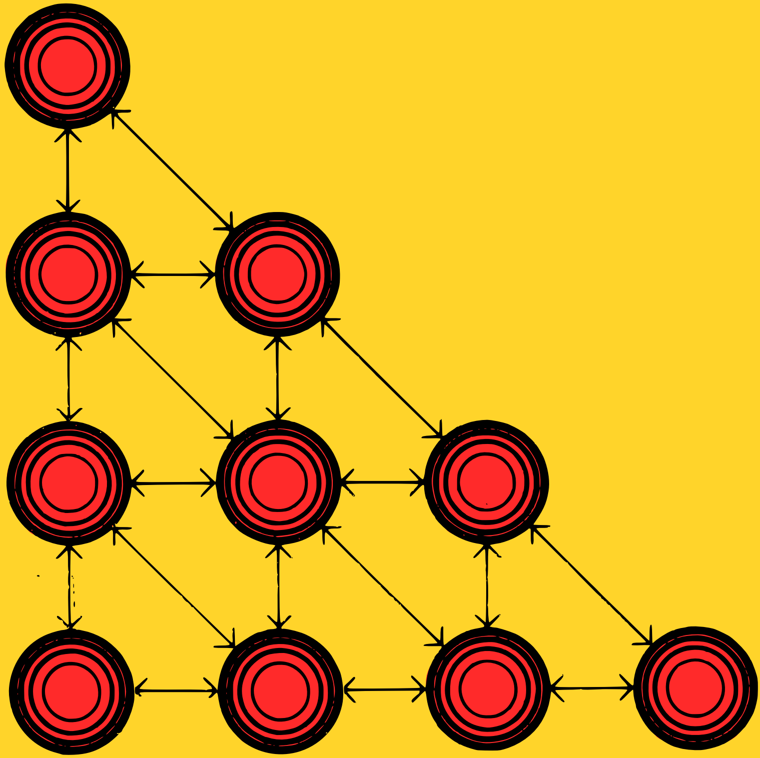
S. V. VONSOVSKY

MAGNETISM of ELEMENTARY PARTICLES



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С. В. ВОНСОВСКИЙ

МАГНЕТИЗМ МИКРОЧАСТИЦ

ИЗДАТЕЛЬСТВО «НАУКА»
МОСКВА

S. V. VONSOVSKY

MAGNETISM of ELEMENTARY PARTICLES

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by O. A. Germogenova, Cand. Sc.

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It may well be that these electrons
Are worlds just like our very own,
With kings and scholars, arts and armies,
And memories of ages flown.

And atoms—cosmic systems, spinning
Around a central spinning sphere,
Where things are just like ours, but smaller,
Or nothing like what we have here.

(V. Bryusov. *The World of the Electron.*
Transl. by L. Zellikoff)

PREFACE

The universal character of magnetism makes it an interesting object of study in investigating the most diversified natural phenomena, from elementary particles to the boundless expanses of space. It is also important that magnetic properties are interesting not only in themselves as a striking natural phenomenon, but also because they are closely related to other physical characteristics of matter. The possibility of their precise measurement offers the greatest opportunity to obtain invaluable information about practically the entire range of physical properties of material bodies, be it a particle of atomic or subatomic dimension, a macroscopic system in different aggregate states, or a cosmic object (a planet, a star, a galaxy, etc.).

This second, informatory, aspect of magnetism is especially important at the present stage of physics because of the unusually fast increase in the sophistication of experimental techniques. "Magnetic information" was also used widely at the dawn of atomic physics. It is sufficient to recall the historic role played by the discovery and analysis of the Zeeman effect, the most important external magnetic phenomenon in atomic spectroscopy, as well as the discovery and analysis of internal atomic magnetic effects such as the magnetic fine and hyperfine structure of spectral lines.

It is coming to be very important in studying the structure of atomic nuclei to determine the magnetic moments of their ground and especially their excited states. No less interesting is the informatory aspect of magnetism in elementary particle physics in view of the problem of the so-called anomalous magnetic moments of the electron, proton, neutron, muon and also the anomalous magnetic moments of more complex systems, such as positronium, muonium, and mesic atoms. In this problem magnetism appeared as the major arbiter in evaluating the applicability of the modern theory of quantized fields, both the electromagnetic and the electron-positron. Measurement of the magnetic moment of a baryon is of considerable interest when it comes to verifying the quarks hypothesis. Finally, the attention of physicists is drawn more and more frequently to the question of whether the Dirac magnetic monopole exists at all. Its discovery or the disproof of its existence would be events of extreme significance for the foundations of physics.

There are some highly interesting problems arising from the effect of very strong magnetic fields on processes of interaction between material fields, processes which result in a variety of non-linear effects (magnetic bremsstrahlung, production of electron-positron pairs, etc.).

It was this growing importance of the informatory aspect of the study of the magnetic properties of matter that suggested the idea of a book about the magnetism of atomic and subatomic particles.

In working on the present monograph the author did not set himself the goal of giving an encyclopedic elaboration of the whole field of magnetism that would include the mathematical apparatus of theory. Rather, this book is meant to give the reader an overview of the present state of study of the magnetic properties of the microcosm, without going into professional subtleties.

The book contains a fairly detailed though, of course, incomplete reference list, which may help to satisfy the reader who wishes to investigate any one question in its entirity.

The book opens with a review of the well-known aspects of magnetism of the elementary particle that was discovered first, the electron (Chapter 1). This is followed by a brief summary of data concerning the magnetic properties of atomic electron shells (Chapter 2). Chapter 3 is devoted to the magnetic properties of atomic nuclei and their constituent nucleons—the proton and the neutron. It also contains a description of the most important experimental techniques of determining the magnetic moments of nuclei and nucleons (detailed tables of measured magnetic moments are given in the Appendix at the end of the book). Chapter 4 deals with the problem of the anomalous magnetic moment of an elementary particle and with the relation of this problem to the quarks hypothesis. Chapter 5 offers a fairly detailed description of the situation arising from the Dirac hypothesis concerning the magnetic monopole. Finally, Chapter 6 gives a very brief presentation of non-linear magnetic effects in strong fields.

As has been noted, the author did not pursue the goal of giving a rigorous mathematical elaboration of theory or a comprehensive review of experimental facts. He confined himself to outlining the general situation, stressing the physical essence of the described phenomena. It is the author's hope that this book will find many readers among physicists and specialists in related branches of the natural sciences and will help them in their practical research.

In conclusion the author takes pleasure in expressing his deep gratitude to M. I. Kaganov for his invaluable comments on the text. During the whole period of work on the manuscript the author was assisted by his wife L. A. Shubina, to whom he expresses his thanks for her continuous support.

S. V. Vonsovsky

CONTENTS

SPIN MAGNETIC MOMENT OF THE ELECTRON

1. Spin, Magnetic Moment, and G-Factor of the Electron

The electron, which was discovered at the end of the nineteenth century, is the most studied elementary particle among those known at present. For a long time it was believed that the basic characteristics of this particle are its electric charge and mass, while the magnetic properties manifest themselves only in the motion of electrons in the orbits of atoms and molecules, in conductors, and in cathode rays. However, studies of the line optical spectra of atoms revealed that the spectral lines have a fine structure*. To explain the nature of this structure it was necessary to revise the former concepts of the basic characteristics of the electron. The fine structure is simplest in hydrogen-like atoms with one valence electron, where it amounts to double splitting of the spectral lines (doublets). It was possible to explain the appearance of doublets and their splitting width by assuming that the electron has its own mechanical angular momentum ***s***—*spin*. It was also necessary to assume that the spin moment has only two possible orientations with respect to an external magnetic field. These orientations are such that two corresponding projections on the direction of a field which coincides with the z-axis, for example, are equal in magnitude to $\hbar/2$ and have opposite signs:

$$s_z = \pm \frac{\hbar}{2} \tag{1.1}$$

* See, for example, [842].

where $2\pi\hbar = h = 6.626196(50) \times 10^{-27}$ erg·s is Planck's constant*. In addition, it was found that the spin of the electron generates a magnetic moment whose two possible projections on the direction of the external magnetic field are**

$$\mu_{sp}^{z} = \pm \frac{e\hbar}{2mc} = \mp \frac{|e|}{mc} s_z \tag{1.2}$$

where $|e| = 4.803250(21) \times 10^{-10}$ esu is the magnitude of the electron electric charge, $m = 9.109558(54) \times 10^{-28}$ g is its rest mass, and $c \cong 3 \times 10^{10}$ cm·s^{-1} ($c = 2.9979250(10) \times 10^{10}$ cm·s^{-1}) is the speed of light. In other words, these projections are equal in absolute value to the Bohr magneton, the natural atomic unit of magnetic moment:

$$\mu_B = \frac{|e|\hbar}{2mc} = 0.9284851\,(65) \times 10^{-20}\ \text{erg}\cdot\text{Gs}^{-1} \tag{1.2a}$$

These new basic properties of the electron were at first given a pictorial classical interpretation. It was assumed that the electron, a charged "ball", revolves around an axis that passes through its centre, and its electric charge in the process generates a current of its own, which in turn produces a magnetic moment. The negative sign of the electron charge determines the antiparallel orientation of the spin and the magnetic moment***.

However, this attempt of a purely classical explanation of the nature of the electron spin lead to fundamental difficulties. First of all, in order to compute the value of the magnetic moment using the laws of classical electrodynamics it was necessary to formulate certain assumptions concerning

* The numerical values of the fundamental constants are taken from [902]. The numbers in parentheses stand for the standard deviation for the last figures in the constants.

** It was established later that the magnitude of the magnetic moment differs somewhat from (1.2). This question will be considered in detail in Chapter 4.

*** The rigorous classical electrodynamic theory of the "spinning" electron was developed by Frenkel [349, 353], Tamm [894], and Thomas [913, 914]. Later classical relativistic equations for a charged particle with a spin were formulated and solved in [98, 228, 229, 230, 241, 300, 316, 407, 583, 601, 762, 763, 783, 892, 893, 964].

the structure of the electron (form, dimensions, spatial distribution of the charge, etc.). Secondly, the classical theory gave no explanation of why the spin in an external field can have only two different orientations (the space quantization of the spin moment). And last, the theory could not provide a rational explanation of the so-called *gyromagnetic anomaly* of the spin. This anomaly consists in the fact that, as can be seen from (1.2), the ratio between the spin magnetic moment and the mechanical moment, i.e., the *magnetomechanical ratio* for spin is equal to

$$g'_{sp} = \left| \frac{\mu^z_{sp}}{s_z} \right| = \frac{|e|}{mc} \tag{1.3}$$

whereas according to the classical laws for an electron moving in an orbit, this ratio should be twice as small, i.e., should equal $|e|/2mc$ [see Eq. (2.9) below]. Recall that a similar anomaly was observed in the measurement of the gyromagnetic effect in ferromagnetics (see, for example, [945]); it was also noted when the laws governing the anomalous Zeeman effect were explained (see Chapter 2). These difficulties indicated the insufficiency of the classical explanation of the nature of the electron spin and prompted scientists at the outset to consider the existence of the spin and the respective magnetic moment simply an experimental fact that as yet had no adequate theoretical explanation. The spin was regarded as the fourth *intrinsic* degree of freedom of the electron.

2. The Stern-Gerlach Experiment: Determination of the Spin and the Magnetic Moment of the Electron

A striking and direct experiment which proved the existence of the electron spin and magnetic moment and gave the rule of their space quantization was the experiment of Stern and Gerlach with the deflection of atomic beams in a non-uniform magnetic field [377]*.

* Independently and simultaneously with these authors the same kind of experiments were conducted by the Soviet physicists Kapitza and Semenov [268, 518].

The schematic representation of the corresponding experimental set-up is shown in Fig. 1.1. The electric furnace A vaporizes the studied substance. From the flux of the vaporized molecules or atoms leaving the furnace through a small opening d a series of diaphragms bb' cuts out a thin, narrow beam that enters the space between the poles BB' of the electromagnet and finally hits the screen C. The pole

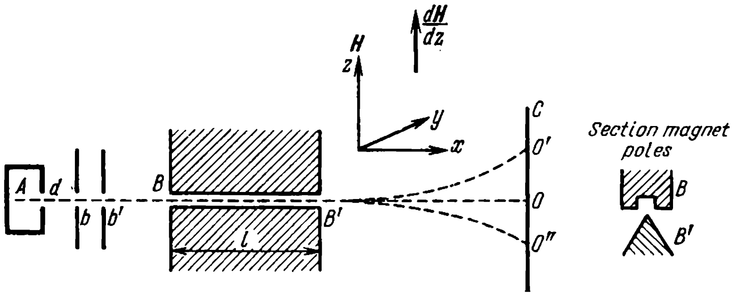

Fig. 1.1. The schematic representation of the experiment with the deflection of atomic and (or) molecular beams in a non-uniform magnetic field.

pieces B and B' of the electromagnet, as shown in Fig. 1.1, have such a cross section that the magnetic field between the pieces has a large gradient in the direction of the z-axis which is normal to the direction of the beam $\left(\frac{dH}{dz} \neq 0\right)$.

Atoms, having a magnetic moment, will be deflected from their initial direction of motion only in a magnetic field that is noticeably non-uniform on the scale of the order of atomic dimensions (10^{-8} cm). If this condition is not satisfied, there will be no deflection of the atoms in the beam at all; instead, the atoms will precess around the direction of the magnetic field with the translational motion being unchanged. If, on the other hand, the non-uniformity is present, the atom with the magnetic moment $\boldsymbol{\mu}$ forming an angle θ with the direction of the field gradient (i.e., with the z-axis) will be subjected to the action of a deflecting force

$$\mu \frac{dH}{dz} \cos\theta \tag{1.4}$$

This force will affect the electron trajectory (along the x-axis) on an interval of length l (see Fig. 1.1), where $dH/dz \neq 0$. By the laws of uniformly accelerated motion, for given values of μ and θ the deflection of an atom at the end of this trajectory is equal in magnitude to

$$z_\theta = \frac{1}{2} \frac{\mu}{M} \left(\frac{dH}{dz}\right)_{\text{av}} t^2 \cos\theta \tag{1.5}$$

where M is the mass of the atom, and $t = l/v$ is the time that the atom spends passing through the region with a non-zero magnetic field. In the last formula v is the average velocity of an atom in the beam, the velocity specified by the conditions of thermal equilibrium for a given temperature in the furnace A.

If all angles θ were equally probable, then one should observe a wide band $O'O''$ on the screen C instead of a narrow image of the slit at point O, with an upper edge O' corresponding to deflected atoms the magnetic moments of which are parallel to the field ($\theta = 0$) and a lower edge O'' corresponding to atoms with a moment antiparallel to the field ($\theta = 180°$). In fact the picture is quite different. If the experiment is performed with the beams of hydrogen-like atoms having one valence electron, then we obtain two separate images of the slit at O' and O'' instead of a continuous band between O' and O''. From the quantum-mechanical point of view the normal state of such atoms is characterized by the absence of both the mechanical angular momentum and the orbital magnetic moment (see Chapter 2). Therefore these atoms should not experience any deflection in a magnetic field. But if we assume the existence of spin and its associated magnetic moment and take the rule of their space quantization into account, the result of the experiment immediately becomes clear. The double splitting of the beam is a consequence of space quantization for the spin magnetic moment for which there are only two possible projections: along the magnetic field ($\theta = 0$) and in the opposite direction ($\theta = 180°$).

These experiments also provide an opportunity to determine the magnitude of the projection of the spin magnetic moment on the direction of the external magnetic field using the measured deflection of the atomic beam z_θ and formula (1.5).

Figure 1.2 gives the photomicrogram on screen C of the deposit obtained in the experiment with the beam of sodium vapour (see [599]). At high furnace temperatures the beam is nearly pure atomic, which leads to the double-humped

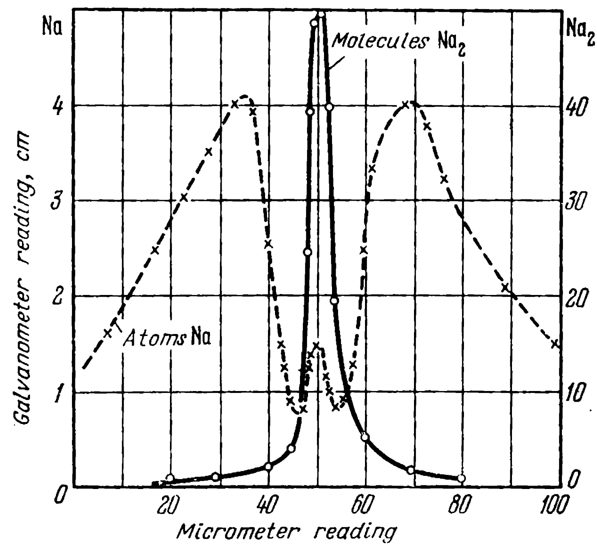

Fig. 1.2. The photomicrogram of the deposit of an atomic beam (the broken line) and a molecular beam (the solid line) of sodium. The abscissa gives the distance (in microns) on the photograph of the deposit, the ordinate gives the galvanometer reading. The left scale corresponds to Na atoms, the right scale to Na_2 molecules (from [599]).

curve (the broken line in Fig. 1.2*). At lower temperatures a greater portion of the atoms combine into pairs to form the molecules of Na_2; since these molecules have neither orbital nor spin magnetic moments they experience no deflection when passing through the field. This fact is illustrated by the solid line in Fig. 1.2 (the photomicrogram

* The small maximum in the central section of the broken line corresponds to the zero deflection due to a small impurity of Na_2 molecules also present at high temperatures.

of the deposit of the undeflected beam of sodium molecules). At the present time the technique of such experiments has been improved to a degree which allows to guarantee the accuracy in measurements up to 0.1-0.2 per cent.

These experiments must be considered fundamental for the entire field of atomic physics since they have lead to the direct determination of the atomic nature of magnetism.

3. Elements of the Quantum Theory of the Electron Spin

With the development of the consistent atomic theory—quantum mechanics—it became necessary to work out a theoretical interpretation of electron spin and its magnetism, which would be more profound than that existing in the classical theory. The solution of this problem, however, could not be obtained within the scope of a non-relativistic quantum mechanics limited to analysis of processes involving particles with velocities small by comparison with that of light. The reason for this lies in the fact that the phenomenon of spin magnetism, as any other magnetic phenomenon, can be classified as a typical relativistic effect the consistent explanation of which should take into account the requirements of the theory of relativity.

Pauli [722] in 1927 formulated an approximate semi-empirical quantum theory of electron spin* in which he simply postulated the existence of spin and its magnetic moment. Then, since there are only two possible spin orientations, the general laws of quantum mechanics stipulate that if the magnitude of the spin component s_z is equal to $\hbar/2$, the absolute value of the spin vector itself is

$$|\mathbf{s}| = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\,\hbar \tag{1.6}$$

The modulus of the vector of the spin magnetic moment is then given by the following equality:

$$|\mu_{\mathrm{sp}}| = \frac{|e|}{mc}\sqrt{s(s+1)} = \sqrt{3}\,\mu_{\mathrm{B}} \tag{1.7}$$

where s is the spin quantum number ($s = 1/2$).

* See, for example, the monographs of Blokhintsev [116], Bohm [126], Frenkel [350], and Landau and Lifshitz [571].

The problem of electron spin received a theoretical justification after Dirac's formulation [253] of a *relativistic* quantum theory of the electron which did not postulate the existence of spin and its magnetic moment. On the contrary, they automatically occurred from the theory and their values were in exact agreement with experiment. Thus, Dirac's theory clearly demonstrated the impossibility (in principle) of the intuitive classical interpretation of the spin associated with the concept of a spinning ball—the electron. Note that even if one tries to come up with a visual illustration of the spin, its appearance should be ascribed to the specifically quantum kinematic properties of the *translational* motion of an electron. Therefore the spin magnetic moment is called *kinematic*, to distinguish from the "true" moment which is observed in the case of some other particles, the proton and the neutron for example (see Chapter 3).

A simple and at the same time rigorous solution of Dirac's relativistic wave equation can be found, for example, in the second volume of the book of Shpol'sky [843] or in Schiff's book [817]. In the papers of Corben [228, 229, 230] and Datzeff [241] mentioned above there is an interesting solution of the classical relativistic equations for a particle with spin; this solution is compared to that of Dirac's. See, in this connection, [217, 601, 690, 889, 992].

Sometimes [350] the spin of Dirac's electron is associated with an additional rapid oscillatory motion of the relativistic electron (the so-called "trembling", or Zitterbewegung, in Schrödinger's terminology). The discussion of these concepts one can find, for example, in the review paper of Kramers [555]; their further development is given in detail in a series of papers of Yamasaki [981-984], Browne [153], and Crowther and Ter Haar [235].

In the case of an electron moving in a field with central symmetry and potential $\varphi(\boldsymbol{r})$, the Dirac wave equation (see, for example, Davydov's monograph [242]) has the form

$$i\hbar \frac{\partial\psi(\boldsymbol{r}, t)}{\partial t} = \hat{H}\psi(\boldsymbol{r}, t) \tag{1.8}$$

where $\psi(\boldsymbol{r}, t)$ is the wave function, and $\hat{H}$ is the energy operator (Hamiltonian) of the relativistic electron in the central field:

$$\hat{H} = c\boldsymbol{\alpha}\boldsymbol{p} + \beta mc^2 + e\varphi(\boldsymbol{r}) \tag{1.9}$$

In this equation $p = -i\hbar\nabla$ is the momentum operator, and α_x, α_y, α_z, and β are pairwise anticommuting matrices ($\alpha_i\alpha_j + \alpha_j\alpha_i = \alpha_i\beta + \beta\alpha_i = 0$, $i \neq j = x, y, z$) the squares of which are equal to :unity $\alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 = 1$. In the simplest representation the explicit form of these matrices is

$$\beta = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}, \quad \alpha_x = \begin{Vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{Vmatrix}$$

$$\alpha_y = \begin{Vmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{Vmatrix}, \quad \alpha_z = \begin{Vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{Vmatrix} \qquad (1.10)$$

or, in more compact notations,

$$\alpha = \begin{Vmatrix} 0 & \sigma \\ \sigma & 0 \end{Vmatrix}, \quad \beta = \begin{Vmatrix} I & 0 \\ 0 & -I \end{Vmatrix} \qquad (1.10a)$$

where σ (σ_x, σ_y, σ_z) are the Pauli matrices:

$$\sigma_x = \begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix}, \quad \sigma_y = \begin{Vmatrix} 0 & -i \\ i & 0 \end{Vmatrix}, \quad \sigma_z = \begin{Vmatrix} 1 & 0 \\ 0 & -1 \end{Vmatrix} \qquad (1.11)$$

and $I = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$ is the unit matrix.

It is well known from classical mechanics that in a centrally symmetric field angular momentum is an integral of motion. Therefore one can expect that the orbital angular momentum $L = [r, p]$ of a relativistic electron is also an integral of motion. However, this is not so. Let us show that the operator $\hat{L} = [\hat{r}, \hat{p}]$ does not commute with the Hamiltonian (1.9). For example, for the z-component of the orbital angular momentum $\hat{L}_z$ from (1.9) and the definition of angular momentum we obtain

$$\begin{aligned} \hat{L}_z\hat{H} - \hat{H}\hat{L}_z &= c\alpha\,[(x\hat{p}_y - y\hat{p}_x)\,\hat{p} - \hat{p}\,(x\hat{p}_y - y\hat{p}_x)] \\ &= i\hbar c\,(\alpha_x\hat{p}_y - \alpha_y\hat{p}_x) \neq 0 \end{aligned} \qquad (1.12)$$

As follows from (1.12), $\hat{L}_z$ does not commute with $\hat{H}$ and therefore cannot be regarded as an integral of motion. It is possible to show that the same is true for $\hat{L}_y$ and $\hat{L}_x$ which means that in the relativistic case in a central field, instead of the conservation of the orbital angular momentum, we have the conservation of the total angular momentum. The latter is equal to the sum of the orbital and some *additional* angular momentum.

