and **open** if it does not contain any of its endpoints. The intervals $[a, b]$, $(-\infty, a]$, and $[b, \infty)$ are closed, and the intervals (a, b), $(-\infty, a)$,

Table 1 **Interval Notation**

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \le x \le b$	
$[a, b)$	$a \le x < b$	
$(a, b]$	$a < x \le b$	
(a, b)	$a < x < b$	
$(-\infty, a]$	$x \le a$	
$(-\infty, a)$	$x < a$	
$[b, \infty)$	$x \ge b$	
(b, ∞)	$x > b$	

and (b, ∞) are open. Note that the symbol ∞ (read infinity) is not a number. When we write $[b, \infty)$, we are simply referring to the interval that starts at b and continues indefinitely to the right. We never refer to ∞ as an endpoint, and we never write $[b, \infty]$. The interval $(-\infty, \infty)$ is the entire real number line.

Note that an endpoint of a line graph in Table 1 has a square bracket through it if the endpoint is included in the interval; a parenthesis through an endpoint indicates that it is not included.

CONCEPTUAL INSIGHT

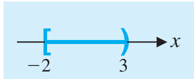
The notation $(2, 7)$ has two common mathematical interpretations: the ordered pair with first coordinate 2 and second coordinate 7, and the open interval consisting of all real numbers between 2 and 7. The choice of interpretation is usually determined by the context in which the notation is used. The notation $(2, -7)$ could be interpreted as an ordered pair but not as an interval. In interval notation, the left endpoint is always written first. So, $(-7, 2)$ is correct interval notation, but $(2, -7)$ is not.

EXAMPLE 5 Interval and Inequality Notation, and Line Graphs

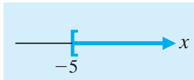
(A) Write $[-2, 3)$ as a double inequality and graph.

(B) Write $x \geq -5$ in interval notation and graph.

SOLUTION (A) $[-2, 3)$ is equivalent to $-2 \leq x < 3$.



(B) $x \geq -5$ is equivalent to $[-5, \infty)$.



Matched Problem 5

(A) Write $(-7, 4]$ as a double inequality and graph.

(B) Write $x < 3$ in interval notation and graph.

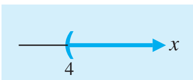
Explore and Discuss 3 The solution to Example 5B shows the graph of the inequality $x \geq -5$. What is the graph of $x < -5$? What is the corresponding interval? Describe the relationship between these sets.

EXAMPLE 6 Solving a Linear Inequality Solve and graph:

$$2(2x + 3) < 6(x - 2) + 10$$

SOLUTION

$$\begin{aligned} 2(2x + 3) &< 6(x - 2) + 10 && \text{Remove parentheses.} \\ 4x + 6 &< 6x - 12 + 10 && \text{Combine like terms.} \\ 4x + 6 &< 6x - 2 && \text{Subtract } 6x \text{ from both sides.} \\ -2x + 6 &< -2 && \text{Subtract 6 from both sides.} \\ -2x &< -8 && \text{Divide both sides by } -2 \text{ and reverse the sense of the inequality.} \\ x > 4 \quad &\text{or} \quad (4, \infty) \end{aligned}$$



Notice that in the graph of $x > 4$, we use a parenthesis through 4, since the point 4 is not included in the graph.

Matched Problem 6 Solve and graph: $3(x - 1) \leq 5(x + 2) - 5$

EXAMPLE 7 Solving a Double Inequality Solve and graph: $-3 < 2x + 3 \le 9$

SOLUTION We are looking for all numbers x such that $2x + 3$ is between -3 and 9, including 9 but not -3. We proceed as before except that we try to isolate x in the middle:

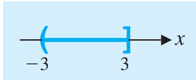
$$-3 < 2x + 3 \le 9$$

$$\boxed{-3 - 3 < 2x + 3 - 3 \le 9 - 3}$$

$$-6 < 2x \le 6$$

$$\boxed{\frac{-6}{2} < \frac{2x}{2} \le \frac{6}{2}}$$

$$-3 < x \le 3 \quad \text{or} \quad (-3, 3]$$



Matched Problem 7 Solve and graph: $-8 \le 3x - 5 < 7$

Note that a linear equation usually has exactly one solution, while a linear inequality usually has infinitely many solutions.

Applications

To realize the full potential of algebra, we must be able to translate real-world problems into mathematics. In short, we must be able to do word problems.

Here are some suggestions that will help you get started:

PROCEDURE For Solving Word Problems

1. Read the problem carefully and introduce a variable to represent an unknown quantity in the problem. Often the question asked in a problem will indicate the unknown quantity that should be represented by a variable.
2. Identify other quantities in the problem (known or unknown), and whenever possible, express unknown quantities in terms of the variable you introduced in Step 1.
3. Write a verbal statement using the conditions stated in the problem and then write an equivalent mathematical statement (equation or inequality).
4. Solve the equation or inequality and answer the questions posed in the problem.
5. Check the solution(s) in the original problem.

EXAMPLE 8 Purchase Price Alex purchases a plasma TV, pays 7% state sales tax, and is charged \$65 for delivery. If Alex's total cost is \$1,668.93, what was the purchase price of the TV?

SOLUTION

Step 1 **Introduce a variable for the unknown quantity.** After reading the problem, we decide to let x represent the purchase price of the TV.

Step 2 **Identify quantities in the problem.**

Delivery charge: \$65

Sales tax: $0.07x$

Total cost: \$1,668.93

Step 3 **Write a verbal statement and an equation.**

$$\begin{array}{ccccccc} \text{Price} & + & \text{Delivery Charge} & + & \text{Sales Tax} & = & \text{Total Cost} \\ x & + & 65 & + & 0.07x & = & 1{,}668.93 \end{array}$$

Step 4 **Solve the equation and answer the question.**

$$
\begin{aligned}
x + 65 + 0.07x &= 1{,}668.93 && \text{Combine like terms.}\\
1.07x + 65 &= 1{,}668.93 && \text{Subtract 65 from both sides.}\\
1.07x &= 1{,}603.93 && \text{Divide both sides by 1.07.}\\
x &= 1{,}499
\end{aligned}
$$

The price of the TV is $1,499.

Step 5 **Check the answer in the original problem.**

$$
\begin{aligned}
\text{Price} &= \$1{,}499.00\\
\text{Delivery charge} &= \$\phantom{1{,}4}65.00\\
\text{Tax} = 0.07 \cdot 1{,}499 &= \$\phantom{1{,}}104.93\\
\hline
\text{Total} &= \$1{,}668.93
\end{aligned}
$$

Matched Problem 8 Mary paid 8.5% sales tax and a $190 title and license fee when she bought a new car for a total of $28,400. What is the purchase price of the car?

The next example involves the important concept of **break-even analysis**, which is encountered in several places in this text. Any manufacturing company has **costs**, $\boldsymbol{C}$, and **revenues**, $\boldsymbol{R}$. The company will have a **loss** if $R < C$, will **break even** if $R = C$, and will have a **profit** if $R > C$. Costs involve **fixed costs**, such as plant overhead, product design, setup, and promotion, and **variable costs**, which are dependent on the number of items produced at a certain cost per item.

EXAMPLE 9 Break-Even Analysis A multimedia company produces DVDs. Onetime fixed costs for a particular DVD are $48,000, which include costs such as filming, editing, and promotion. Variable costs amount to $12.40 per DVD and include manufacturing, packaging, and distribution costs for each DVD actually sold to a retailer. The DVD is sold to retail outlets at $17.40 each. How many DVDs must be manufactured and sold in order for the company to break even?

SOLUTION

Step 1 Let x = number of DVDs manufactured and sold.

Step 2

$$
\begin{aligned}
C &= \text{cost of producing } x \text{ DVDs}\\
R &= \text{revenue (return) on sales of } x \text{ DVDs}\\
\text{Fixed costs} &= \$48{,}000\\
\text{Variable costs} &= \$12.40x\\
C &= \text{Fixed costs} + \text{variable costs}\\
&= \$48{,}000 + \$12.40x\\
R &= \$17.40x
\end{aligned}
$$

Step 3 The company breaks even if $R = C$; that is, if

$$\$17.40x = \$48{,}000 + \$12.40x$$

Step 4

$$
\begin{aligned}
17.4x &= 48{,}000 + 12.4x && \text{Subtract } 12.4x \text{ from both sides.}\\
5x &= 48{,}000 && \text{Divide both sides by 5.}\\
x &= 9{,}600
\end{aligned}
$$

The company must make and sell 9,600 DVDs to break even.

Step 5 Check:

Costs	**Revenue**
$48{,}000 + 12.4(9{,}600)$	$17.4(9{,}600)$
$= \$167{,}040$	$= \$167{,}040$

Matched Problem 9 How many DVDs would a multimedia company have to make and sell to break even if the fixed costs are \$36,000, variable costs are \$10.40 per DVD, and the DVDs are sold to retailers for \$15.20 each?

Table 2 **CPI (1982–1984 = 100)**

Year	Index
1960	29.6
1973	44.4
1986	109.6
1999	156.9
2012	229.6

EXAMPLE 10 Consumer Price Index The Consumer Price Index (CPI) is a measure of the average change in prices over time from a designated reference period, which equals 100. The index is based on prices of basic consumer goods and services. Table 2 lists the CPI for several years from 1960 to 2012. What net annual salary in 2012 would have the same purchasing power as a net annual salary of \$13,000 in 1960? Compute the answer to the nearest dollar. (*Source*: U.S. Bureau of Labor Statistics)

SOLUTION

Step 1 Let $x =$ the purchasing power of an annual salary in 2012.

Step 2 Annual salary in 1960 = \$13,000

$$\text{CPI in 1960} = 29.6$$
$$\text{CPI in 2012} = 229.6$$

Step 3 The ratio of a salary in 2012 to a salary in 1960 is the same as the ratio of the CPI in 2012 to the CPI in 1960.

$$\frac{x}{13{,}000} = \frac{229.6}{29.6} \qquad \text{Multiply both sides by 13,000.}$$

Step 4

$$x = 13{,}000 \cdot \frac{229.6}{29.6}$$
$$= \$100{,}838 \text{ per year}$$

Step 5 To check the answer, we confirm that the salary ratio agrees with the CPI ratio:

Salary Ratio	**CPI Ratio**
$\frac{100{,}838}{13{,}000} = 7.757$	$\frac{229.6}{29.6} = 7.757$

Matched Problem 10 What net annual salary in 1973 would have had the same purchasing power as a net annual salary of \$100,000 in 2012? Compute the answer to the nearest dollar.

Exercises 1.1

Solve Problems 1–6.

1. $2m + 9 = 5m - 6$

2. $3y - 4 = 6y - 19$

3. $2x + 3 < -4$

4. $5x + 2 > 1$

5. $-3x \ge -12$

6. $-4x \le 8$

Solve Problems 7–10 and graph.

7. $-4x - 7 > 5$

8. $-2x + 8 < 4$

9. $2 \le x + 3 \le 5$

10. $-4 < 2y - 3 < 9$

Solve Problems 11–24.

11. $\frac{x}{4} + \frac{1}{2} = \frac{1}{8}$

12. $\frac{m}{3} - 4 = \frac{2}{3}$

13. $\frac{y}{-5} > \frac{3}{2}$

14. $\frac{x}{-4} < \frac{5}{6}$

15. $2u + 4 = 5u + 1 - 7u$

16. $-3y + 9 + y = 13 - 8y$

17. $10x + 25(x - 3) = 275$

18. $-3(4 - x) = 5 - (x + 1)$

19. $3 - y \le 4(y - 3)$

20. $x - 2 \ge 2(x - 5)$

21. $\frac{x}{5} - \frac{x}{6} = \frac{6}{5}$

22. $\frac{y}{4} - \frac{y}{3} = \frac{1}{2}$

23. $\frac{m}{5} - 3 < \frac{3}{5} - \frac{m}{2}$

24. $\frac{u}{2} - \frac{2}{3} < \frac{u}{3} + 2$

Solve Problems 25–28 and graph.

25. $2 \le 3x - 7 < 14$

26. $-4 \le 5x + 6 < 21$

27. $-4 \le \frac{9}{5}C + 32 \le 68$

28. $-1 \le \frac{2}{3}t + 5 \le 11$

Solve Problems 29–34 for the indicated variable.

29. $3x - 4y = 12$; for y

30. $y = -\frac{2}{3}x + 8$; for x

31. $Ax + By = C$; for y $(B \neq 0)$

32. $y = mx + b$; for m

33. $F = \frac{9}{5}C + 32$; for C

34. $C = \frac{5}{9}(F - 32)$; for F

Solve Problems 35 and 36 and graph.

35. $-3 \leq 4 - 7x < 18$

36. $-10 \leq 8 - 3u \leq -6$

37. What can be said about the signs of the numbers a and b in each case?

(A) $ab > 0$

(B) $ab < 0$

(C) $\frac{a}{b} > 0$

(D) $\frac{a}{b} < 0$

38. What can be said about the signs of the numbers a, b, and c in each case?

(A) $abc > 0$

(B) $\frac{ab}{c} < 0$

(C) $\frac{a}{bc} > 0$

(D) $\frac{a^2}{bc} < 0$

39. If both a and b are positive numbers and b/a is greater than 1, then is $a - b$ positive or negative?

40. If both a and b are negative numbers and b/a is greater than 1, then is $a - b$ positive or negative?

In Problems 41–46, discuss the validity of each statement. If the statement is true, explain why. If not, give a counterexample.

41. If the intersection of two open intervals is nonempty, then their intersection is an open interval.

42. If the intersection of two closed intervals is nonempty, then their intersection is a closed interval.

43. The union of any two open intervals is an open interval.

44. The union of any two closed intervals is a closed interval.

45. If the intersection of two open intervals is nonempty, then their union is an open interval.

46. If the intersection of two closed intervals is nonempty, then their union is a closed interval.

Applications

47. Ticket sales. A rock concert brought in \$432,500 on the sale of 9,500 tickets. If the tickets sold for \$35 and \$55 each, how many of each type of ticket were sold?

48. Parking meter coins. An all-day parking meter takes only dimes and quarters. If it contains 100 coins with a total value of \$14.50, how many of each type of coin are in the meter?

49. IRA. You have \$500,000 in an IRA (Individual Retirement Account) at the time you retire. You have the option of investing this money in two funds: Fund A pays 5.2% annually and Fund B pays 7.7% annually. How should you divide your money between Fund A and Fund B to produce an annual interest income of \$34,000?

50. IRA. Refer to Problem 49. How should you divide your money between Fund A and Fund B to produce an annual interest income of \$30,000?

51. Car prices. If the price change of cars parallels the change in the CPI (see Table 2 in Example 10), what would a car sell for (to the nearest dollar) in 2012 if a comparable model sold for \$10,000 in 1999?

52. Home values. If the price change in houses parallels the CPI (see Table 2 in Example 10), what would a house valued at \$200,000 in 2012 be valued at (to the nearest dollar) in 1960?

53. Retail and wholesale prices. Retail prices in a department store are obtained by marking up the wholesale price by 40%. That is, the retail price is obtained by adding 40% of the wholesale price to the wholesale price.

(A) What is the retail price of a suit if the wholesale price is \$300?

(B) What is the wholesale price of a pair of jeans if the retail price is \$77?

54. Retail and sale prices. Sale prices in a department store are obtained by marking down the retail price by 15%. That is, the sale price is obtained by subtracting 15% of the retail price from the retail price.

(A) What is the sale price of a hat that has a retail price of \$60?

(B) What is the retail price of a dress that has a sale price of \$136?

55. Equipment rental. A golf course charges \$52 for a round of golf using a set of their clubs, and \$44 if you have your own clubs. If you buy a set of clubs for \$270, how many rounds must you play to recover the cost of the clubs?

56. Equipment rental. The local supermarket rents carpet cleaners for \$20 a day. These cleaners use shampoo in a special cartridge that sells for \$16 and is available only from the supermarket. A home carpet cleaner can be purchased for \$300. Shampoo for the home cleaner is readily available for \$9 a bottle. Past experience has shown that it takes two shampoo cartridges to clean the 10-foot-by-12-foot carpet in your living room with the rented cleaner. Cleaning the same area with the home cleaner will consume three bottles of shampoo. If you buy the home cleaner, how many times must you clean the living-room carpet to make buying cheaper than renting?

57. Sales commissions. One employee of a computer store is paid a base salary of \$2,000 a month plus an 8% commission on all sales over \$7,000 during the month. How much must the employee sell in one month to earn a total of \$4,000 for the month?

58. Sales commissions. A second employee of the computer store in Problem 57 is paid a base salary of \$3,000 a month plus a 5% commission on all sales during the month.

(A) How much must this employee sell in one month to earn a total of \$4,000 for the month?

(B) Determine the sales level at which both employees receive the same monthly income.

(C) If employees can select either of these payment methods, how would you advise an employee to make this selection?

59. **Break-even analysis.** A publisher for a promising new novel figures fixed costs (overhead, advances, promotion, copy editing, typesetting) at \$55,000, and variable costs (printing, paper, binding, shipping) at \$1.60 for each book produced. If the book is sold to distributors for \$11 each, how many must be produced and sold for the publisher to break even?

60. **Break-even analysis.** The publisher of a new book figures fixed costs at \$92,000 and variable costs at \$2.10 for each book produced. If the book is sold to distributors for \$15 each, how many must be sold for the publisher to break even?

61. **Break-even analysis.** The publisher in Problem 59 finds that rising prices for paper increase the variable costs to \$2.10 per book.

(A) Discuss possible strategies the company might use to deal with this increase in costs.

(B) If the company continues to sell the books for \$11, how many books must they sell now to make a profit?

(C) If the company wants to start making a profit at the same production level as before the cost increase, how much should they sell the book for now?

62. **Break-even analysis.** The publisher in Problem 60 finds that rising prices for paper increase the variable costs to \$2.70 per book.

(A) Discuss possible strategies the company might use to deal with this increase in costs.

(B) If the company continues to sell the books for \$15, how many books must they sell now to make a profit?

(C) If the company wants to start making a profit at the same production level as before the cost increase, how much should they sell the book for now?

63. **Wildlife management.** A naturalist estimated the total number of rainbow trout in a certain lake using the capture–mark–recapture technique. He netted, marked, and released 200 rainbow trout. A week later, allowing for thorough mixing, he again netted 200 trout, and found 8 marked ones among them. Assuming that the proportion of marked fish in the second sample was the same as the proportion of all marked fish in the total population, estimate the number of rainbow trout in the lake.

64. **Temperature conversion.** If the temperature for a 24-hour period at an Antarctic station ranged between $-49°F$ and $14°F$ (that is, $-49 \le F \le 14$), what was the range in degrees Celsius? [*Note:* $F = \frac{9}{5}C + 32$.]

65. **Psychology.** The IQ (intelligence quotient) is found by dividing the mental age (MA), as indicated on standard tests, by the chronological age (CA) and multiplying by 100. For example, if a child has a mental age of 12 and a chronological age of 8, the calculated IQ is 150. If a 9-year-old girl has an IQ of 140, compute her mental age.

66. **Psychology.** Refer to Problem 65. If the IQ of a group of 12-year-old children varies between 80 and 140, what is the range of their mental ages?

Answers to Matched Problems

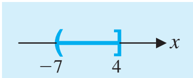
1. $x = 4$

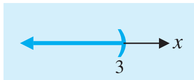
2. $x = 2$

3. (A) $L = \dfrac{S - 2WH}{2W + 2H}$ (B) $H = \dfrac{S - 2LW}{2L + 2W}$

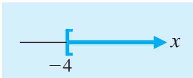
4. (A) $<$ (B) $<$ (C) $>$

5. (A) $-7 < x \le 4$; (B) $(-\infty, 3)$

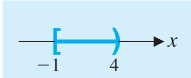




6. $x \ge -4$ or $[-4, \infty)$



7. $-1 \le x < 4$ or $[-1, 4)$



8. \$26,000 **9.** 7,500 DVDs **10.** \$19,338

1.2 Graphs and Lines

- Cartesian Coordinate System
- Graphs of $Ax + By = C$
- Slope of a Line
- Equations of Lines: Special Forms
- Applications

In this section, we will consider one of the most basic geometric figures—a line. When we use the term *line* in this book, we mean *straight line*. We will learn how to recognize and graph a line, and how to use information concerning a line to find its equation. Examining the graph of any equation often results in additional insight into the nature of the equation's solutions.

Cartesian Coordinate System

Recall that to form a **Cartesian** or **rectangular coordinate system**, we select two real number lines—one horizontal and one vertical—and let them cross through their origins as indicated in Figure 1. Up and to the right are the usual choices for the positive directions. These two number lines are called the **horizontal axis** and the **vertical**

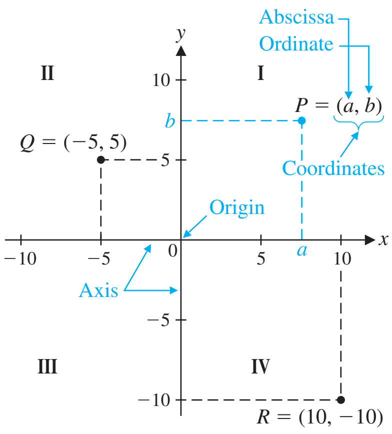


Figure 1 **The Cartesian (rectangular) coordinate system**

axis, or, together, the **coordinate axes**. The horizontal axis is usually referred to as the ***x* axis** and the vertical axis as the ***y* axis**, and each is labeled accordingly. The coordinate axes divide the plane into four parts called **quadrants**, which are numbered counterclockwise from I to IV (see Fig. 1).

Now we want to assign *coordinates* to each point in the plane. Given an arbitrary point P in the plane, pass horizontal and vertical lines through the point (Fig. 1). The vertical line will intersect the horizontal axis at a point with coordinate a, and the horizontal line will intersect the vertical axis at a point with coordinate b. These two numbers, written as the **ordered pair** (a, b),* form the **coordinates** of the point P. The first coordinate, a, is called the **abscissa** of P; the second coordinate, b, is called the **ordinate** of P. The abscissa of Q in Figure 1 is -5, and the ordinate of Q is 5. The coordinates of a point can also be referenced in terms of the axis labels. The ***x* coordinate** of R in Figure 1 is 10, and the ***y* coordinate** of R is -10. The point with coordinates $(0, 0)$ is called the **origin**.

The procedure we have just described assigns to each point P in the plane a unique pair of real numbers (a, b). Conversely, if we are given an ordered pair of real numbers (a, b), then, reversing this procedure, we can determine a unique point P in the plane. Thus,

There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

This is often referred to as the **fundamental theorem of analytic geometry**.

Graphs of $Ax + By = C$

In Section 1.1, we called an equation of the form $ax + b = 0$ $(a \neq 0)$ a linear equation in one variable. Now we want to consider linear equations in two variables:

DEFINITION Linear Equations in Two Variables

A **linear equation in two variables** is an equation that can be written in the **standard form**

$$Ax + By = C$$

where A, B, and C are constants (A and B not both 0), and x and y are variables.

A **solution** of an equation in two variables is an ordered pair of real numbers that satisfies the equation. For example, $(4, 3)$ is a solution of $3x - 2y = 6$. The **solution set** of an equation in two variables is the set of all solutions of the equation. The **graph** of an equation is the graph of its solution set.

Explore and Discuss 1 (A) As noted earlier, $(4, 3)$ is a solution of the equation

$$3x - 2y = 6$$

Find three more solutions of this equation. Plot these solutions in a Cartesian coordinate system. What familiar geometric shape could be used to describe the solution set of this equation?

*Here we use (a, b) as the coordinates of a point in a plane. In Section 1.1, we used (a, b) to represent an interval on a real number line. These concepts are not the same. You must always interpret the symbol (a, b) in terms of the context in which it is used.

(B) Repeat part (A) for the equation $x = 2$.
(C) Repeat part (A) for the equation $y = -3$.

In Explore and Discuss 1, you may have recognized that the graph of each equation is a (straight) line. Theorem 1 confirms this fact.

THEOREM 1 Graph of a Linear Equation in Two Variables

The graph of any equation of the form

$$Ax + By = C \quad (A \text{ and } B \text{ not both } 0) \tag{1}$$

is a line, and any line in a Cartesian coordinate system is the graph of an equation of this form.

If $A \neq 0$ and $B \neq 0$, then equation (1) can be written as

$$y = -\frac{A}{B}x + \frac{C}{B} = mx + b, m \neq 0$$

If $A = 0$ and $B \neq 0$, then equation (1) can be written as

$$y = \frac{C}{B}$$

and its graph is a **horizontal line**. If $A \neq 0$ and $B = 0$, then equation (1) can be written as

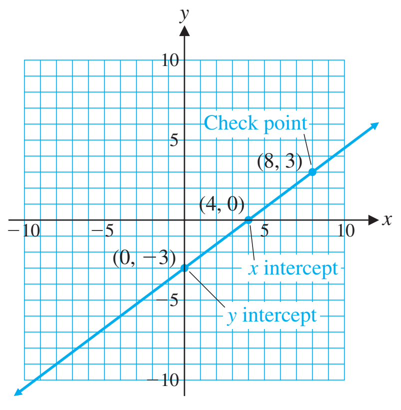
$$x = \frac{C}{A}$$

and its graph is a **vertical line**. To graph equation (1), or any of its special cases, plot any two points in the solution set and use a straightedge to draw the line through these two points. The points where the line crosses the axes are often the easiest to find. The *y* intercept* is the *y* coordinate of the point where the graph crosses the *y* axis, and the *x* intercept is the *x* coordinate of the point where the graph crosses the *x* axis. To find the *y* intercept, let $x = 0$ and solve for y. To find the *x* intercept, let $y = 0$ and solve for x. It is a good idea to find a third point as a check point.

EXAMPLE 1 Using Intercepts to Graph a Line Graph: $3x - 4y = 12$

SOLUTION

x	y	
0	-3	*y* intercept
4	0	*x* intercept
8	3	Check point



Matched Problem 1 Graph: $4x - 3y = 12$

*If the *x* intercept is *a* and the *y* intercept is *b*, then the graph of the line passes through the points $(a, 0)$ and $(0, b)$. It is common practice to refer to both the numbers *a* and *b* and the points $(a, 0)$ and $(0, b)$ as the *x* and *y* intercepts of the line.

The icon in the margin is used throughout this book to identify optional graphing calculator activities that are intended to give you additional insight into the concepts under discussion. You may have to consult the manual for your calculator* for the details necessary to carry out these activities.

EXAMPLE 2 Using a Graphing Calculator Graph $3x - 4y = 12$ on a graphing calculator and find the intercepts.

SOLUTION First, we solve $3x - 4y = 12$ for y.

$$3x - 4y = 12 \qquad \text{Add } -3x \text{ to both sides.}$$
$$-4y = -3x + 12 \qquad \text{Divide both sides by } -4.$$
$$y = \frac{-3x + 12}{-4} \qquad \text{Simplify.}$$
$$y = \frac{3}{4}x - 3 \tag{2}$$

Now we enter the right side of equation (2) in a calculator (Fig. 2A), enter values for the window variables (Fig. 2B), and graph the line (Fig. 2C). (The numerals to the left and right of the screen in Figure 2C are Xmin and Xmax, respectively. Similarly, the numerals below and above the screen are Ymin and Ymax.)

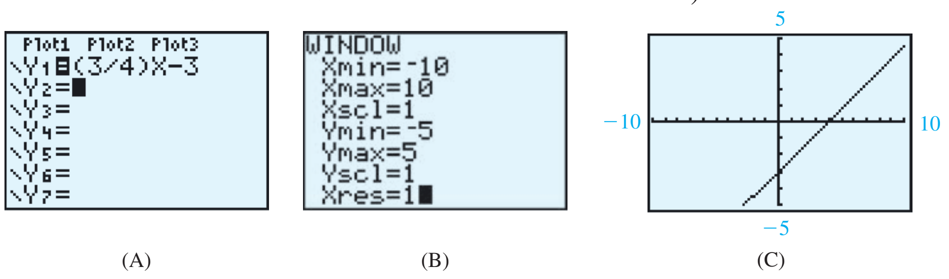


Figure 2 **Graphing a line on a graphing calculator**

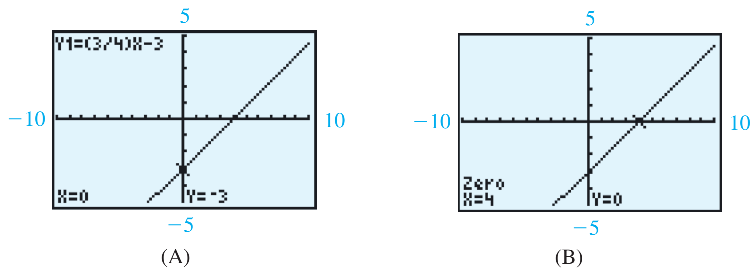


Figure 3 **Using `TRACE` and `zero` on a graphing calculator**

Next we use two calculator commands to find the intercepts: `TRACE` (Fig. 3A) and `zero` (Fig. 3B). The y intercept is -3 (Fig. 3A) and the x intercept is 4 (Fig. 3B).

Matched Problem 2 Graph $4x - 3y = 12$ on a graphing calculator and find the intercepts.

EXAMPLE 3 Horizontal and Vertical Lines

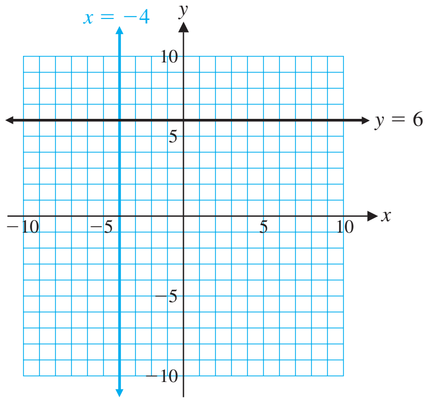
(A) Graph $x = -4$ and $y = 6$ simultaneously in the same rectangular coordinate system.

(B) Write the equations of the vertical and horizontal lines that pass through the point $(7, -5)$.

*We used a Texas Instruments graphing calculator from the TI-83/84 family to produce the graphing calculator screens in the book. Manuals for most graphing calculators are readily available on the Internet.

SOLUTION

(A)



(B) Horizontal line through $(7, -5)$: $y = -5$
Vertical line through $(7, -5)$: $x = 7$

Matched Problem 3

(A) Graph $x = 5$ and $y = -3$ simultaneously in the same rectangular coordinate system.

(B) Write the equations of the vertical and horizontal lines that pass through the point $(-8, 2)$.

Slope of a Line

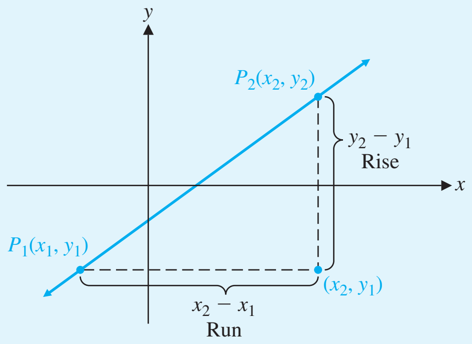
If we take two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, on a line, then the ratio of the change in y to the change in x as the point moves from point P_1 to point P_2 is called the **slope** of the line. In a sense, slope provides a measure of the "steepness" of a line relative to the x axis. The change in x is often called the **run**, and the change in y is the **rise**.

DEFINITION Slope of a Line

If a line passes through two distinct points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then its slope is given by the formula

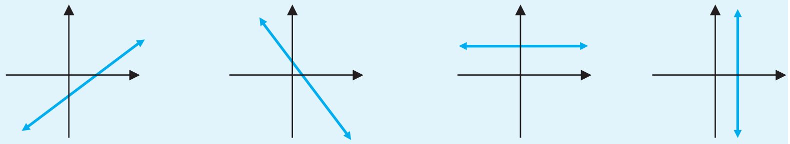
$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2$$

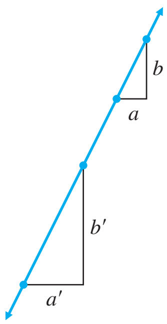
$$= \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$



For a horizontal line, y does not change; its slope is 0. For a vertical line, x does not change; $x_1 = x_2$ so its slope is not defined. In general, the slope of a line may be positive, negative, 0, or not defined. Each case is illustrated geometrically in Table 1.

Table 1 **Geometric Interpretation of Slope**

Line	Rising as x moves from left to right	Falling as x moves from left to right	Horizontal	Vertical
Slope	Positive	Negative	0	Not defined
Example				



$$m = \frac{b}{a} = \frac{b'}{a'}$$

Figure 4

CONCEPTUAL INSIGHT

One property of real numbers discussed in Appendix A, Section A.1, is

$$\frac{-a}{-b} = -\frac{-a}{b} = -\frac{a}{-b} = \frac{a}{b}, \quad b \neq 0$$

This property implies that it does not matter which point we label as P_1 and which we label as P_2 in the slope formula. For example, if $A = (4, 3)$ and $B = (1, 2)$, then

$$B = P_2 = (1, 2) \qquad A = P_2 = (4, 3)$$
$$A = P_1 = (4, 3) \qquad B = P_1 = (1, 2)$$
$$m = \frac{2 - 3}{1 - 4} = \frac{-1}{-3} = \frac{1}{3} = \frac{3 - 2}{4 - 1}$$

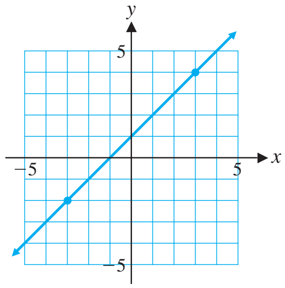
A property of similar triangles (see Table I in Appendix C) ensures that the slope of a line is the same for any pair of distinct points on the line (Fig. 4).

EXAMPLE 4 Finding Slopes Sketch a line through each pair of points, and find the slope of each line.

(A) $(-3, -2), (3, 4)$ (B) $(-1, 3), (2, -3)$
(C) $(-2, -3), (3, -3)$ (D) $(-2, 4), (-2, -2)$

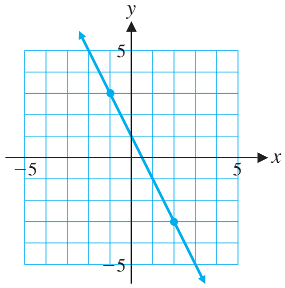
SOLUTION

(A)



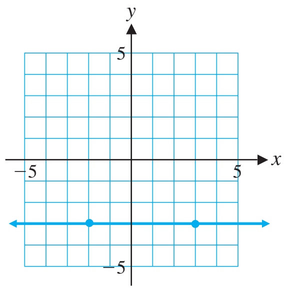
$$m = \frac{4 - (-2)}{3 - (-3)} = \frac{6}{6} = 1$$

(B)



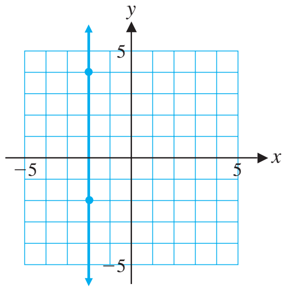
$$m = \frac{-3 - 3}{2 - (-1)} = \frac{-6}{3} = -2$$

(C)



$$m = \frac{-3 - (-3)}{3 - (-2)} = \frac{0}{5} = 0$$

(D)



$$m = \frac{-2 - 4}{-2 - (-2)} = \frac{-6}{0}$$

Slope is not defined.

Matched Problem 4 Find the slope of the line through each pair of points.

(A) $(-2, 4), (3, 4)$ (B) $(-2, 4), (0, -4)$

(C) $(-1, 5), (-1, -2)$ (D) $(-1, -2), (2, 1)$

Equations of Lines: Special Forms

Let us start by investigating why $y = mx + b$ is called the *slope-intercept form* for a line.

Explore and Discuss 2

(A) Graph $y = x + b$ for $b = -5, -3, 0, 3$, and 5 simultaneously in the same coordinate system. Verbally describe the geometric significance of b.

(B) Graph $y = mx - 1$ for $m = -2, -1, 0, 1$, and 2 simultaneously in the same coordinate system. Verbally describe the geometric significance of m.

(C) Using a graphing calculator, explore the graph of $y = mx + b$ for different values of m and b.

As you may have deduced from Explore and Discuss 2, constants m and b in $y = mx + b$ have the following geometric interpretations.

If we let $x = 0$, then $y = b$. So the graph of $y = mx + b$ crosses the y axis at $(0, b)$. The constant b is the *y intercept*. For example, the y intercept of the graph of $y = -4x - 1$ is -1.

To determine the geometric significance of m, we proceed as follows: If $y = mx + b$, then by setting $x = 0$ and $x = 1$, we conclude that $(0, b)$ and $(1, m + b)$ lie on its graph (Fig. 5). The slope of this line is given by:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(m + b) - b}{1 - 0} = m$$

So m is the slope of the line given by $y = mx + b$.

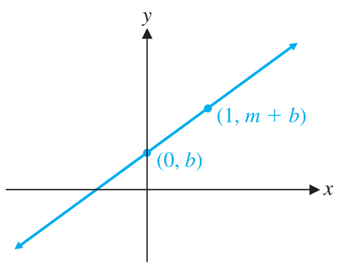


Figure 5

DEFINITION Slope-Intercept Form

The equation

$$y = mx + b \qquad m = \text{slope},\ b = y \text{ intercept} \tag{3}$$

is called the **slope-intercept form** of an equation of a line.

EXAMPLE 5 Using the Slope-Intercept Form

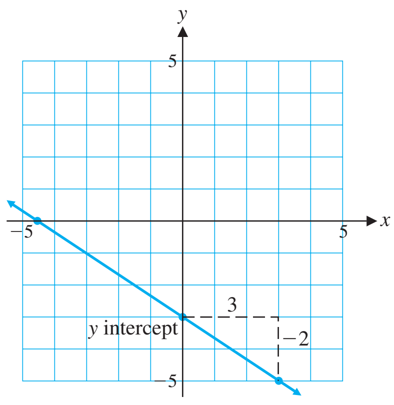
(A) Find the slope and y intercept, and graph $y = -\frac{2}{3}x - 3$.

(B) Write the equation of the line with slope $\frac{2}{3}$ and y intercept -2.

SOLUTION

(A) Slope $= m = -\frac{2}{3}$
y intercept $= b = -3$

(B) $m = \frac{2}{3}$ and $b = -2$;
so, $y = \frac{2}{3}x - 2$



Matched Problem 5 Write the equation of the line with slope $\frac{1}{2}$ and y intercept -1. Graph.