Indeed, it can be demonstrated that the quantity which is conserved is $\hat{\boldsymbol{L}} + \hat{\boldsymbol{s}}$, where the additional angular momentum $\hat{\boldsymbol{s}}$ is related to the Pauli matrices [722] through a simple identity:

$$\hat{\boldsymbol{s}} = \frac{1}{2}\hbar\boldsymbol{\sigma} \tag{1.13}$$

In order to compute the commutation relation, for example, between $\hat{s}_z$ and $\hat{H}$, one has to take into account the commutation relations for the Pauli matrices (1.11):

$$\sigma_k\sigma_l = -\sigma_l\sigma_k = i\sigma_m, \quad \sigma_k^2 = I \, (k \neq l \neq m = x, y, z) \tag{1.14}$$

and the definition of the Dirac matrices (1.10). This yields

$$\sigma_z\alpha_x = -\alpha_x\sigma_z = i\alpha_y, \quad \sigma_z\alpha_y = -\alpha_y\sigma_z = -i\alpha_x,$$
$$\sigma_z\alpha_z = \alpha_z\sigma_z$$

Finally we obtain

$$\hat{s}_z\hat{H} - \hat{H}\hat{s}_z = \frac{\hbar c}{2}(\sigma_z\boldsymbol{\alpha}\hat{\boldsymbol{p}} - \boldsymbol{\alpha}\hat{\boldsymbol{p}}\sigma_z) = ic\hbar(\alpha_y\hat{p}_x - \alpha_x\hat{p}_y) \tag{1.15}$$

Comparing (1.15) and (1.12), we can see that in the general case only the *total* angular momentum is conserved. So is its projection on the z-axis:

$$\hat{J}_z = \hat{L}_z + \frac{\hbar}{2}\sigma_z \tag{1.16}$$

The projection $\hat{s}_z$ of the operator of the spin angular momentum is not, generally speaking, an integral of motion (it is an integral of motion only in the state with a certain value of the electron momentum directed along the z-axis, i.e., when $\boldsymbol{\alpha}\hat{\boldsymbol{p}} = \alpha_z\hat{p}_z$).

This striking result of Dirac's theory, which correctly predicted the value of the g-factor of the electron with spin 1/2, can be easily extended to include particles with spin 1 (see [74, 989]). However, the generalization for cases corresponding to $s > 1$ is associated with considerable mathematical difficulties since for such values of spin the number of possible values $2s + 1$ of its components increases rapidly. Belinfante [75] carried out the computations for $s = 3/2$, Tumanov [929] for $s = 2$, and Moldauer and Case [656] for $s = N + 1/2$ (N is a positive integer). According to these authors, the value of the g-factor is inversely proportional to s:

$$g = \frac{e}{2mc} \frac{1}{s}$$

A general solution for the case of arbitrary spin which takes into account the requirements of Galilean invariance up to the accuracy of the first order of all the derivatives appearing in the wave equations of the respective particles with spin s is given in a more recent paper of Hagen and Harley [429]. This solution confirms the above-mentioned results (see also [318, 556]).

In connection with the problem of the existence of elementary particles with higher values of spin ($s > 1/2$), one should mention the papers of Tamm and Ginzburg [386, 389, 391, 896]; the second paper contains a recent review of this field. In addition, Vonsovsky and Svirsky [946] have demonstrated that in Dirac's theory [253] the relativistic electron has not only spin, but also pseudospin with two possible projections corresponding to the two projections v and $-v$ of the electron's velocity on the direction of the electron momentum.

4. The Feasibility of the Measurement of Spin Angular Momentum for the Free Electron

Strictly speaking, from the experiments with the deflection of a molecular beam in a magnetic field it only follows that the atom as a whole has a certain value of the magnetic moment. In some cases, on the basis of indirect assumptions about the nature of the orbital states, this observed atomic magnetism is ascribed to the electron spin, although this is not an immediate consequence of experimental results. Thus, the molecular beam experiments considered by themselves are not sufficient for separating the magnetic effects associated with the spin of the electron from those related to its orbital motion. Therefore it appears very desirable to conduct a similar study with a beam of *free* electrons.

However, as was shown by Bohr, such an experiment on the determination of the magnetic moment of a free electron is doomed to failure (see [445, 723]). This is a simple consequence of the uncertainty principle of quantum mechanics. Indeed, since the spin magnetism of the electron is of kinematic origin, it cannot be separated from the magnetic effects associated with the translational motion of the electron as a charged particle. It was found that any attempt to determine the spin magnetic moment introduces an inevitable uncertainty in the magnitude of the electron momentum, which causes the uncertainty in the magnetic effects related to the translational motion always to excess the total magnetic effect of the spin.

Bohr's approach can be briefly formulated as follows. Let the position of the electron be defined up to the accuracy Δr; we want to determine the magnetic moment of spin μ at a distance r from the field $H \propto \mu/r^3$ generated by it using, for example, a certain measuring device (magnetometer). This can be done only if $\Delta r \ll r$. According to the uncertainty principle of Heisenberg (see, for example, [116]) ($\Delta r \Delta p \geqslant \hbar/2$), one can never be sure that the electron the position of which is determined up to the accuracy Δr does not have a momentum $\Delta p \geqslant \hbar/2\Delta r$. But if the electron has such momentum, then as it is moving it will generate at a distance r a magnetic field

$$H_{\text{orb}} \propto \frac{e\Delta p}{mcr^2} = \frac{e\hbar}{2mcr^2\Delta r}$$

In order to separate the effect of the spin magnetic moment of the electron from the magnetic effect of its translational motion it is necessary that the field H_{sp} be much greater than H_{orb}, i.e.,

$$\frac{\mu}{r^3} \gg \frac{e\hbar}{2mcr^2\Delta r} \quad \text{or} \quad \Delta r \gg r$$

since $\mu \cong e\hbar/2mc$. However, the requirement $\Delta r \gg r$ is in contradiction with the initial condition of the possibility of the observation of H_{sp}. This means that such measurements of the spin magnetic moment of the electron are impossible. A more detailed discussion of the specific conditions of the impossibility of setting up experiments of Stern-Gerlach type for a beam of free electrons can be found in Mott and Messey's monograph [661].

However, this does not mean that it is impossible in principle to observe the electron spin. The spin magnetic moment of a free electron can never be separated in a unique way from the orbital magnetic moment only when it concerns the experiments allowing a classical interpretation of the particle trajectory, which is another evidence of the fact that an intuitive approach to spin from the concepts of classical mechanics is doomed to failure. Other experiments not based on the concept of a classical trajectory of a particle can be used for the proof of the presence of spin in the case of a free electron. This can be done if one studies the so-called polarization of electron waves (beams of free electrons), which is the direct consequence of the existence of electron spin.

Mott [659] was the first to compute the effect of polarization of electrons in the field of an atomic nucleus unshielded by the electron shell. It is essential that the electric field scattering the polarized electron beam be strongly non-uniform on the microscale (on intervals $\cong 10^{-8}$ cm). The physical factor that leads to the phenomenon of polarization is that the scattering of the electron depends on the spin-orbit interaction of the electron spin magnetic moment with the magnetic field generated by its motion in the electric field of the scattering object (for instance, the atomic nucleus). These experiments are very involved and have led to a positive result only after a series of failures (15 years later than the theoretical prediction of the effect by Mott in [659]). Further theoretical analysis was performed by Landau [567] and Sokolov [866].

A detailed review of this interesting part of electronic physics can be found in [341, 710, 824, 918] to which we refer the interested reader.

We shall just mention three more very interesting papers by Farago [315] and Kalckar [514, 515] which contain the details of the discussion on the question concerning the possibility to measure the spin and magnetic moment of a free electron.

In his first article Kalckar states that the impossibility of the measurement of the spin magnetic moment of a free electron by means of the experiment of the Stern-Gerlach type does not imply any limitation on the measurability of spin (i. e., the mechanical angular momentum) in this situation. Kalckar suggests a purely classical method for determining the electron spin based on the use of a measuring device in the form of a rigid macroscopic rotator with a fixed axis. The measurement occurs through the transfer of the electron spin component to the rotational degrees of freedom of the macroscopic body.

In his paper Farago completely agrees with Kalckar when it comes to the assertion that the determination of the spin and spin magnetic moment for a free electron can be considered as two different problems (this circumstance is demonstrated very clearly and in detail in Kalckar's article).

However, Farago argues very convincingly against the statement to the effect that the spin of a free electron can be measured in an experiment based completely on classical concepts. He shows that the "thought experiment" suggested by Kalckar is unacceptable since it lacks the decisive element of reality, namely, the interaction between the electron and the macroscopic system which is responsible for the transfer of the mechanical angular momentum from the electron to the classical body, the rotator. Thus, it is only in non-classical experiments that one can find the possibility to measure not only the intrinsic magnetic moment but also the intrinsic spin moment.

However, the second article of Kalckar [515] subjects to dispute the arguments of Farago.

The details of the discussion between Kalckar and Farago are very interesting; the reader can find these in the above-mentioned articles. On the quantum methods of the determination of electron spin see also the review paper of Farago [314].

5. Spin and Magnetic Moment of the Positron

All facts concerning the magnetism of the electron can be applied to another elementary particle—the *positron*, which differs from the electron only by the positive sign of its electric charge. In other words, the positron is the *antiparticle* with respect to the electron. This particle is much more scarce than the electron. It is not a component of an atomic shell, and it annihilates easily in collisions with an electron.

As yet, there were no direct experiments on the registration and measurement of the spin magnetism of the positron (see the review of Tolhock [918]). There is a possibility to obtain indirect information on the positron spin and magnetic moment. As follows from the relativistic quantum theory of the electron and the positron, their interaction (collision) can lead not only to their annihilation accompanied by the photon emission, but also to formation of metastable bound states of atomic type—*positronium* (see, for example, [79, 80, 489, 567, 750, 796]).

The theory [248, 249] predicts the existence of two close states: the ground state ^1S corresponding to the antiparallel spins of the electron and the positron (*parapositronium*), and the excited state ^3S with the energy of excitation 8.2×10^{-4} eV (*orthopositronium*). The lifetime of parapositronium (which decays into two photons) is of the order of 10^{-10} s, whereas the lifetime of orthopositronium (which decays into three photons) is of the order of 10^{-7} s, which is quite accessible for observation. The first experimental discovery of positronium has occurred in 1951-1952 [248, 249].

At the present time there are many publications on the experimental determination of the lifetime and the spectrum of positronium, including the fine structure (FS) and the hyperfine structure (HfS). In the case of positronium, when both the particle playing the role of the nucleus of the atom and the particle moving on the orbit have the same mass and therefore the same magnetic moment, the splitting of the energy levels corresponding to fine and hyperfine structure (see Chapters 2 and 3) is of the same order of magnitude.

Good agreement of the experimental data and the theory in positronium studies can be regarded as a convincing proof of the equality of the magnetic moments of the electron, on one hand, and of its antiparticle, the positron, on the other.

More detailed information on the properties of positronium can be found in the monographs on quantum electrodynamics (see, for example, [9, 86, 867]).

Chapter 4 contains a new refinement of the relativistic quantum theory, which has found striking confirmation in experiment. It gives a more accurate value for the magnetic moment and the g-factor of the electron (and also of the positron).

MAGNETISM OF THE ATOMIC ELECTRON SHELL

Now that we are familiar with the magnetic properties of the electron as an elementary particle, it would be natural to consider the magnetism of the simplest association of this particle, the atomic electron shell.

The magnetic properties of atomic electron shells are dependent on three factors: the orbital motion of the electrons, the spin of the electrons, and the magnetism of the atomic nucleus.

1. The Magnetism of the Orbital Motion of the Electron in a One-Electron Atom

1.1. The Quasi-Classical Method

Let us first consider the magnetic properties of the orbital motion of the electron. It is possible to establish the relationship between the magnetic moment and the mechanical angular momentum by applying the classical theory of the orbital motion of the electron.

Indeed, the motion of the electron in an elliptic orbit with the period of revolution T is equivalent to a circulating current of intensity

$$i = \frac{e}{cT} \tag{2.1}$$

The magnetic moment generated by this current is equal to the product of the current intensity and the area of the orbit (see Fig. 2.1)

$$S=\frac{1}{2}\int_{0}^{2\pi} r^2\,d\varphi \qquad (2.2)$$

where φ is the angle formed by the larger diameter of the ellipse and the radius vector with the origin in a focus, i.e.

$$\mu_{\text{orb}}=iS=\frac{eS}{cT} \qquad (2.3)$$

As follows from the law of conservation of angular momentum, p_φ (the angular momentum of the electron) is constant and is by definition equal to

$$p_\varphi=mr^2\frac{d\varphi}{dt} \qquad (2.4)$$

Here $d\varphi/dt$ is the angular velocity of the electron orbital motion. Let us replace r^2 in (2.2) with its equivalent in (2.4). Then

$$S=\frac{p_\varphi}{2m}\int_{0}^{T} dt=\frac{p_\varphi T}{2m} \qquad (2.5)$$

For the magnetic moment μ_{orb} from (2.5) and (2.3) we obtain

$$\mu_{\text{orb}}=\frac{e}{2mc}p_\varphi \qquad (2.6)$$

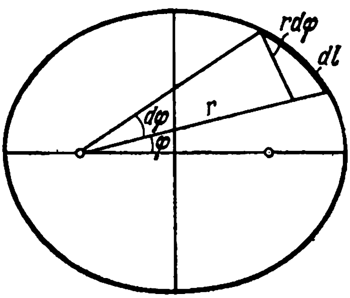

Fig. 2.1. The elliptic orbit of an electron (for the computation of orbital magnetic moments).

From Bohr's quantization rule for orbits [842] it follows that

$$p_\varphi = l\hbar \quad (l = 1, 2, 3, \ldots, n) \qquad (2.7)$$

(l is the azimuthal or angular-momentum quantum number, and n is the principal quantum number). Equation (2.6) combined with (1.2a) and (2.7) leads to the following:

$$\mu_{\text{orb}}=l\frac{e\hbar}{2mc}=l\mu_{\text{B}} \qquad (2.8)$$

Thus, as follows from the early version of quantum mechanics, the magnetic moment associated with the orbital motion of the electron in an atom is a multiple of the Bohr magneton, whereas the ratio of this magnetic moment to the mechanical angular momentum is, according to (2.6), equal to

$$g'_{\text{orb}} = \frac{\mu_{\text{orb}}}{p_\varphi} = \frac{e}{2mc} \tag{2.9}$$

This ratio is twice as small as the corresponding ratio for the spin moments (see (1.3)). Usually this *magnetomechanical ratio* is expressed in units of $e/2mc$. Therefore, instead of (1.3) and (2.9) we have

$$g_{\text{sp}} = 2, \quad g_{\text{orb}} = 1 \tag{2.9a}$$

*1.2. The Quantum-Mechanical Method (Space Quantization of Orbits)**

The quantum-mechanical theory allowed a refinement of this problem. The rigorous solution of the Schrödinger equation in the case of a centrally symmetric field of the atomic nucleus has demonstrated that for the stationary state of the electron in a one-electron atom** (which can be called orbital motion only conditionally) the absolute value of the vector of angular momentum instead of (2.7) is given by

$$|l| = \sqrt{l(l+1)}\,\hbar \tag{2.10}$$

whereas for the set of possible values of the azimuthal quantum number l for a given principal number n we have

$$l = 0,\ 1,\ 2,\ \ldots,\ (n-1) \tag{2.10a}$$

rather than (2.7).

Similarly, for the magnetic moment instead of (2.8) we obtain

$$\mu_{\text{orb}} = \sqrt{l(l+1)}\,\frac{e\hbar}{2mc} = \sqrt{l(l+1)}\,\mu_{\text{B}} \tag{2.11}$$

The gyromagnetic ratio g_{obr}, on the other hand, remains the same as specified in (2.9).

However, it follows from (2.1), (2.10a), and (2.11) that there may be stationary atomic states with $l = 0$ which

* See, for example, [571].

** E.g. the hydrogen atom or the hydrogen-like ions He^+, Li^{2+}, Be^{3+}, etc.

correspond to the zero values of the mechanical angular momentum l and the magnetic moment μ_{orb}. These states with the "static" distribution of the charge density of the electron cloud are called the s-states. The states with $l = 1, 2, 3, \ldots$ are denoted respectively as p-, d-, f-, ... states.

From the equations of quantum mechanics it follows without any additional assumptions that the values of the projection l_z of the mechanical angular-momentum vector $\boldsymbol{l}$ (in units of $\hbar$) on the direction of an external magnetic field $\boldsymbol{H}$ (the z-axis) must form a discrete set (*space quantization*):

$$l_z = m_l \hbar \tag{2.12}$$

The magnitude of the possible projections of vector $\boldsymbol{l}$ is given by the *magnetic orbital quantum number* m_l which for a given l takes on $2l + 1$ values:

$$m_l = -l, \quad -(l-1), \ldots, \quad -1, 0, 1, \ldots, l-1, l \tag{2.12a}$$

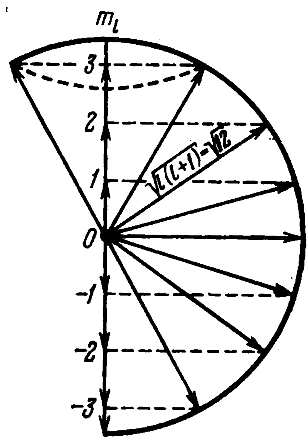

Fig. 2.2. Space quantization (and precession) of the orbital mechanical angular momentum of the electron (for $l = 3$).

Figure 2.2 provides a graphical illustration of the space quantization of the orbital angular momentum for the f-state ($l = 3$).

The same rule of space quantization applies to magnetic moment whose projections in units of μ_B are given by the quantum number m_l, i.e.,

$$(\mu_{\text{orb}})_H = m_l \mu_B \tag{2.13}$$

Thus, in quantum mechanics, as in the quasi-classical method, the projections of spin and orbital magnetic moment are multiples of the Bohr magneton. This is not true, however, as far as the absolute values of the vectors μ_{sp} and μ_{obr} are concerned. The reason is the appearance of the factors $\sqrt{s(s+1)}$ in (1.7) and $\sqrt{l(l+1)}$ in (2.11).

Quantum mechanics does not make it possible to determine the vectors of mechanical angular momentum or magnetic moment, that is specify the absolute values and directions simultaneously. The possibility of simultaneous determination exists only for the absolute value of the vector and of any of its projections on the direction of some external field (the field of quantization). It does not make sense to talk about the two other components of this vector. This conclusion follows directly from the uncertainty relation in quantum mechanics.

To obtain a pictorial classical interpretation of the uncertainty relation for the vectors of mechanical angular momentum and magnetic moment, we can say that these vectors can be defined to an accuracy of the *precession* around the direction of a magnetic field while the time average of their projections onto a plane normal to the field is zero.

2. Orbital and Spin Magnetism of the Shell of a Multi-Electron Atom

2.1. The Vector Model of the Electron Shell

a) Summation Rules for Angular Momentum. In the case of the shell of a multi-electron atom (see, for example, [116, 550, 571, 817]) described in terms of the centrally symmetric self-consistent field approximation we can preserve the same quantum characteristics of states of individual electrons as in the one-electron atom, i.e., the quantum numbers n, l, m_l, m_s. The quantum state of a multi-electron atom is defined, first of all, by its electron configuration, in other words, by specification of the numbers of electrons with given n and l. According to Pauli's principle, there can be no more than $2(2l + 1)$ electrons in each equivalent state (with given n and l); when this number is reached, we obtain a *filled electron shell* $nl^{2(2l+1)}$. Table 2.1 gives a scheme of the sequential filling of the electron shells in atoms.

For a complete description of the quantum state of the shell in a multi-electron atom it is necessary to specify, in addition to electron configuration, the total momenta: the total orbital angular momentum $\boldsymbol{L}$ and the total spin $\boldsymbol{S}$.

Table 2.1

Sequential Filling of Electron Shells in Atoms

n	Configuration with given n and l							Total number of electrons in the shell	Shell symbol
	s $(l=0)$	p $(l=1)$	d $(l=2)$	f $(l=3)$	g $(l=4)$	h $(l=5)$	k $(l=6)$		
1	$1s^2$							2	K
2	$2s^2$	$2p^6$						8	L
3	$3s^2$	$3p^6$	$3d^{10}$					18	M
4	$4s^2$	$4p^6$	$4d^{10}$	$4f^{14}$				32	N
5	$5s^2$	$5p^6$	$5d^{10}$	$5f^{14}$	$5g^{18}$			50	O
6	$6s^2$	$6p^6$	$6d^{10}$	$6f^{14}$	$6g^{18}$	$6h^{22}$		72	P
7	$7s^2$	$7p^6$	$7d^{10}$	$7f^{14}$	$7g^{18}$	$7h^{22}$	$7k^{26}$	98	Q

For example, in the case of two electrons with orbital quantum numbers l_1 and l_2 there is a set of possible values for the orbital quantum number L corresponding to the total orbital angular momentum $\boldsymbol{L}$:

$$L = l_1 + l_2,\ l_1 + l_2 - 1,\ \ldots,\ l_1 - l_2\ (l_1 \geqslant l_2) \tag{2.14}$$

The absolute values of the vector $\boldsymbol{L}$ and the vector of the total orbital *magnetic* moment $\boldsymbol{\mu}_L$ ar

$$|\boldsymbol{L}| = \sqrt{L(L+1)}\,\hbar \text{ and } |\boldsymbol{\mu}_L| = \sqrt{L(L+1)}\,\mu_B \tag{2.15}$$

respectively.

Quantization rules for the projections of these vectors on the direction of an external field $\boldsymbol{H}$ are the same as in the case of a single electron. In units of $\hbar$ the $(2L+1)$ values of the projections are given by the total magnetic orbital quantum number

$$m_L = -L,\quad -(L-1),\ \ldots,\ (L-1),\ L$$

Similar summation rules hold for the total spin $\boldsymbol{S}$ and the corresponding magnetic moment $\boldsymbol{\mu}_S$, the absolute values of which are equal to

$$|\boldsymbol{S}| = \sqrt{S(S+1)}\,\hbar \quad \text{and} \quad |\boldsymbol{\mu}_S| = 2\sqrt{S(S+1)}\,\mu_B \tag{2.16}$$

As for the projections of these vectors on the direction of the magnetic field $\boldsymbol{H}$, their $2S + 1$ values are also proportional to $\hbar$ and μ_B and are given by the total spin magnetic quantum number

$$m_S = -S, \quad -(S-1), \ldots, (S-1), S$$

The total angular momentum $\boldsymbol{J}$ of an atomic electron shell is equal to the vector sum of the total orbital angular momentum $\boldsymbol{L}$ and the total spin $\boldsymbol{S}$:

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S} \tag{2.17}$$

This is the Russell-Saunders rule for the coupling of angular momenta (see the books of Blokhintsev [116], Condon and Shortley [223], Herzberg [454], Shpol'sky [842], and Sommerfeld [870]).

If $L \geqslant S$, then for the total angular-momentum quantum number J we have the following set of possible values:

$$J = L + S,\ L + S - 1, \ldots, L - S + 1,\ L - S \tag{2.17a}$$

$2S + 1$ values in all;
if $L < S$, then J attains the following values:

$$J = S + L,\ S + L - 1, \ldots, S - L + 1,\ S - L \tag{2.17b}$$

$2L + 1$ values in all.
For the magnitude of vector $\boldsymbol{J}$ we obtain

$$|\boldsymbol{J}| = \sqrt{J(J+1)}\,\hbar. \tag{2.18}$$

The situation with the projections of vector $\boldsymbol{J}$ on the direction of an external field is the same as that with the projections of vectors $\boldsymbol{L}$ and $\boldsymbol{S}$: they are also multiples of $\hbar$ and are given by the total magnetic quantum number m_J acquiring $2J + 1$ different values:

$$m_J = -J, \quad -(J-1), \ldots, (J-1), J \tag{2.18a}$$

The direction of vector $\boldsymbol{J}$ is specified by

$$\cos(\widehat{\boldsymbol{J}, \boldsymbol{H}}) = \frac{m_J}{\sqrt{J(J+1)}} \tag{2.18b}$$

In a one-electron atom the total angular momentum is denoted by $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$. If $l = 0$, there is only one value of the total angular-momentum quantum number j: $j = s = 1/2$; if $l > 0$, we have two values of j: $j = l + 1/2$, $l - 1/2$. Thus, the set of possible values of j is that of half-

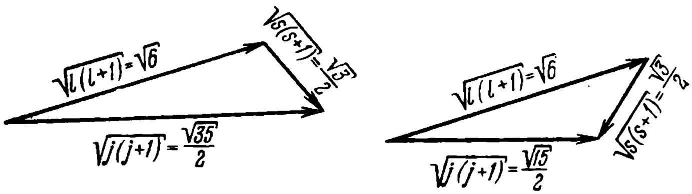

Fig. 2.3. Summation of orbital (l) and spin (s) mechanical angular momenta of the electron in an atom (for $l = 2$ and $s = 1/2$).

integers: 1/2, 3/2, 5/2, etc. Figure 2.3 illustrates the summation rule for vectors $\boldsymbol{l}$ and $\boldsymbol{s}$ when $l = 2$, $s = 1/2$ and $j = 3/2, 5/2$.

b) LS- and jj-coupling. According to the above approximate rules of vector summation of spins and angular momenta in an atomic shell, this summation is done in the following order: first, orbital angular momenta and spins of individual electrons are added up to form vectors $\boldsymbol{L}$ and $\boldsymbol{S}$ respectively; in the next step the two are added together to form the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. Such summation corresponds to the *LS-coupling of Russell-Saunders* already mentioned above, or the *normal coupling*. It is an approximation valid when *the magnitude of the electrostatic interaction between the electrons of an atomic shell is much more than the magnetic* (spin-orbit) *interaction**. In this situation the energy intervals between the states of the electron shell with different $\boldsymbol{L}$ and $\boldsymbol{S}$ considerably exceed the energy intervals for the states with given $\boldsymbol{L}$ and $\boldsymbol{S}$ but

* Strictly speaking, we refer to the so-called residual electrostatic interaction the operator of which is given by the difference between the energy of total interaction $\sum\limits_k e^2/r_{ik}$ and the centrally symmetric self-consistent potential $V(r_i)$ acting on the i-th electron. Summation in the expression for the total interaction is over all values of k except $k = i$.

with different values of $\boldsymbol{J}$, in other words, with different mutual orientations of vectors $\boldsymbol{L}$ and $\boldsymbol{S}$ of specified magnitude. These adjacent levels (i.e. with given L and S but with different J) form the *fine structure* of a multiplet in the energy spectrum of an atomic shell (see Chapter 1 and also [223]).