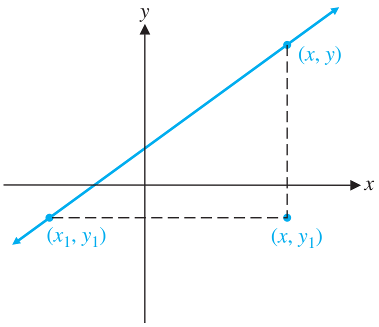


Figure 6

Suppose that a line has slope m and passes through a fixed point (x_1, y_1). If the point (x, y) is any other point on the line (Fig. 6), then

$$\frac{y - y_1}{x - x_1} = m$$

That is,

$$y - y_1 = m(x - x_1) \tag{4}$$

We now observe that (x_1, y_1) also satisfies equation (4) and conclude that equation (4) is an equation of a line with slope m that passes through (x_1, y_1).

DEFINITION Point-Slope Form

An equation of a line with slope m that passes through (x_1, y_1) is

$$y - y_1 = m(x - x_1) \tag{4}$$

which is called the **point-slope form** of an equation of a line.

The point-slope form is extremely useful, since it enables us to find an equation for a line if we know its slope and the coordinates of a point on the line or if we know the coordinates of two points on the line.

EXAMPLE 6 Using the Point-Slope Form

(A) Find an equation for the line that has slope $\frac{1}{2}$ and passes through $(-4, 3)$. Write the final answer in the form $Ax + By = C$.

(B) Find an equation for the line that passes through the points $(-3, 2)$ and $(-4, 5)$. Write the resulting equation in the form $y = mx + b$.

SOLUTION

(A) Use $y - y_1 = m(x - x_1)$. Let $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 3)$. Then

$$y - 3 = \tfrac{1}{2}[x - (-4)]$$

$$y - 3 = \tfrac{1}{2}(x + 4) \qquad \text{Multiply both sides by 2.}$$

$$2y - 6 = x + 4$$

$$-x + 2y = 10 \quad \text{or} \quad x - 2y = -10$$

(B) First, find the slope of the line by using the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{-4 - (-3)} = \frac{3}{-1} = -3$$

Now use $y - y_1 = m(x - x_1)$ with $m = -3$ and $(x_1, y_1) = (-3, 2)$:

$$\begin{aligned} y - 2 &= -3[x - (-3)] \\ y - 2 &= -3(x + 3) \\ y - 2 &= -3x - 9 \\ y &= -3x - 7 \end{aligned}$$

Matched Problem 6

(A) Find an equation for the line that has slope $\frac{2}{3}$ and passes through $(6, -2)$. Write the resulting equation in the form $Ax + By = C, A > 0$.

(B) Find an equation for the line that passes through $(2, -3)$ and $(4, 3)$. Write the resulting equation in the form $y = mx + b$.

The various forms of the equation of a line that we have discussed are summarized in Table 2 for quick reference.

Table 2 **Equations of a Line**

Standard form	$Ax + By = C$	A and B not both 0
Slope-intercept form	$y = mx + b$	Slope: m; y intercept: b
Point-slope form	$y - y_1 = m(x - x_1)$	Slope: m; point: (x_1, y_1)
Horizontal line	$y = b$	Slope: 0
Vertical line	$x = a$	Slope: undefined

Applications

We will now see how equations of lines occur in certain applications.

EXAMPLE 7 Cost Equation The management of a company that manufactures skateboards has fixed costs (costs at 0 output) of \$300 per day and total costs of \$4,300 per day at an output of 100 skateboards per day. Assume that cost C is linearly related to output x.

(A) Find the slope of the line joining the points associated with outputs of 0 and 100; that is, the line passing through $(0, 300)$ and $(100, 4{,}300)$.

(B) Find an equation of the line relating output to cost. Write the final answer in the form $C = mx + b$.

(C) Graph the cost equation from part (B) for $0 \le x \le 200$.

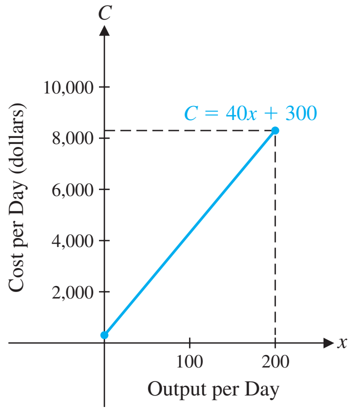
SOLUTION

(A)
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4{,}300 - 300}{100 - 0} \\ &= \frac{4{,}000}{100} = 40 \end{aligned}$$

(B) We must find an equation of the line that passes through $(0, 300)$ with slope 40. We use the slope-intercept form:

$$\begin{aligned} C &= mx + b \\ C &= 40x + 300 \end{aligned}$$

(C)



In Example 7, the *fixed cost* of \$300 per day covers plant cost, insurance, and so on. This cost is incurred whether or not there is any production. The *variable cost* is $40x$, which depends on the day's output. Since increasing production from x to $x + 1$ will increase the cost by \$40 (from $40x + 300$ to $40x + 340$), the slope 40 can be interpreted as the **rate of change** of the cost function with respect to production x.

Matched Problem 7 Answer parts (A) and (B) in Example 7 for fixed costs of \$250 per day and total costs of \$3,450 per day at an output of 80 skateboards per day.

In a free competitive market, the price of a product is determined by the relationship between supply and demand. If there is a surplus—that is, the supply is greater than the demand—the price tends to come down. If there is a shortage—that is, the demand is greater than the supply—the price tends to go up. The price tends to move toward an equilibrium price at which the supply and demand are equal. Example 8 introduces the basic concepts.

EXAMPLE 8 Supply and Demand At a price of \$9.00 per box of oranges, the supply is 320,000 boxes and the demand is 200,000 boxes. At a price of \$8.50 per box, the supply is 270,000 boxes and the demand is 300,000 boxes.

(A) Find a price–supply equation of the form $p = mx + b$, where p is the price in dollars and x is the corresponding supply in thousands of boxes.

(B) Find a price–demand equation of the form $p = mx + b$, where p is the price in dollars and x is the corresponding demand in thousands of boxes.

(C) Graph the price–supply and price–demand equations in the same coordinate system and find their point of intersection.

SOLUTION

(A) To find a price–supply equation of the form $p = mx + b$, we must find two points of the form (x, p) that are on the supply line. From the given supply data, $(320, 9)$ and $(270, 8.5)$ are two such points. First, find the slope of the line:

$$m = \frac{9 - 8.5}{320 - 270} = \frac{0.5}{50} = 0.01$$

Now use the point-slope form to find the equation of the line:

$$\begin{aligned} p - p_1 &= m(x - x_1) && (x_1, p_1) = (320, 9) \\ p - 9 &= 0.01(x - 320) \\ p - 9 &= 0.01x - 3.2 \\ p &= 0.01x + 5.8 && \text{Price–supply equation} \end{aligned}$$

(B) From the given demand data, $(200, 9)$ and $(300, 8.5)$ are two points on the demand line.

$$m = \frac{8.5 - 9}{300 - 200} = \frac{-0.5}{100} = -0.005$$

$$\begin{aligned} p - p_1 &= m(x - x_1) && (x_1, p_1) = (200, 9) \\ p - 9 &= -0.005(x - 200) \\ p - 9 &= -0.005x + 1 \\ p &= -0.005x + 10 && \text{Price–demand equation} \end{aligned}$$

(C) From part (A), we plot the points $(320, 9)$ and $(270, 8.5)$ and then draw the line through them. We do the same with the points $(200, 9)$ and $(300, 8.5)$ from part (B) (Fig. 7). (Note that we restricted the axes to intervals that contain these data points.) To find the intersection point of the two lines, we equate the right-hand sides of the price–supply and price–demand equations and solve for x:

Price–supply Price–demand

$$\begin{aligned} 0.01x + 5.8 &= -0.005x + 10 \\ 0.015x &= 4.2 \\ x &= 280 \end{aligned}$$

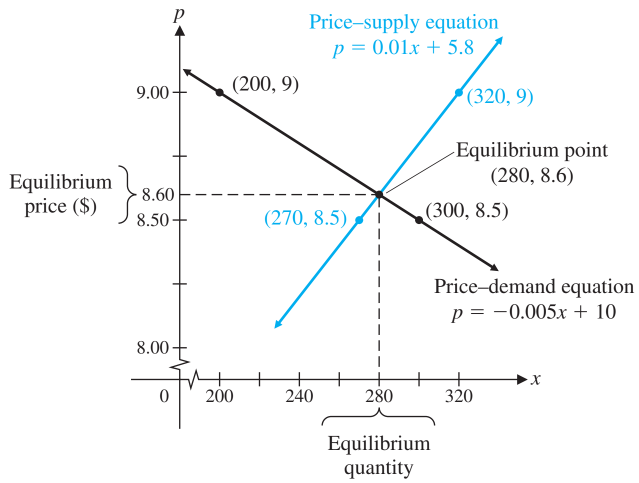


Figure 7 **Graphs of price–supply and price–demand equations**

Now use the price–supply equation to find p when $x = 280$:

$$\begin{aligned} p &= 0.01x + 5.8 \\ p &= 0.01(280) + 5.8 = 8.6 \end{aligned}$$

As a check, we use the price–demand equation to find p when $x = 280$:

$$\begin{aligned} p &= -0.005x + 10 \\ p &= -0.005(280) + 10 = 8.6 \end{aligned}$$

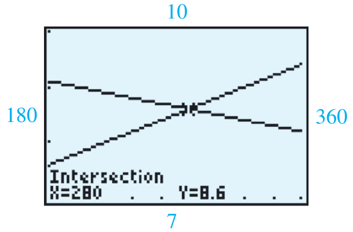


Figure 8 **Finding an intersection point**

The lines intersect at $(280, 8.6)$. The intersection point of the price–supply and price–demand equations is called the **equilibrium point**, and its coordinates are the **equilibrium quantity** (280) and the **equilibrium price** ($8.60). These terms are illustrated in Figure 7. The intersection point can also be found by using the INTERSECT command on a graphing calculator (Fig. 8). To summarize, the price of a box of oranges tends toward the equilibrium price of $8.60, at which the supply and demand are both equal to 280,000 boxes.

Matched Problem 8 At a price of \$12.59 per box of grapefruit, the supply is 595,000 boxes and the demand is 650,000 boxes. At a price of \$13.19 per box, the supply is 695,000 boxes and the demand is 590,000 boxes. Assume that the relationship between price and supply is linear and that the relationship between price and demand is linear.

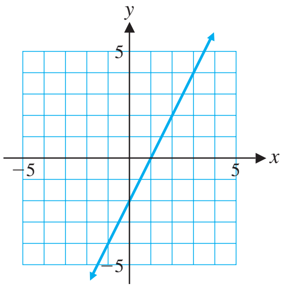
(A) Find a price–supply equation of the form $p = mx + b$.

(B) Find a price–demand equation of the form $p = mx + b$.

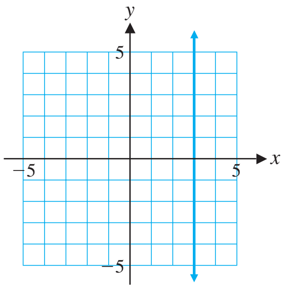
(C) Find the equilibrium point.

Exercises 1.2

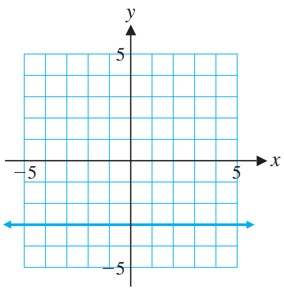
Problems 1–4 refer to graphs (A)–(D).



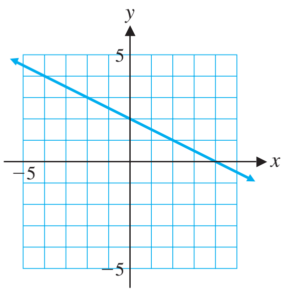
(A)



(B)



(C)



(D)

1. Identify the graph(s) of lines with a negative slope.

2. Identify the graph(s) of lines with a positive slope.

3. Identify the graph(s) of any lines with slope zero.

4. Identify the graph(s) of any lines with undefined slope.

In Problems 5–8, sketch a graph of each equation in a rectangular coordinate system.

5. $y = 2x - 3$

6. $y = \frac{x}{2} + 1$

7. $2x + 3y = 12$

8. $8x - 3y = 24$

In Problems 9–14, find the slope and y intercept of the graph of each equation.

9. $y = 5x - 7$

10. $y = 3x + 2$

11. $y = -\frac{5}{2}x - 9$

12. $y = -\frac{10}{3}x + 4$

13. $y = \frac{x}{4} + \frac{2}{3}$

14. $y = \frac{x}{5} - \frac{1}{2}$

In Problems 15–18, write an equation of the line with the indicated slope and y intercept.

15. Slope $= 2$
y intercept $= 1$

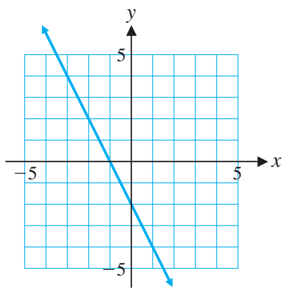
16. Slope $= 1$
y intercept $= 5$

17. Slope $= -\frac{1}{3}$
y intercept $= 6$

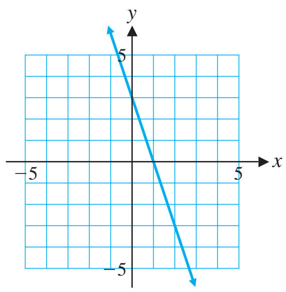
18. Slope $= \frac{6}{7}$
y intercept $= -\frac{9}{2}$

In Problems 19–22, use the graph of each line to find the x intercept, y intercept, and slope. Write the slope-intercept form of the equation of the line.

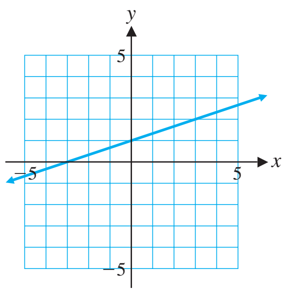
19.



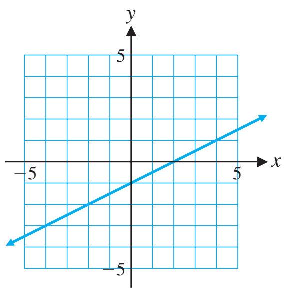
20.



21.



22.



Sketch a graph of each equation or pair of equations in Problems 23–28 in a rectangular coordinate system.

23. $y = -\frac{2}{3}x - 2$

24. $y = -\frac{3}{2}x + 1$

25. $3x - 2y = 10$

26. $5x - 6y = 15$

27. $x = 3; y = -2$

28. $x = -3; y = 2$

In Problems 29–34, find the slope of the graph of each equation.

29. $4x + y = 3$

30. $5x - y = -2$

31. $3x + 5y = 15$

32. $2x - 3y = 18$

33. $-4x + 2y = 9$

34. $-x + 8y = 4$

35. Given $Ax + By = 12$, graph each of the following three cases in the same coordinate system.

(A) $A = 2$ and $B = 0$

(B) $A = 0$ and $B = 3$

(C) $A = 3$ and $B = 4$

36. Given $Ax + By = 24$, graph each of the following three cases in the same coordinate system.

(A) $A = 6$ and $B = 0$

(B) $A = 0$ and $B = 8$

(C) $A = 2$ and $B = 3$

37. Graph $y = 25x + 200$, $x \geq 0$.

38. Graph $y = 40x + 160$, $x \geq 0$.

39. (A) Graph $y = 1.2x - 4.2$ in a rectangular coordinate system.

(B) Find the x and y intercepts algebraically to one decimal place.

(C) Graph $y = 1.2x - 4.2$ in a graphing calculator.

(D) Find the x and y intercepts to one decimal place using `TRACE` and the `zero` command.

40. (A) Graph $y = -0.8x + 5.2$ in a rectangular coordinate system.

(B) Find the x and y intercepts algebraically to one decimal place.

(C) Graph $y = -0.8x + 5.2$ in a graphing calculator.

(D) Find the x and y intercepts to one decimal place using `TRACE` and the `zero` command.

(E) Using the results of parts (A) and (B), or (C) and (D), find the solution set for the linear inequality

$$-0.8x + 5.2 < 0$$

In Problems 41–44, write the equations of the vertical and horizontal lines through each point.

41. $(4, -3)$

42. $(-5, 6)$

43. $(-1.5, -3.5)$

44. $(2.6, 3.8)$

In Problems 45–52, write the slope-intercept form of the equation of the line with the indicated slope that goes through the given point.

45. $m = 5$; $(3, 0)$

46. $m = 4$; $(0, 6)$

47. $m = -2$; $(-1, 9)$

48. $m = -10$; $(2, -5)$

49. $m = \frac{1}{3}$; $(-4, -8)$

50. $m = \frac{2}{7}$; $(7, 1)$

51. $m = -3.2$; $(5.8, 12.3)$

52. $m = 0.9$; $(2.3, 6.7)$

In Problems 53–60,

(A) *Find the slope of the line that passes through the given points.*

(B) *Find the standard form of the equation of the line.*

(C) *Find the slope-intercept form of the equation of the line.*

53. $(2, 5)$ and $(5, 7)$

54. $(1, 2)$ and $(3, 5)$

55. $(-2, -1)$ and $(2, -6)$

56. $(2, 3)$ and $(-3, 7)$

57. $(5, 3)$ and $(5, -3)$

58. $(1, 4)$ and $(0, 4)$

59. $(-2, 5)$ and $(3, 5)$

60. $(2, 0)$ and $(2, -3)$

61. Discuss the relationship among the graphs of the lines with equation $y = mx + 2$, where m is any real number.

62. Discuss the relationship among the graphs of the lines with equation $y = -0.5x + b$, where b is any real number.

Applications

63. **Cost analysis.** A donut shop has a fixed cost of \$124 per day and a variable cost of \$0.12 per donut. Find the total daily cost of producing x donuts. How many donuts can be produced for a total daily cost of \$250?

64. **Cost analysis.** A small company manufactures picnic tables. The weekly fixed cost is \$1,200 and the variable cost is \$45 per table. Find the total weekly cost of producing x picnic tables. How many picnic tables can be produced for a total weekly cost of \$4,800?

65. **Cost analysis.** A plant can manufacture 80 golf clubs per day for a total daily cost of \$7,647 and 100 golf clubs per day for a total daily cost of \$9,147.

(A) Assuming that daily cost and production are linearly related, find the total daily cost of producing x golf clubs.

(B) Graph the total daily cost for $0 \leq x \leq 200$.

(C) Interpret the slope and y intercept of this cost equation.

66. **Cost analysis.** A plant can manufacture 50 tennis rackets per day for a total daily cost of \$3,855 and 60 tennis rackets per day for a total daily cost of \$4,245.

(A) Assuming that daily cost and production are linearly related, find the total daily cost of producing x tennis rackets.

(B) Graph the total daily cost for $0 \leq x \leq 100$.

(C) Interpret the slope and y intercept of this cost equation.

67. **Business—Markup policy.** A drugstore sells a drug costing \$85 for \$112 and a drug costing \$175 for \$238.

(A) If the markup policy of the drugstore is assumed to be linear, write an equation that expresses retail price R in terms of cost C (wholesale price).

(B) What does a store pay (to the nearest dollar) for a drug that retails for \$185?

68. **Business—Markup policy.** A clothing store sells a shirt costing \$20 for \$33 and a jacket costing \$60 for \$93.

(A) If the markup policy of the store is assumed to be linear, write an equation that expresses retail price R in terms of cost C (wholesale price).

(B) What does a store pay for a suit that retails for \$240?

69. **Business—Depreciation.** A farmer buys a new tractor for \$157,000 and assumes that it will have a trade-in value of \$82,000 after 10 years. The farmer uses a constant rate of depreciation (commonly called **straight-line**

depreciation—one of several methods permitted by the IRS) to determine the annual value of the tractor.

(A) Find a linear model for the depreciated value V of the tractor t years after it was purchased.

(B) What is the depreciated value of the tractor after 6 years?

(C) When will the depreciated value fall below $70,000?

(D) Graph V for $0 \le t \le 20$ and illustrate the answers from parts (B) and (C) on the graph.

70. **Business—Depreciation.** A charter fishing company buys a new boat for $224,000 and assumes that it will have a trade-in value of $115,200 after 16 years.

(A) Find a linear model for the depreciated value V of the boat t years after it was purchased.

(B) What is the depreciated value of the boat after 10 years?

(C) When will the depreciated value fall below $100,000?

(D) Graph V for $0 \le t \le 30$ and illustrate the answers from (B) and (C) on the graph.

71. **Boiling point.** The temperature at which water starts to boil is called its **boiling point** and is linearly related to the altitude. Water boils at 212°F at sea level and at 193.6°F at an altitude of 10,000 feet. (*Source*: biggreenegg.com)

(A) Find a relationship of the form $T = mx + b$ where T is degrees Fahrenheit and x is altitude in thousands of feet.

(B) Find the boiling point at an altitude of 3,500 feet.

(C) Find the altitude if the boiling point is 200°F.

(D) Graph T and illustrate the answers to (B) and (C) on the graph.

72. **Boiling point.** The temperature at which water starts to boil is also linearly related to barometric pressure. Water boils at 212°F at a pressure of 29.9 inHg (inches of mercury) and at 191°F at a pressure of 28.4 inHg. (*Source*: biggreenegg.com)

(A) Find a relationship of the form $T = mx + b$, where T is degrees Fahrenheit and x is pressure in inches of mercury.

(B) Find the boiling point at a pressure of 31 inHg.

(C) Find the pressure if the boiling point is 199°F.

(D) Graph T and illustrate the answers to (B) and (C) on the graph.

73. **Flight conditions.** In stable air, the air temperature drops about 3.6°F for each 1,000-foot rise in altitude. (*Source*: Federal Aviation Administration)

(A) If the temperature at sea level is 70°F, write a linear equation that expresses temperature T in terms of altitude A in thousands of feet.

(B) At what altitude is the temperature 34°F?

74. **Flight navigation.** The airspeed indicator on some aircraft is affected by the changes in atmospheric pressure at different altitudes. A pilot can estimate the true airspeed by observing the indicated airspeed and adding to it about 1.6% for every 1,000 feet of altitude. (*Source:* Megginson Technologies Ltd.)

(A) A pilot maintains a constant reading of 200 miles per hour on the airspeed indicator as the aircraft climbs from sea level to an altitude of 10,000 feet. Write a linear equation that expresses true airspeed T (in miles per hour) in terms of altitude A (in thousands of feet).

(B) What would be the true airspeed of the aircraft at 6,500 feet?

75. **Demographics.** The average number of persons per household in the United States has been shrinking steadily for as long as statistics have been kept and is approximately linear with respect to time. In 1980 there were about 2.76 persons per household, and in 2012 about 2.55. (*Source:* U.S. Census Bureau)

(A) If N represents the average number of persons per household and t represents the number of years since 1980, write a linear equation that expresses N in terms of t.

(B) Use this equation to estimate household size in the year 2030.

76. **Demographics.** The **median** household income divides the households into two groups: the half whose income is less than or equal to the median, and the half whose income is greater than the median. The median household income in the United States grew from about $30,000 in 1990 to about $53,000 in 2010. (*Source*: U.S. Census Bureau)

(A) If I represents the median household income and t represents the number of years since 1990, write a linear equation that expresses I in terms of t.

(B) Use this equation to estimate median household income in the year 2030.

77. **Cigarette smoking.** The percentage of female cigarette smokers in the United States declined from 21.0% in 2000 to 17.3% in 2010. (*Source*: Centers for Disease Control)

(A) Find a linear equation relating percentage of female smokers (f) to years since 2000 (t).

(B) Find the year in which the percentage of female smokers falls below 12%.

78. **Cigarette smoking.** The percentage of male cigarette smokers in the United States declined from 25.7% in 2000 to 21.5% in 2010. (*Source:* Centers for Disease Control)

(A) Find a linear equation relating percentage of male smokers (m) to years since 2000 (t).

(B) Find the year in which the percentage of male smokers falls below 12%.

79. **Supply and demand.** At a price of $2.28 per bushel, the supply of barley is 7,500 million bushels and the demand is 7,900 million bushels. At a price of $2.37 per bushel, the supply is 7,900 million bushels and the demand is 7,800 million bushels.

(A) Find a price–supply equation of the form $p = mx + b$.

(B) Find a price–demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.

(D) Graph the price–supply equation, price–demand equation, and equilibrium point in the same coordinate system.

80. Supply and demand. At a price of \$1.94 per bushel, the supply of corn is 9,800 million bushels and the demand is 9,300 million bushels. At a price of \$1.82 per bushel, the supply is 9,400 million bushels and the demand is 9,500 million bushels.

(A) Find a price–supply equation of the form $p = mx + b$.

(B) Find a price–demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.

(D) Graph the price–supply equation, price–demand equation, and equilibrium point in the same coordinate system.

81. Physics. Hooke's law states that the relationship between the stretch s of a spring and the weight w causing the stretch is linear. For a particular spring, a 5-pound weight causes a stretch of 2 inches, while with no weight, the stretch of the spring is 0.

(A) Find a linear equation that expresses s in terms of w.

(B) What is the stretch for a weight of 20 pounds?

(C) What weight will cause a stretch of 3.6 inches?

82. Physics. The distance d between a fixed spring and the floor is a linear function of the weight w attached to the bottom of the spring. The bottom of the spring is 18 inches from the floor when the weight is 3 pounds, and 10 inches from the floor when the weight is 5 pounds.

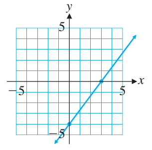
(A) Find a linear equation that expresses d in terms of w.

(B) Find the distance from the bottom of the spring to the floor if no weight is attached.

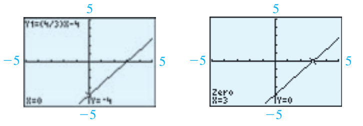
(C) Find the smallest weight that will make the bottom of the spring touch the floor. (Ignore the height of the weight.)

Answers to Matched Problems

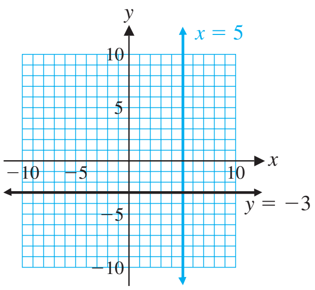
1.



2. y intercept $= -4$, x intercept $= 3$



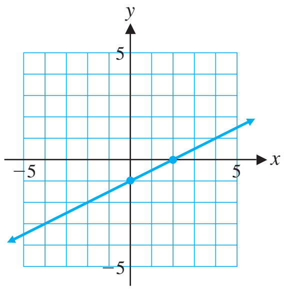
3. (A)



(B) Horizontal line: $y = 2$; vertical line: $x = -8$

4. (A) 0 (B) -4 (C) Not defined (D) 1

5. $y = \frac{1}{2}x - 1$



6. (A) $2x - 3y = 18$ (B) $y = 3x - 9$

7. (A) $m = 40$ (B) $C = 40x + 250$

8. (A) $p = 0.006x + 9.02$ (B) $p = -0.01x + 19.09$ (C) $(629, 12.80)$

1.3 Linear Regression

- Slope as a Rate of Change
- Linear Regression

Mathematical modeling is the process of using mathematics to solve real-world problems. This process can be broken down into three steps (Fig. 1):

Step 1 *Construct* the **mathematical model** (that is, a mathematics problem that, when solved, will provide information about the real-world problem).

Step 2 *Solve* the mathematical model.

Step 3 *Interpret* the solution to the mathematical model in terms of the original real-world problem.

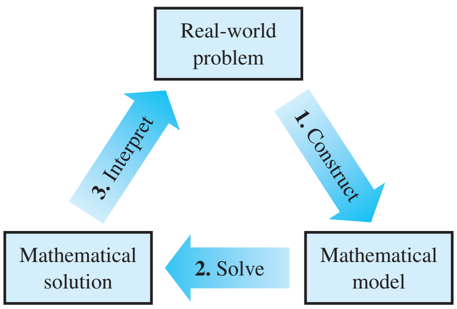


Figure 1

In more complex problems, this cycle may have to be repeated several times to obtain the required information about the real-world problem. In this section, we will discuss one of the simplest mathematical models, a linear equation. With the aid of a graphing calculator or computer, we also will learn how to analyze a linear model based on real-world data.

Slope as a Rate of Change

If x and y are related by the equation $y = mx + b$, where m and b are constants with $m \neq 0$, then x and y are **linearly related**. If (x_1, y_1) and (x_2, y_2) are two distinct points on this line, then the slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} \qquad (1)$$

In applications, ratio (1) is called the **rate of change** of y with respect to x. Since the slope of a line is unique, **the rate of change of two linearly related variables is constant**. Here are some examples of familiar rates of change: miles per hour, revolutions per minute, price per pound, passengers per plane, and so on. If the relationship between x and y is not linear, ratio (1) is called the **average rate of change** of y with respect to x.

EXAMPLE 1 Estimating Body Surface Area Appropriate doses of medicine for both animals and humans are often based on body surface area (BSA). Since weight is much easier to determine than BSA, veterinarians use the weight of an animal to estimate BSA. The following linear equation expresses BSA for canines in terms of weight:

$$a = 16.12w + 375.6$$

where a is BSA in square inches and w is weight in pounds. (*Source*: Veterinary Oncology Consultants, PTY LTD)

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a one-pound increase in weight?

SOLUTION

(A) The rate-of-change BSA with respect to weight is 16.12 square inches per pound.

(B) Since slope is the ratio of rise to run, increasing w by 1 pound (run) increases a by 16.12 square inches (rise).

Matched Problem 1 The equation $a = 28.55w + 118.7$ expresses BSA for felines in terms of weight, where a is BSA in square inches and w is weight in pounds.

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a one-pound increase in weight?

Explore and Discuss 1 As illustrated in Example 1A, the slope m of a line with equation $y = mx + b$ has two interpretations:

1. m is the rate of change of y with respect to x.
2. Increasing x by one unit will change y by m units.

How are these two interpretations related?

Parachutes are used to deliver cargo to areas that cannot be reached by other means. The **rate of descent** of the cargo is the rate of change of altitude with respect to time. The absolute value of the rate of descent is called the **speed** of the cargo. At low altitudes, the altitude of the cargo and the time in the air are linearly related. The appropriate rate of descent varies widely with the item. Bulk food (rice, flour, beans, etc.) and clothing can tolerate nearly any rate of descent under 40 ft/sec. Machinery and electronics (pumps, generators, radios, etc.) should generally be dropped at 15 ft/sec or less. Butler Tactical Parachute Systems in Roanoke, Virginia, manufactures a variety of canopies for dropping cargo. The following example uses information taken from the company's brochures.

EXAMPLE 2 Finding the Rate of Descent A 100-pound cargo of delicate electronic equipment is dropped from an altitude of 2,880 feet and lands 200 seconds later. (*Source*: Butler Tactical Parachute Systems)

(A) Find a linear model relating altitude a (in feet) and time in the air t (in seconds).

(B) How fast is the cargo moving when it lands?

SOLUTION

(A) If $a = mt + b$ is the linear equation relating altitude a and time in air t, then the graph of this equation must pass through the following points:

$$(t_1, a_1) = (0, 2{,}880) \quad \text{Cargo is dropped from plane.}$$
$$(t_2, a_2) = (200, 0) \quad \text{Cargo lands.}$$

The slope of this line is

$$m = \frac{a_2 - a_1}{t_2 - t_1} = \frac{0 - 2{,}880}{200 - 0} = -14.4$$

and the equation of this line is

$$a - 0 = -14.4(t - 200)$$
$$a = -14.4t + 2{,}880$$

(B) The rate of descent is the slope $m = -14.4$, so the speed of the cargo at landing is $|-14.4| = 14.4$ ft/sec.

Matched Problem 2 A 400-pound load of grain is dropped from an altitude of 2,880 feet and lands 80 seconds later.

(A) Find a linear model relating altitude a (in feet) and time in the air t (in seconds).

(B) How fast is the cargo moving when it lands?

Linear Regression

In real-world applications, we often encounter numerical data in the form of a table. **Regression analysis** is a process for finding a function that provides a useful model for a set of data points. Graphs of equations are often called **curves**, and regression analysis is also referred to as **curve fitting**. In the next example, we use a linear model obtained by using **linear regression** on a graphing calculator.

Table 1 **Round-Shaped Diamond Prices**

Weight (carats)	Price
0.5	\$2,790
0.6	\$3,191
0.7	\$3,694
0.8	\$4,154
0.9	\$5,018
1.0	\$5,898

Source: www.tradeshop.com

EXAMPLE 3 Diamond Prices Prices for round-shaped diamonds taken from an online trader are given in Table 1.

(A) A linear model for the data in Table 1 is given by

$$p = 6{,}140c - 480 \tag{2}$$

where p is the price of a diamond weighing c carats. (We will discuss the source of models like this later in this section.) Plot the points in Table 1 on a Cartesian coordinate system, producing a *scatter plot*, and graph the model on the same axes.

(B) Interpret the slope of the model in (2).

(C) Use the model to estimate the cost of a 0.85-carat diamond and the cost of a 1.2-carat diamond. Round answers to the nearest dollar.

(D) Use the model to estimate the weight of a diamond (to two decimal places) that sells for \$4,000.

SOLUTION

(A) A **scatter plot** is simply a graph of the points in Table 1 (Fig. 2A). To add the graph of the model to the scatter plot, we find any two points that satisfy equation (2) [we choose $(0.4, 1{,}976)$ and $(1.1, 6{,}274)$]. Plotting these points and drawing a line through them gives us Figure 2B.

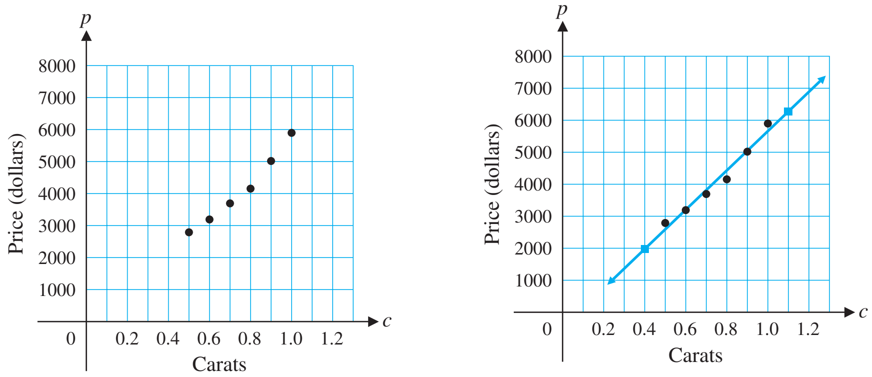


Figure 2 (A) Scatter plot (B) Linear model

(B) The rate of change of the price of a diamond with respect to its weight is 6,140. Increasing the weight by one carat will increase the price by about \$6,140.

(C) The graph of the model (Fig. 2B) does not pass through any of the points in the scatter plot, but it comes close to all of them. [Verify this by evaluating equation (2) at $c = 0.5, 0.6, \ldots, 1$.] So we can use equation (2) to approximate points not in Table 1.

$$c = 0.85 \qquad\qquad c = 1.2$$
$$p \approx 6{,}140(0.85) - 480 \qquad p \approx 6{,}140(1.2) - 480$$
$$= \$4{,}739 \qquad\qquad = \$6{,}888$$

A 0.85-carat diamond will cost about \$4,739, and a 1.2-carat diamond will cost about \$6,888.

(D) To find the weight of a \$4,000 diamond, we solve the following equation for c:

$$6{,}140c - 480 = 4{,}000 \qquad \text{Add 480 to both sides.}$$
$$6{,}140c = 4{,}480 \qquad \text{Divide both sides by 6,140.}$$
$$c = \frac{4{,}480}{6{,}140} \approx 0.73 \qquad \text{Rounded to two decimal places.}$$

A \$4,000 diamond will weigh about 0.73 carat.

Matched Problem 3 Prices for emerald-shaped diamonds from an online trader are given in Table 2. Repeat Example 3 for this data with the linear model

$$p = 5{,}600c - 1{,}100$$

where p is the price of an emerald-shaped diamond weighing c carats.

Table 2 **Emerald-Shaped Diamond Prices**

Weight (carats)	Price
0.5	\$1,677
0.6	\$2,353
0.7	\$2,718
0.8	\$3,218
0.9	\$3,982
1.0	\$4,510

Source: www.tradeshop.com

The model we used in Example 3 was obtained using a technique called **linear regression**, and the model is called the **regression line**. This technique produces a line that is the **best fit*** for a given data set. Although you can find a linear regression line

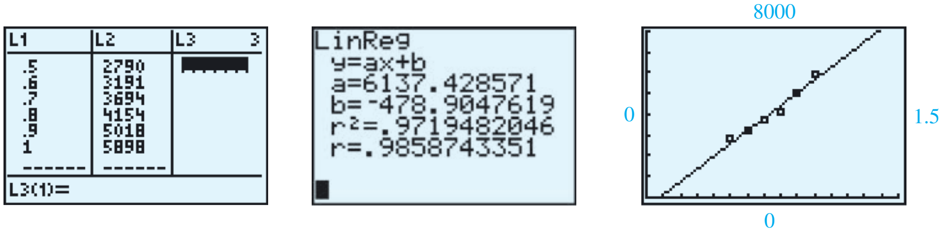
*The line of best fit is the line that minimizes the sum of the squares of the vertical distances from the data points to the line.

by hand, we prefer to leave the calculations to a graphing calculator or a computer. Don't be concerned if you don't have either of these electronic devices. We will supply the regression model in most of the applications we discuss, as we did in Example 3.

Explore and Discuss 2 As stated previously, we used linear regression to produce the model in Example 3. If you have a graphing calculator that supports linear regression, then you can find this model. The linear regression process varies greatly from one calculator to another. Consult the user's manual for the details of linear regression. The screens in Figure 3 are related to the construction of the model in Example 3 on a Texas Instruments TI-84 Plus.

(A) Produce similar screens on your graphing calculator.

(B) Do the same for Matched Problem 3.



(A) Entering the data (B) Finding the model (C) Graphing the data and the model

Figure 3 Linear regression on a graphing calculator

In Example 3, we used the regression model to approximate points that were not given in Table 1 but would fit between points in the table. This process is called **interpolation**. In the next example, we use a regression model to approximate points outside the given data set. This process is called **extrapolation**, and the approximations are often referred to as **predictions**.

Table 3 Atmospheric Concentration of CO_2 (parts per million)

2000	2005	2007	2009	2011
369	380	384	387	392

Source: U.S. Department of Energy

EXAMPLE 4 **Atmospheric Concentration of Carbon Dioxide** Table 3 contains information about the concentration of carbon dioxide (CO_2) in the atmosphere. The linear regression model for the data is

$$C = 369 + 2.05t$$

where C is the concentration (in parts per million) of carbon dioxide and t is the time in years with $t = 0$ corresponding to the year 2000.

(A) Interpret the slope of the regression line as a rate of change.

(B) Use the regression model to predict the concentration of CO_2 in the atmosphere in 2025.