Thus, in the case of Russell-Saunders LS-coupling we have, in addition to the total angular momentum $\boldsymbol{J}$ of all the electrons, the following integrals of motion: (1) their total orbital angular momentum with the operator

$$\hat{\boldsymbol{L}} = \sum_k \hat{\boldsymbol{l}}_k$$

and (2) the total spin with the operator

$$\hat{\boldsymbol{S}} = \sum_k \hat{\boldsymbol{s}}_k$$

The sums correspond to vector summation.

In the LS-coupling scheme the atomic states are characterized by four quantum numbers: L, S, m_L, m_S or L, S, J, m_J, while the energy of the states depends only on L and S.

The magnitude of relativistic spin-orbit interaction is a function of the angle between vectors $\boldsymbol{L}$ and $\boldsymbol{S}$. In atomic shells of heavy elements the magnetic energy of spin-orbit coupling may exceed the energy of residual electrostatic interaction since the electron velocities in the inner shells of these atoms are close to the velocity of light, and therefore relativistic magnetic effects are very significant. In this case another approximation is valid: vectors $\boldsymbol{l}_k$ and $\boldsymbol{s}_k$ of individual electrons are coupled in the first stage not with the same vectors of other electrons but between themselves, to form the vector of the total angular momentum of a given electron $\boldsymbol{j}_k = \boldsymbol{l}_k + \boldsymbol{s}_k$. It is only after this stage that the summation of individual vectors $\boldsymbol{j}_k$ is performed to obtain the total angular momentum $\boldsymbol{J}$. This approximation is called *jj-coupling** (see [223]).

We can depict the above rules in the following way. In the case of the LS-coupling the angular velocity of the pre-

* In real atoms we do not come across jj-coupling in its pure form. Actually one usually deals with a case of intermediate coupling when the magnitude of the energy of residual interaction is comparable to that of spin-orbit energy.

cession of vectors $\boldsymbol{L}$ and $\boldsymbol{S}$ around the resultant vector $\boldsymbol{J}$ (see, for instance, Figs. 2.2 and 2.4) is small. Therefore the quantities $\boldsymbol{L}$ and $\boldsymbol{S}$ to a high degree of approximation are conserved even when spin-orbit interaction is taken into account. With jj-coupling $\boldsymbol{L}$ and $\boldsymbol{S}$ can no longer be interpreted as integrals of motion even in the approximate

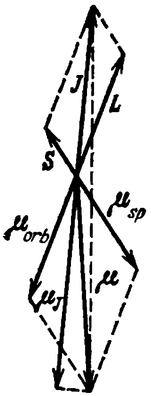

Fig. 2.4. Summation of mechanical angular (spin $\boldsymbol{S}$ and orbital $\boldsymbol{L}$) moments and magnetic ($\boldsymbol{\mu}_{sp}$ and $\boldsymbol{\mu}_{orb}$) moments of the atomic electron shell. One can see from the diagram that because of the negative sign of the electron charge ($e < 0$) the magnetic moments $\boldsymbol{\mu}_{sp}$ and $\boldsymbol{\mu}_{orb}$ are antiparallel with respect to the corresponding mechanical angular momenta, and due to the gyromagnetic anomaly of the spin the vector of the total magnetic moment $\boldsymbol{\mu}$ forms with the total angular momentum $\boldsymbol{J}$ an angle smaller than 180°. The notation $\boldsymbol{\mu}_J$ stands for the projection of the total magnetic moment on the direction of vector $\boldsymbol{J}$.

sense, and the only meaningful vector quantity is that of total angular momentum $\boldsymbol{J}$, which is conserved and is given by the true quantum number of the total angular momentum J.

c) Multiplicity of levels' degeneracy and fine-structure multiplets. As has been noted, in order to achieve a complete description of the quantum states of the electron shell in a multi-electron atom it is necessary to specify, in addition to the electron configuration, the spin and the orbital angular momentum of the shell*. States with identical configurations but with different $\boldsymbol{L}$ and $\boldsymbol{S}$ have different energies because of the electrostatic interaction between the electrons. Such an energy difference usually falls in the interval

* In addition, one should also know the *parity* of states (see, for example, Sec. 31 in [550]). It will be recalled that a state is even if its wave function does not change sign in the coordinates transformation $x = -x'$, $y = -y'$, $z = -z'$; it is odd when the sign changes. In a multi-electron atom the parity of the wave function is determined by that of the sum $\sum_k \boldsymbol{l}_k$.

0.1-1 eV. As a rule, this is several times less than the energy interval between levels corresponding to different electron configurations (these energy intervals are of the order of several eV). If in the atomic shell there were no other types of interaction (besides electrostatic), the multiplicity of degeneracy of each atomic level with given $\boldsymbol{L}$ and $\boldsymbol{S}$ would be $(2L+1)(2S+1)$ in accordance with the possible spatial orientations of these vectors.

However, if the magnetic interaction between electrons (spin-orbit and the like) in the atom is small compared to the electrostatic interaction (which is always the case except in the above-mentioned jj-coupling in the atoms of heavy elements of the Periodic Table), it may cause a splitting of the $(2L+1)(2S+1)$-multiple degenerate level with given L and S into a set (a fine-structure multiplet) of closely located energy levels with different values of the total angular momentum J. The number of levels in such a multiplet, according to (2.17a) or (2.17b), is equal to $2S+1$ or $2L+1$ respectively*.

Each of the levels of the multiplet with a given J is degenerate with respect to the direction of vector $\boldsymbol{J}$. The multiplicity of this degeneracy is $2J+1$. The sum of numbers $2J+1$ with all possible values of J for given L and S is equal to

$$\sum_{L+S}^{|L-S|}(2J+1)$$

$$=\begin{cases}\frac{1}{2}(2L+2S+1+2L-2S+1)(2S+1) & \text{for } L\geqslant S\\ \frac{1}{2}(2S+2L+1-2L+2S+1)(2L+1) & \text{for } S>L\end{cases}$$

* The order of magnitude of the energy splitting in such a multiplet can be found from the classical formula for the energy of interaction of two magnetic dipoles with moments μ, located at a distance r; the magnitude of this energy is μ^2/r^3. According to (1.2a) $\mu\cong10^{-20}$ emu, and the average distance $r\cong0.5\times10^{-8}$ cm. Thus $\mu^2/r^3\cong$ $\cong(10^{-40}\times10)/10^{-24}=10^{-15}$ erg $\cong0.001$ eV. These energy values are indeed much smaller than those from the electrostatic energy interval 0.1-1 eV, which is true practically for all atoms except those of heavy elements, where because of large values of μ and small effective radii the "magnetic" energies can be of the same order of magnitude as the electrostatic ones.

i.e. coincides with the mentioned above total multiplicity of degeneracy $(2L + 1)(2S + 1)$.

In conclusion it may be said that the vector model used here is only a way of describing rigorous quantum-mechanical results, based on a purely pictorial analogy.

2.2. Hund Rules

A sequence of levels with *identical* configuration but with different L and S can be determined from a well-known empirical *rule of highest multiplicity* established by Hund [476], according to which the term with the lowest energy (for a given configuration) has the greatest value of the total spin S and the greatest (for this value of S) total orbital angular momentum L.

This is the so-called *first* Hund rule. There is also a *second* Hund rule: if L and S are not equal to zero and if in a shell with the given values of l and n there is less than half of the maximal possible number of electrons ($< 2l + 1$), the lowest energy corresponds to the level of the multiplet with $J = |L - S|$; if the number of electrons is greater than $2l + 1$, it corresponds to the level with $J = L + S$.

For a more detailed discussion of Hund rules see [454, 476, 513, 519, 546, 631, 851] and also the text after formula (2.61).

Hund rules can be also formulated in the following way:

1a) In the ground state the total spin magnetic quantum number $m_S = \sum\limits_k (m_s)_k$ is maximal under limitations of Pauli's principle.

1b) The total magnetic orbital quantum number $m_L = \sum\limits_k (m_l)_k$ in the ground state is maximal under limitations of the rule (1a).

2) The total angular-momentum quantum number J in the case of an incompletely populated shell is given by the expressions:

$J = L - S$, if less than one half of the shell is populated,

$J = L + S$, if more than one half of the shell is populated.

Let us consider the values of L and S that arise in the filling of the electron shells for the first two elements in the Periodic Table. In a helium atom with two electrons in the shell the total spin and orbital angular momentum of the ground state are equal to zero: $S_{\mathrm{He}} = L_{\mathrm{He}} = 0$. If two more electrons are added, i.e. in the table of elements we skip Li with $Z = 3$ and consider Be with $Z = 4$, the ground state again has a magnetically neutral shell with $S_{\mathrm{Be}} = L_{\mathrm{Be}} = 0$.

However, if we add two more electrons, in other words, consider a carbon atom C ($Z = 6$) with six electrons, the filling is different. The ground state of the C atom with three pairs of electrons has $S_{\mathrm{C}} = 1$ and $L_{\mathrm{C}} = 1$. Thus, for example, if we analyse the first 18 elements of the Periodic Table with even numbers of electrons (up to the element krypton with $Z = 36$), only eight of them will be magnetically neutral (with $S = L = 0$) in the ground state. Out of these four are noble gases: He, Ar, Ne, Kr; four others have filled valence shells Be—$2s^2$, Mg—$3s^2$, Ca—$3p^6 4s^2$, and Zn—$3d^{10} 4s^2$. The ground state of every one of these eight atoms is ${}^1S_0$.

For spectral terms the following notations are adopted: the main symbol is the total orbital quantum number L in the form of a letter S, P, D, F, G, H, etc. which corresponds to $L = 0, 1, 2, 3, 4, 5,$ etc. The value of the left-hand superscript is $2S + 1$ which corresponds to the number of states in a multiplet with given J (for $S < L$), the right-hand subscript is J, and the right-hand superscript indicates the parity of the state (g for even, u for odd). For instance, the notation for the principal term of the iron atom is ${}^5D_4^{g,u}$ which means that $L = 2$, $J = 4$, $S = 2$.

As another example, let us consider how one should apply the Hund rules to determine the principal terms of some ions. In the case of a Fe^{2+} ion with the unfilled $3d$-shell having six electrons we obtain

$$S = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = 2$$

$$L = 2 + 1 + 0 - 1 - 2 + 2 = 2, \quad J = 2 + 2 = 4$$

and therefore, the principal term will be $^5D_4$. In the ion Eu^{2+} there are seven electrons in the unfilled $4f$-shell:

$$S=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{7}{2}$$

$$L=3+2+1+0-1-2-3=0,\ J=\frac{7}{2}$$

and the principal term is $^8S_{7/2}$.

In a qualitative sense this rule follows from the requirement of a minimum of energy of the electrostatic interaction between electrons in an atom, namely, of the *exchange* part of this interaction (of course, one must take into account the Pauli principle). Hund rules are of considerable importance in determining the magnetic characteristics of atomic electron shells.

2.3. Filling of Atomic Electron Shells: Normal and Transition Elements

Before listing the values of magnetic moments for atoms of all the elements in the Periodic Table, let us recall that the sequence of configurations given in Table 2.1, that arises in the filling of electron shells, is in fact violated beginning with potassium (K; $Z = 19$). In the atom of potassium and the next element calcium (Ca; $Z = 20$) we observe the filling of the $4s$-states instead of the $3d$-states which follow "in order" after the $3p$-states are filled in argon (Ar; $Z = 18$). It is only in scandium (Sc; $Z = 21$) that we find the late filling of the ten-place $3d$-shell completed in copper (Cu; $Z = 29$).

In filling of the $3d$-shell (the *group of iron*) there are other violations of the simple sequence. For example, in the chromium atom (Cr; $Z = 24$) we find the configuration $3d^54s$ instead of the "regular" configuration $3d^44s^2$, while the configuration $3d^84s^2$ of the nickel atom (Ni; $Z = 28$) is replaced in the atom of copper by the configuration $3d^{10}4s$ and not by the configuration $3d^94s^2$. The late filling of the $4d$-shell is found in the elements from yttrium (Y; $Z = 39$) to palladium (Pd; $Z = 46$) (the *group of palladium*), that of the $4f$-shell in elements from lanthanum (La; $Z = 57$) to ytterbium (Yb; $Z = 70$) (the *group of rare earths* or lanthanides), that of the $5d$-shell from lutetium (Lu; $Z = 71$) to platinum (Pt; $Z = 78$) (the *group of platinum*), and, finally, of the shells $6d$ and $5f$ from actinium (Ac; $Z = 89$) to uranium (U; $Z = 92$) and also transuranium elements (the *group of actinides*) (see Table I in Appendix).

Thus, it is in the filling of the d- and f-shells that we observe the violation of the normal filling sequence. Elements in which the completion of these shells is "late" are called *transition elements*. Altogether there are 42 transition elements in the Periodic Table (excluding transuranium elements); out of these 24 are d-metals from the group of iron, palladium, and platinum, 14 are the rare earths*, and 4 are actinides. Table I in the Appendix contains certain data concerning the atomic electron shells of transition elements.

The physical cause of the existence of vacant inner shells in a multi-electron atom is the fact that the electron energy in these shells depends essentially not only on quantum number n but also on l. For a given n energy increases with the increase of l. The greater the difference between the self-consistent field of the shell of a multi-electron atom and the Coulomb field of the atom of hydrogen, the stronger is the dependence of the electron energy on l. Therefore, from the point of view of energy it may be more advantageous if in transition from element Z to element $Z + 1$ the added electron is characterized not by a greater value of l (consistent with a given n) but by an increased n and at the same time by a decreased l. For example, the states $(n + 1)\, s$ and $(n + 1)\, p$ may have lower energy than states nd or nf.

Making use of the approximate statistical method of Thomas-Fermi, we can quantitatively predict the minimal atomic number $[Z(l)]_{\min}$, which marks the beginning of the filling of a shell with a given l (see, for example, [401, 550, 571]). A simple calculation shows that $[Z(p)]_{\min} = 5$, $[Z(d)]_{\min} = 21$, $[Z(f)]_{\min} = 58$, etc., which is in excellent agreement with experiment.

Table I contains, in addition to electron configurations, the spectral terms of the ground state, Lande factors g_J,

* Strictly speaking, the number of elements with an incomplete $4f$-shell is equal to 12 (see Table Id) from Ce ($Z = 58$) to Tm ($Z = 69$), but usually La ($Z = 57$) and Yb ($Z = 70$) are added to this group.

and the maximal values of the projections of the total magnetic moment for transition elements.

Figure 2.5 presents a diagram for the maximal projection of the total magnetic moment of atoms as a function of Z. As follows from this diagram, the general periodic structure

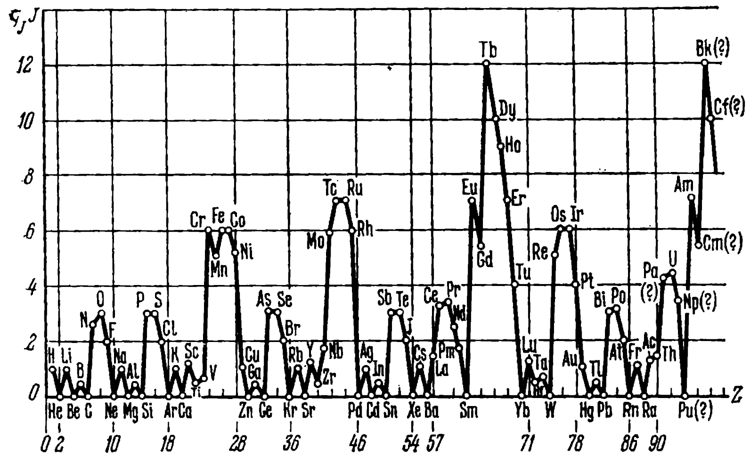

Fig. 2.5. The maximal projection of the total magnetic moment of the atomic electron shell $g_J J$ (in units of μ_B) as a function of the ordinal number Z in the Periodic Table.

of this dependence is influenced by certain "bursts" corresponding to groups of transition elements having the greatest values of magnetic moments.

3. Determination of the Total Magnetic Moment and the Atomic Lande Factor

Let us now consider the question of the determination of the magnetic moment of the atom.

The resultant magnetic moment μ of the atomic electron shell [233, 454], due to the gyromagnetic anomaly of the spin ($g_{sp} = 2g_{orb}$), is not directed along the total angular momentum $\boldsymbol{J}$. Figure 2.4 provides a graphical illustration of this statement. The scale is chosen in such a way that the length of vector μ_{orb} is equal to that of vector $\boldsymbol{L}$; therefore the length of vector μ_{sp} should be equal to the doubled length of vector $\boldsymbol{S}$. Since the electron is negatively charged, the directions of μ_{orb} and μ_{sp} are antiparallel to those of $\boldsymbol{L}$ and $\boldsymbol{S}$, respectively. The total magnetic moment μ forms with vector $\boldsymbol{J}$ an angle not equal to 180°. In classical terms we can say that because vectors $\boldsymbol{L}$ and $\boldsymbol{S}$ precess around the direction of vector $\boldsymbol{J}$, vectors μ_{orb} and μ_{sp} will also precess around $\boldsymbol{J}$. If each of these vectors is resolved into two components, one parallel to $\boldsymbol{J}$ and the other normal to it, the time average (over the period of precession) of the values of the normal components $(\mu_{orb})_\perp$ and $(\mu_{sp})_\perp$ will be equal to zero, the reason for this being that these vectors continuously change their direction. Therefore, the effective magnetic moment of the atomic electron shell will equal the sum of the projections $(\mu_{orb})_\parallel$ and $(\mu_{sp})_\parallel$ on the direction of vector $\boldsymbol{J}$, i.e.

$$\mu_J = \mu_{orb} \cos(\widehat{\boldsymbol{L}, \boldsymbol{J}}) + \mu_{sp} \cos(\widehat{\boldsymbol{S}, \boldsymbol{J}}) \tag{2.19}$$

Applying usual trigonometric formulas to the triangle formed by the vectors $\boldsymbol{L}$, $\boldsymbol{S}$, and $\boldsymbol{J}$, we obtain

$$\begin{aligned} \cos(\widehat{\boldsymbol{L}, \boldsymbol{J}}) &= \frac{L(L+1)+J(J+1)-S(S+1)}{2\sqrt{L(L+1)}\sqrt{J(J+1)}}, \\ \cos(\widehat{\boldsymbol{S}, \boldsymbol{J}}) &= \frac{S(S+1)+J(J+1)-L(L+1)}{2\sqrt{S(S+1)}\sqrt{J(J+1)}} \end{aligned} \tag{2.19a}$$

Substituting into (2.19) these values of cosines and of the values of μ_{orb} from (2.15) and μ_{sp} from (2.16), we find

$$\begin{aligned} \mu_J &= \left[1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}\right] \sqrt{J(J+1)}\, \mu_B \\ &= g_J \sqrt{J(J+1)}\, \mu_B \end{aligned} \tag{2.20}$$

where

$$\begin{aligned} g_J &= 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)} \\ &= g_{orb}\alpha_L + g_{sp}\alpha_S \end{aligned} \tag{2.21}$$

is the *Lande factor* of the electron shell,

$$\alpha_L = \frac{[J(J+1)+L(L+1)-S(S+1)]}{2J(J+1)},$$
$$\alpha_S = \frac{[J(J+1)+S(S+1)-L(L+1)]}{2J(J+1)} \qquad (2.21a)$$

If $L = 0$, then $J = S$ and $g_J = g_{sp} = 2$, i.e. in the case of a spin the Lande factor, as we have seen, is equal to two; if, on the contrary, $S = 0$, then $J = L$ and $g_J = g_{orb} = 1$, i.e. for a purely orbital angular momentum the Lande factor is equal to unity. Thus, the Lande factor for a spin is twice as big as for an orbital angular momentum, which is in complete agreement with (2.9a) and is a direct consequence of the gyromagnetic anomaly of spin.

As follows from (2.21), the magnitude of the Lande factor varies in certain limits for the various components of a given multiplet; the limits for given L and S correspond to the extremal values of the quantum number J: if $L > S$, then $J = L \pm S$:

$$1 + \frac{S}{L+S} \geqslant g_J \geqslant 1 - \frac{S}{L-S+1} \qquad (2.21b)$$

if $L < S$, the extremal values of J are $S \pm L$:

$$1 + \frac{S}{L+S} \geqslant g_J \geqslant \frac{S+1}{S-L+1} \qquad (2.21c)$$

It is important to note that when $L = S$, there may be two cases: if $J \neq 0$, it follows directly from (2.21) that

$$g_J(L=S) = \frac{3}{2} \qquad (2.21d)$$

but if $J = 0$, then the Lande factor g_J becomes indefinite (0/0). The magnetic moment μ_J, however, in this case is zero since $J = 0$. On the other hand, for some atomic energy levels (for example, ${}^4D_{1/2}$, ${}^5F_1$, ${}^6G_{3/2}$) the Lande factor is zero, and therefore $\mu_J = 0$ although $J \neq 0$. In an external magnetic field the total magnetic moment of an atom, as well as its total angular momentum, can have $2J + 1$ possible projections. The components of the magnetic moment in the direction of the field are equal to

$$\mu_J = m_J g_J \mu_B \qquad (2.20a)$$

where m_J is given by equation (2.18a). Often the magnetic moment of the atom is defined not as the projection of vector $\boldsymbol{\mu}$ on the direction of vector $\boldsymbol{J}$, but as the maximal positive value of the projection ($|(m_j)_{\max}| = J$) on the direction of the magnetic field, that is, the value $Jg_J\mu_B$ (Fig. 2.5).

Here we are not going to discuss the general theory of summation of angular momenta in the multi-electron atom. Exhaustive treatment of this topic can be found in the fundamental monographs on atomic spectra (see [223, 870] and especially [858], and also the monograph on the theory of ions of transition elements of Griffith [414] and the book of Edmonds [286]).

4. Interaction of Electron Magnetic Moments with Each Other and with External Magnetic Fields

Once we have considered the basic magnetic characteristics of the electron (free and in an atomic shell), i.e. its orbital and spin magnetic moments, we face a natural question. How do these characteristics reveal themselves? Obviously, the magnetic properties of electrons, as has already been mentioned, manifest themselves during the magnetic interaction of electrons (spin-spin, spin-orbit, and orbit-orbit) inside the atomic shell. Besides, there is the possibility of magnetic interaction between the electrons of the atomic shell and the magnetic moment of the atomic nuclei (see Chapter 3). Finally, there is also the magnetic interaction of the electrons of the atomic shell with the magnetic fields produced by the electron shells, by atomic nuclei of surrounding atoms or, in general, by other sources of external fields (for instance, by a solenoid with electric current flowing in its coil, by an electromagnet or by a permanent magnet).

4.1. Zeeman Effect

a) Definition of the Effect. One of the most direct manifestations of magnetic properties of an atom is the Zeeman effect [993], consisting of the splitting of lines in atomic spectra when the emitting atoms are placed in an external magnetic field (see [116, 126, 454, 571, 817, 842, 843]).