SOLUTION

(A) The slope $m = 2.05$ is the rate of change of concentration of CO_2 with respect to time. Since the slope is positive, the concentration of CO_2 is increasing at a rate of 2.05 parts per million per year.

(B) If $t = 25$, then

$$C = 369 + 2.05(25) \approx 420$$

So the model predicts that the atmospheric concentration of CO_2 will be approximately 420 parts per million in 2025.

Matched Problem 4 Using the model of Example 4, estimate the concentration of carbon dioxide in the atmosphere in the year 1990.

Forest managers estimate growth, volume, yield, and forest potential. One common measure is the diameter of a tree at breast height (Dbh), which is defined as the diameter of the tree at a point 4.5 feet above the ground on the uphill side of the tree. Example 5 uses Dbh to estimate the height of balsam fir trees.

EXAMPLE 5 Forestry A linear regression model for the height of balsam fir trees is

$$h = 3.8d + 18.73$$

where d is Dbh in inches and h is the height in feet.

(A) Interpret the slope of this model.

(B) What is the effect of a 1-inch increase in Dbh?

(C) Estimate the height of a balsam fir with a Dbh of 8 inches. Round your answer to the nearest foot.

(D) Estimate the Dbh of a balsam fir that is 30 feet tall. Round your answer to the nearest inch.

SOLUTION

(A) The rate of change of height with respect to breast height diameter is 3.8 feet per inch.

(B) Height increases by 3.8 feet.

(C) We must find h when $d = 8$:

$$\begin{aligned} h &= 3.8d + 18.73 && \text{Substitute } d = 8. \\ h &= 3.8(8) + 18.73 && \text{Evaluate.} \\ h &= 49.13 \approx 49 \text{ ft} \end{aligned}$$

(D) We must find d when $h = 30$:

$$\begin{aligned} h &= 3.8d + 18.73 && \text{Substitute } h = 30. \\ 30 &= 3.8d + 18.73 && \text{Subtract 18.73 from both sides.} \\ 11.27 &= 3.8d && \text{Divide both sides by 3.8.} \\ d &= \frac{11.27}{3.8} \approx 3 \text{ in.} \end{aligned}$$

The data used to produce the regression model in Example 5 are from the Jack Haggerty Forest at Lakehead University in Canada (Table 4). We used the popular

Table 4 **Height and Diameter of the Balsam Fir**

Dbh (in.)	Height (ft)	Dbh (in.)	Height (ft)	Dbh (in.)	Height (ft)	Dbh (in.)	Height (ft)
6.5	51.8	6.4	44.0	3.1	19.7	4.6	26.6
8.6	50.9	4.4	46.9	7.1	55.8	4.8	33.1
5.7	49.2	6.5	52.2	6.3	32.8	3.1	28.5
4.9	46.3	4.1	46.9	2.4	26.2	3.2	29.2
6.4	44.3	8.8	51.2	2.5	29.5	5.0	34.1
4.1	46.9	5.0	36.7	6.9	45.9	3.0	28.2
1.7	13.1	4.9	34.1	2.4	32.8	4.8	33.8
1.8	19.0	3.8	32.2	4.3	39.4	4.4	35.4
3.2	20.0	5.5	49.2	7.3	36.7	11.3	55.4
5.1	46.6	6.3	39.4	10.9	51.5	3.7	32.2

(*Source:* Jack Haggerty Forest, Lakehead University, Canada) spreadsheet Excel to produce a scatter plot of the data in Table 4 and to find the regression model (Fig. 4).

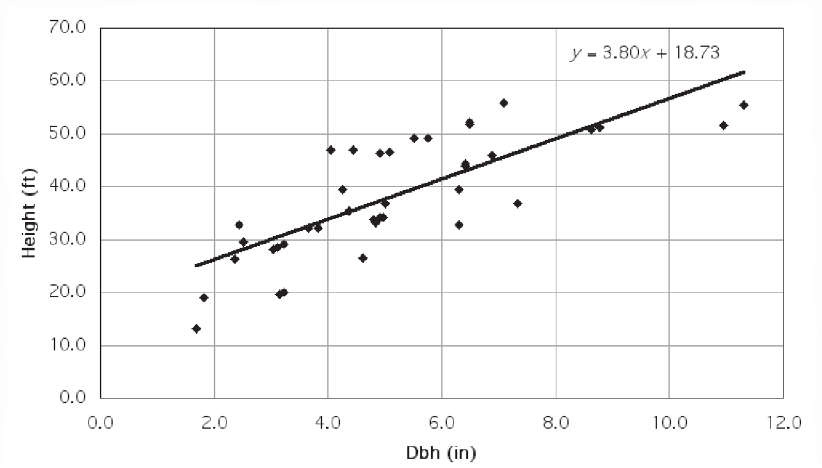


Figure 4 **Linear regression with a spreadsheet**

Matched Problem 5 Figure 5 shows the scatter plot for white spruce trees in the Jack Haggerty Forest at Lakehead University in Canada. A regression model produced by a spreadsheet (Fig. 5), after rounding, is

$$h = 1.8d + 34$$

where d is Dbh in inches and h is the height in feet.

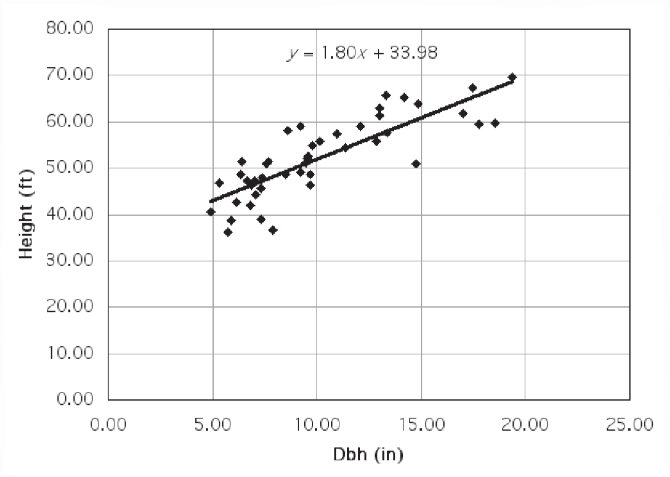


Figure 5 **Linear regression for white spruce trees**

(A) Interpret the slope of this model.

(B) What is the effect of a 1-inch increase in Dbh?

(C) Estimate the height of a white spruce with a Dbh of 10 inches. Round your answer to the nearest foot.

(D) Estimate the Dbh of a white spruce that is 65 feet tall. Round your answer to the nearest inch.

Exercises 1.3

Applications

1. **Ideal weight.** Dr. J. D. Robinson published the following estimate of the ideal body weight of a woman:

$$49 \text{ kg} + 1.7 \text{ kg for each inch over 5 ft}$$

(A) Find a linear model for Robinson's estimate of the ideal weight of a woman using w for ideal body weight (in kilograms) and h for height over 5 ft (in inches).

(B) Interpret the slope of the model.

(C) If a woman is 5′4″ tall, what does the model predict her weight to be?

(D) If a woman weighs 60 kg, what does the model predict her height to be?

2. **Ideal weight.** Dr. J. D. Robinson also published the following estimate of the ideal body weight of a man:

$$52 \text{ kg} + 1.9 \text{ kg for each inch over 5 ft}$$

(A) Find a linear model for Robinson's estimate of the ideal weight of a man using w for ideal body weight (in kilograms) and h for height over 5 ft (in inches).

(B) Interpret the slope of the model.

(C) If a man is 5′8″ tall, what does the model predict his weight to be?

(D) If a man weighs 70 kg, what does the model predict his height to be?

3. **Underwater pressure.** At sea level, the weight of the atmosphere exerts a pressure of 14.7 pounds per square inch, commonly referred to as 1 **atmosphere of pressure**. As an object descends in water, pressure P and depth d are linearly related. In salt water, the pressure at a depth of 33 ft is 2 atms, or 29.4 pounds per square inch.

(A) Find a linear model that relates pressure P (in pounds per square inch) to depth d (in feet).

(B) Interpret the slope of the model.

(C) Find the pressure at a depth of 50 ft.

(D) Find the depth at which the pressure is 4 atms.

4. **Underwater pressure.** Refer to Problem 3. In fresh water, the pressure at a depth of 34 ft is 2 atms, or 29.4 pounds per square inch.

(A) Find a linear model that relates pressure P (in pounds per square inch) to depth d (in feet).

(B) Interpret the slope of the model.

(C) Find the pressure at a depth of 50 ft.

(D) Find the depth at which the pressure is 4 atms.

5. **Rate of descent—Parachutes.** At low altitudes, the altitude of a parachutist and time in the air are linearly related. A jump at 2,880 ft using the U.S. Army's T-10 parachute system lasts 120 secs.

(A) Find a linear model relating altitude a (in feet) and time in the air t (in seconds).

(B) Find the rate of descent for a T-10 system.

(C) Find the speed of the parachutist at landing.

6. **Rate of descent—Parachutes.** The U.S Army is considering a new parachute, the Advanced Tactical Parachute System (ATPS). A jump at 2,880 ft using the ATPS system lasts 180 secs.

(A) Find a linear model relating altitude a (in feet) and time in the air t (in seconds).

(B) Find the rate of descent for an ATPS system parachute.

(C) Find the speed of the parachutist at landing.

7. **Speed of sound.** The speed of sound through air is linearly related to the temperature of the air. If sound travels at 331 m/sec at 0°C and at 343 m/sec at 20°C, construct a linear model relating the speed of sound (s) and the air temperature (t). Interpret the slope of this model. (*Source:* Engineering Toolbox)

8. **Speed of sound.** The speed of sound through sea water is linearly related to the temperature of the water. If sound travels at 1,403 m/sec at 0°C and at 1,481 m/sec at 20°C, construct a linear model relating the speed of sound (s) and the air temperature (t). Interpret the slope of this model. (*Source:* Engineering Toolbox)

9. **Energy production.** Table 5 lists U.S. fossil fuel production as a percentage of total energy production for selected years. A linear regression model for this data is

$$y = -0.3x + 84.4$$

where x represents years since 1985 and y represents the corresponding percentage of total energy production.

Table 5 **U.S. Fossil Fuel Production**

Year	Production (%)
1985	85
1990	83
1995	81
2000	80
2005	79
2010	78

Source: Energy Information Administration

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Interpret the slope of the model.

(C) Use the model to predict fossil fuel production in 2025.

(D) Use the model to estimate the first year for which fossil fuel production is less than 70% of total energy production.

10. Energy consumption. Table 6 lists U.S. fossil fuel consumption as a percentage of total energy consumption for selected years. A linear regression model for this data is

$$y = -0.09x + 85.8$$

where x represents years since 1985 and y represents the corresponding percentage of fossil fuel consumption.

Table 6 U.S. Fossil Fuel Consumption

Year	Consumption (%)
1985	86
1990	85
1995	85
2000	84
2005	85
2010	83

Source: Energy Information Administration

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Interpret the slope of the model.

(C) Use the model to predict fossil fuel consumption in 2025.

(D) Use the model to estimate the first year for which fossil fuel consumption is less than 80% of total energy consumption.

11. Cigarette smoking. The data in Table 7 shows that the percentage of female cigarette smokers in the U.S. declined from 22.1% in 1997 to 17.3% in 2010.

Table 7 Percentage of Smoking Prevalence among U.S. Adults

Year	Males (%)	Females (%)
1997	27.6	22.1
2000	25.7	21.0
2003	24.1	19.2
2006	23.9	18.0
2010	21.5	17.3

Source: Centers for Disease Control

(A) Applying linear regression to the data for females in Table 7 produces the model

$$f = -0.39t + 21.93$$

where f is percentage of female smokers and t is time in years since 1997. Draw a scatter plot of the female smoker data and a graph of the regression model on the same axes for $0 \le t \le 15$.

(B) Estimate the first year in which the percentage of female smokers is less than 10%.

12. Cigarette smoking. The data in Table 7 shows that the percentage of male cigarette smokers in the U.S. declined from 27.6% in 1997 to 21.5% in 2010.

(A) Applying linear regression to the data for males in Table 7 produces the model

$$m = -0.44t + 27.28$$

where m is percentage of male smokers and t is time in years since 1997. Draw a scatter plot of the male smoker data and a graph of the regression model for $0 \le t \le 15$.

(B) Estimate the first year in which the percentage of male smokers is less than 15%.

13. Undergraduate enrollment. Table 8 lists enrollment in U.S. degree-granting institutions for both undergraduate and graduate students. A linear regression model for undergraduate enrollment is

$$y = 0.23x + 9.56$$

where x represents years since 1980 and y is undergraduate enrollment in millions of students.

Table 8 Fall Undergraduate and Graduate Enrollment (millions of students)

Year	Undergraduate	Graduate
1980	10.48	1.34
1985	10.60	1.38
1990	11.96	1.59
1995	12.23	1.73
2000	13.16	1.85
2005	14.96	2.19
2010	18.08	2.94

Source: National Center for Education Statistics

(A) Draw a scatter plot of the undergraduate enrollment data and a graph of the model on the same axes.

(B) Predict the undergraduate student enrollment in 2025 (to the nearest 100,000).

(C) Interpret the slope of the model.

14. Graduate student enrollment. A linear regression model for the graduate student enrollment in Table 8 is

$$y = 0.048x + 1.14$$

where x represents years since 1980 and y is graduate enrollment in millions of students.

(A) Draw a scatter plot of the graduate enrollment data and a graph of the model on the same axes.

(B) Predict the graduate student enrollment in 2025 (to the nearest 100,000).

(C) Interpret the slope of the model.

15. Licensed drivers. Table 9 contains the state population and the number of licensed drivers in the state (both in millions) for the states with population under 1 million in 2010. The regression model for this data is

$$y = 0.75x$$

where x is the state population and y is the number of licensed drivers in the state.

Table 9 **Licensed Drivers in 2010**

State	Population	Licensed Drivers
Alaska	0.71	0.52
Delaware	0.90	0.70
Montana	0.99	0.74
North Dakota	0.67	0.48
South Dakota	0.81	0.60
Vermont	0.63	0.51
Wyoming	0.56	0.42

Source: Bureau of Transportation Statistics

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) If the population of Idaho in 2010 was about 1.6 million, use the model to estimate the number of licensed drivers in Idaho in 2010 to the nearest thousand.

(C) If the number of licensed drivers in Rhode Island in 2010 was about 0.75 million, use the model to estimate the population of Rhode Island in 2010 to the nearest thousand.

16. Licensed drivers. Table 10 contains the state population and the number of licensed drivers in the state (both in millions) for the states with population over 10 million in 2010. The regression model for this data is

$$y = 0.63x + 0.31$$

where x is the state population and y is the number of licensed drivers in the state.

Table 10 **Licensed Drivers in 2010**

State	Population	Licensed Drivers
California	37	24
Florida	19	14
Illinois	13	8
New York	19	11
Ohio	12	8
Pennsylvania	13	9
Texas	25	15

Source: Bureau of Transportation Statistics

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) If the population of Minnesota in 2010 was about 5.3 million, use the model to estimate the number of licensed drivers in Minnesota in 2010 to the nearest thousand.

(C) If the number of licensed drivers in Wisconsin in 2010 was about 4.1 million, use the model to estimate the population of Wisconsin in 2010 to the nearest thousand.

17. Net sales. A linear regression model for the net sales data in Table 11 is

$$S = 15.8t + 251$$

where S is net sales and t is time since 2000 in years.

Table 11 **Walmart Stores, Inc.**

Billions of U.S. Dollars	2008	2009	2010	2011	2012
Net sales	374	401	405	419	444
Operating income	21.9	22.8	24.0	25.5	26.6

Source: Walmart Stores, Inc.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Predict Walmart's net sales for 2022.

18. Operating income. A linear regression model for the operating income data in Table 11 is

$$I = 1.21t + 12.06$$

where I is operating income and t is time since 2000 in years.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Predict Walmart's annual operating income for 2024.

19. Freezing temperature. Ethylene glycol and propylene glycol are liquids used in antifreeze and deicing solutions. Ethylene glycol is listed as a hazardous chemical by the Environmental Protection Agency, while propylene glycol is generally regarded as safe. Table 12 lists the freezing temperature for various concentrations (as a percentage of total weight) of each chemical in a solution used to deice airplanes. A linear regression model for the ethylene glycol data in Table 12 is

$$E = -0.55T + 31$$

where E is the percentage of ethylene glycol in the deicing solution and T is the temperature at which the solution freezes.

Table 12 **Freezing Temperatures**

Freezing Temperature (°F)	Ethylene Glycol (% Wt.)	Propylene Glycol (% Wt.)
−50	56	58
−40	53	55
−30	49	52
−20	45	48
−10	40	43
0	33	36
10	25	29
20	16	19

Source: T. Labuza, University of Minnesota

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Use the model to estimate the freezing temperature to the nearest degree of a solution that is 30% ethylene glycol.

(C) Use the model to estimate the percentage of ethylene glycol in a solution that freezes at 15°F.

20. Freezing temperature. A linear regression model for the propylene glycol data in Table 12 is

$$P = -0.54T + 34$$

where P is the percentage of propylene glycol in the deicing solution and T is the temperature at which the solution freezes.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Use the model to estimate the freezing temperature to the nearest degree of a solution that is 30% propylene glycol.

(C) Use the model to estimate the percentage of propylene glycol in a solution that freezes at 15°F.

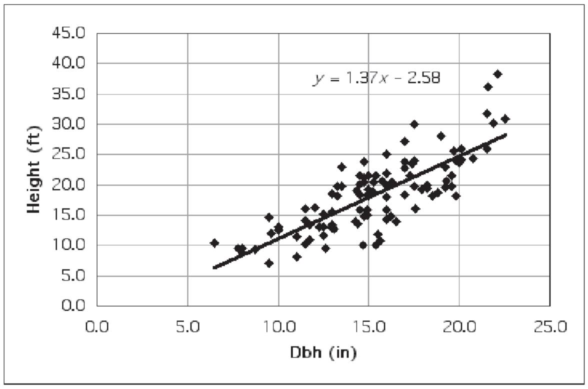
21. Forestry. The figure contains a scatter plot of 100 data points for black spruce trees and the linear regression model for this data.

(A) Interpret the slope of the model.

(B) What is the effect of a 1-in. increase in Dbh?

(C) Estimate the height of a black spruce with a Dbh of 15 in. Round your answer to the nearest foot.

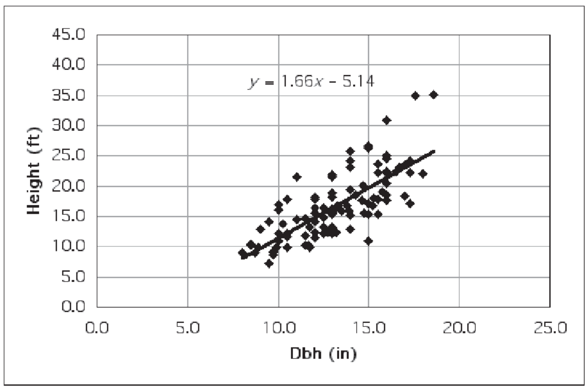
(D) Estimate the Dbh of a black spruce that is 25 ft tall. Round your answer to the nearest inch.



black spruce
Source: Lakehead University

22. Forestry. The figure contains a scatter plot of 100 data points for black walnut trees and the linear regression model for this data.

(A) Interpret the slope of the model.



black walnut
Source: Kagen Research

(B) What is the effect of a 1-in. increase in Dbh?

(C) Estimate the height of a black walnut with a Dbh of 12 in. Round your answer to the nearest foot.

(D) Estimate the Dbh of a black walnut that is 25 ft tall. Round your answer to the nearest inch.