In a laboratory reproduction of this phenomenon the light source (sodium flame, mercury arc, etc.) is located between the poles of an electromagnet while the radiation itself is directed into a spectroscope of high resolving power (Fig. 2.6). With this setup, one can analyse the spectral composition of light emitted both parallel to the magnetic

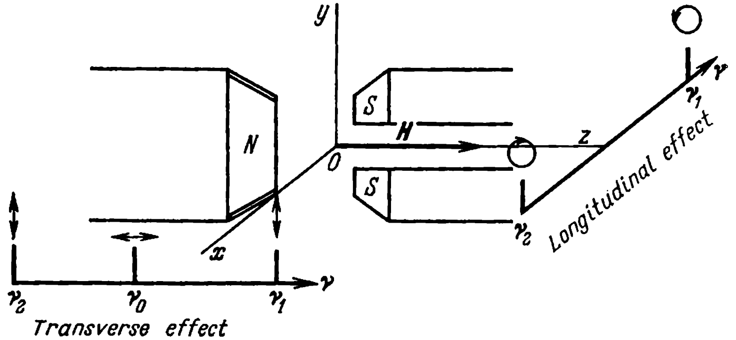

Fig. 2.6. A schematic representation of the experiment for the observation of the transverse and the longitudinal Zeeman effect. N and S are the magnet poles, $\boldsymbol{H}$ is the vector of the magnetic field, ν_0 is the frequency of the main spectral line, and ν_1 and ν_2 are the frequencies of the shifted spectral lines.

field (the *longitudinal effect*) and normal to it (the *transverse effect*). If the field is strong (see below), we observe the *normal Zeeman effect* (or the *Paschen-Back effect*). In the normal longitudinal effect, instead of the single spectral line of frequency ν_0 observed in the absence of the field (a line is not polarized, as a rule), we find two lines symmetrically displaced with respect to ν_0: one with a lower frequency $\nu_2 < \nu_0$, and one with a higher frequency $\nu_1 > \nu_0$, such that

$$\nu_0 - \nu_2 = \nu_1 - \nu_0$$

Both these lines are circularly polarized (see Fig. 2.6). In the normal transverse effect we observe three lines: the unshifted line ν_0 linearly polarized in the direction of the field and two lines ν_1 and ν_2 displaced in the same way as those in the longitudinal effect but with different polarizations (in contrast to the preceding case, these two are linearly polarized in the direction normal to that of the magnetic field (see Fig. 2.6)).

In the case of a weak field this phenomenon becomes much more complex: additional lines appear. Explanation of this *anomalous Zeeman effect* will be given below.

b) A classical theory of the normal Zeeman effect; the Larmor frequency. The normal effect can be explained with sufficient completeness on the basis of the elementary classical electron theory of Lorentz [608]. In order to simplify calculations, let us assume that the electron in the atom of hydrogen, for instance, moves with angular velocity ω_0 in a circular orbit of radius r, whose plane is normal to the direction of the external magnetic field $\boldsymbol{H}$. In the absence of a magnetic field the electron is subject to the effect of a centripetal Coulomb force e^2/r^2; its equation of motion has the form

$$m\omega_0^2 r = \frac{e^2}{r^2} \tag{2.22}$$

whence

$$\omega_0 = \left(\frac{e^2}{mr^3}\right)^{1/2} \tag{2.22a}$$

If the magnetic field is switched on, the magnetic flux through the area of the orbit will experience temporal variation in the process. This in turn will generate an additional induced electric field tangential to the orbit. The additional field will change the velocity of the electron moving in its orbit. At the same time, since the electron is moving in magnetic field H, it will be affected by the Lorentz force $\boldsymbol{F}_H$ ditected along the radius:

$$\boldsymbol{F}_H = \frac{e}{c}\,[\boldsymbol{vH}] \tag{2.23}$$

The magnitude and direction of $\boldsymbol{F}_H$ will be such that the radius of the orbit will stay constant. Therefore, the switching on of the magnetic field will result only in an increase or decrease of the angular velocity of the electron, depending on the direction of its motion relative to that of the magnetic field.

If the new value of the angular velocity of the electron is ω, then according to (2.23) the additional radial Lorentz force due to the magnetic field is equal to

$$F_H = \pm \frac{1}{c} He\omega r$$

The two signs correspond to the two possible antiparallel orientations of the normal vector of the plane of the electron orbit, the electron having two opposite directions of its orbital motion with respect to the given direction of vector $\boldsymbol{H}$.

In this case, instead of (2.22), we have the following equation of motion*:

$$m\omega^2 r = m\omega_0^2 r \pm \frac{1}{c} He\omega r \tag{2.24}$$

From this equation we can easily find ω. Since in the atom $\omega_0 \cong 10^{16}$ s^{-1} and $eH/cm \cong 10^{12}$ s^{-1}, one can approximately write

$$\omega = \omega_0 \pm \frac{eH}{2mc} \tag{2.25}$$

even if H is of the order of the strongest magnetic fields available ($\cong 10^5$ Oe).

The quantity

$$\omega_L = 2\pi\nu_L = \frac{eH}{2mc} \tag{2.26}$$

is called the *Larmor frequency*. It determines the magnitude of the effect of a magnetic field on the orbital motion of the electron in the atom. If the magnetic field $\boldsymbol{H}$ is not normal to the plane of the orbit, the effect of the field is also determined by ω_L, which coincides in the general case with the angular velocity of the Larmor precession of the electron orbit around the direction of the magnetic field (Fig. 2.7).

The change of the electron energy caused by the additional angular velocity is equal to the energy of the interaction of the orbital magnetic moment and the external field, i.e., to the scalar product of these vectors taken with an opposite sign: $-\boldsymbol{\mu}_{\text{orb}}\boldsymbol{H}$. According to (2.11) and (2.13), the absolute value of this energy increment is

$$\Delta\mathscr{E}_H = \mu_{\text{orb}} H \cos\theta = m_l \frac{|e|\hbar}{2mc} H = m_l \hbar |\omega_L| \tag{2.27}$$

* When $\boldsymbol{H}$ is normal to the plane of the orbit.

We shall demonstrate that when the external magnetic field is not very strong this quantity coincides (for a given value of the magnetic moment of the atom) with the increment of the kinetic energy of the electron in the atom, due to the effect produced by the external magnetic field [897]. Indeed, the Larmor precession with an angular frequency

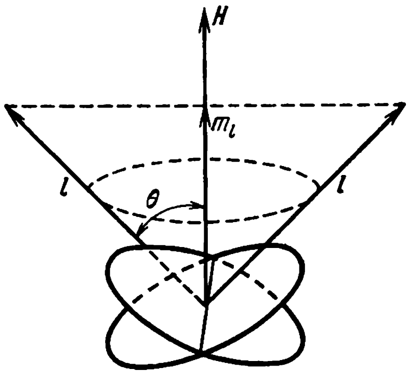

Fig. 2.7. Precession of the electron orbit (l) around the magnetic field ($\boldsymbol{H}$).

ω_L (2.26), caused by the field $\boldsymbol{H}$, increases the linear velocity of the electron in the atom by a quantity $\Delta \boldsymbol{v}_L = [\boldsymbol{\omega}_L \boldsymbol{r}]$, where $\boldsymbol{r}$ is the radius vector of the electron with respect to the atomic nucleus. The corresponding change of the electron kinetic energy is given by

$$\Delta \mathscr{E}_{\text{kin}} = \frac{m}{2} [(\boldsymbol{v}_0 + \Delta \boldsymbol{v}_L)^2 - \boldsymbol{v}_0^2] = m(\boldsymbol{v}_0 \Delta \boldsymbol{v}_L) + \frac{m}{2} (\Delta \boldsymbol{v}_L)^2$$

Since for all available magnetic fields ($H \leqslant 10^5$ Oe) the velocity $\Delta \boldsymbol{v}_L$ is small, the term containing $(\Delta \boldsymbol{v}_L)^2$ can be neglected. Then

$$\Delta \mathscr{E}_{\text{kin}} \cong m(\boldsymbol{v}_0 \Delta \boldsymbol{v}_L) = m(\boldsymbol{v}_0 [\boldsymbol{\omega}_L \boldsymbol{r}]) = m(\boldsymbol{\omega}_L [\boldsymbol{r} \boldsymbol{v}_0]) = \boldsymbol{\omega}_L \boldsymbol{p}_\varphi$$

where $\boldsymbol{p}_\varphi = m [\boldsymbol{r} \boldsymbol{v}_0]$ is the mechanical angular momentum of the electron in the atom (see (2.4)). Substituting the value of the frequency ω_L for (2.26) and taking into account (2.6), we find

$$\Delta\mathscr{E}_{\text{kin}} = -\frac{e}{2mc}(\boldsymbol{p}_\varphi \boldsymbol{H}) = -\mu_{\text{orb}}\boldsymbol{H}$$

Thus, we have shown that the kinetic energy of the electron in the field $\boldsymbol{H}$ is equal to the potential energy of a magnetic dipole with the moment μ_{orb} in the same field (a special case of the equivalence theorem of Ampere).

The effect of the Larmor precession is a particular case of the induction law of Lenz (one can easily see that the additional motion of the electron resulting from the switching on of the magnetic field creates a magnetic field antiparallel to the first one) and forms the basis of a universal phenomenon of *diamagnetism* typical of all atoms*.

Let us explain the *normal* Zeeman effect using the obtained result. Assume that the magnetic field is directed along the z-axis (see Fig. 2.6). In the light source O the atomic orbits have all kinds of orientations. Under the effect of the magnetic field they start precessing around the z-axis. Because of the transverse nature of light waves, the waves propagating in a given direction result only from those components of the electron acceleration which are normal to this direction. Therefore, in longitudinal observation (along the z-axis) we catch only the light produced by the components of electron motion lying in the plane xy, i.e., from projections of electron orbits on this plane. Owing to the chaotic nature of the orientations of electron orbits in the different atoms of the source, half of the number of the projections will be characterized by the angular velocity $+\omega_0$, and the other half by $-\omega_0$. Therefore, when $H \neq 0$, the absolute values of the resultant angular velocities will be equal to $\omega_0 + \omega_L$ and $\omega_0 - \omega_L$, respectively, which means that the linear frequencies of the two shifted spectral lines will be

$$\nu_1 = \nu_0 + \frac{eH}{4\pi mc} = \nu_0 + \Delta\nu, \quad \nu_2 = \nu_0 - \frac{eH}{4\pi mc} = \nu_0 - \Delta\nu \qquad (2.28)$$

---

* It should be noted that the "classical" deduction of equation (2.26) leads to the same result as the rigorous quantum-mechanical approach. This coincidence is due to the fact that the expression for the Larmor frequency does not contain Planks's constant $\hbar$, a characteristic of phenomena of the microworld.

In complete agreement with experiment, these lines are shifted with respect to the original line ν_0 in a symmetrical way (by $\Delta\nu$) and are circularly polarized in the opposite directions (in Fig. 2.6 this is shown by circles).

If we observe in the direction normal to the field (along the x-axis), we see three lines. The undisplaced line ν_0 is produced by the z-component of the electron motion, which the magnetic field does not affect (since the Lorentz force is equal to zero if $\boldsymbol{H} \parallel \boldsymbol{v}$). Therefore the undisplaced line ν_0 will be linearly polarized along the z-axis in agreement with observation.

On the other hand, the y-component of the motion normal to the field $\boldsymbol{H}$ (there is no light generated by the x-component, that propagates along the x-axis) changes its frequency by the quantity ω_L. This results in the appearance of two shifted lines ν_1 and ν_2 linearly polarized in the direction of the y-axis (in Fig. 2.6 it is shown by double arrows).

From comparison of theoretical and experimental values for $\Delta\nu$ one can find the specific charge of the electron $e/m = 1.7588028(54) \times 10^7$ emu [902], which agrees perfectly with the data on the specific charge obtained in experiments on the deflection of cathode rays in electric and magnetic fields. Besides, the direction of the circular polarization of the shifted lines in the longitudinal effect provided the first experimental evidence of the fact that it is the *negative* electric charge in the atom (i.e., the electron) that is responsible for the emission of light.

c) Elementary quantum theory of the anomalous and the normal Zeeman effect. Let us now consider the quantum-mechanical refinement of the explanation of these magnetic effects. We shall first discuss the elementary theory of the *anomalous* effect.

When an atom is placed in a relatively weak magnetic field $\boldsymbol{H}$ (weak in comparison with the internal magnetic fields of the fine structure), this field does not break the coupling between the vectors $\boldsymbol{L}$ and $\boldsymbol{S}$ and we shall observe the precession of the resultant vector $\boldsymbol{J}$ around the direction of the magnetic field. The energy change $\Delta\mathscr{E}_H$ (the *Zeeman splitting interval*) caused by the effect of the magnetic field on the total magnetic moment $\boldsymbol{\mu}_J$, owing to (2.20), will be equal to

$$\Delta\mathscr{E}_H = \mu_J H \cos(\widehat{\boldsymbol{J}, \boldsymbol{H}}) = g_J \mu_B H \sqrt{J(J+1)} \cos(\widehat{\boldsymbol{J}, \boldsymbol{H}}) \quad (2.29)$$

But the projection of vector $\boldsymbol{J}$ on the direction of $\boldsymbol{H}$, i. e. $\sqrt{J(J+1)} \cos(\widehat{\boldsymbol{J}, \boldsymbol{H}})$, according to (2.18b), is equal to the total magnetic quantum number m_J. Thus,

$$\Delta\mathscr{E}_H = m_J g_J \mu_B H \quad (2.30)$$

Therefore, in the external magnetic field each n-th atomic level with the energy $\mathscr{E}_n$ is split into $2J+1$ levels with energies $\mathscr{E}_n + (\Delta\mathscr{E}_H)_n$. The number $2J+1$ specifies the multiplicity of the level (see above). The frequencies of radiation emitted by an atom in a magnetic field will be

$$\nu_{ik} + (\Delta\nu)_{ik} = \{[\mathscr{E}_i + (\Delta\mathscr{E}_H)_i] - [\mathscr{E}_k + (\Delta\mathscr{E}_H)_k]\} h^{-1}$$

Taking into account that $h\nu_{ik} = \mathscr{E}_i - \mathscr{E}_k$ and also equations (2.30), (2.26), and (1.2a), we find

$$(\Delta\nu)_{ik} = (m_J)_i (g_J)_i - (m_J)_k (g_J)_k \quad (2.31)$$

As an illustration let us apply the last formula for the explanation of the anomalous Zeeman effect of the fine structure of the D-line in the spectrum of sodium. Table 2.2 gives the values of L, S, J, g_J, m_J, and $m_J g_J$ for three unperturbed energy levels, the transitions between which correspond to the two D-lines. Figure 2.8 contains a graphic representation of the splitting of the energy levels; it also gives the wavelengths for the components of the doublet ($\lambda_{D_2} = 5890$ Å and $\lambda_{D_1} = 5895.9$ Å) as well as the energy interval of the doublet fine structure $\Delta\mathscr{E}_{FS} = \mathscr{E}_{^2P_{3/2}} - \mathscr{E}_{^2P_{1/2}} \cong 3.4 \times 10^{-15}$ erg. From comparison of this energy difference with the energy $\Delta\mathscr{E}_{D_1}$ or $\Delta\mathscr{E}_{D_2}$ corresponding to the wavelengths of the doublet (for example, with $\Delta\mathscr{E}_{D_2} \cong 3.4 \times 10^{-12}$ erg) one can easily see that the latter (caused by electrostatic interaction between electrons) is three orders of magnitude greater than the energy interval of the fine structure (caused by spin-orbit interaction).

Table 2.2

Data for Computating Pattern Characteristics of the Anomalous Zeeman Effect for the Fundamental Doublet (D-line) in the Atomic Spectrum of Sodium

States	L	S	J	g_J	m_J	$g_J m_J$
$3^2S_{1/2}$	0	1/2	1/2	2	1/2, —1/2	1, —1
$3^2P_{1/2}$	1	1/2	1/2	2/3	1/2, —1/2	1/3, —1/3
$3^2P_{3/2}$	1	1/2	3/2	4/3	3/2, 1/2 —1/2, —3/2	2, 2/3 —2/3, —2

Finally, Fig. 2.8 shows the Zeeman components for the transverse anomalous effect. The lines denoted by σ are polarized normally to the magnetic field, and those denoted by π, parallel to the field. The polarization of these lines can be predicted theoretically. Theory also gives the *selection rules* according to which only part of the transitions between various levels are possible. These rules require that the total magnetic quantum number either remains constant in the transition ($\Delta m_J = 0$, π-components) or changes by unity ($\Delta m_J = \pm 1$, σ-components). In the longitudinal effect only σ-components are left ($\Delta m_J = \pm 1$), and they exhibit circular polarization.

It is necessary to point out that these selection rules are not absolute: they are valid only for the electric dipole radiation of the atom. But in addition to this radiation there is also the quadrupole electric, dipole magnetic, and other types of radiation, which are forbidden by the simplest dipole selection rules (the probability of all magnetic transitions is smaller than that of the corresponding electric transitions by a factor v^2/c^2; for the light elements this ratio is about 10^{-5}). Dipole electric radiation is only the most probable one (the transition rate for this process is about 10^8 s^{-1}) and it produces spectral lines of the highest intensity.

In quantum systems the selection rules, in general, follow from the conservation laws for quantum transitions which means that they can be obtained within the scope of the group-theoretic method. For more details see [223, 454, 858]. Theory shows that each subsequent order of electric or magnetic multiplicity corresponds to the decrease in transition probability by a factor given by the ratio of the square of atomic dimensions (10^{-16} cm^2) to the square of the wavelength of light ($\simeq 10^{-8}$ cm^2), i.e. 10^8 times.

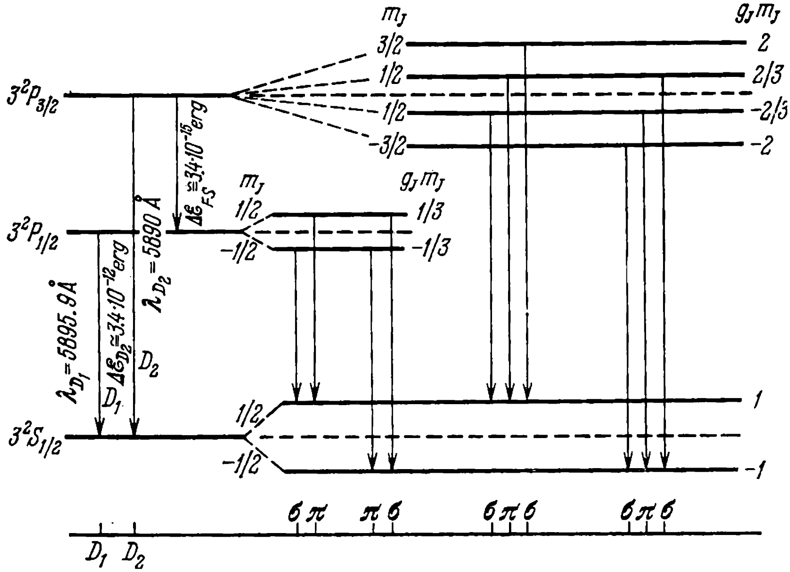

Fig. 2.8. The scheme of levels and transitions in the anomalous Zeeman effect for the fundamental doublet (D-line) of the line spectrum of the sodium atom.

Let us now consider the elementary theory of the *normal* effect. If the external magnetic field becomes *strong* in comparison with the internal fields of fine structure but the splitting of energy levels produced by it is still small compared to the energy intervals between adjacent multiplets, and if this field breaks the coupling between vectors $\boldsymbol{L}$ and $\boldsymbol{S}$, the quantum number J loses its meaning and vectors $\boldsymbol{S}$ and $\boldsymbol{L}$ start precessing independently around the direction of the magnetic field. The magnitude of the projections of these vectors on the direction of the field is given by the orbital and spin magnetic quantum numbers, m_L and m_S. The energy increment in the magnetic field ($g_{\text{orb}} = 1$ and $g_{\text{sp}} = 2$) will be given by the following equation:

$$\Delta\mathscr{E}_H = \Delta\mathscr{E}_L + \Delta\mathscr{E}_S = (m_L + 2m_S)\mu_B H \qquad (2.32a)$$

instead of (2.30).

Taking into account (2.26) and (1.2a), we obtain

$$\Delta\nu = (\Delta m_L + 2\Delta m_S)\,\nu_L \qquad (2.32b)$$

The selection rules in this case are $\Delta m_L = 0, \pm 1$, and $\Delta m_S = 0$. Therefore, according to (2.32b), there are only three possibilities:

$$\Delta\nu = 0, \quad \pm\nu_L$$

which is the normal Zeeman effect.

It should be noted that during the whole course of the development of electron physics, this magneto-optical effect* invariably served as a powerful experimental criterion of the validity of the theoretical interpretations of the Zeeman effect.

4.2. Quantum-Mechanical Theory of the Effect of Magnetic Field on the Atomic Electron Shell

a) The Hamiltonian of the problem. Let us now present a more accurate quantum-mechanical description of the effect of magnetic field on the orbital and spin magnetic moments of electrons in the atomic shell.

As is well known from classical electrodynamics (see, for example, [570]), the momentum $\mathscr{P}$ of a particle with a charge e, moving in an electromagnetic field with vector potential $\boldsymbol{A}(\boldsymbol{r})$, is defined as a sum of the ordinary mechanical momentum $\boldsymbol{P} = mv/\sqrt{1 - v^2/c^2}$ and a term from the

---

* For example, in the above case of positronium (Ch. 1, Sec. 5) the Zeeman effect exhibits distinctive characteristics. These are caused by the fact that the total orbital magnetic moment of positronium is zero while its spin magnetic moment is not proportional to the spin mechanical angular momentum [9, 77, 81].

external field:

$$\mathscr{P} = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{e}{c} A(\boldsymbol{r}) \qquad (2.33)$$

If there is, in addition, a potential field with a scalar potential $\varphi(r)$, then in the non-relativistic approximation (up to the accuracy of the terms $\sim v^2/c^2$) the Hamiltonian (energy) of the particle is equal to

$$\mathscr{H} = \frac{1}{2m}\left(\mathscr{P} - \frac{e}{c} A\right)^2 + e\varphi \qquad (2.34)$$

In passing over to the quantum-mechanical description we must write, instead of the classical Hamiltonian function (2.34), the respective Hamiltonian operator $\hat{\mathscr{H}}$ in which generalized momenta and coordinates are interpreted already not as classical quantities but as operators $\hat{\mathscr{P}}$ and $\hat{\boldsymbol{r}}$ (on the conformity principle see Ch. 1 and [116, 126, 550, 571, 817, 842, 843]). In particular, if one adopts the so-called coordinate representation of quantum mechanics, when coordinate operators coincide with their classical expressions, the momentum operator will be given by

$$\hat{\mathscr{P}} = -i\hbar\nabla \left(\hat{\mathscr{P}}_x = -i\hbar\frac{\partial}{\partial x}, \quad \hat{\mathscr{P}}_y = -i\hbar\frac{\partial}{\partial y}, \quad \hat{\mathscr{P}}_z = -i\hbar\frac{\partial}{\partial z}\right)$$

If the elementary particle has also spin, then, since there is no classical analog of spin, the conformity principle does not allow to guess the form of the Hamiltonian operator with spin. In this situation one must appeal either to experiment or to Dirac's relativistic quantum theory (see Ch. 1, Sec. 3), which make it possible to generalize the classical Hamiltonian and to write the energy operator of an elementary particle with spin $\hbar\hat{\boldsymbol{s}}$ in the non-relativistic approximation [722] * as

$$\hat{\mathscr{H}} = \frac{1}{2m}\left[\hat{\mathscr{P}} - \frac{e}{c} A(\boldsymbol{r})\right]^2 + e\varphi(\boldsymbol{r}) - \frac{\hbar e}{mc}(\hat{\boldsymbol{s}}\boldsymbol{H}) \qquad (2.35)$$

* The wave equation with the Hamiltonian (2.35) is known as Pauli's equation. The last member in the right-hand side of (2.35) gives the relativistic correction for the motion of the electron in an electromagnetic field.

where $\hat{s}$ is the spin operator in units of $\hbar/2$ (this is described in details in Chapter 1). Replacing the square of the binom in the first member of the right-hand side of (2.35) by its explicit expression, we find

$$\hat{\mathscr{H}} = \frac{1}{2m} \hat{\mathscr{P}}^2 + e\varphi(\boldsymbol{r}) + \frac{e}{2mc} (\boldsymbol{A}\hat{\mathscr{P}} + \hat{\mathscr{P}}\boldsymbol{A}) + \frac{e^2}{2mc^2} A^2 - \frac{\hbar e}{mc} (\hat{\boldsymbol{s}}\boldsymbol{H}) \tag{2.36}$$

In deriving this formula we have taken into account that generally the operators $\hat{\mathscr{P}}$ and $\boldsymbol{A}$ do not commute, i.e. $\boldsymbol{A}\hat{\mathscr{P}} - \hat{\mathscr{P}}\boldsymbol{A} \neq 0$.