23. Cable television. Table 13 shows the increase in both price and revenue for cable television in the United States. The figure shows a scatter plot and a linear regression model for the average monthly price data in Table 13.

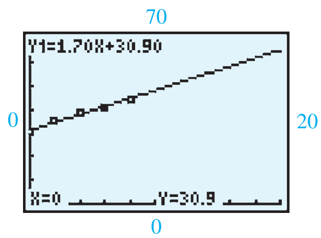
Table 13 **Cable Television Price and Revenue**

Year	Average Monthly Price (dollars)	Annual Total Revenue (billions of dollars)
2000	30.37	36.43
2002	34.71	47.99
2004	38.14	58.59
2006	41.17	71.89
2008	44.28	85.23
2010	47.89	93.37

Source: SNL Kagan

(A) Interpret the slope of the model.

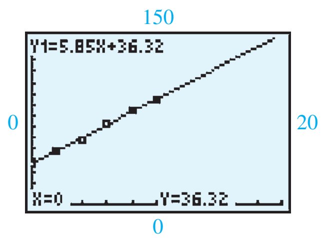
(B) Use the model to predict the average monthly price (to the nearest dollar) in 2024.



24. Cable television. The figure shows a scatter plot and a linear regression model for the annual revenue data in Table 13.

(A) Interpret the slope of the model.

(B) Use the model to predict the annual revenue (to the nearest billion dollars) in 2024.



25. College enrollment. Table 14 lists the fall enrollment in degree-granting institutions by gender, and the figure contains a scatter plot and a regression line for each data set.

(A) Interpret the slope of each model.

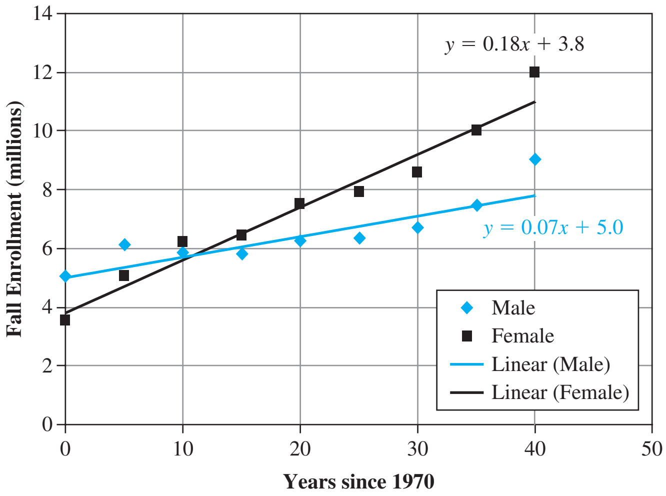
(B) Predict both the male enrollment and the female enrollment in 2025.

Table 14 Fall Enrollment (millions of students)

Year	Male	Female
1970	5.04	3.54
1975	6.15	5.04
1980	5.87	6.22
1985	5.82	6.43
1990	6.28	7.53
1995	6.34	7.92
2000	6.72	8.59
2005	7.46	10.03
2010	9.04	11.97

Source: National Center for Education Statistics

(C) Estimate the first year for which female enrollment will exceed male enrollment by at least 5 million.

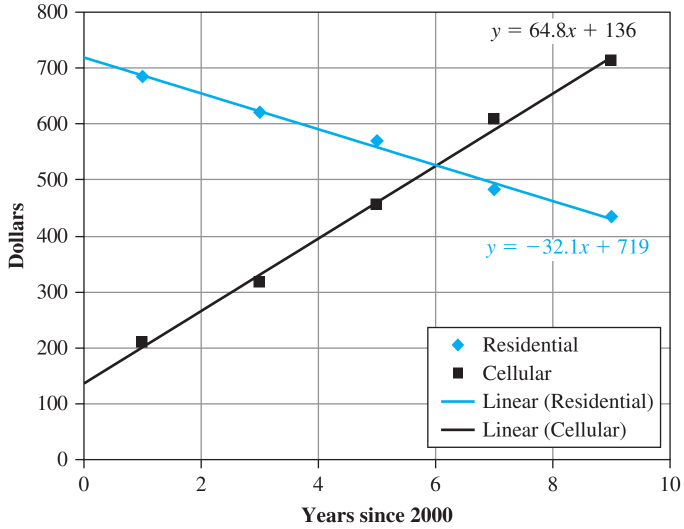


26. Telephone expenditures. Table 15 lists average annual telephone expenditures (in dollars) per consumer unit on residential phone service and cellular phone service, and the figure contains a scatter plot and regression line for each data set.

Table 15 Telephone Expenditures

Year	Residential Service ($)	Cellular Service ($)
2001	686	210
2003	620	316
2005	570	455
2007	482	608
2009	434	712

Source: Bureau of Labor Statistics



(A) Interpret the slope of each model.

(B) Predict (to the nearest dollar) the average annual residential and cellular expenditures in 2020.

(C) Would the linear regression models give reasonable predictions for the year 2025? Explain.

Problems 27–30 require a graphing calculator or a computer that can calculate the linear regression line for a given data set.

27. Olympic Games. Find a linear regression model for the men's 100-meter freestyle data given in Table 16, where x is years since 1990 and y is winning time (in seconds). Do the same for the women's 100-meter freestyle data. (Round regression coefficients to three decimal places.) Do these models indicate that the women will eventually catch up with the men?

Table 16 Winning Times in Olympic Swimming Events

	100-Meter Freestyle		200-Meter Backstroke	
	Men	Women	Men	Women
1992	49.02	54.65	1:58.47	2:07.06
1996	48.74	54.50	1.58.54	2:07.83
2000	48.30	53.83	1:56.76	2:08.16
2004	48.17	53.84	1:54.76	2:09.16
2008	47.21	53.12	1:53.94	2:05.24
2012	47.52	53.00	1:53.41	2:04.06

Source: www.infoplease.com

28. Olympic Games. Find a linear regression model for the men's 200-meter backstroke data given in Table 16, where x is years since 1990 and y is winning time (in seconds). Do the same for the women's 200-meter backstroke data. (Round regression coefficients to three decimal places.) Do these models indicate that the women will eventually catch up with the men?

29. Supply and demand. Table 17 contains price–supply data and price–demand data for corn. Find a linear regression model for the price–supply data where x is supply (in billions of bushels) and y is price (in dollars). Do the same for the price–demand data. (Round regression coefficients to two decimal places.) Find the equilibrium price for corn.

Table 17 **Supply and Demand for U.S. Corn**

Price ($/bu)	Supply (billion bu)	Price ($/bu)	Demand (billion bu)
2.15	6.29	2.07	9.78
2.29	7.27	2.15	9.35
2.36	7.53	2.22	8.47
2.48	7.93	2.34	8.12
2.47	8.12	2.39	7.76
2.55	8.24	2.47	6.98

Source: www.usda.gov/nass/pubs/histdata.htm

30. Supply and demand. Table 18 contains price–supply data and price–demand data for soybeans. Find a linear regression model for the price–supply data where x is supply (in billions of bushels) and y is price (in dollars). Do the same for the price–demand data. (Round regression coefficients to two decimal places.) Find the equilibrium price for soybeans.

Table 18 **Supply and Demand for U.S. Soybeans**

Price ($/bu)	Supply (billion bu)	Price ($/bu)	Demand (billion bu)
5.15	1.55	4.93	2.60
5.79	1.86	5.48	2.40
5.88	1.94	5.71	2.18
6.07	2.08	6.07	2.05
6.15	2.15	6.40	1.95
6.25	2.27	6.66	1.85

Source: www.usda.gov/nass/pubs/histdata.htm

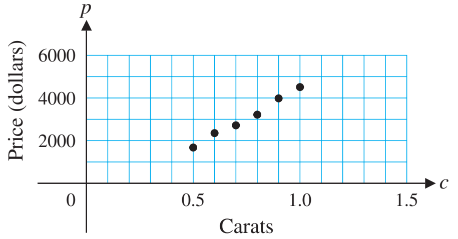
Answers to Matched Problems

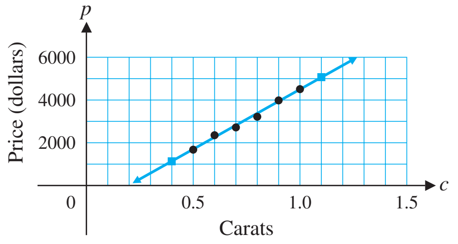
1. (A) The rate of change BSA with respect to weight is 28.55 square inches per pound.

(B) Increasing w by 1 pound increases a by 28.55 square inches.

2. (A) $a = -36t + 2{,}880$ (B) 36 ft/sec

3. (A)





(B) The rate of change of the price of a diamond with respect to its weight is $5,600. Increasing the weight by one carat will increase the price by about $5,600.

(C) $3,660; $5,620 (D) 0.91 carat

4. Approximately 349 parts per million.

5. (A) The slope is 1.8, so the rate of change of height with respect to breast height diameter is 1.8 feet per inch.

(B) Height increases by 1.8 feet.

(C) 52 ft (D) 17 in.

Chapter 1 Summary and Review

Important Terms, Symbols, and Concepts

1.1 Linear Equations and Inequalities

EXAMPLES

- A **first-degree, or linear, equation** in one variable is any equation that can be written in the form

$$\textbf{Standard form:}\quad ax + b = 0 \quad a \neq 0$$

If the equality sign in the standard form is replaced by $<, >, \leq$, or $\geq$, the resulting expression is called a **first-degree, or linear, inequality**.

- A **solution** of an equation (or inequality) involving a single variable is a number that when substituted for the variable makes the equation (inequality) true. The set of all solutions is called the **solution set**.
- If we perform an operation on an equation (or inequality) that produces another equation (or inequality) with the same solution set, then the two equations (or inequalities) are **equivalent**. Equations are solved by adding or subtracting the same quantity to both sides, or by multiplying both sides by the same *nonzero* quantity until an equation with an obvious solution is obtained. Ex. 1, p. 19 Ex. 2, p. 20
- The **interval notation** $[a, b)$, for example, represents the solution of the **double inequality** $a \leq x < b$. Ex. 5, p. 23

1.1 Linear Equations and Inequalities (*Continued*)

- Inequalities are solved in the same manner as equations with one important exception. If both sides of an inequality are multiplied by the same *negative* number or divided by the same *negative* number, then the direction or sense of the inequality will reverse ($<$ becomes $>$, $\geq$ becomes $\leq$, and so on). Ex. 6, p. 23 Ex. 7, p. 24
- A suggested strategy (p. 24) can be used to solve many word problems. Ex. 8, p. 24
- A company breaks even if revenues R = costs C, makes a profit if $R > C$, and incurs a loss if $R < C$. Ex. 9, p. 25

1.2 Graphs and Lines

- A **Cartesian or rectangular coordinate system** is formed by the intersection of a horizontal real number line, usually called the ***x* axis**, and a vertical real number line, usually called the ***y* axis**, at their origins. The axes determine a plane and divide this plane into four **quadrants**. Each point in the plane corresponds to its **coordinates**—an ordered pair (a, b) determined by passing horizontal and vertical lines through the point. The **abscissa** or ***x* coordinate** a is the coordinate of the intersection of the vertical line and the x axis, and the **ordinate** or ***y* coordinate** b is the coordinate of the intersection of the horizontal line and the y axis. The point with coordinates $(0, 0)$ is called the **origin**. Fig 1, p. 29
- The **standard form** for a linear equation in two variables is $Ax + By = C$, with A and B not both zero. The graph of this equation is a line, and every line in a Cartesian coordinate system is the graph of a linear equation. Ex. 1, p. 30 Ex. 2, p. 31
- The graph of the equation $x = a$ is a **vertical line** and the graph of $y = b$ is a **horizontal line**. Ex. 3, p. 31
- If (x_1, y_1) and (x_2, y_2) are two distinct points on a line, then $m = (y_2 - y_1)/(x_2 - x_1)$ is the **slope** of the line. Ex. 4, p. 33
- The equation $y = mx + b$ is the **slope-intercept form** of the equation of the line with slope m and y intercept b. Ex. 5, p. 34
- The **point-slope form** of the equation of the line with slope m that passes through (x_1, y_1) is $y - y_1 = m(x - x_1)$. Ex. 6, p. 35
- In a competitive market, the intersection of the supply equation and the demand equation is called the **equilibrium point**, the corresponding price is called the **equilibrium price**, and the common value of supply and demand is called the **equilibrium quantity**. Ex. 8, p. 37

1.3 Linear Regression

- A **mathematical model** is a mathematics problem that, when solved, will provide information about a real-world problem.
- If the variables x and y are related by the equation $y = mx + b$, then x and y are **linearly related** and the slope m is the **rate of change** of y with respect to x. Ex. 1, p. 43 Ex. 2, p. 44
- A graph of the points in a data set is called a **scatter plot**. **Linear regression** is used to find the line that is the **best fit** for a data set. A regression model can be used to **interpolate** between points in a data set or to **extrapolate** or predict points outside the data set. Ex. 3, p. 45 Ex. 4, p. 46 Ex. 5, p. 47

Review Exercises

Work through all the problems in this chapter review and check answers in the back of the book. Following each answer you will find a number in italics indicating the section where that type of problem is discussed. Where weaknesses show up, review appropriate sections in the text.

1. Solve $2x + 3 = 7x - 11$.

2. Solve $\dfrac{x}{12} - \dfrac{x-3}{3} = \dfrac{1}{2}$.

3. Solve $3x - 4y = -10$, for y.

4. Solve $3x - 4y = 7$ for x.

Solve Problems 5–7 and graph on a real number line.

5. $4y - 3 < 10$

6. $-1 < -2x + 5 \leq 3$

7. $1 - \dfrac{x-3}{3} \leq \dfrac{1}{2}$

8. Sketch a graph of $3x + 2y = 9$.

9. Write an equation of a line with x intercept 6 and y intercept 4. Write the final answer in the form $Ax + By = C$.

10. Sketch a graph of $2x - 3y = 18$. What are the intercepts and slope of the line?

11. Write the equation of a line with each indicated slope and intercept in the form $y = mx + b$.

(A) $m = \frac{1}{4}$ and x intercept 5

(B) $m = -\frac{2}{5}$ and y intercept -2

12. Write the equations of the vertical line and the horizontal line that pass through $(-6, 5)$.

13. Write the equation of a line passing through the point $(3, -2)$ with the slope 2, in the form $y = mx + b$.

14. Write the equation of the line through the two indicated points. Write the final answer in the form $Ax + By = C$.

(A) $(-3, 5), (1, -1)$ (B) $(-1, 5), (4, 5)$

(C) $(-2, 7), (-2, -2)$

Solve Problems 15–19.

15. $3x + 25 = 5x$ **16.** $\frac{u}{5} = \frac{u}{6} + \frac{6}{5}$

17. $\frac{5x}{3} - \frac{4 + x}{2} = \frac{x - 2}{4} + 1$

18. $\frac{1}{8}x + \frac{5}{2}(2x - 3) = 6$

19. $0.2(x - 3) + 0.05x = 0.4$

Solve Problems 20–24 and graph on a real number line.

20. $2(x + 4) > 5x - 4$ **21.** $3(2 - x) - 2 \leq 2x - 1$

22. $\frac{x + 3}{8} - \frac{4 + x}{2} > 5 - \frac{2 - x}{3}$

23. $-5 \leq 3 - 2x < 1$ **24.** $-1.5 \leq 2 - 4x \leq 0.5$

25. Given $Ax + By = 30$, graph each of the following cases on the same coordinate axes.

(A) $A = 5$ and $B = 0$ (B) $A = 0$ and $B = 6$

(C) $A = 6$ and $B = 5$

26. Describe the graphs of $x = -3$ and $y = 2$. Graph both simultaneously in the same coordinate system.

27. Describe the lines defined by the following equations:

(A) $3x + 4y = 0$ (B) $3x + 4 = 0$

(C) $4y = 0$ (D) $3x + 4y - 36 = 0$

Solve Problems 28 and 29 for the indicated variable.

28. $A = \frac{1}{2}(a + b)h$; for a $(h \neq 0)$

29. $S = \frac{P}{1 - dt}$; for d $(dt \neq 1)$

30. For what values of a are the following inequalities true?

(A) $a < a$, and (B) $a \leq a$.

31. If a and b are negative numbers and $a > b$, then is a/b greater than 1 or less than 1?

32. Graph $y = mx + b$ and $y = -\frac{1}{m}x + b$ simultaneously in the same coordinate system for b fixed and several different values of m, $m \neq 0$. Describe the apparent relationship between the graphs of the two equations.

Applications

33. Investing. An investor has $300,000 to invest. If part is invested at 5% and the rest at 9%, how much should be invested at 5% to yield 8% on the total amount?

34. Break-even analysis. A producer of educational DVDs is producing an instructional DVD. She estimates that it will cost $90,000 to record the DVD and $5.10 per unit to copy and distribute the DVD. If the wholesale price of the DVD is $14.70, how many DVDs must be sold for the producer to break even?

35. Sports medicine. A simple rule of thumb for determining your maximum safe heart rate (in beats per minute) is to subtract your age from 220. While exercising, you should maintain a heart rate between 60% and 85% of your maximum safe rate.

(A) Find a linear model for the minimum heart rate m that a person of age x years should maintain while exercising.

(B) Find a linear model for the maximum heart rate M that a person of age x years should maintain while exercising.

(C) What range of heartbeats should you maintain while exercising if you are 20 years old?

(D) What range of heartbeats should you maintain while exercising if you are 50 years old?

36. Psychology. The results of a psychological survey showed that there is a relationship between listening to one's favorite music and one's happiness index. It was observed that when the number of favorite songs is 5, the happiness index of a person is 20. When the number of songs increases to 15, the happiness index goes up to 50. Assuming the relationship to be linear

(A) Find the linear equation expressing the above relationship.

(B) How many songs would give one a happiness index of 25?

37. Business—Pricing. A sporting goods store sells tennis rackets that cost $130 for $208 and court shoes that cost $50 for $80.

(A) If the markup policy of the store for items that cost over $10 is linear and is reflected in the pricing of these two items, write an equation that expresses retail price R in terms of cost C.

(B) What would be the retail price of a pair of in-line skates that cost $120?

(C) What would be the cost of a pair of cross-country skis that had a retail price of $176?

(D) What is the slope of the graph of the equation found in part (A)? Interpret the slope relative to the problem.

38. Income. A salesperson receives a base salary of $400 per week and a commission of 10% on all sales over $6,000 during the week. Find the weekly earnings for weekly sales of $4,000 and for weekly sales of $10,000.

39. Health. Five hundred units of a particular health food can fulfill the nutritional requirement of 25 people. If the supply of the food is increased to 550 units, the nutritional requirement of 30 people can be met. Assuming the relationship between the supply of food (s) and the nutritional requirement (r) per person to be linear, express s as a function of r. How many units of food will be required if the nutritional requirement of 65 people is to be met?

40. Freezing temperature. Methanol, also known as wood alcohol, can be used as a fuel for suitably equipped vehicles. Table 1 lists the freezing temperature for various concentrations (as a percentage of total weight) of methanol in water. A linear regression model for the data in Table 1 is

$$T = 40 - 2M$$

where M is the percentage of methanol in the solution and T is the temperature at which the solution freezes.

Table 1

Methanol (%Wt)	Freezing temperature (°F)
0	32
10	20
20	0
30	−15
40	−40
50	−65
60	−95

Source: Ashland Inc.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Use the model to estimate the freezing temperature to the nearest degree of a solution that is 35% methanol.

(C) Use the model to estimate the percentage of methanol in a solution that freezes at −50°F.