In the case of a uniform magnetic field $\boldsymbol{H}$ the vector potential $\boldsymbol{A}$ can be written in the following form*:

$$\boldsymbol{A} = \frac{1}{2} [\boldsymbol{H}\boldsymbol{r}] \tag{2.37}$$

If a uniform field $\boldsymbol{H}$ is directed along the z-axis, it follows from (2.37) that

$$A_x = -\frac{1}{2} Hy, \quad A_y = \frac{1}{2} Hz, \quad A_z = 0 \tag{2.38}$$

It is obvious that in the case of a uniform magnetic field the operators $\hat{\mathscr{P}}$ and $\boldsymbol{A}$ commute which means that in the Hamiltonian (2.36) the third and the fourth members in the right-hand side are simply added up. For a system of all the electrons in the atomic shell the Hamiltonian will consist of a sum of Hamiltonians (2.36) for individual

* The vector potential is determined only to the accuracy of the gradient of an arbitrary scalar function of coordinates and time:

$$\boldsymbol{A} \rightarrow \boldsymbol{A} + \nabla f(\boldsymbol{r}, t)$$

while the scalar potential to the accuracy of the time derivative of the function:

$$\varphi \rightarrow \varphi - \frac{1}{c} \frac{\partial f}{\partial t}$$

These transformations do not change physical quantities (the gauge invariance in field theory; see [570]).

electrons:

$$\hat{\mathscr{H}} = \sum_k \left(\frac{\hat{\mathscr{P}}_k^2}{2m} + e\varphi_k\right) - \frac{eH}{2mc}\sum_k [\boldsymbol{r}_k\hat{\mathscr{P}}_k] + \frac{e^2}{8mc^2}\sum_k [\boldsymbol{H}\boldsymbol{r}_k]^2$$

$$-\frac{e\hbar}{mc}\sum_k (\hat{\boldsymbol{s}}_k\boldsymbol{H}) = \hat{\mathscr{H}}_0 - \frac{eH}{2mc}\sum_k [\boldsymbol{r}_k\hat{\mathscr{P}}_k]$$

$$+\frac{e^2}{8mc^2}\sum_k [\boldsymbol{H}\boldsymbol{r}_k]^2 - \frac{e\hbar}{mc}\sum_k (\hat{\boldsymbol{s}}_k\boldsymbol{H}) \tag{2.39}$$

In this expression the summation is extended over all the electrons of the shell. Besides, vector $\boldsymbol{A}$ is replaced according to (2.37), and the following identity is used:

$$[\boldsymbol{H}\boldsymbol{r}_k]\,\hat{\mathscr{P}}_k = \boldsymbol{H}\,[\boldsymbol{r}_k\hat{\mathscr{P}}_k]$$

The first two sums in (2.39), denoted by $\hat{\mathscr{H}}_0$, form the so-called "zero" Hamiltonian of the free atom for $H = 0$. It should be noted that the operator of the vector product $[\boldsymbol{r}_k\hat{\mathscr{P}}_k]$ is by definition the operator of the orbital angular momentum of the k-th electron in the atom, $\hbar\hat{\boldsymbol{l}}_k$, and a sum of them over all values of k gives the operator of the total orbital angular momentum $\hbar\boldsymbol{L}$. In quite the same way the sum $\sum_k \hat{\boldsymbol{s}}_k$ represents the total spin of the atomic shell, $\hat{\boldsymbol{S}}$. Taking this into account, we obtain, instead of (2.39),

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \frac{e\hbar}{2mc}(\hat{\boldsymbol{L}} + 2\hat{\boldsymbol{S}})\,\boldsymbol{H} + \frac{e^2}{8mc^2}\sum_k [\boldsymbol{H}\boldsymbol{r}_k]^2 \tag{2.40}$$

Introducing explicitly the sign of the electron charge $e_{el} = -e$, we have changed the sign of the second member; the symbol e is used below in the sense of the absolute value of the elementary charge.

The solution of the Schrödinger equation with the operator $\hat{\mathscr{H}}_0$ allows us to find the energy spectrum of a free atom. (In the general case the operator $\hat{\mathscr{H}}_0$ may contain, in addition to the scalar potential describing the electrostatic interaction of electrons with the atomic nucleus and between each other, the internal magnetic interactions which determine the fine and hyperfine structure. For this see the material below.)

The other members in the right-hand side of (2.40) specify the effects produced by an external uniform magnetic field acting on the atomic shell. Note that it is this portion of the Hamiltonian (2.40) that is responsible for the description of the two basic physical mechanisms of interaction between the magnetic field and the moving particles with the charge e. As will be seen from further discussion, the first term of this "field" Hamiltonian gives the *paramagnetic* action of the field (i.e. the orientation and polarization effects) while the second gives the precession *diamagnetism*.

b) The case of weak external magnetic fields. Atomic dia- and paramagnetism. When external magnetic fields are weak (in comparison with the effective field of the inner electron magnetic interactions), both "magnetic" terms in (2.40) can be regarded as small perturbations when compared with $\hat{\mathscr{H}}_0$. The major role belongs to the paramagnetic term depending linearly on the field (in this case the field can be interpreted as the small parameter of the perturbation theory) (see [116, 126, 571, 817, 843]), whereas the diamagnetic term appears to be weaker since it contains the square of the field $\boldsymbol{H}$. Neglecting the diamagnetic term, we can now consider only the *paramagnetic effect* of the field.

[It follows from the general structure of the paramagnetic operator (2.40) that the perturbation arising from a uniform magnetic field selects a certain direction in space (the axis of quantization). It is due to this that the magnetic field removes the degeneration of the levels in a free atom with respect to the magnetic quantum number m_J, i.e. to the directions of the total atomic angular momentum $\boldsymbol{J}$, and leads to the *Zeeman splitting* of levels already familiar to us from (2.29). The axial symmetry of the uniform field means that the projection of vector $\boldsymbol{J}$ on the direction of the field preserves its meaning; this projection is determined by the values of the quantum number m_J. In the case of the Russell-Saunders coupling the energy of splitting in a uniform magnetic field $\boldsymbol{H}$ in the first approximation of the perturbation theory is given by the mean value of the operator of the "proper" magnetic moment of the atom (i.e., when

$H = 0$):

$$\frac{e\hbar}{2mc}(\hat{\boldsymbol{L}}+2\hat{\boldsymbol{S}}) = \mu_B(\hat{\boldsymbol{J}}+\hat{\boldsymbol{S}}) \tag{2.41}$$

In deriving this formula we made use of the obvious equality $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$. Since the field is directed along the z-axis, the magnitude of the Zeeman splitting in the first approximation is

$$(\Delta\mathscr{E})_1 = \mu_B(\bar{J}_z+\bar{S}_z)H \tag{2.42}$$

The bar above the symbols in (2.42) means quantum-mechanical averaging. When determining the mean values $\bar{J}_z$ and $\bar{S}_z$, one has to take into account the fact that the projection J_z in this case is an integral of motion, and its mean value is equal to the exact eigenvalue m_J. The projection S_z of the spin, on the contrary, is not an integral of motion, and for this reason it is necessary to find exactly its *mean* value. Since here we are dealing with the Russell-Saunders coupling, the mean value of the projection of vector $\boldsymbol{S}$ on the plane normal to vector $\boldsymbol{J}$ is zero when vector $\boldsymbol{S}$ precesses very rapidly around vector $\boldsymbol{J}$ (see Fig. 2.4). The only non-zero projection of $\boldsymbol{S}$ is that on the direction of vector $\boldsymbol{J}$; its value, obviously, is

$$\boldsymbol{S}_J = (\boldsymbol{S}\boldsymbol{J})\frac{\boldsymbol{J}}{J^2} \tag{2.43}$$

The quantity that we are interested in is the mean value of the projection of the vector (2.43) on the z-axis (i.e. the direction of the magnetic field $\boldsymbol{H}$); this quantity is equal to $(\boldsymbol{S}\boldsymbol{J})\,J_z/\boldsymbol{J}^2$. Therefore, formula (2.42) takes the form

$$(\Delta\mathscr{E})_1 = -\mu_B H J_z\left(1+\frac{\overline{\boldsymbol{S}\boldsymbol{J}}}{\boldsymbol{J}^2}\right) \tag{2.44}$$

Thus, the problem is reduced to that of determining the mean value of the scalar product $\overline{\boldsymbol{S}\boldsymbol{J}}$. It can be easily found if one makes use of an obvious identity

$$\boldsymbol{L}^2 = (\boldsymbol{J}-\boldsymbol{S})^2 = \boldsymbol{J}^2+\boldsymbol{S}^2-2\boldsymbol{S}\boldsymbol{J} \tag{2.45}$$

which, after taking the average, results in

$$\frac{\overline{\boldsymbol{S}\boldsymbol{J}}}{\boldsymbol{J}^2} = \frac{J(J+1)-L(L+1)+S(S+1)}{2J(J+1)} \tag{2.46}$$

Substituting (2.46) in (2.44), we find

$$(\Delta\mathscr{E})_1 = \mu_B m_J g_J H \qquad (2.47)$$

where g_J is the Lande factor (see (2.21)).

Using the general definition of magnetic moment

$$\mu = -\frac{\partial \mathscr{E}}{\partial H} \qquad (2.47a)$$

($\mathscr{E}$ is the energy of the system) and equation (2.47), we obtain for magnetic moment of the atomic shell a formula similar to that of Sec. 3 (equation (2.20a)):

$$\mu_J = -m_J g_J \mu_B \qquad (2.48)$$

The minus in (2.48) means that this magnetic moment is oriented in the opposite direction with respect to the corresponding mechanical angular momentum $m_J\hbar$ (see Fig. 2.4). As follows from (2.48), for an atom being in the quantum state with a definite value of the total magnetic quantum number m_J in the direction of the external magnetic field $\boldsymbol{H}$, the mean value of the magnetic moment in this direction is μ_J.

In connection with the obtained result we must formulate a certain classification of atoms according to their magnetic properties. Formula (2.48) shows that in the first approximation the condition for the paramagnetic effect is that g_J and J are different from zero (since when $J = 0$, the only value of m_J is also zero).

If an atom in the free state with $J = 0$ is completely neutral from the point of view of its magnetic properties, i.e., has neither spin nor orbital angular momentum ($\boldsymbol{S} = 0$, $\boldsymbol{L} = 0$)*, there is no orientation paramagnetism, and the second member in the right-hand side of operator (2.40) causes no splitting of the "zero" level of the atom in the first as well as in the higher approximations of the perturbation theory (the reason for this is that since operators $\hat{\boldsymbol{L}}$ and $\hat{\boldsymbol{S}}$ are themselves equal to zero, all their matrix elements that determine the higher perturbation approxi-

* Here, of course, we assume that the external magnetic field is not sufficiently strong to break the inner electric coupling between individual electrons of the atomic shell.

mations are also equal to zero (see, for example, [116, 223, 858]).

Thus, the only effect observed in magnetically neutral atoms is that due to the "diamagnetic" member in (2.40). In the first approximation the respective shift of the energy level will be given by the mean value of the last term in (2.40):

$$(\Delta\mathscr{E})_1 = \frac{e^2}{8mc^2} \sum_k \overline{[\boldsymbol{H}\boldsymbol{r}_k]^2} \tag{2.49}$$

Using the explicit expression for the square of the vector product of vectors $\boldsymbol{H}$ and $\boldsymbol{r}_k$, we find $[\boldsymbol{H}\boldsymbol{r}_k]^2 = \boldsymbol{H}^2\boldsymbol{r}_k^2 \sin^2\theta$, where θ is the angle between $\boldsymbol{r}$ and $\boldsymbol{H}$. Since in a magnetically neutral S-state ($L = S = 0$) the distribution of electron charge in the atomic shell is spherically symmetric, $\overline{\sin^2\theta} = 2/3$. Then instead of (2.49) we obtain

$$(\Delta\mathscr{E})_1 = \frac{e^2}{12mc^2} \sum_k \overline{r_k^2 H^2} \tag{2.50}$$

From the general formula for magnetic moment (2.47a) we find that the latter is equal to

$$\mu_{\text{diam}} = -\frac{e^2}{6mc^2} \sum_k \overline{r_k^2} H$$

Transformation of this expression into that of a product of the diamagnetic susceptibility χ_{diam} and the field H results in the following formula for the negative diamagnetic susceptibility of a magnetically neutral atom (with $L = S = 0$):

$$\chi_{\text{diam}} = -\frac{e^2}{6mc^2} \sum_k \overline{r_k^2} \tag{2.51}$$

This formula was first obtained by Langevin [573, 574]. Pauli [721] introduced a correction into the numerical factor.

More complex is the case when the total angular momentum of the atom is equal to zero ($J = 0$), but both spin and orbital angular momentum (in this situation necessarily equal in magnitude) have non-zero values ($L = S \neq 0$). Equations (2.47) and (2.48) predict in this case that both the level shift and the total magnetic moment of the atom are equal to zero. However, this result is valid only in the first approximation of the perturbation theory. Since in this case operators $\hat{\boldsymbol{L}}$ and $\hat{\boldsymbol{S}}$ differ from zero, the energy corrections in the second and higher approximations of the perturbation theory may also differ from zero. Generally speaking, these corrections are not negligible compared to the contribution from the diamagnetic term (2.49) in the first approximation (since the latter is quadratic with respect to the field).

In some situations the second approximation correction may even exceed the diamagnetic effect of the first approximation. This is caused by the fact that, according to the perturbation theory, the energy correction of the second approximation is equal to the sum of expressions whose denominators contain differences of the energies of unperturbed levels, i. e., the intervals of the fine structure of the multiplet, and these differences are small quantities. Besides, the perturbation theory also specifies that the second approximation correction to the *ground* level of the system is always *negative*. Therefore the atomic magnetic moment resulting from this correction, $\mu = -\frac{\partial (\Delta \mathscr{E})_2}{\partial H}$, will be always positive. This means that an atom whose ground state is characterized by $J = 0$ and $L = S \neq 0$ is always paramagnetic provided, of course, that this paramagnetism is not suppressed by the diamagnetic effect.

The paramagnetism obtained in the second approximation is called *polarization paramagnetism* (or paramagnetism of Van Vleck). Its susceptibility has the following form (see, for example, [945]):

$$\chi_{\text{Van Vleck}} = 2 \sum_{n'}{}' \frac{|(n \mid \hat{M}_z^{(0)} \mid n')|^2}{\mathscr{E}_{n'}^{(0)} - \mathscr{E}_n^{(0)}} \tag{2.52}$$

In order to obtain this expression let us somewhat change equation (2.41) on the basis of the following notation:

$$-H_z \frac{e\hbar}{2mc} (\hat{L}_z + 2\hat{S}_z) = -H_z \frac{e\hbar}{2mc} \sum_k (\hat{l}_{zk} + 2\hat{s}_{zk}) = -H_z \hat{M}_z$$

where $\hat{M}_z = \hat{J}_z + \hat{S}_z = \sum_k (\hat{j}_{zk} + \hat{s}_{zk})$.

According to the general statistical-thermodynamic definition of average magnetic moment $\langle \hat{M}_z \rangle$, we have

$$\langle \hat{M}_z \rangle = (n \,|\, \hat{M}_z \,|\, n) = -\frac{\partial (n \,|\, \hat{\mathscr{H}} \,|\, n)}{\partial H_z} \qquad (2.53)$$

where $(n \,|\hat{\mathscr{H}}|\, n)$ and $(n \,|\hat{M}_z|\, n)$ are the diagonal matrix elements of operators $\hat{\mathscr{H}}$ and $\hat{M}_z$, respectively, in an n-th atomic state. In order to compute the mean value (2.53) in the case of weak fields one can use the perturbation theory in its standard form assuming that the role of a small parameter in (2.40) belongs to the strength of the external magnetic field H. Then operator $\hat{\mathscr{H}}$ can be written as a series:

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + H_z \hat{W}^{(1)} + H_z^2 \hat{W}^{(2)} + \ldots = \hat{\mathscr{H}}_0 + \hat{\mathscr{H}}'$$

where $\hat{\mathscr{H}}_0$ is the energy operator in the zero approximation (when $H = 0$), and $\hat{\mathscr{H}}'$ is the energy operator of perturbation

$$\hat{\mathscr{H}}' = H_z \hat{W}^{(1)} + H_z^2 \hat{W}^{(2)} + \ldots \qquad (2.53a)$$

Thus,

$$(n \,|\, \hat{M}_z \,|\, n) = -(n \,|\, \hat{W}^{(1)} \,|\, n) - 2H_z (n \,|\, \hat{W}^{(2)} \,|\, n) + \ldots$$

From the formulas of the standard perturbation theory (see, for example, [571]) we find

$$(n \,|\, \hat{W}^{(1)} \,|\, n) = -(n \,|\, \hat{M}_z^{(0)} \,|\, n), \qquad (2.54a)$$

$$(n \,|\, \hat{W}^{(2)} \,|\, n) = -\sum_{n'}{}' \frac{|(n \,|\, \hat{M}_z^{(0)} \,|\, n')|^2}{\mathscr{E}_{n'}^{(0)} - \mathscr{E}_n^{(0)}} + \sum_k \frac{e^2}{8mc^2} (n \,|\, x_k^2 + y_k^2 \,|\, n) \qquad (2.54b)$$

where the primed summation symbol means that the term with $n = n'$ is excluded. The expression $(n \,|\, \hat{M}_z^{(0)} \,|\, n')$ stands for a non-diagonal matrix element of operator (2.53) computed between unperturbed states n and n' (when $H = 0$), and $\mathscr{E}_n^{(0)}$ and $\mathscr{E}_{n'}^{(0)}$ are the energies of these unperturbed states.

On the basis of (2.54a) and (2.54b) the expression (2.53) for the average magnetic moment of the atom can be transformed as follows:

$$(n\,|\,\hat{M}_z\,|\,n) = (n\,|\,\hat{M}_z^{(0)}\,|\,n) + 2H_z \sum_{n'}{}' \frac{|(n\,|\,\hat{M}_z^{(0)}\,|\,n')|^2}{\mathscr{E}_{n'}^{(0)} - \mathscr{E}_n^{(0)}} - H_z \sum_k \frac{e^2}{4mc^2}(n\,|\,x_k^2 + y_k^2\,|\,n) \qquad (2.55)$$

If the electric field of the atom is spherically symmetric, all non-diagonal matrix elements $(n\,|\,\hat{M}_z^{(0)}\,|\,n')$ (for $n' \neq n$) are equal to zero, which means that the second member in (2.55) also vanishes. In this case magnetic properties are determined only by the last member in the right-hand side of (2.55), and therefore the susceptibility will be characteristic of that of precession diamagnetism (2.51).

If the condition of spherical symmetry is violated, in the expression for the susceptibility, owing to (2.55), there appears a positive (paramagnetic) additional term which somewhat lowers the absolute value given by (2.51). This paramagnetic contribution corresponds to the Van Vleck susceptibility (2.52).

c) The case of strong external magnetic fields. As we have seen from the elementary theory of the Zeeman effect, when the external magnetic field is *strong*, i.e., the energy of this field $\mu_B H$ is comparable with or exceeds the energy of the fine structure intervals, we observe the *Paschen-Back effect*. The quantitative theory of this phenomenon is very simple. If the energy of the interaction of the atom with the external magnetic field is much greater than the energy of the intra-atomic spin-orbit and spin-spin interactions but, of course, smaller than the energy intervals between the different multiplets, we can (in the first approximation) neglect the effect of the intra-atomic interactions. Besides, in this case it is not just the projections of the total angular momentum J_z that are preserved but also the projections of the orbital angular momentum L_z and spin S_z. The perturbation operator (2.40) has an eigenvalue equal to the energy of splitting

$$\Delta\mathscr{E} = \mu_B\,(m_L + 2m_S)\,H$$

This expression is the same as the formula (2.32a).

In connection with the problem of the effect of strong and superstrong magnetic fields we should mention the paper of Newton [680]. This problem became quite important as the result of the latest discoveries in astrophysics (like the discovery of neutron stars; see Ch. 6).

4.3. Fine Structure of the Electron Spectrum of Atoms

Let us now present a somewhat more detailed description of the *natural fine structure* of the atomic energy spectrum which is determined by the internal magnetic (relativistic) interactions between electrons. From the most general considerations these interactions can be divided into two classes. The first class contains those interactions whose energy operators are linear with respect to the operators of the electron spin s_i, i.e. *spin-orbit* interactions. The second class includes interactions with energy operators quadratic with respect to the operators of the electron spin, i.e., *spin-spin* interactions. The general qualitative analysis shows that the terms in the atomic shell Hamiltonian, corresponding to spin-orbit and spin-spin interactions, will have the form of scalar products

$$\hat{A}_{ik}(\boldsymbol{r})(\hat{\boldsymbol{l}}_i\hat{\boldsymbol{s}}_k) \quad \text{and} \quad \hat{B}_{ik}(\boldsymbol{r})(\hat{\boldsymbol{s}}_i\hat{\boldsymbol{s}}_k) \tag{2.56}$$

where $\hat{\boldsymbol{l}}_i = [\boldsymbol{r}_i\hat{\boldsymbol{p}}_i]$ is the operator of the orbital angular momentum, and the quantities $\hat{A}_{ik}$ and $\hat{B}_{ik}$ are operators acting (together with operators $\hat{\boldsymbol{l}}_i$) on the electron coordinates.

Let us consider a simple and obvious derivation of the fact that spin-orbit energy is proportional to $(\boldsymbol{ls})$. An electron with spin $\boldsymbol{s}$ moves in the electron shell of an atom in an electrostatic field $\boldsymbol{E}$ of the shielded nucleus. Therefore the electron spin, owing to the movement of the electron with velocity $\boldsymbol{v}$, will be subjected to an effective magnetic field (resulting from induction) $\boldsymbol{B}_{\text{eff}} = c^{-1}[\boldsymbol{vE}]$ which interacts with the spin with energy $-\mu_s\boldsymbol{B}_{\text{eff}}$. Under the assumption of a centrally symmetric potential $\boldsymbol{E} = -\nabla\varphi(\boldsymbol{r}) = -\left(\frac{\partial\varphi}{\partial r}\right)\left(\frac{\boldsymbol{r}}{r}\right)$, and therefore $\boldsymbol{B}_{\text{eff}} \propto [\boldsymbol{r}\, m\boldsymbol{v}]$. An immediate consequence of this is that $-\mu_s\boldsymbol{B}_{\text{eff}} = \lambda(\boldsymbol{ls})$.

The expression for the energy of spin-orbit interaction can also be obtained on the basis of the principles of symmetry. Indeed, in the non-relativistic approximation the operator of this interaction should be a scalar quantity, i.e., the invariant with respect to rotations and spatial reflections. Moreover, this operator can contain only the spin operator $\hat{\boldsymbol{s}}$, the operator of momentum $\hat{\boldsymbol{p}}$, and that of the scalar potential energy $V(r)$. Since $\hat{\boldsymbol{p}}$ is a polar vector and $\hat{\boldsymbol{s}}$ an axial [see (1.13)], the only possible scalar expression bilinear in $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{p}}$ is

$$\hat{\mathscr{H}}_{\text{sp-orb}} = A\,(\hat{\boldsymbol{s}}\,[\nabla V\hat{\boldsymbol{p}}])$$

where the constant A, according to Dirac's relativistic equation, is equal to $1/2m^2c^2$. In the case of a centrally symmetric field

$$\nabla V = \frac{\partial V}{\partial r}\frac{\boldsymbol{r}}{r}$$

Next we substitute this expression for ∇V into that for operator $\hat{\mathscr{H}}_{\text{sp-orb}}$ yielding

$$\hat{\mathscr{H}}_{\text{sp-orb}} = \frac{1}{2m^2c^2r}\frac{\partial V}{\partial r}(\hat{\boldsymbol{s}}\hat{\boldsymbol{l}})$$

where $\hat{\boldsymbol{l}} = [\boldsymbol{r}\hat{\boldsymbol{p}}]$ is the operator of orbital angular momentum.

It is a known experimental fact that spin-orbit interaction, as a rule, is much greater than the spin-spin (relativistic) interaction ($\overline{A}_{ik} > \overline{B}_{ik}$); on the other hand, among the spin-orbit terms the dominant ones are those having identical indices ($\overline{A}_{ii} > \overline{A}_{ik}$, $i \neq k$), which means that the interaction of spin with its "own" orbit is always more intense than with an "alien" one. In the case of an electron in a centrally symmetric field with potential energy $V(r)$ Dirac's theory (see [243, 817]) in the first approximation predicts

$$\overline{A_{ii}(r)} \cong \frac{\hbar^2}{2m^2c^2}\left(\frac{1}{r}\frac{\partial V(r)}{\partial r}\right) \tag{2.57}$$

The same result can be obtained from pictorial classical considerations [349, 353, 913]. For hydrogen-like atoms the potential energy $V(r) = -Z_{\text{eff}}e^2/r$, and instead of (2.57) we can therefore write

$$\overline{A_{ii}(r)} \cong \frac{Z_{\mathrm{eff}}\hbar^2 e^2}{2m^2c^2}\overline{\frac{1}{r^3}}$$

If we only consider the case of the Russell-Saunders coupling, the summation of expressions of the type (2.56) results in that vectors of spin and orbital angular momentum of individual electrons, due to strong electrostatic interaction, add up to form the vectors of total atomic spin $\boldsymbol{S}$ and total orbital angular momentum $\boldsymbol{L}$. Besides, the spin-orbit interaction in this case is regarded as a perturbation of the zero Hamiltonian (see (2.40)). Therefore, in computing the energy corrections in the first approximation of the perturbation theory it is necessary to find the mean value of the operator of spin-orbit interaction (when $\overline{A}_{ii} \gg \overline{A}_{ik}$)

$$A(S, L)(\hat{\boldsymbol{L}}\hat{\boldsymbol{S}}) \tag{2.58}$$

In this expression the averaging over the atomic states with given absolute values of the total angular momentum L and total spin S has, in essence, already been performed. What is left is to compute the average of the scalar product in (2.58) over the states with given L and S and with a fixed value of the total angular momentum J*.

By analogy with (2.45) and (2.46) we have

$$\overline{\hat{\boldsymbol{L}}\hat{\boldsymbol{S}}} = \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)] \tag{2.59}$$

Therefore, spin-orbit energy of the multiplet splitting will be

$$\mathscr{E}_{\text{sp-orb}} = \frac{1}{2}A(S, L)[J(J+1) - L(L+1) - S(S+1)] \tag{2.60}$$

* Strictly speaking, one should solve the problem of the perturbation theory for a degenerate level with given L and S. However, we can make use of the fact that the *correct* functions of the zero approximation, for which operator (2.58) is diagonal, are known beforehand. In other words, we know the functions corresponding to definite values of the total angular momentum J. This is just what was used in deriving formula (2.59).

Thus, in a fine-structure multiplet the energy intervals between adjacent levels with J and $J-1$ are given by

$$\Delta\mathscr{E}_{\text{sp-orb}} = AJ \tag{2.61}$$

(the Lande rule for intervals) [223, 454, 476, 870].

If constant A in (2.60) is positive, we are dealing with a *normal* multiplet, and its lowest energy level will correspond to the smallest value of J (see 2.17a) and (2.17b), i.e. $J = |L - S|$.

On the other hand, if $A < 0$, the multiplet, as we have mentioned above, is called *reversed*, and the lowest level corresponds to the maximal value of J, i.e., $J = L + S$.

There is an empirical rule to the effect that normal multiplets are observed when less than half of the outer shell of an atom is filled, and reversed multiplets are observed when the outer shell is filled more than its half. (See Hund's second rule above.)

The relativistic operator of spin-spin interaction after averaging over states with given L and S will contain terms proportional to S^2 and $(\boldsymbol{LS})^2$. Corrections resulting from terms of the first group are obviously independent of J and thus make no contribution to splitting of the multiplets. The terms of the second group, on the other hand, produce corrections of two types (see (2.59)). Corrections of the first type are proportional to $J(J+1)$ and may be included in (2.59) if we modify the value of the coefficient A. The corrections of the second type are specific for spin-spin interactions: they are proportional to $J^2(J+1)^2$. Their contribution, however, is much smaller than that of spin-orbit interactions (2.59).

As an example, let us mention the paper of Breene [145] which contains a concrete computation of spin-orbit and spin-spin corrections for the case of the oxygen atom. The spin-spin corrections appear to be about 20-40 times smaller (for various states) than the spin-orbit. As to corrections resulting from the interaction of spin with "alien" orbits, they prove to be of the same order of magnitude as those corresponding to spin-spin interaction.

4.4. The Effect of a Variable Magnetic Field: Magnetic Resonance

Up to this point we discussed the effect of a constant magnetic field on an electron in the atom (or on a free electron). Of considerable interest, however, is the influence of *variable* electromagnetic fields on elementary particles and atomic shells. If the variable external field is interpreted as perturbation, one should distinguish between two extreme cases of adiabatic and sudden perturbations.

In the first case the characteristic perturbation time (the period of the variable external field, the time interval of "switching on" or "switching off" of the constant field, the transit time of a particle passing through non-uniform segments where the field increases from zero to its maximal value, etc.) is much greater than the characteristic time of the state of the elementary particle (for example, the period of revolution in the atomic orbit, the period of the Larmor precession, etc.). Therefore the particle has enough time to "adapt" itself to the new conditions of a perturbed potential.

In the second case (of a sudden perturbation) there is a considerable probability of quantum transitions accompanied by the absorption or emission of finite quanta of energy. If the frequency of the external magnetic field is ω, the probability of such quantum transitions becomes maximal (close to unity) in resonance, i.e., when the frequency of the external field coincides with the natural frequency of the particle itself, ω_0. A more detailed discussion of the phenomenon of magnetic resonance is given below, in Chapter 3, in connection with the nuclear magnetic resonance (NMR).

5. Certain Methods of Measurement of the Magnetic Moment of an Atom

5.1. Stern Method

In connection with the problem of measuring atomic magnetic moments let us point out that Stern [879] suggested another way of determining them with a high degree of accuracy. His techniques are based on establishing an equilibrium between the force of gravity acting on a very long molecular beam and a magnetic force directed against the force of gravity. For example, this can be achieved by using a non-uniform magnetic field generated by a current flowing in a conductor which is placed exactly above the molecular beam, parallel to the line of propagation. It can be easily seen that the condition of force equilibrium per one mole of the analysed substance has the form

$$N\mu \frac{\partial H}{\partial z} = NMg$$

where N is Avogadro's number, M is the mass of the atom, g is the acceleration of gravity, and z is the vertical coordinate. Since the quantities g, N, M, and $\partial H/\partial z$ can be determined very accurately, it becomes possible to find the value of the molar magneton

$$N\mu = Ne\hbar/2mc$$

with a high degree of accuracy if, for example, the total magnetic moment of the atom is formed by the spin of one valence electron. On the other hand, the degree of accuracy of the determination of the Faraday constant $F = Ne$ and the velocity of light c is also very high. Therefore this method enables us to find an accurate value of $\hbar/m$.

5.2. Methods of Measuring Atomic Magnetic Moments in the Condensed Phase

Let us note that in addition to the analysis of the magnetic properties of atoms in isolated states, when it is known beforehand that the magnitude of interaction between various atoms or molecules is small (molecular beams, atomic spectra), one can formulate the problem of the investigation of magnetic moments of atomic electron shells in solids and liquids. Although, strictly speaking, in this case we can only talk of the magnetism of a collective of atoms from a condensed phase, still certain information concerning *individual* magnetic properties of single atoms can be obtained. In particular, through the well-known experiments on the gyromagnetic effect in ferromagnetics it has already been established that in many cases the electron spins are responsible for the magnetism in ferromagnetic bodies. Using the magnetic resonance method (for more details see Chapter 3), one can find not only the g-factors but also the mechanical angular momentum and magnetic moment of atoms in a solid or liquid phase.

For example, Zavoysky [981] (see also [11, 266, 267, 610]), working with paramagnetic compounds $MnSO_4 \cdot 3H_2O$ and $CuCl_2 \cdot 2H_2O$, found for the spins of ions Mn^{2+} and Cu^{2+} the values $\sqrt{5/2\,(5/2+1)}\,\hbar$ and $\sqrt{1/2\,(1/2+1)}\,\hbar$, respectively. The Lande factor in the first case proved to be equal to 1.96, and, therefore, the magnetic moment $\mu = 4.90\mu_B$. Zavoysky's experiments provided the first direct proof of the space quantization of an atomic spin in a solid body.

MAGNETISM OF NUCLEONS (PROTONS AND NEUTRONS) AND ATOMIC NUCLEI*

1. Magnetic Moments of Protons and Neutrons

Heavy elementary particles (protons and neutrons, as well as atomic nuclei consisting of them**) possess their own magnetic moments. Therefore it is possible to talk about *nuclear magnetism*. However, since any nuclear mass exceeds the electron mass by three orders of magnitude [see (1.2)], the magnetic moments of nucleons and composite nuclei are about a thousand times less than the spin or orbital magnetic moments of an atomic electron shell. Because of this the nuclear magnetism manifests itself in the microcosm in a much more subtle way (for example, in the hyperfine structure of spectral lines); thus, its detection and analysis requires the application of complex experimental techniques.

1.1. Nuclear Magneton

Nevertheless, the success in the development of modern experimental methods is so great that it has been possible not only to detect, but also to measure with great accuracy the magnetic moments of individual nucleons as well as of composite nuclei, and also of such elementary particles as mesons (see below Sec. 6 and also Ch. 4).

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* A modern presentation of the general aspects of nuclear magnetism can be found in [108, 268, 551, 646, 765, 766, 853, 948].

** The idea of the proton-neutron structure of atomic nuclei belongs to Ivanenko and Ambartsumyan [487]; see also [446, 483, 484, 617].

On the basis of Dirac's relativistic quantum theory (see Ch. 1) which correctly predicted the magnetic properties of the electron, one would think that the proton, being a positively charged elementary particle with a spin equal to $\frac{1}{2}\hbar$, should have a spin magnetic moment $e\hbar/2Mc$, where M is the mass of the proton which exceeds the electron mass m by a factor of 1836.109(11)*. Thus, it would seem that in nuclear magnetism it is the magnitude of the projection of the proton magnetic moment on the direction of an external magnetic field, equal to

$$\mu_N = \frac{e\hbar}{2Mc} = \frac{1}{1836.109}\mu_B = 5.050951\,(50) \times 10^{-24}\ \mathrm{erg \cdot Gs^{-1}} \quad (3.1)$$

which plays the role of the elementary magnetic moment.

The quantity μ_N is called the *nuclear magneton*. It is less than Bohr's magneton μ_B by a factor of 1836.109, and this is why the effect of nuclear magnetism is small compared to that of the magnetism of the electron shell.

As far as the neutron, a particle without an electric charge, is concerned, the usual conceptions of the theory make it seem possible to assume that the neutron does not have a magnetic moment.

In this connection we can make the following remark (see [106]). The electric neutrality of a system only means that the integral over the distribution of charge density is zero. If, for example, we are dealing with a system which consists of a negatively charged sphere with a particle at the centre having a positive charge of the same magnitude, this system as a whole is neutral. In spite of that, its rotation generates a negative magnetic moment. Although this example is not equivalent to a model of the neutron, it provides an illustration of the fact that an electrically neutral system can in principle have magnetic properties.

Tamm and Altschuler [89] were the first to formulate the theoretical conclusion of the existence of a magnetic moment for the neutron. They performed a detailed analysis of the

* Let us recall that the numbers in parentheses appearing at the end of numerical expressions as, for example, in the number 1836.109 (11) stand for the root-mean-square deviation in the last digits of the expression.

experimental data available at that time on the values of magnetic moments of atomic nuclei; their result was that the sign of the neutron magnetic moment is negative (which means that the vector of magnetic moment is antiparallel to the spin) whereas its absolute value is of the same order of magnitude as nuclear magneton (more precisely, they predicted the value $-0.5\mu_N$). Theoretical conclusions of Tamm and Altschuler were in good agreement with experimental results [47, 823]. It is also necessary to note that the response to Tamm and Altschuler's conclusions was quite sceptical, even sharply negative. The future, however, totally confirmed the validity of their hypothesis.

1.2. Anomalous Magnetic Moments of Nucleons

As follows from measurements, the proton magnetic moment μ_p is nearly three times as great as the nuclear magneton or, more precisely,

$$\mu_p = 2.792782(17)\,\mu_N = \gamma_p\mu_N \tag{3.2}$$

whereas the respective quantity for the neutron is, instead of being zero, equal to [216]

$$\mu_n = -(1.913148 \pm 0.00066)\,\mu_N = \gamma_n\mu_N \tag{3.3}$$

Data concerning nuclear magnetic moments one can find, for example, in Smith's review [853]. Cohen and DuMond [210] compiled a critical review concerning the proton magnetic moment μ_p. A very accurate determination of μ_p/μ_N is given in [810, 811]. These papers made use of the equality between the ratio of the angular frequency of the proton spin precession ω_{NMR} to the cyclotron angular frequency ω_H on one side, and the ratio μ_p/μ_N on the other. Indeed, on the basis of equation (3.1), and also from the fact that

$$\omega_{\text{NMR}} = \frac{2\mu_p H}{\hbar}, \quad \omega_H = \frac{eH}{Mc}$$

we immediately obtain

$$\frac{\mu_p}{\mu_N} = \frac{\omega_{\text{NMR}}}{\omega_H}$$

From the measured values of ω_{NMR} and ω_H the above authors found that

$$\frac{\mu_p}{\mu_N} = 2.79277 \pm 0.00005$$

Mamyrin and Frantsuzov [620, 621] suggested a new way of defining μ_p/μ_N, based on the use of a mass spectrograph. Their result is

$$\frac{\mu_p}{\mu_N} = 2.79279 \pm 0.00002$$

Cohen and DuMond [210] give a value

$$\frac{\mu_p}{\mu_N} = 2.79276 \pm 0.00002$$

Finally, Marion and Winkler [622] have found, on the basis of the results of the measurements of proton energies in nuclear reactions Al^{27} (p, γ) Si^{28} and Li^7 (p, n) Be^7 [795, 801, 802] (these have been obtained by the magnetic method and by the method of transit time), apparently the most accurate value yet obtained:

$$\frac{\mu_p}{\mu_N} = 2.79267\ (12)$$

For a critical analysis of data for the ratios μ_p/μ_B, μ_p/μ_N, and γ_p see a fundamental review of Taylor et al. [902]. A more recent determination of μ_p/μ_N is described in [365]; this value is

$$\frac{\mu_p}{\mu_N} = 2.792783 \pm 0.000016$$

See also [517, 613, 619, 974].

Button and Maglić [159] were the first to measure the magnetic moment of the antiproton; their result was

$$\mu_{\bar{p}} \cong -2.79\mu_N$$

Experimental techniques were based on the determination of the asymmetry in the double scattering of antiprotons with energy 960 MeV in a hydrogen bubble chamber (see also below, p. 190). A more recent paper on the experimental determination of the antiproton magnetic moment is that of Frauenfelder [345].

As follows from (3.2), the proton magnetic moment can be presented (in units of μ_N) as a sum

$$\gamma_p = 1 + (\gamma_p - 1) = 1 + \gamma_p^{\text{anom}}$$

where the first member in the right-hand side (+1) corresponds to Dirac's prediction for the magnitude of magnetic moment (μ_N), and the second (γ_p^{anom}) gives the so-called anomalous term of μ_p, whose absolute value with great accuracy equals the quantity $|\gamma_n|$. When one attemps to explain the nature of nucleon magnetism, such poor agreement between the natural assumptions based on Dirac's theory, on one hand, and experiment, on the other, leads at the very beginning to a difficulty which did not arise in the analysis of magnetic properties of the electron (see also Chapter 4). Since modern quantum theory of nucleons and atomic nuclei is far from being complete, it cannot supply an exhaustive quantitative interpretation of magnetism either in the case of individual nucleons or in the case of their associations, the atomic nuclei. Nevertheless it is possible to come up with some kind of qualitative analysis of the nature of anomalous magnetic moments of nucleons.

The point is that the concept of a free elementary particle (electron, proton, etc.) is a crude approximation. Actually, when a single particle is considered, it exists in material relation with the surrounding physical vacuum that is characterized by quite definite physical properties. It is through interaction with this vacuum that particles relate to each other. Therefore each type of such interactions corresponds to its own physical vacuum.

In addition, interaction with vacuum leads to certain specific vacuum effects in the properties of elementary particles. Thus, the material of Chapter 4 will demonstrate that the electromagnetic interaction of electrons and positrons with electromagnetic vacuum results in this kind of vacuum effects, namely, in the shift of spectral lines in atomic spectra, and also in the change of the magnitude of the elementary magnetic moment (Bohr magneton) of the free electron. In the case of electrons these effects are very insignificant, a fact that can be explained by the weakness of electromagnetic interactions (Chapter 4). On the other hand, with nucleons there are, in addition to electromagnetic interaction (associated with the electric charge of the proton), specific powerful nuclear forces *unrelated* to electromagnetism. The carrier of this nuclear interaction is, therefore, not the electromagnetic vacuum but what is called a mesonic vacuum.

Unlike electromagnetic interactions, nuclear interactions cannot be classified as weak. As a matter of fact they are so strong that the very conception of a free nucleon without the surrounding mesonic field (vacuum) is an inadmissible approximation. The last circumstance presents the greatest difficulty in the development of a quantitative theory of nuclear (mesonic) interactions, the reason for this being that the large magnitude of nuclear interactions does not allow to make use of the perturbation method. Meanwhile, we have to be satisfied with a semiclassical theory of the Pauli type in the equations of which the experimentally observed magnetic moments of nucleons given by (3.2) and (3.3) are substituted.

In order to visualize the qualitative aspects of the physical mechanism responsible for the appearance of the anomalous term in the nucleon magnetic moment γ_n, and also for the anomalous value of γ_p^{anom}, let us recall that, for example, the easily calculable electromagnetic interactions between charges allow a pictorial interpretation in terms of the constant circulation of quanta of electromagnetic field, the *virtual photons*. Electric charges interacting with each other are continuously emitting and absorbing virtual quanta of electromagnetic field thus establishing electromagnetic links between them, and also with the field. The specific nuclear interaction between nucleons can be presented as a similar kind of interchange of virtual quanta belonging to the meson field of nuclear forces, the π-mesons, or pions*.

Unlike photons which have a zero rest mass and no electric charge, the pions possess a finite rest mass (about 300

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* The idea of transfer of nuclear interaction by particles was first formulated by Tamm and Ivanenko [899].

times the electron mass, i.e. 273 m_e for charged pions and 264 m_e for neutral pions) and may have charges 0, $+e$, or $-e$*. The pions are described by the Bose statistics, and their spin $s = 0$.

It is possible to say that as a result of continuous emission and absorption of pions the nucleon is surrounded by a mesonic field: an electric charge in a similar way creates an electromagnetic field around itself through the process of emission and absorption of photons. A proton, for example, after emitting a positively charged pion or absorbing a negatively charged pion turns into a neutron. And vice versa, a neutron which has absorbed a positively charged pion or emitted a negatively charged pion becomes a proton.

There is a definite possibility that the mechanism of this kind of "quasi-splitting" of nucleons is more complicated: for example, the emitted pions can still undergo the process of decay into an electron (positron) and neutrino. But this does not affect the qualitative picture. It is an established fact that experiment has confirmed the validity (at least in the qualitative sense) of theoretical conceptions of the exchange nature of at least part of the nuclear interaction between the proton and the neutron (see, for example, [155]).

Thus, nucleons, which are detected in experiment either as protons or as neutrons, go through a continuous process of transformation into each other. The nucleons detected as protons, however, exist most of the time in the proton state, whereas those detected as neutrons in the neutron one. Since the process of exchange of pions does not occur instantaneously (it requires a certain time), there are continuously emerging (for very short time intervals) pions in the nucleus. This is the reason why the resulting magnetic moment of the proton is greater than nuclear magneton, and the neutron, due to the mesonic field exclusively, acquires a negative magnetic moment.

Indeed, as follows from comparison of experimental values for magnetic moments of the proton (3.2) and the

* The neutral pion cannot be observed directly but only by means of its disintegration into two photons or into an electron-positron pair and a photon [603].

neutron (3.3), the difference in their moduli is very close to one nuclear magneton while their signs are opposite.

A more detailed qualitative explanation of the appearance of anomalous magnetic moments in nucleons is as follows [529]. Although a charged virtual pion is a particle with a spin of zero, if it is produced in an orbital p-state, it will have an orbital magnetic moment equal to one mesonic magneton:

$$\mu_{\text{mes}} = \frac{e\hbar}{2m_\pi c} = \frac{M\mu_N}{m_\pi} \cong 7\mu_N$$

since the ratio of masses of the nucleon and the pion $M/m_\pi \cong \cong 7$.

According to the conservation law for the projection of mechanical angular momentum, the orbital angular momentum of a virtual pion in a p-state can have only two values of its projection on the direction of the nucleon spin (0 or $+1$, but not -1) because it is the necessary condition for the resulting projection of the angular momentum of the system (nucleon + pion) to stay conserved in the process of emission of the pion. A consequence of this law is that for a proton emitting a positive pion the additional magnetic moment associated with the orbital angular momentum of the emitted pion will be directed along the nucleon spin. On the other hand, for a neutron which emits a negative pion the additional magnetic moment will be negative, a fact which is observed in experiment. The absolute value of the additional magnetic moment arising from the orbital motion of the pion is approximately $7\mu_N$.

Of course, the above intuitive picture does not allow to interpret the proton and the neutron as some excited states emitting mesons. It is always necessary to have in mind that in the quantitative theory these are strictly stationary states equivalent to a superposition of several states: the "bare" proton, the neutron, the positive pion, etc.

The question of the magnetic moments of nucleons will be considered in greater detail in Chapter 4. There we shall also give a qualitative explanation of the anomality in the values of nucleon magnetic moments, based on new concepts of quarks.

2. Magnetic Moments of Nuclei (Analysis of Experimental Data)

The studies of magnetic properties of atomic nuclei also reveal a more complicated pattern than that of the electron shell. Table II in the Appendix gives the values of spin and dipole magnetic moment of nuclei of all stable and some radioactive isotopes in the ground state, measured by now (see also [362, 576, 614, 836, 837, 853]).

By the present time spins and magnetic moments of most stable and radioactive isotopes with very large half-lives $\tau_{1/2}$ have been already measured (although the process of making numerical data more precise still continues). Therefore, the attention of investigators is concentrated mainly on the development of new methods of determining the magnetic moments of short-lived radioactive isotopes, and also of the excited states of atomic nuclei (see below). For the most part, the measuring techniques make use of the radioactivity of the short-lived excited states of the studied isotopes. The simplest and most convenient method of determining the moments of excited nuclei is based on the Mössbauer effect.

As follows from Table II, there is a simple rule of additivity for the nuclear spin, which obviously does not hold for nuclear magnetic moments. Even in the case of the simplest composite nucleus, the deuteron (the nucleus of deuterium), consisting of one proton and one neutron with the spin equal to unity (this indicates parallel orientation of the proton and the neutron mechanical angular momenta in the deuteron), the resulting magnetic moment is not precisely equal to the algebraic sum of the proton and neutron magnetic moments*. Using numerical values, we obtain

$$\mu_{{}_1\mathrm{H}^2} - (\mu_p + \mu_n) = -0.022228\mu_N$$

The accuracy of measurement guarantees four significant figures, i.e. the error does not exceed 0.0001, which constitutes less than 0.5 per cent of the observed difference.

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* The first measurement of the magnetic moment of the deuteron was carried out by Estermann and Stern [310].

If one assumes the existence of central forces acting between the neutron and the proton in the deuteron, with a potential depending only on the absolute value of the distance between them, $V(r)$, the ground state should necessarily be an S-state (in other words, the deuteron should not have any orbital angular momentum: $L = 0$). Since the total spin is equal to unity, the ground state is a triplet $^3S_1$. At the same time the obvious violation of the additivity of nucleon magnetic moments in the deuteron, as well as its electric quadrupole moment* which has been experimentally observed, clearly indicate that the state $^3S_1$ cannot be the ground state (since the state $^3S_1$ necessarily implies the additivity of magnetic spin moments, and the complete absence of electric quadrupole moment). This contradiction between theory and experiment can be resolved if one assumes that the ground state of the deuteron is not a purely S-state, but a mixture of states $^3S_1$ ($L = 0$) and $^3D_1$ ($L = 2$). The contribution from the state $^3D_1$, as determined from the observed deviation of the value of magnetic moment from that given by the rule of additivity as well as on the basis of the magnitude of the quadrupole moment, should not exceed 4 per cent.