41. High school dropout rates. Table 2 gives U.S. high school dropout rates as percentages for selected years since 1980. A linear regression model for the data is

$$r = -0.198t + 14.2$$

where t represents years since 1980 and r is the dropout rate.

Table 2 **High School Dropout Rates (%)**

1980	1985	1990	1995	2000	2005	2010
14.1	12.6	12.1	12.0	10.9	9.4	7.4

(A) Interpret the slope of the model.

(B) Draw a scatter plot of the data and the model in the same coordinate system.

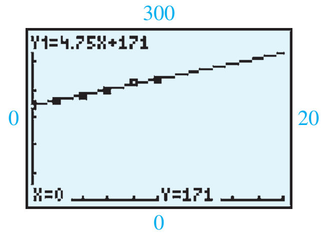
(C) Use the model to predict the first year for which the dropout rate is less than 5%.

42. Consumer Price Index. The U.S. Consumer Price Index (CPI) in recent years is given in Table 3. A scatter plot of the data and linear regression line are shown in the figure, where x represents years since 2000.

Table 3 **Consumer Price Index (1982–1984 = 100)**

Year	CPI
2000	172.2
2002	179.9
2004	188.9
2006	198.3
2008	211.1
2010	218.1

Source: U.S. Bureau of Labor Statistics



(A) Interpret the slope of the model.

(B) Predict the CPI in 2024.

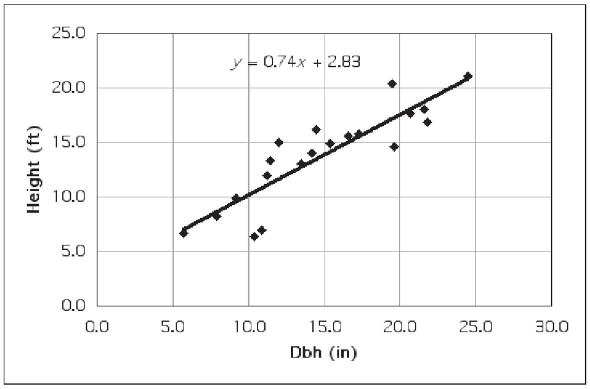
43. Forestry. The figure contains a scatter plot of 20 data points for white pine trees and the linear regression model for this data.

(A) Interpret the slope of the model.

(B) What is the effect of a 1-in. increase in Dbh?

(C) Estimate the height of a white pine tree with a Dbh of 25 in. Round your answer to the nearest foot.

(D) Estimate the Dbh of a white pine tree that is 15 ft tall. Round your answer to the nearest inch.



2 Functions and Graphs

Introduction

The function concept is one of the most important ideas in mathematics. The study of mathematics beyond the elementary level requires a firm understanding of a basic list of elementary functions, their properties, and their graphs. See the inside back cover of this book for a list of the functions that form our library of elementary functions. Most functions in the list will be introduced to you by the end of Chapter 2. For example, in Section 2.3 you will learn how to apply quadratic functions to model the rate of blood flow in an artery (see Problems 71 and 72 on page 99).

2.1 Functions

- Equations in Two Variables
- Definition of a Function
- Functions Specified by Equations
- Function Notation
- Applications

We introduce the general notion of a *function* as a correspondence between two sets. Then we restrict attention to functions for which the two sets are both sets of real numbers. The most useful are those functions that are specified by equations in two variables. We discuss the terminology and notation associated with functions, graphs of functions, and applications.

Equations in Two Variables

In Chapter 1, we found that the graph of an equation of the form $Ax + By = C$, where A and B are not both zero, is a line. Because a line is determined by any two of its points, such an equation is easy to graph: Just plot *any* two points in its solution set and sketch the unique line through them.

More complicated equations in two variables, such as $y = 9 - x^2$ or $x^2 = y^4$, are more difficult to graph. To **sketch the graph** of an equation, we plot enough points from its solution set in a rectangular coordinate system so that the total graph is apparent, and then we connect these points with a smooth curve. This process is called **point-by-point plotting**.

EXAMPLE 1 Point-by-Point Plotting Sketch the graph of each equation.

(A) $y = 9 - x^2$ (B) $x^2 = y^4$

SOLUTION

(A) Make up a table of solutions—that is, ordered pairs of real numbers that satisfy the given equation. For easy mental calculation, choose integer values for x.

x	-4	-3	-2	-1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

After plotting these solutions, if there are any portions of the graph that are unclear, plot additional points until the shape of the graph is apparent. Then join all the plotted points with a smooth curve (Fig. 1). Arrowheads are used to indicate that the graph continues beyond the portion shown here with no significant changes in shape.

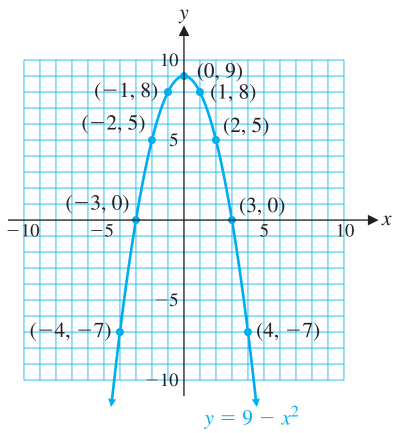


Figure 1 $y = 9 - x^2$

(B) Again we make a table of solutions—here it may be easier to choose integer values for y and calculate values for x. Note, for example, that if $y = 2$, then $x = \pm 4$; that is, the ordered pairs $(4, 2)$ and $(-4, 2)$ are both in the solution set.

x	± 9	± 4	± 1	0	± 1	± 4	± 9
y	-3	-2	-1	0	1	2	3

We plot these points and join them with a smooth curve (Fig. 2).

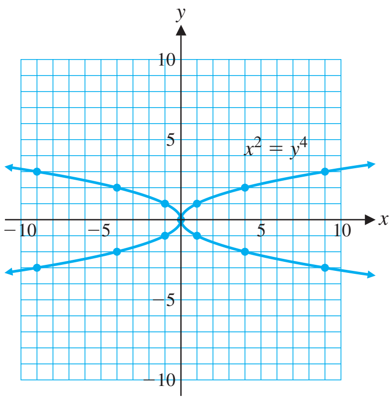


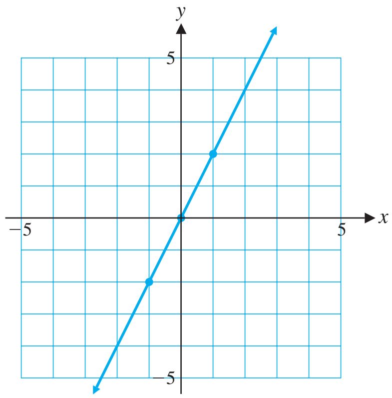
Figure 2 $x^2 = y^4$

Matched Problem 1 Sketch the graph of each equation.

(A) $y = x^2 - 4$ (B) $y^2 = \dfrac{100}{x^2 + 1}$

Explore and Discuss 1 To graph the equation $y = -x^3 + 3x$, we use point-by-point plotting to obtain

x	y
-1	-2
0	0
1	2

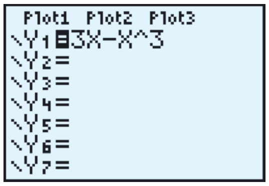


(A) Do you think this is the correct graph of the equation? Why or why not?

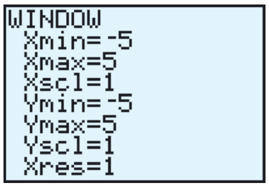
(B) Add points on the graph for $x = -2, -1.5, -0.5, 0.5, 1.5$, and 2.

(C) Now, what do you think the graph looks like? Sketch your version of the graph, adding more points as necessary.

(D) Graph this equation on a graphing calculator and compare it with your graph from part (C).



(A)



(B)

Figure 3

The icon in the margin is used throughout this book to identify optional graphing calculator activities that are intended to give you additional insight into the concepts under discussion. You may have to consult the manual for your graphing calculator for the details necessary to carry out these activities. For example, to graph the equation in Explore and Discuss 1 on most graphing calculators, you must enter the equation (Fig. 3A) and the window variables (Fig. 3B).

As Explore and Discuss 1 illustrates, the shape of a graph may not be apparent from your first choice of points. Using point-by-point plotting, it may be difficult to find points in the solution set of the equation, and it may be difficult to determine when you have found enough points to understand the shape of the graph. We will supplement the technique of point-by-point plotting with a detailed analysis of several basic equations, giving you the ability to sketch graphs with accuracy and confidence.

Definition of a Function

Central to the concept of function is correspondence. You are familiar with correspondences in daily life. For example,

To each person, there corresponds an annual income.

To each item in a supermarket, there corresponds a price.

To each student, there corresponds a grade-point average.

To each day, there corresponds a maximum temperature.

For the manufacture of x items, there corresponds a cost.

For the sale of x items, there corresponds a revenue.

To each square, there corresponds an area.

To each number, there corresponds its cube.

One of the most important aspects of any science is the establishment of correspondences among various types of phenomena. Once a correspondence is known, predictions can be made. A cost analyst would like to predict costs for various levels of output in a manufacturing process; a medical researcher would like to know the correspondence between heart disease and obesity; a psychologist would like to predict the level of performance after a subject has repeated a task a given number of times; and so on.

What do all of these examples have in common? Each describes the matching of elements from one set with the elements in a second set.

Consider Tables 1–3. Tables 1 and 2 specify functions, but Table 3 does not. Why not? The definition of the term *function* will explain.

Table 1

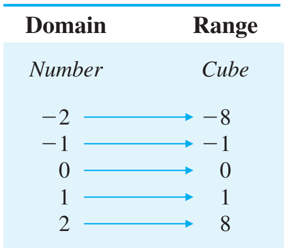


Table 2

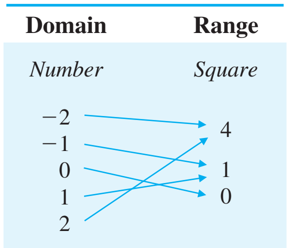
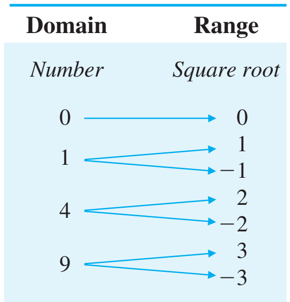


Table 3



DEFINITION Function

A **function** is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set.

The first set is called the **domain**, and the set of corresponding elements in the second set is called the **range**.

Tables 1 and 2 specify functions since to each domain value, there corresponds exactly one range value (for example, the cube of -2 is -8 and no other number). On the other hand, Table 3 does not specify a function since to at least one domain value, there corresponds more than one range value (for example, to the domain value 9, there corresponds -3 and 3, both square roots of 9).

Explore and Discuss 2

Consider the set of students enrolled in a college and the set of faculty members at that college. Suppose we define a correspondence between the two sets by saying that a student corresponds to a faculty member if the student is currently enrolled in a course taught by that faculty member. Is this correspondence a function? Discuss.

Functions Specified by Equations

Most of the functions in this book will have domains and ranges that are (infinite) sets of real numbers. The **graph** of such a function is the set of all points (x, y) in the Cartesian plane such that x is an element of the domain and y is the corresponding element in the range. The correspondence between domain and range elements is often specified by an equation in two variables. Consider, for example, the equation for the area of a rectangle with width 1 inch less than its length (Fig. 4). If x is the length, then the area y is given by

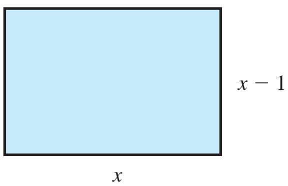


Figure 4

$$y = x(x - 1) \qquad x \geq 1$$

For each **input** x (length), we obtain an **output** y (area). For example,

$$\begin{array}{llll} \text{If} & x = 5, & \text{then} & y = 5(5 - 1) = 5 \cdot 4 = 20. \\ \text{If} & x = 1, & \text{then} & y = 1(1 - 1) = 1 \cdot 0 = 0. \\ \text{If} & x = \sqrt{5}, & \text{then} & y = \sqrt{5}(\sqrt{5} - 1) = 5 - \sqrt{5} \\ & & & \approx 2.7639. \end{array}$$

The input values are domain values, and the output values are range values. The equation assigns each domain value x a range value y. The variable x is called an *independent variable* (since values can be "independently" assigned to x from the domain), and y is called a *dependent variable* (since the value of y "depends" on the value assigned to x). In general, any variable used as a placeholder for domain values is called an **independent variable**; any variable that is used as a placeholder for range values is called a **dependent variable**.

When does an equation specify a function?

DEFINITION Functions Specified by Equations

If in an equation in two variables, we get exactly one output (value for the dependent variable) for each input (value for the independent variable), then the equation specifies a function. The graph of such a function is just the graph of the specifying equation.

If we get more than one output for a given input, the equation does not specify a function.

EXAMPLE 2 Functions and Equations Determine which of the following equations specify functions with independent variable x.

(A) $4y - 3x = 8$, x a real number (B) $y^2 - x^2 = 9$, x a real number

SOLUTION

(A) Solving for the dependent variable y, we have

$$\begin{aligned} 4y - 3x &= 8 \\ 4y &= 8 + 3x \qquad (1) \\ y &= 2 + \frac{3}{4}x \end{aligned}$$

Since each input value x corresponds to exactly one output value $\left(y = 2 + \frac{3}{4}x\right)$, we see that equation (1) specifies a function.

(B) Solving for the dependent variable y, we have

$$\begin{aligned} y^2 - x^2 &= 9 \\ y^2 &= 9 + x^2 \qquad (2) \\ y &= \pm\sqrt{9 + x^2} \end{aligned}$$

Since $9 + x^2$ is always a positive real number for any real number x, and since each positive real number has two square roots,* then to each input value x there corresponds two output values $\left(y = -\sqrt{9 + x^2} \text{ and } y = \sqrt{9 + x^2}\right)$. For example, if $x = 4$, then equation (2) is satisfied for $y = 5$ and for $y = -5$. So equation (2) does not specify a function.

*Recall that each positive real number N has two square roots: $\sqrt{N}$, the principal square root; and $-\sqrt{N}$, the negative of the principal square root (see Appendix A, Section A.6).

Matched Problem 2 Determine which of the following equations specify functions with independent variable x.

(A) $y^2 - x^4 = 9$, x a real number (B) $3y - 2x = 3$, x a real number

Since the graph of an equation is the graph of all the ordered pairs that satisfy the equation, it is very easy to determine whether an equation specifies a function by examining its graph. The graphs of the two equations we considered in Example 2 are shown in Figure 5.

In Figure 5A, notice that any vertical line will intersect the graph of the equation $4y - 3x = 8$ in exactly one point. This shows that to each x value, there corresponds exactly one y value, confirming our conclusion that this equation specifies a function. On the other hand, Figure 5B shows that there exist vertical lines that intersect the graph of $y^2 - x^2 = 9$ in two points. This indicates that there exist x values to which there correspond two different y values and verifies our conclusion that this equation does not specify a function. These observations are generalized in Theorem 1.

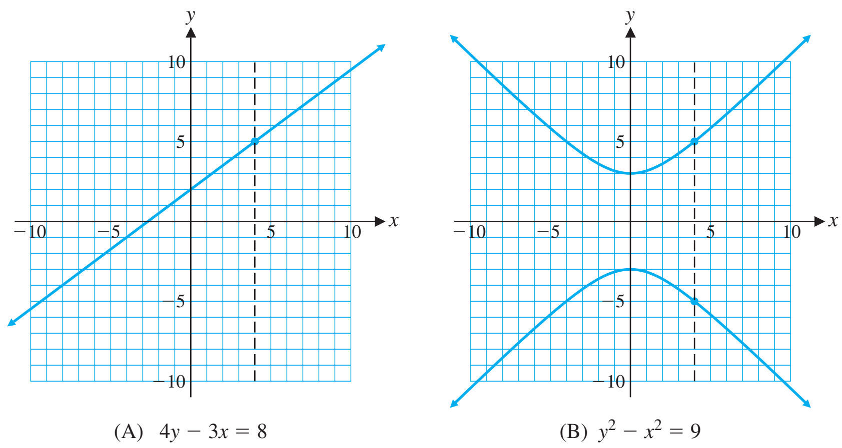


Figure 5

THEOREM 1 Vertical-Line Test for a Function

An equation specifies a function if each vertical line in the coordinate system passes through, at most, one point on the graph of the equation.

If any vertical line passes through two or more points on the graph of an equation, then the equation does not specify a function.

The function graphed in Figure 5A is an example of a *linear function*. The vertical-line test implies that equations of the form $y = mx + b$, where $m \neq 0$, specify functions; they are called **linear functions**. Similarly, equations of the form $y = b$ specify functions; they are called **constant functions**, and their graphs are horizontal lines. The vertical-line test implies that equations of the form $x = a$ do not specify functions; note that the graph of $x = a$ is a vertical line.

In Example 2, the domains were explicitly stated along with the given equations. In many cases, this will not be done. Unless stated to the contrary, we shall adhere to the following convention regarding domains and ranges for functions specified by equations:

If a function is specified by an equation and the domain is not indicated, then we assume that the domain is the set of all real-number replacements of the independent variable (inputs) that produce real values for the dependent variable (outputs). The range is the set of all outputs corresponding to input values.

EXAMPLE 3 Finding a Domain Find the domain of the function specified by the equation $y = \sqrt{4 - x}$, assuming that x is the independent variable.

SOLUTION For y to be real, $4 - x$ must be greater than or equal to 0; that is,

$$\begin{aligned} 4 - x &\geq 0 \\ -x &\geq -4 \\ x &\leq 4 \end{aligned}$$

Sense of inequality reverses when both sides are divided by -1.

Domain: $x \leq 4$ (inequality notation) or $(-\infty, 4]$ (interval notation)

Matched Problem 3 Find the domain of the function specified by the equation $y = \sqrt{x - 2}$, assuming x is the independent variable.

Function Notation

We have seen that a function involves two sets, a domain and a range, and a correspondence that assigns to each element in the domain exactly one element in the range. Just as we use letters as names for numbers, now we will use letters as names for functions. For example, f and g may be used to name the functions specified by the equations $y = 2x + 1$ and $y = x^2 + 2x - 3$:

$$\begin{aligned} f&: \quad y = 2x + 1 \\ g&: \quad y = x^2 + 2x - 3 \end{aligned} \tag{3}$$

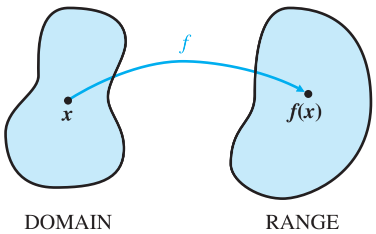


Figure 6

If x represents an element in the domain of a function f, then we frequently use the symbol

$$f(x)$$

in place of y to designate the number in the range of the function f to which x is paired (Fig. 6). This symbol does *not* represent the product of f and x. The symbol $f(x)$ is read as "f of x," "f at x," or "the value of f at x." Whenever we write $y = f(x)$, we assume that the variable x is an independent variable and that both y and $f(x)$ are dependent variables.

Using function notation, we can now write functions f and g in equation (3) as

$$f(x) = 2x + 1 \qquad \text{and} \qquad g(x) = x^2 + 2x - 3$$

Let us find $f(3)$ and $g(-5)$. To find $f(3)$, we replace x with 3 wherever x occurs in $f(x) = 2x + 1$ and evaluate the right side:

$$\begin{aligned} f(x) &= 2x + 1 \\ f(\mathbf{3}) &= 2 \cdot \mathbf{3} + 1 \\ &= 6 + 1 = 7 \end{aligned}$$

For input 3, the output is 7.