The fact that the ground state of the deuteron is a mixture of two orbital states demonstrates that in this case orbital angular momentum is not conserved and the number L is not a quantum number which characterizes the state of a system. This, in turn, means that nuclear forces cannot be central, i.e. their potential is non-central (tensor) by nature.

Let us consider this problem in greater detail (see, for example, [106]). In the general case the magnetic moment

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* The electric quadrupole moment of an atomic nucleus is the quantity

$$Q = \int \rho(\boldsymbol{r})(3z^2 - r^2)\, d\boldsymbol{r}$$

where $\rho(\boldsymbol{r})$ is the electric charge density of the nucleus at a point $\boldsymbol{r}$. The quadrupole moment is a characteristic of deviation of the charge distribution in the nucleus from the spherically symmetric one (in spherically symmetric nuclei $Q = 0$). For more details, see, for example, [108, 242, 551].

operator of the system "neutron + proton" has the form

$$\hat{\mu} = \mu_n \hat{\sigma}_n + \mu_p \hat{\sigma}_p + \hat{L}_p \qquad (3.4)$$

where $\hat{\sigma}_n$ and $\hat{\sigma}_p$ are spin operators of the neutron and proton, respectively, μ_n and μ_p are their magnetic moments in units of μ_N, and $\hat{L}_p$ is the operator of the proton orbital angular momentum, equal to half the total angular orbital momentum $\hat{L}$ of the deuteron. (It is assumed that there is no contribution from the neutron into the orbital magnetic moment since the neutron has no charge*.) The coefficient at $\hat{L}_p$ is equal to unity because in (3.4) the magnetic moment is given in units of μ_N. Introducing the total spin of the system $\hat{S} = \frac{1}{2}(\hat{\sigma}_n + \hat{\sigma}_p)$, we obtain, instead of (3.4),

$$\hat{\mu} = (\mu_n + \mu_p)\hat{S} + \frac{1}{2}(\mu_n + \mu_p)(\hat{\sigma}_n - \hat{\sigma}_p) + \frac{1}{2}\hat{L} \qquad (3.4a)$$

The eigenvalue of the operator $(\hat{\sigma}_n - \hat{\sigma}_p)$ in a triplet state is zero (because of parallel orientation of the proton and the neutron spins). If we introduce the total angular momentum (2.17), equation (3.4a) can be written as

$$\mu = (\mu_n + \mu_p)\hat{J} - \left(\mu_n + \mu_p - \frac{1}{2}\right)\hat{L} \qquad (3.4b)$$

The observed magnetic moment is equal to the average of expression (3.4b) in the state with $\hat{J}_z = J$. Therefore, replacing in (3.4b) $\hat{L}$ by $\hat{L}_z$ according to**

$$L \to L_z = \frac{(LJ)\,J_z}{J^2} = \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)}\,J_z$$

and taking into account the fact that for the deuteron $J(J+1) = S(S+1)$ and in a mixture of S- and D-states

$$\langle L(L+1)\rangle_{\text{av}} = 0 \times \mathcal{P}_S + 6 \times \mathcal{P}_D = 6\mathcal{P}_D$$

* Strictly speaking, this is not so. As was indicated by Migdal [645], the nuclear interaction between neutrons and protons in a nucleus results in the generation of a proton current by the orbital motion of neutrons. This, in turn, leads to the appearance of an orbital Lande factor for the neutrons.

** See Chapter 2.

where $\mathscr{P}_S$ and $\mathscr{P}_D$ are the statistical weights of S- and D-states respectively, we obtain ($J = 1$)

$$\mu_{{}_1\mathrm{H}^2} = \mu_n + \mu_p - \frac{3}{2}\left(\mu_n + \mu_p - \frac{1}{2}\right)\mathscr{P}_D \tag{3.4c}$$

Comparing (3.4c) with the experimental value $\mu_{{}_1\mathrm{H}^2}$, we get a figure of 3.93 per cent for the concentration of the ${}^3D_1$-state admixture.

For more details we refer the reader to any textbook on nuclear physics (in particular see Sec. 14 in the monograph of Bethe and Morrison [93]). Here we shall just note that in addition to the trivial supplementary term for the nuclear magnetic moment, which is associated with the orbital motion of nucleons (in the states with $L \neq 0$), there may be other contributions caused by the exchange interaction between the nucleons in a nucleus, i.e. the interaction due to the emission and absorption of pions.

The reality of these additional exchange terms follows from the values of the magnetic moments of the next two complex nuclei after the deuteron: the triton ${}_1\mathrm{H}^3$ and the light isotope of helium ${}_2\mathrm{He}^3$ (see Table II in the Appendix). These two are *mirror* nuclei which differ from one another in that all the protons are replaced by neutrons and vice versa. As can be seen from Table II, the magnetic moment of the triton is very close to that of the proton, and the magnetic moment of the light isotope of helium to that of the neutron. This is an evidence of the fact that the ground states of these nuclei for the most part correspond to the state ${}^2S_{1/2}$. The difference between the value of magnetic moment for this state and the observed one indicates that the ground state contains at least one more state with a non-zero value of the orbital angular momentum ($L \neq 0$). A computation similar to that carried out in this paragraph for the deuteron shows that there is practically one more state in the admixture, i.e. the state ${}^4D_{1/2}$ with the weight of 4 per cent. The presence of such an admixture fits very well the data on the total magnetic moment of the above two nuclei:

$$\mu({}_1\mathrm{H}^3) + \mu({}_2\mathrm{He}^3) = 0.8512\mu_N$$

(the theoretical value is 0.849), rather than the values of their individual moments. For magnetic moments of individual nuclei we have

$$\mu_{\text{theor}}({}_1H^3) - \mu_{\text{exp}}({}_1H^3) = -0.27\mu_N$$

$$\mu_{\text{theor}}({}_2He^3) - \mu_{\text{exp}}({}_2He^3) = +0.27\mu_N$$

These discrepancies are associated most probably with the difference in the values of the magnetic moments of free nucleons, on one hand, and of the nucleons which participate in the orbital motion inside the nucleus, on the other, i.e. due to virtual mesonic currents discussed earlier.

Unfortunately, because of the absence of a consistent theory of nuclear interactions, it is still impossible to obtain a detailed qualitative estimate of the exchange contribution into the nuclear magnetic moment.

As an example of the computation of the deuteron magnetic moment which takes into account the inner structure of the proton and neutron (a bare nucleus and a pion cloud) in a bound state let us mention the paper of Young and Cutkosky [988]. The problem of the magnetic moment of nuclei ${}_2He^3$ and ${}_1H^3$ has also been treated by Gerstenberger and Nogami [381], who attempted to improve the agreement between the theory and the experiments considering the difference in the electromagnetic structure of bound and of free nucleons. We refer the interested reader to the original paper.

Data in Table II lead immediately to the following general conclusions concerning the spins and magnetic moments of atomic nuclei:

1) All nuclei with an even number of protons (even Z) and neutrons (even $A - Z$, where A is the mass number of the nucleus of an isotope) have spin zero.

2) Nuclei with odd A have spin equal to $(n + 1/2)\hbar$, where $n = 0, 1, \ldots$.

3) Nuclei with odd Z and odd $A - Z$, i.e. with even A, have spin equal to $n\hbar$, where $n = 0, 1, \ldots$.

By the spin I of a nucleus we mean the total angular momentum of a nucleus equal to a vector sum of the orbital angular momenta and the spins of constituent nucleons. Orbital angular momenta of nucleons are given by multiples of $\hbar$, and spins are equal to $\hbar/2$. This is exactly the reason why in nuclei with odd A the total spin is half-integral, whereas in those with even A it is integral (in units of $\hbar$). The fact that the spin of a nucleus having an even number of both protons and neutrons (case 1) is *always* zero in the ground state is due, of course, to the specific nature of nuclear forces.

Another conclusion that can be drawn from Table II is that all nuclei with a non-zero spin have also a non-zero dipole magnetic moment $\mu = g_N I \mu_N$, where I is the total spin quantum number and g_N is the magnetomechanical ratio for a nucleus.

Nuclei with $I \geqslant 1$ can also possess an electric quadrupole moment. On the other hand, nuclei with $I \geqslant 3/2$ can have an octupole magnetic moment. The experimental investigation of these magnetic moments of higher multiplicity is however still in its rudimentary stage.

The fact that the major part of nuclei of both stable and radioactive isotopes, studied with respect to their magnetic properties, have spin numbers less than 9/2 (see Table 3.1 compiled on the basis of data of Table II) allows to make an assumption that nucleons, like electrons in an atomic shell, form closed shells with zero values of spin and magnetic moment. Table II contains many examples of nuclei with such closed shells. Therefore, the nuclear spin is a result of the addition of the spins of only a few nucleons outside the closed shells. For instance, in nuclei with $I = 0$: ${}_2He^4$, ${}_6C^{12}$, ${}_8O^{16}$, etc. all nucleons belong to closed shells, while in the nucleus ${}_7N^{14}$ there are six neutrons and six protons which form closed shells, and the seventh pair

Table 3.1

Spin of isotope I	Number of nuclei with given spin	Spin of isotope I	Number of nuclei with given spin
0	172	4	5
1/2	57	9/2	18
1	24	5	8
3/2	59	11/2	2
2	21	6	4
5/2	51	13/2	1
3	10	7	1
7/2	31	8	2

of a neutron and a proton is responsible for the total nuclear spin being equal to the sum of its constituents: $I = 1$. The corresponding magnetic moment of this nucleus, however, differs very significantly from that of the deuteron, for which I is also equal to unity. The magnetic moment of ${}_7N^{14}$ is only $0.403562\mu_N$.

In heavier nuclei spin quantum numbers can reach large values. For instance, in the case of the indium isotopes ${}_{49}In^{113}$ and ${}_{49}In^{115}$ we have $I = 9/2$; their magnetic moments are also large: $5.52317\mu_N$ and $5.53441\mu_N$. The niobium isotope ${}_{41}Nb^{93}$ has $I = 9/2$ and $\mu = 6.16719\mu_N$.

From Table II it also follows that there is no complete parallelism between the magnitudes of I and μ. For example, the caesium isotope ${}_{55}Cs^{134m}$ has an anomalously large spin $I = 8$ while its magnetic moment $\mu = +1.10\mu_N$ is quite small. At the same time the praseodymium isotope ${}_{59}Pr^{141}$ is characterized by $I = 5/2$ and $\mu = +4.09\mu_N$, etc.

Frenkel [352] pointed out that the comparatively small values of nuclear spin and magnetic moment invite an analogy with the magnetism of atomic shells, and also with the paramagnetism of alkali metals where, because of the requirements of Pauli's principle, we also encounter the phenomenon of closed atomic shells, i.e. the fact that in the absence of excitation ($T = 0$ K and $H = 0$) the total spin of the group of conduction electrons is always zero. Frenkel compared the appearance of nuclear spin with the presence of a non-zero total spin and magnetic moment in the atoms of transition elements having incomplete inner electron shells, and even with the spontaneous magnetization of ferromagnetics.

3. The Theory of Magnetic Moments of Atomic Nuclei

3.1. The Shell Model of Atomic Nucleus

The modern quantitative theory of nuclear forces is still so far from the stage of completion that it is impossible to perform an accurate computation of the magnetic moments of nucleons and complex atomic nuclei. Nevertheless, there is a number of qualitative interpretations that we are going to discuss.

As mentioned above, it is the smallness of the value of nuclear spin that determines the structure in the nucleus. Development of this conception has lead to the formulation of the well-known *shell model of the nucleus* (details concerning the concrete structure of this model as well as its numerous applications one can find in the monograph of Davydov [242], in a paper of Elliot and Lane [298], in the book of Blatt and Weisskopf [106], and in lectures of Landau and Smorodinskii [572]; see also the monograph of Migdal [646]). Although it was only some time after its formulation that the shell model acquired sufficient theoretical grounds, a number of important qualitative considerations were advanced at the time of its appearance, which allowed to adopt this model in spite of seemingly substantial objections*. These objections, generally, can be reduced to the statement that nuclear interaction is so strong that there is no support whatsoever for interpreting nucleons as free particles, even in the approximate sense. The atomic nucleus should be regarded as a system of strongly interacting particles, i.e. a certain united association similar to molecules, crystals, or liquids (the *liquid-drop model of the nucleus*).

However, in opposition to this generally correct and seemingly absolute objection one can cite a number of contra-objections which render the shell model not so unacceptable. First of all, we should have in mind that the dimensions of individual nucleons (4.5×10^{-14} cm) are smaller than the average distance between them ($1.8 \times \times 10^{-13}$ cm). Even this relative spatial freedom of nucleons' motion inside a nucleus alone allows (in principle) to imagine them as preserving their individual properties. The "field" acting on individual nucleons in a nucleus is not as smooth as the atomic Coulomb field; instead, due

* The reader interested in a more rigorous justification of the shell model can be recommended to look over, for example, a very interesting paper of Bethe [91]; see also an article of Brandow [143] and the books of Migdal [646, 648], where one can find a rigorous formulation and revision of the shell model. For those who are interested in the details of the application and proof of the nuclear shell model we can recommend the following works: [413, 472, 528, 589, 632, 709, 968, 977].

to the small value of the characteristic radius of nuclear forces, its potential exhibits strong local fluctuations. It seems as if these should completely exclude the possibility of emergence of nuclear shells that would be similar to electron ones. However, the shell model is saved by Pauli's principle (the fact that the wave function of a system consisting of fermions—since this is what protons and neutrons with their spin equal to 1/2 are—is antisymmetric with respect to the transposition of coordinates of the particles), according to which no two particles can exist in the same quantum state. Therefore, even in the presence of a strongly fluctuating nuclear potential affecting the motion of the nucleons, these will move along sufficiently smooth trajectories due to the absence of free states with lower energy in the system, which should have been available to nucleons after their energy is lost in the process of scattering on the strongly fluctuating potential.

From the point of view of the shell model, the states of atomic nuclei can be characterized in a way to the quantum-mechanical description of states of atomic electron shells. If it is assumed, for instance, that the magnetic spin-orbit interaction for an individual nucleon in a nucleus is weaker than interaction of the nucleon with the averaged nuclear field, the coupling will be of the Russell-Saunders type (LS-coupling). In this case the orbital angular momenta of nucleons are added up to form the total orbital angular momentum of the nucleus:

$$\boldsymbol{L} = \sum_k \boldsymbol{l}_k$$

while nucleons' spins make up the total spin of the nucleus:

$$\boldsymbol{S} = \sum_k \boldsymbol{s}_k$$

The total nuclear angular momentum is equal to the sum of these two angular momenta:

$$\boldsymbol{I} = \boldsymbol{L} + \boldsymbol{S} \tag{3.5}$$

Another possibility is that of the jj-coupling, when the total angular momenta of each nucleon are added up to form the total nuclear angular momentum:

$$\boldsymbol{I} = \sum_k \boldsymbol{j}_k \quad (\boldsymbol{j}_k = \boldsymbol{l}_k + \boldsymbol{s}_k)$$

In contrast to the case of electron shells, in nuclei the relative contribution of spin-orbit interaction is significant. Therefore, the true coupling in nuclei is much closer to the *jj*-type than to the *LS*-coupling (at least as far as medium-weight and heavy nuclei are concerned). In general, the situation in nuclei is much more complex than what is observed in the electron shell, exactly because of this intermediate* nature of coupling (between *jj*- and *LS*-scheme).

In the case of the *jj*-coupling the state of each nucleon is specified by its total angular momentum and parity. Since the spin of the nucleon is known (it is equal to 1/2), we can find its orbital angular momentum $l = j = \pm 1/2$. This means that in a nucleus the following states with different values of j and l are possible:

$$s_{1/2},\ p_{1/2},\ p_{3/2},\ d_{3/2},\ d_{5/2},\ f_{5/2},\ f_{7/2},\ \ldots$$

Here s, d, ... are even states, and p, f, ... are odd ones. The numbering of these states corresponds to the magnitude of their energy (since they represent energy levels). In contrast to the case of electron shell where the principal quantum number n ($\geqslant l + 1$) is given before the symbol of orbital and spin characteristics (e.g. $1s$, $2s$, . . ., $2p$, $3p$, etc.), the numeration of nucleon levels corresponding to different orbital characteristics always starts with unity:

$$1s_{1/2},\ 2s_{1/2},\ 3s_{1/2},\ \ldots;\ 1p_{1/2},\ 2p_{1/2},\ \ldots;$$
$$1d_{3/2},\ 2d_{3/2},\ 3d_{3/2};\ \text{etc.}$$

Naturally, insufficient information on the inner structure of the nucleus does not allow to predict the order of these levels. That must be found from experiments. The shell model can be used only to establish certain regulari-

* See, for example, the paper of Wybourne [980], which contains a computation for the case of intermediate coupling in two rare-earth isotopes $_{59}Pr^{141}$ and $_{67}Ho^{165}$. See also a more general treatment of heavy deformed nuclei in the recent paper of Ratha Raju and Hecht in ***Proc. Int. Conf. on Nucl. Moments and Nucl. Structure***, **Tokyo, 1973.**

ties in the level sequence (see, for example, [522]). Namely, it has been found that with the increase of the orbital quantum number the energy of the level also increases (because of the growth of the centrifugal potential of the nucleon). Experiment also demonstrates that the nature of spin-orbit interaction is such that the energy of the level with parallel spin and orbital angular momentum, i.e. with $j = l + 1/2$, is always lower than that of the level with antiparallel spin and orbital angular momentum ($j = l - 1/2$).

It is of special interest to us to consider the rules governing the magnitude of the spin of the ground state in atomic nuclei. We shall assume that only the nucleons outside closed shells contribute to nuclear spin while those belonging to these shells may be disregarded. As has been noted, spin of nuclei with an even number of both protons (Z) and neutrons ($A - Z$) is zero: $\sum_j j_j = 0$. For nuclei having an odd mass number A which corresponds to an odd number of either protons (Z) or neutrons ($N = A - Z$), the total spin is determined by the angular momentum of a single particle (either a proton or a neutron). In the case of nuclei with an even A but with both Z and N odd the nucleons outside closed shells have the same values of the total angular momentum and parity, and so each proton-neutron pair contributes a double nucleon angular momentum into the total spin. There are only four nuclei of this kind: ${}_1H^2$, ${}_3Li^6$, ${}_5B^{10}$, and ${}_7N^{14}$; all the remaining nuclei of this kind are radioactive (see Table II in the Appendix).

3.2. Calculation of Nuclear Magnetic Moments. Schmidt Diagrams

There is in general a satisfactory agreement (in the qualitative sense) between the shell model and the observed values of nuclear magnetic moments, at least as far as the light nuclei (with A not exceeding 25) are concerned. We are usually interested in the value of magnetic moment averaged over the motion of nucleons in the nucleus. Namely, in the mean value of the maximal projection of magnetic moment, i.e.

$$\mu_s = g_j^{\text{nuc}} j \mu_N \tag{3.6}$$

where g_j^{nuc} is a quantity similar to the Lande factor for the electron shell. Since the total angular momentum of a nucleon is equal to the sum of its orbital angular momentum and spin ($\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$), the magnetic moment can be written as

$$\mu = (g_l^{\text{nuc}}\boldsymbol{l} + g_s^{\text{nuc}}\boldsymbol{s})\,\mu_N \tag{3.7}$$

In this equation g_l and g_s are the orbital and spin magnetomechanical ratios for nucleons, respectively. Taking into account the fact that the spin of proton and neutron is equal to $\hbar/2$ while their magnetic moments are given by formulas (3.2) and (3.3), we obtain

	g_l^{nuc}	g_s^{nuc}
Protons	1	5.5854
Neutrons	0	−3.8262

The problem now consists in determining g_j^{nuc} for nuclei with odd numbers of protons and neutrons through the magnetomechanical ratios g_l^{nuc} and g_s^{nuc} and j (or l). To do this let us form a scalar product of both sides of (3.7) and vector $\boldsymbol{j}$*. With (3.6) and (2.45) taken into consideration, this leads to

$$\begin{aligned}\mu\boldsymbol{j} &= g_j^{\text{nuc}}\boldsymbol{j}^2 = g_l^{\text{nuc}}\boldsymbol{l}\boldsymbol{j} + g_s^{\text{nuc}}\boldsymbol{s}\boldsymbol{j}\\ &= \frac{1}{2}\left[g_l^{\text{nuc}}(\boldsymbol{j}^2 + \boldsymbol{l}^2 - \boldsymbol{s}^2) + g_s^{\text{nuc}}(\boldsymbol{j}^2 + \boldsymbol{s}^2 - \boldsymbol{l}^2)\right]\end{aligned}$$

Now replace the squares of vectors by their eigenvalues $j(j+1)$, $l(l+1)$, and $s(s+1)$. After simple transformations we obtain (substituting $1/2$ for s and $l \mp 1/2$ for j)

$$\mu = g_j^{\text{nuc}} j = \left(g_l^{\text{nuc}} \mp \frac{g_s^{\text{nuc}} - g_l^{\text{nuc}}}{2l+1}\right) j \tag{3.8}$$

---

* In what follows we shall omit factors μ_N and $\hbar$ with the understanding that nuclear magnetic moments are usually measured in units of μ_N, and nuclear spin, in units of $\hbar$.

Finally,

for the odd neutron

$$\mu = -\frac{\mu_n}{j+1}\, j \qquad \left(j = l - \frac{1}{2}\right) \tag{3.9a}$$

$$\mu = +\mu_n \qquad \left(j = l + \frac{1}{2}\right) \tag{3.9b}$$

for the odd proton

$$\mu = \frac{j}{j+1}\left[\left(j + \frac{3}{2}\right) - \mu_p\right] \quad \left(j = l - \frac{1}{2}\right) \tag{3.9c}$$

$$\mu = \left(j - \frac{1}{2}\right) + \mu_p \qquad \left(j = l + \frac{1}{2}\right) \tag{3.9d}$$

Table 3.2 gives the values of magnetic moments computed by the Schmidt method for certain quantized states of proton and neutron in nuclei.

Table 3.2

Schmidt Values of Magnetic Moments

Odd-proton nucleus				Odd-neutron nucleus			
$j = l + 1/2$		$j = l - 1/2$		$j = l + 1/2$		$j = l - 1/2$	
State	μ	State	μ	State	μ	State	μ
$s_{1/2}$	+2.793			$s_{1/2}$	−1.913		
$p_{3/2}$	+3.793	$p_{1/2}$	−0.264	$p_{3/2}$	−1.913	$p_{1/2}$	+0.638
$d_{5/2}$	+4.793	$d_{3/2}$	+0.124	$d_{5/2}$	−1.913	$d_{3/2}$	+1.148
$f_{7/2}$	+5.793	$f_{5/2}$	+0.862	$f_{7/2}$	−1.913	$f_{5/2}$	+1.306
$g_{9/2}$	+6.793	$g_{7/2}$	+1.717	$g_{9/2}$	−1.913	$g_{7/2}$	+1.488
$h_{11/2}$	+7.793	$h_{9/2}$	+2.624	$h_{11/2}$	−1.913	$h_{9/2}$	+1.565
$i_{13/2}$	+8.793	$i_{11/2}$	+3.560	$i_{13/2}$	−1.913	$i_{11/2}$	+1.619

The curves in Figs. 3.1 and 3.2 (the *Schmidt curves*; see [820]) correspond to equations (3.9a)-(3.9d); the points correspond to the values of the magnetic moments of odd nuclei obtained by measurement. These diagrams permit the following conclusions:

1) Although a great majority of the experimental values of the magnetic moments of nuclei with odd A miss the Schmidt curves, they all lie between them (except those

of four nuclei ${}_1H^3$, ${}_2He^3$, ${}_7N^{15}$, and ${}_6C^{13}$ which fall outside of this region).