Therefore,

$$f(3) = 7$$

The function f assigns the range value 7 to the domain value 3.

To find $g(-5)$, we replace each x by -5 in $g(x) = x^2 + 2x - 3$ and evaluate the right side:

$$\begin{aligned} g(x) &= x^2 + 2x - 3 \\ g(\mathbf{-5}) &= (\mathbf{-5})^2 + 2(\mathbf{-5}) - 3 \\ &= 25 - 10 - 3 = 12 \end{aligned}$$

For input -5, the output is 12.

Therefore,

$$g(-5) = 12$$

The function g assigns the range value 12 to the domain value -5.

It is very important to understand and remember the definition of $f(x)$:

For any element x in the domain of the function f, the symbol $f(x)$ represents the element in the range of f corresponding to x in the domain of f. If x is an input value, then $f(x)$ is the corresponding output value. If x is an element that is not in the domain of f, then f is not defined at x and $f(x)$ does not exist.

EXAMPLE 4 Function Evaluation For $f(x) = 12/(x - 2)$, $g(x) = 1 - x^2$, and $h(x) = \sqrt{x - 1}$, evaluate:

(A) $f(6)$ (B) $g(-2)$ (C) $h(-2)$ (D) $f(0) + g(1) - h(10)$

SOLUTION (A) $f(6) \boxed{= \frac{12}{6 - 2}}^{*} = \frac{12}{4} = 3$

(B) $g(-2) \boxed{= 1 - (-2)^2} = 1 - 4 = -3$

(C) $h(-2) \boxed{= \sqrt{-2 - 1}} = \sqrt{-3}$

But $\sqrt{-3}$ is not a real number. Since we have agreed to restrict the domain of a function to values of x that produce real values for the function, -2 is not in the domain of h, and $h(-2)$ does not exist.

$$\begin{aligned}(D)\ f(0) + g(1) - h(10) &\boxed{= \frac{12}{0 - 2} + (1 - 1^2) - \sqrt{10 - 1}}\\ &= \frac{12}{-2} + 0 - \sqrt{9}\\ &= -6 - 3 = -9\end{aligned}$$

Matched Problem 4 Use the functions in Example 4 to find

(A) $f(-2)$ (B) $g(-1)$ (C) $h(-8)$ (D) $\frac{f(3)}{h(5)}$

EXAMPLE 5 Finding Domains Find the domains of functions f, g, and h:

$$f(x) = \frac{12}{x - 2} \qquad g(x) = 1 - x^2 \qquad h(x) = \sqrt{x - 1}$$

SOLUTION *Domain of f:* $12/(x - 2)$ represents a real number for all replacements of x by real numbers except for $x = 2$ (division by 0 is not defined). Thus, $f(2)$ does not exist, and the domain of f is the set of all real numbers except 2. We often indicate this by writing

$$f(x) = \frac{12}{x - 2} \qquad x \neq 2$$

Domain of g: The domain is R, the set of all real numbers, since $1 - x^2$ represents a real number for all replacements of x by real numbers.

Domain of h: The domain is the set of all real numbers x such that $\sqrt{x - 1}$ is a real number, so

$$\begin{aligned}x - 1 &\geq 0\\ x &\geq 1 \qquad \text{or, in interval notation,} \qquad [1, \infty)\end{aligned}$$

Matched Problem 5 Find the domains of functions F, G, and H:

$$F(x) = x^2 - 3x + 1 \qquad G(x) = \frac{5}{x + 3} \qquad H(x) = \sqrt{2 - x}$$

*Dashed boxes are used throughout the book to represent steps that are usually performed mentally.

In addition to evaluating functions at specific numbers, it is important to be able to evaluate functions at expressions that involve one or more variables. For example, the **difference quotient**

$$\frac{f(x+h)-f(x)}{h} \qquad x \text{ and } x+h \text{ in the domain of } f,\ h \neq 0$$

is studied extensively in calculus.

CONCEPTUAL INSIGHT

In algebra, you learned to use parentheses for grouping variables. For example,

$$2(x+h) = 2x + 2h$$

Now we are using parentheses in the function symbol $f(x)$. For example, if $f(x) = x^2$, then

$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

Note that $f(x) + f(h) = x^2 + h^2 \neq f(x+h)$. That is, the function name f does not distribute across the grouped variables $(x+h)$, as the "2" does in $2(x+h)$ (see Appendix A, Section A.2).

EXAMPLE 6 Using Function Notation For $f(x) = x^2 - 2x + 7$, find

(A) $f(a)$ (B) $f(a+h)$ (C) $f(a+h) - f(a)$ (D) $\dfrac{f(a+h)-f(a)}{h}, \quad h \neq 0$

SOLUTION

(A) $f(a) = a^2 - 2a + 7$

(B) $f(a+h) = (a+h)^2 - 2(a+h) + 7 = a^2 + 2ah + h^2 - 2a - 2h + 7$

(C) $f(a+h) - f(a) = (a^2 + 2ah + h^2 - 2a - 2h + 7) - (a^2 - 2a + 7)$
$= 2ah + h^2 - 2h$

(D) $$\frac{f(a+h)-f(a)}{h} = \frac{2ah + h^2 - 2h}{h} = \boxed{\frac{h(2a+h-2)}{h}} \qquad \text{Because } h \neq 0,\ \frac{h}{h} = 1.$$
$$= 2a + h - 2$$

Matched Problem 6 Repeat Example 6 for $f(x) = x^2 - 4x + 9$.

Applications

We now turn to the important concepts of **break-even** and **profit–loss** analysis, which we will return to a number of times in this book. Any manufacturing company has **costs**, C, and **revenues**, R. The company will have a **loss** if $R < C$, will **break even** if $R = C$, and will have a **profit** if $R > C$. Costs include **fixed costs** such as plant overhead, product design, setup, and promotion; and **variable costs**, which are dependent on the number of items produced at a certain cost per item. In addition, **price–demand** functions, usually established by financial departments using historical data or sampling techniques, play an important part in profit–loss analysis. We will let x, the number of units manufactured and sold, represent the independent variable. Cost functions, revenue functions, profit functions, and price–demand functions are often stated in the following forms, where a, b, m, and n are constants determined from the context of a particular problem:

Cost Function

$$C = (\text{fixed costs}) + (\text{variable costs})$$
$$= a + bx$$

Price–Demand Function

$$p = m - nx \quad x \text{ is the number of items that can be sold at } \$p \text{ per item.}$$

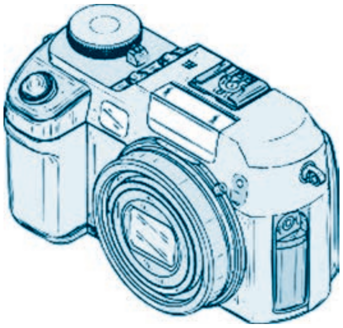
Revenue Function

$$\begin{aligned} R &= (\text{number of items sold}) \times (\text{price per item}) \\ &= xp = x(m - nx) \end{aligned}$$

Profit Function

$$\begin{aligned} P &= R - C \\ &= x(m - nx) - (a + bx) \end{aligned}$$

Example 7 and Matched Problem 7 explore the relationships among the algebraic definition of a function, the numerical values of the function, and the graphical representation of the function. The interplay among algebraic, numeric, and graphic viewpoints is an important aspect of our treatment of functions and their use. In Example 7, we will see how a function can be used to describe data from the real world, a process that is often referred to as *mathematical modeling*. Note that the domain of such a function is determined by practical considerations within the problem.



EXAMPLE 7 Price–Demand and Revenue Modeling A manufacturer of a popular digital camera wholesales the camera to retail outlets throughout the United States. Using statistical methods, the financial department in the company produced the price–demand data in Table 4, where p is the wholesale price per camera at which x million cameras are sold. Notice that as the price goes down, the number sold goes up.

Table 4 **Price–Demand**

x (Millions)	p (\$)
2	87
5	68
8	53
12	37

Using special analytical techniques (regression analysis), an analyst obtained the following price–demand function to model the Table 4 data:

$$p(x) = 94.8 - 5x \qquad 1 \le x \le 15 \tag{4}$$

(A) Plot the data in Table 4. Then sketch a graph of the price–demand function in the same coordinate system.

(B) What is the company's revenue function for this camera, and what is its domain?

(C) Complete Table 5, computing revenues to the nearest million dollars.

(D) Plot the data in Table 5. Then sketch a graph of the revenue function using these points.

(E) Plot the revenue function on a graphing calculator.

Table 5 **Revenue**

x (Millions)	$R(x)$ (Million \$)
1	90
3	
6	
9	
12	
15	

SOLUTION (A)

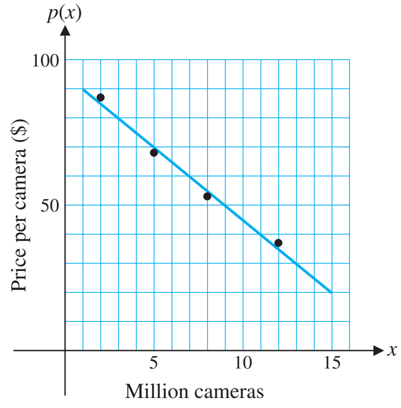


Figure 7 **Price–demand**

In Figure 7, notice that the model approximates the actual data in Table 4, and it is assumed that it gives realistic and useful results for all other values of x between 1 million and 15 million.

(B) $R(x) = xp(x) = x(94.8 - 5x)$ million dollars

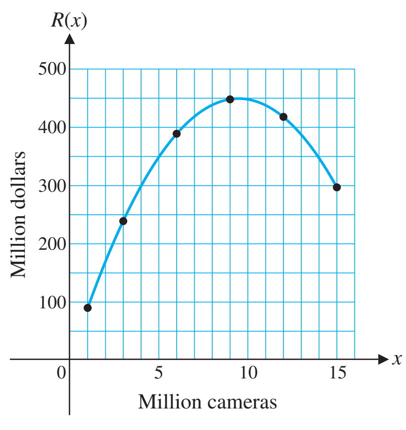
Domain: $1 \le x \le 15$

[Same domain as the price-demand function, equation (4).]

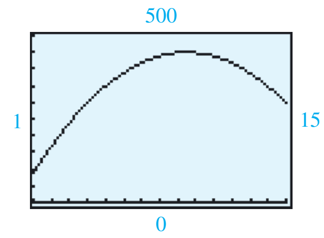
(C) Table 5 **Revenue**

x (Millions)	$R(x)$ (Million $)
1	90
3	239
6	389
9	448
12	418
15	297

(D)



(E)



Matched Problem 7 The financial department in Example 7, using statistical techniques, produced the data in Table 6, where $C(x)$ is the cost in millions of dollars for manufacturing and selling x million cameras.

Table 6 **Cost Data**

x (Millions)	$C(x)$ (Million $)
1	175
5	260
8	305
12	395

Using special analytical techniques (regression analysis), an analyst produced the following cost function to model the Table 6 data:

$$C(x) = 156 + 19.7x \qquad 1 \le x \le 15 \qquad (5)$$

(A) Plot the data in Table 6. Then sketch a graph of equation (5) in the same coordinate system.

(B) What is the company's profit function for this camera, and what is its domain?

(C) Complete Table 7, computing profits to the nearest million dollars.

Table 7 Profit

x (Millions)	$P(x)$ (Million $)
1	−86
3	
6	
9	
12	
15	

(D) Plot the data in Table 7. Then sketch a graph of the profit function using these points.

(E) Plot the profit function on a graphing calculator.

Exercises 2.1

In Problems 1–8, use point-by-point plotting to sketch the graph of each equation.

1. $y = x + 1$

2. $x = y + 1$

3. $x = y^2$

4. $y = x^2$

5. $y = x^3$

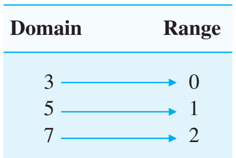
6. $x = y^3$

7. $xy = -6$

8. $xy = 12$

Indicate whether each table in Problems 9–14 specifies a function.

9.



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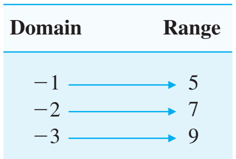

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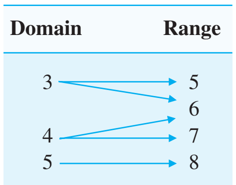

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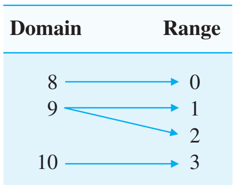

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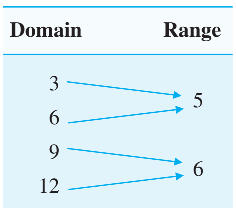

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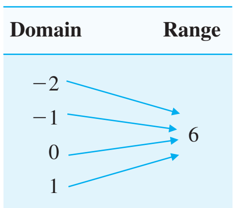
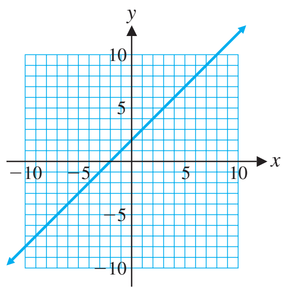
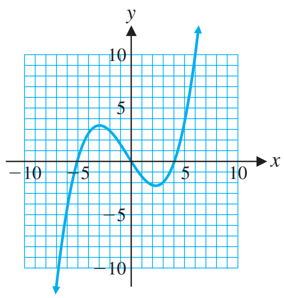


Indicate whether each graph in Problems 15–20 specifies a function.

15.



16.



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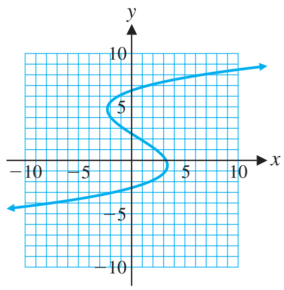

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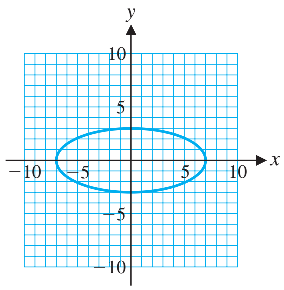

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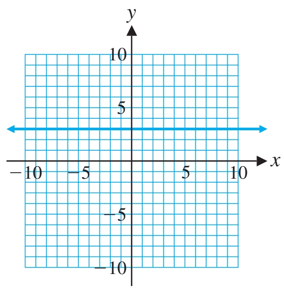

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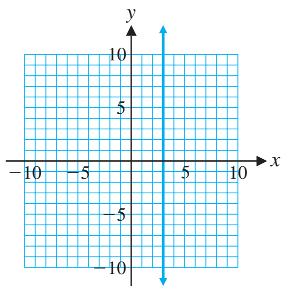


In Problems 21–28, each equation specifies a function with independent variable x. Determine whether the function is linear, constant, or neither.

21. $y - 2x = 7$

22. $y = 10 - 3x$

23. $xy - 4 = 0$

24. $x^2 - y = 8$

25. $y = 5x + \frac{1}{2}(7 - 10x)$

26. $y = \frac{2 + x}{3} + \frac{2 - x}{3}$

27. $3x + 4y = 5$

28. $9x - 2y + 6 = 0$

In Problems 29–36, use point-by-point plotting to sketch the graph of each function.

29. $f(x) = 1 - x$

30. $f(x) = \frac{x}{2} - 3$

31. $f(x) = x^2 - 1$

32. $f(x) = 3 - x^2$

33. $f(x) = 4 - x^3$

34. $f(x) = x^3 - 2$

35. $f(x) = \frac{8}{x}$

36. $f(x) = \frac{-6}{x}$

In Problems 37 and 38, the three points in the table are on the graph of the indicated function f. Do these three points provide sufficient information for you to sketch the graph of $y = f(x)$? Add more points to the table until you are satisfied that your sketch is a good representation of the graph of $y = f(x)$ on the interval $[-5, 5]$.

37.

x	-1	0	1
$f(x)$	-1	0	1

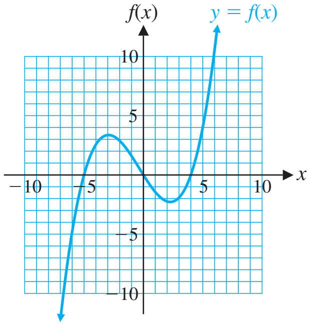
$f(x) = \frac{2x}{x^2 + 1}$

38.

x	0	1	2
$f(x)$	0	1	2

$f(x) = \frac{3x^2}{x^2 + 2}$

In Problems 39–46, use the following graph of a function f to determine x or y to the nearest integer, as indicated. Some problems may have more than one answer.



39. $y = f(-5)$

40. $y = f(4)$

41. $y = f(5)$

42. $y = f(-2)$

43. $0 = f(x)$

44. $3 = f(x), x < 0$

45. $-4 = f(x)$

46. $4 = f(x)$

In Problems 47–52, find the domain of each function.

47. $F(x) = 2x^3 - x^2 + 3$

48. $H(x) = 7 - 2x^2 - x^4$

49. $f(x) = \frac{x - 2}{x + 4}$

50. $g(x) = \frac{x + 1}{x - 2}$

51. $g(x) = \sqrt{7 - x}$

52. $F(x) = \frac{1}{\sqrt{5 + x}}$

In Problems 53–60, does the equation specify a function with independent variable x? If so, find the domain of the function. If not, find a value of x to which there corresponds more than one value of y.

53. $2x + 5y = 10$

54. $6x - 7y = 21$

55. $y(x + y) = 4$

56. $x(x + y) = 4$

57. $x^{-3} + y^3 = 27$

58. $x^2 + y^2 = 9$

59. $x^3 - y^2 = 0$

60. $\sqrt{x} - y^3 = 0$

In Problems 61–72, find and simplify the expression if $f(x) = x^2 - 4$.

61. $f(4)$

62. $f(-5)$

63. $f(x + 1)$

64. $f(x - 2)$

65. $f(-6x)$

66. $f(10x)$

67. $f(x^3)$

68. $f(\sqrt{x})$

69. $f(2) + f(h)$

70. $f(-3) + f(h)$

71. $f(2 + h)$

72. $f(-3 + h)$

73. $f(2 + h) - f(2)$

74. $f(-3 + h) - f(-3)$

In Problems 75–80, find and simplify each of the following, assuming $h \neq 0$ in (C).

(A) $f(x + h)$

(B) $f(x + h) - f(x)$

(C) $\frac{f(x + h) - f(x)}{h}$

75. $f(x) = 4x - 3$

76. $f(x) = -3x + 9$

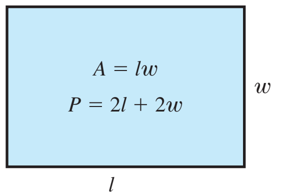
77. $f(x) = 4x^2 - 7x + 6$

78. $f(x) = 3x^2 + 5x - 8$

79. $f(x) = x(20 - x)$

80. $f(x) = x(x + 40)$

Problems 81–84 refer to the area A and perimeter P of a rectangle with length l and width w (see the figure).



81. The area of a rectangle is 25 sq. in. Express the perimeter $P(w)$ as a function of the width w, and state the domain of this function.

82. The area of a rectangle is 81 sq. in. Express the perimeter $P(l)$ as a function of the length l, and state the domain of this function.

83. The perimeter of a rectangle is 100 m. Express the area $A(l)$ as a function of the length l, and state the domain of this function.

84. The perimeter of a rectangle is 160 m. Express the area $A(w)$ as a function of the width w, and state the domain of this function.