2) Although for many nuclei the dispersion in observed values of magnetic moments falls somewhere between (1/2) μ_N and (3/2) μ_N, it is none the less possible to indicate with a reasonable degree of certainty by which of the

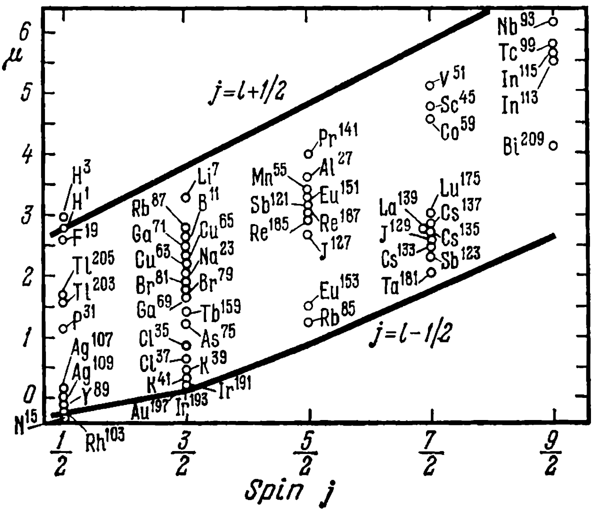

Fig. 3.1. Schmidt diagram [820] for odd-proton nuclei.

Schmidt lines should a given nucleus be classified, and thus to determine its parity. The lines corresponding to mean deviations will be approximately parallel to Schmidt curves.

3) Mean deviations of observed magnetic moments from the values specified by Schmidt lines are somewhat greater (by about 20 per cent) for nuclei with an odd number of protons than for those with an odd number of neutrons.

4) In most cases magnetic moments of nuclei which have only one nucleon in addition to the closed shell (except for the nucleus ${}_{83}Bi^{209}$ whose magnetic moment differs from the calculated quantity by $1.4\mu_N$), are nearly equal to those given by the Schmidt lines.

Still quite significant discrepancy between the observed values of magnetic moments and Schmidt values can be understood if one takes into account the fact that the quantum state of a nucleon in the nucleus (i.e. its wave function with quantum numbers n, l, j) is highly approximately

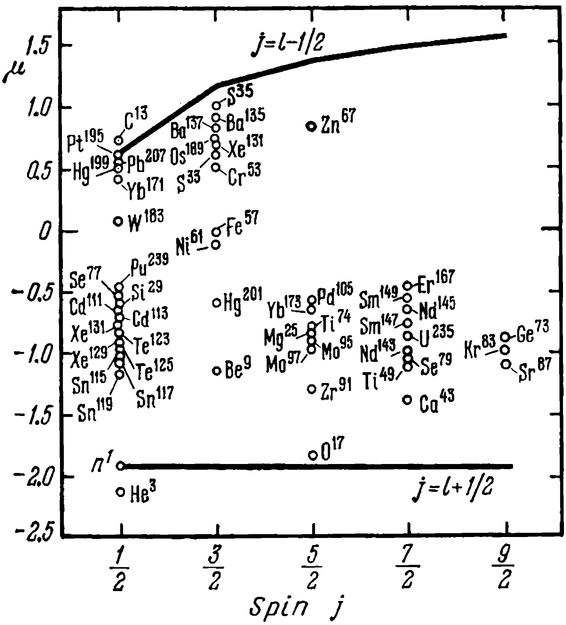

Fig. 3.2. Schmidt diagram [820] for odd-neutron nuclei.

defined in the shell model. Besides, it is necessary to take into account the possibility of the deviation of spin magnetic moments for the nucleons inside a nucleus from the respective values for free nucleons given by (3.2) and (3.3), due to the presence of exchange mesonic currents in the nucleus. In addition, it may be that the magnitude of magnetic moments is strongly influenced by the difference between the actual coupling inside the nucleus and the coupling of the *LS*- or *jj*-scheme. The interaction between nucleons outside closed shells can also be of importance, and in certain cases of heavy nuclei their magnetic properties may be affected by the nucleons from the closed shells (see below).

In spite of its shortcomings the shells model of atomic nuclei applied to the case of light nuclei allowed, on the basis of the analysis of the observed values of spins and magnetic moments, to use these data for determination of nuclear quantum states themselves. A more complete presentation of this question one can find in the following references: [93, 106, 242, 298, 572].

3.3. Medium and Heavy Nuclei

The theory of medium and heavy nuclei, as distinct from that of light nuclei, has to take into account the fact that in addition to the nuclear interaction between nucleons there is also the Coulomb repulsion between protons. The reason for this is that although the electrostatic interaction is weak compared to the nuclear one, its range is large and the relative intensity increases in direct proportion to the square of the number of protons. Therefore, in heavy nuclei we observe a separately dependence of their properties on the number of protons (Z) and the number of neutrons ($N = A - Z$).

A very interesting fact associated with the dependence of the state of heavy nuclei on Z and N was the discovery of the *magic numbers* and their associated *magic nuclei* in which either the number of protons or the number of neutrons corresponds to that of the closed shells. It has been universally accepted by now that the magic numbers are 2, 8, 20, 28, 50, 82, and 126. A most striking example is provided by the nucleus ${}_{82}Pb^{208}$—a double-magic nucleus since both the number of neutrons (126) and the number of protons (82) in it are magic.

The magic nuclei are especially stable. Indeed, if in an ordinary heavy nucleus the difference between the first excited energy level and the ground one is about 200 keV, in a magic nucleus, for example in ${}_{82}Pb^{208}$, this energy interval exceeds 2.5 MeV, i.e. 12 times as great as the respective quantity in an ordinary nucleus.

Magic nuclei, as well as light ones, are spherically symmetric. However, since in the case of a large number of protons there is an equilibrium between the Coulomb forces with their tendency to move the electric charges apart, and the surface tension tending to preserve the spherical shape, the latter in magic nuclei is quite unstable. Therefore, the smallest perturbation can lead to a modification of the spherical shape of the nucleus. A fair evidence of the instability of heavy nuclei is their decay observed for nuclei of uranium, thorium, plutonium, etc.

Thus, one of the most important differences in the structure of heavy nuclei is associated with the shape factor (the spherical shape in the case of light nuclei; the non-spherical, elongated one in heavy nuclei). This, in turn, leads to the appearance of rotational levels which, according to quantum mechanics, are absent in systems possessing spherical symmetry. In this sense heavy non-spherical nuclei resemble non-spherical molecules with their rotational spectra. If one assumes, for the sake of simplicity, that the shape of a heavy nucleus can be approximated by an "elongated sphere", it follows that there is a possibility for such a nucleus to rotate around any axis normal to its axis of symmetry, i.e. to the spheroid axis of rotation. In a nucleus of this kind the total angular momentum of an individual nucleon is no longer preserved: it is only the magnitude of its projection on the axis of symmetry of the nucleus, which is equal to the sum of the projections of the nucleon orbital angular momentum and spin, that is an integral of motion. The total angular momentum of the nucleon in this case has no meaning.

Thus, the total spin of a rotating nucleus is

$$\boldsymbol{I} = \Omega \boldsymbol{n} + \boldsymbol{K} \tag{3.10}$$

where Ω is the sum of the projections of the total angular momentum of individual nucleons on the axis of symmetry of the nucleus, $\boldsymbol{n}$ is the unit vector along this axis, and $\boldsymbol{K}$ is the nucleus angular momentum, normal to the axis of symmetry ($\boldsymbol{K}\boldsymbol{n} = 0$). As follows from (3.10), the projection of the spin of the nucleus on its axis of symmetry is equal to $\boldsymbol{I}\boldsymbol{n} = \Omega$, i.e. to the sum of the projections of the total angular momentum of individual nucleons. Quantum mechanics predicts for the energy of rotational levels

$$\mathscr{E}_I = -\frac{\hbar^2}{2J'} I(I+1)$$

Here J' is the moment of inertia of the nucleus. We must note that this moment of inertia cannot be identified with that of a solid body, because when the spheroidal shape modifies to the spherical one the rotational spectrum should disappear. Therefore a non-spherical nucleus is closer to a liquid drop than to a solid body (the liquid-drop model of the nucleus was introduced in [129, 130, 351, 566]). Consequently, at the present time heavy nuclei are described in terms of the *generalized*, or *collective* model suggested by A. Bohr [127, 128]*. This model of the nucleus combines the positive features of the liquid-drop and shell conceptions. Its main characteristic is that it takes into consideration the fact that the motion of nucleons outside closed shells causes the deformation of the latter; this deformation contributes to the parameters of the nucleus.

A nucleon moving in the axially symmetric field of a heavy nucleus is described by a quantum number of the projection of its orbital angular momentum (λ) on the axis of the spheroid. There are only two possible values for the projection of the nucleon spin ($\pm 1/2$) which means that the total projection $\omega = \lambda \pm 1/2$. Notations for the states with $\lambda = 0, 1\ \ 2, \ldots$ are the same as in the case of a molecule, namely, σ, π, Δ, etc. The multiplicity of the degeneration of levels with a given ω is two, i.e. there may be states with $\pm\omega$. In the case of a centrally symmetric field the multiplicity of degeneration is as a rule much higher; it is determined by that of the level with a given j (the multiplicity in this case is $2\,(2j+1)$). Therefore, in heavy nuclei each pair of protons or neutrons forms a closed shell (instead of $2\,(2j+1)$ nucleons in a light nucleus).

Thus, we arrive at a seemingly strange conclusion: the model of a single particle (i.e. a particle without a match) suits heavy nuclei better than the light ones. In heavy

* An improved version of this model based on the introduction of intermediate coupling schemes has been recently suggested by Ratha Raju and Hecht (see reference on p. 90).

nuclei containing one unmatched proton and one neutron of the same kind these particles occupy *different* levels (in contrast to the case of light nuclei where they belonged to the same level). Therefore the properties of such nuclei are more complex as compared to those of nuclei having one external proton or one external neutron.

It is obvious that the spin of a heavy nuclei coincides with the projection of the total angular momentum of the nucleon on the axis of the nucleus, while the projection of the orbital angular momentum is $\lambda = \omega \pm 1/2$.

The magnetic moment of a heavy nucleus results from those of external nucleons and the magnetic moment associated with the rotation of an axially symmetric nucleus. In the case of a nucleus spinning as a whole the magnetomechanical ratio is $Ze/2AMc$, where Ze is the charge of the nucleus and AM is its mass. If expressed in units of μ_N, the magnetomechanical ratio, obviously, is

$$g_K = \frac{Z}{A} \tag{3.11}$$

Therefore, for the total nuclear magnetic moment we have

$$\boldsymbol{\mu} = g_\Omega \Omega \boldsymbol{n} + g_K \boldsymbol{K} \tag{3.12}$$

where g_Ω is the magnetomechanical ratio for a nucleon in an axially symmetric field. Substituting the moment $\boldsymbol{K}$ as specified in (3.10) into (3.12), we obtain

$$\boldsymbol{\mu} = (g_\Omega - g_K)\,\Omega \boldsymbol{n} + g_K \boldsymbol{I} \tag{3.13}$$

We must find the projection of the magnetic moment on the direction of the spin of the nucleus. The nuclear magnetic moment, by definition, is $\boldsymbol{\mu} = \mu_I \boldsymbol{I}/I$.
Thus,

$$\mu_I = \frac{\boldsymbol{\mu I}}{I+1}. \tag{3.14}$$

As follows from (3.13) the scalar product $\boldsymbol{\mu I}$ is given by

$$\boldsymbol{\mu I} = (g_\Omega - g_K)\,\Omega \boldsymbol{nI} + g_K \boldsymbol{I}^2 = (g_\Omega I + g_K) I \tag{3.15}$$

In derivation of the last formula we made use of the already familiar relation $\boldsymbol{nI} = \Omega$ and also of the fact that in the ground state $\Omega = I$. Substituting (3.15) into (3.14), we find the final expression for the magnetic moment of a heavy nuclei:

$$\mu_I = (g_\Omega I + g_K)\frac{I}{I+1} \tag{3.16}$$

One can distinguish two cases corresponding to small and large values of deviation from the spherical symmetry of the nucleus. In the first case (weak coupling) it is possible to assume that $g_\Omega \cong g_j^{\text{nuc}}$, where g_j^{nuc} is defined in (3.8). Then equation (3.16) takes the form

$$\mu_I = (g_j^{\text{nuc}} I + g_K)\frac{I}{I+1} \tag{3.16a}$$

This formula is meaningful only for $j > 3/2$ since for $j = 1/2$ there is no coupling between the nucleon and the surface of the nucleus and, therefore, $\mu_I \cong g_j^{\text{nuc}} I$; the case $j = 3/2$ has its own peculiarities (see [128]).

As an example consider a heavy nucleus with an odd number of nucleons, the nucleus having spin $I = 5/2$ and positive parity. According to the elementary shell model, such a nucleus should have a magnetic moment $\mu = 4.793$ for an odd number of protons and $\mu = -1.913$ for an odd number of neutrons. On the other hand formula (3.16a) with the values for g_j^{nuc} taken from Table 3.2 and with the assumption that $g_K \cong Z/A$ leads to quite different results for the magnetic moments. In Table 3.3 the magnetic moments computed from formula (3.16) are compared with experimentally obtained values. As follows from this

Table 3.3

Comparison of Measured and Theoretical (with Rotation Effects Taken into Account) Values of Non-spherical Atomic Nuclei

Odd-proton nuclei ($I = 5/2$)			Odd-neutron nuclei ($I = 5/2$)		
Nucleus	Experiment	Theory	Nucleus	Experiment	Theory
Al^{27}	3.641421	3.77	Mg^{25}	−0.85532	−0.67
Sb^{121}	3.35292	3.73	Mo^{95}	−0.93270	−0.81
Cs^{131}	3.517	3.73	Pd^{105}	−0.57	−0.81
Pr^{141}	4.09	3.73	Cd^{111}	−0.59499	−0.81
Re^{187}	3.17591	3.71			

table, taking into account the rotation of non-spherical nuclei results in an essential improvement of the agreement between theoretical and experimental values of magnetic moments.

For those nuclei which differ substantially from the spherical ones the approximation $g_\Omega \cong g_j^{\mathrm{nuc}}$ is much too rough, and in this case it is necessary to use another approximate expression for g_Ω, namely,

$$g_\Omega = \frac{1}{\Omega} \langle g_s s_\zeta + g_l l_\zeta \rangle \tag{3.16b}$$

where g_s and g_l are the spin and the orbital magnetomechanical ratios, respectively, and s_ζ and l_ζ are the projections of the corresponding mechanical angular momenta on the axis of the nucleus. Naturally, in order to perform the averaging of the right-hand side of (3.16b), we have to know the wave functions of the nucleons moving in the axially symmetric field of the nucleus.

An interesting fact is that the probabilities of the magnetic dipole transitions between the rotational levels of nuclei depend on $(g_\Omega - g_K)^2$. Therefore, from the experimentally defined values of the magnetic moment of the nucleus and the intensity of the corresponding spectral lines, using an assumption about the sign of the difference $(g_\Omega - g_K)$, one can determine separately the values of magnetomechanical ratios g_Ω and g_K. Table 3.4 presents certain data obtained in the works of Huus and Bjerregaard [477], Huus and Zupancic [478], and Stelson and McGowan [875] concerning the determination of g_Ω and g_K (see also

Table 3.4

Gyromagnetic Factors

Nucleus	I	μ	$(g_\Omega - g_K)^2$	g_Ω	g_K	g_j^{nuc}
Ta^{181}	7/2	2.35	0.202	0.70	0.25	0.49
Au^{197}	3/2	0.14349	0.149	—0.001	0.32	0.12
Re^{185}	5/2	3.17156	0.17	1.53	0.53	1.99
Re^{187}	5/2	3.17591	1.32	1.63	0.52	1.99
Ir^{193}	3/2	0.1568	$4\cdot10^{-4}$	0.12	0.10	0.12

[108, 242]). As can be clearly seen from this table, the approximation $g_\Omega \cong g_K$ cannot be used here at all. If these data are regarded as sufficiently accurate, it follows that the approximation for the magnetomechanical ratio $g_K \cong Z/A \cong 0.40\text{-}0.50$ is also very rough.

Let us complete the presentation of the theory of nuclear magnetic moments from the point of view of the shell model and the liquid-drop model by saying that there were attempts to compute corrections to the theoretical values of magnetic moments, the corrections arising from mesonic exchange currents in nuclei and also from interactions depending on the nucleons' velocities*. The reader can find some relevant data and references to the original papers, for example, in Sec. 7.4 of Blin-Stoyle's review [108]. An interesting analysis of the magnetic moments of nuclei with odd A is contained in [135, 136, 137, 641]. We shall present a more consistent theory of nuclear moments in Sec. 5 and also in Chapter 4.

4. Magnetic Moments of Excited Nuclei

A more recent method of obtaining information on nuclear magnetic moments is associated with their measurement in the *excited states* of atomic nuclei. This information is quite valuable, in the first place, from the point of view of clarifying the reasons for the discrepancy between theoretical and observed values of nuclear moments, and secondly, in view of the possibility of studying the structure of rotational and oscillatory quantum states of nuclei, coupling between individual and collective orbital motions of nucleons in the nucleus, etc.

The problem of measuring the characteristics of excited nuclei presents considerable technical difficulties since we are dealing with short-lived isomers whose lifetimes are about 10^{-11}s. The various methods of investigation are based on the angular correlation of photons emitted after the nuclei are exposed to the external magnetic field, on the Mössbauer effect, on the experiments with atomic

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* Recently [144, 669] attention has been drawn to corrections arising from the polarization of the core in single-particle states.

beams, etc. At the present time there are data concerning the measurements of the spin and g-factor of nearly a hundred of isomers. A part of these data is given in Table III in the Appendix. A thorough review paper of Bodenstedt [120] presents a most detailed description and interpretation of these investigations; it also contains an extensive bibliography. See on this question also [119, 122]. A detailed review of Khrynkevich and Ogaza [532] is devoted to the magnetic properties of considerably deformed nuclei. Among other review papers and monographs we must mention [12, 121, 417, 673, 682, 696].

Results of the measurements of nuclear magnetic moments of higher multiplicity are given in [240] (octupole moments of nuclei $_{31}Ga^{69}$ and $_{33}As^{75}$), [599] (nuclei $_{49}In^{115}$), and [491] (nuclei $_{53}I^{127}$). For more details see, for example, Chapter IX in [108] and also [415, 769].

5. Migdal's Theory of Nuclear Magnetic Moments

As follows from the Schmidt diagrams (see Figs. 3.1 and 3.2) the experimental values of the magnetic moments of nuclei in most cases exhibit considerable deviations from the Schmidt lines. These deviations are caused by the residual interactions of nucleons, which are not taken into consideration in the self-consistent field of the single-particle shell model. There were attempts to use the perturbation theory in order to account for these interactions (see, for example, [31, 32, 348, 616]). However, if in this kind of formalism a dimensionless small parameter is not specified, the computation, as a rule, leads to erroneous results.

In this connection we must discuss an important consistent theory of nuclear magnetic moments formulated by Migdal [645, 646, 648]; earlier, Migdal [644] and Migdal and Larkin [649] developed methods in the Landau theory of [569] fermi-liquid for systems of finite dimensions, where the interparticle interaction can be accurately accounted for. Employing the methods of quantum field theory [3], Migdal [645, 646, 647] obtained the renormalized values for nuclear magnetic moments. The renormalization was associated mainly with the spin-orbit interaction between

Fermi quasi-particles representing individual nucleons inside the nucleus, rather than with the modification of nucleons' properties. Without going into details of the complex mathematical apparatus of this theory, let us consider briefly from the intuitive point of view its physical meaning and the comparison with experiment.

As has been noted, the magnetic interaction of an electron shell of the atom with its nucleus leads to the appearance of the hyperfine structure (HfS), or line splitting in optical spectra. Energy intervals of this splitting are given by

$$\Delta\mathscr{E}_{\mathrm{HfS}} = -\boldsymbol{\mu}^{\mathrm{nuc}}\boldsymbol{B}_{\mathrm{el}}$$

where $\boldsymbol{B}_{\mathrm{el}}$ is the effective magnetic induction on the nucleus produced by an external magnetic field $\boldsymbol{H}_0$ of the electron shell. Using the experimental values for $\Delta\mathscr{E}_{\mathrm{Hfs}}$ and determining $\boldsymbol{B}_{\mathrm{el}}$, one can find the magnetic moment $\boldsymbol{\mu}^{\mathrm{nuc}}$. In the general case B_{el} and $\boldsymbol{H}_0$ are related through the paramagnetic susceptibility tensor $\tau_{\alpha\beta}(\boldsymbol{r})$:

$$B_\alpha(\boldsymbol{r}) = \sum_\beta \tau_{\alpha\beta}(\boldsymbol{r}) H_{0\beta} \quad (\alpha,\ \beta = x,\ y,\ z) \tag{3.17}$$

In the shell model the transition from an even-even nucleus with completed neutron and proton shells to the next odd nucleus is described as addition of one quasi-particle to the single-particle state $\varkappa$. The change in magnetic energy corresponding to this transition is equal to the integral of the product of the change in potential energy of a single quasi-particle at a point $\boldsymbol{r}$, equal to $-\boldsymbol{\mu}_0\boldsymbol{B}(\boldsymbol{r})$ ($\boldsymbol{\mu}_0$ being the magnetic moment of the quasi-particle), and the probability for the particle to be at this point, equal to the square of the modulus of the wave function $\varphi_\varkappa(\boldsymbol{r})$ in state $\varkappa$:

$$-\mu^{\mathrm{nuc}}H_0 = -\mu_0 \sum_\varkappa \int \boldsymbol{B}(\boldsymbol{r})\,|\,\varphi_\varkappa(\boldsymbol{r})\,|^2\,\delta n_\varkappa\,d\boldsymbol{r} \tag{3.18}$$

where $\delta n_\varkappa$ is the variation for quasi-particle population numbers in state $\varkappa$ ($\sum_\varkappa \delta n_\varkappa = n$, where n is the total number of nucleons added to the nucleus).

As follows from (3.17) and (3.18), the problem of the determination of the magnetic moments of nuclei is reduced to that of finding (1) the susceptibility tensor $\tau_{\alpha\beta}(\boldsymbol{r})$ and (2) the variation in population numbers. The second of these is solved very easily. If one nucleon in a state $\varkappa$ is added

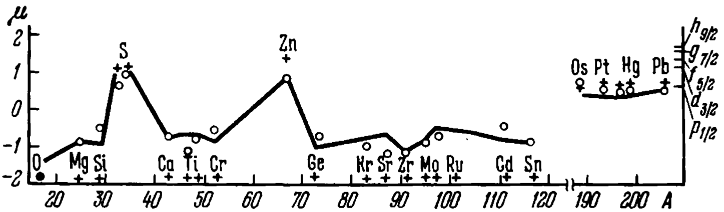

Fig. 3.3. Magnetic moments of spherical nuclei (with an odd neutron) computed by Migdal's theory [645] (solid line). The circles correspond to experimental values and the crosses to the values given by Schmidt curves (see Fig. 3.2).

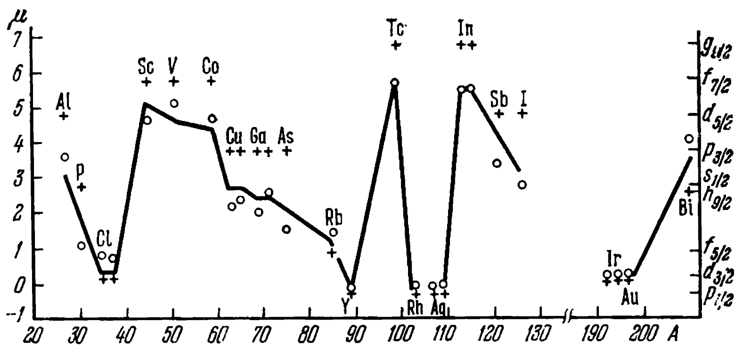

Fig. 3.4. Magnetic moments of spherical nuclei (with an odd proton) computed by Migdal's theory [645] (solid line); the circles correspond to experimental values; the crosses, to the values given by Schmidt curves (see Fig. 3.1).

to the nucleus over its closed shell, then $\delta n_\varkappa = 0$ for $\varkappa \neq \varkappa_0$; $\delta n_{\varkappa_0} = 1$.

In order to find the tensor $\tau_{\alpha\beta}(\boldsymbol{r})$ one must solve complicated equations (see Eq. (34) in [645]*). Migdal's formalism

* We cannot afford to go into details of this involved computation and refer the reader to the original paper of Migdal [645] and to his book [646].

allowed to compute the spin and orbital components of the magnetic moment for nuclei with small or large deviations from the spherical shape. It is important that the equation for the effective field contains only constants g^{nn}, g^{pp}, and g^{np} responsible for the spin-spin interactions between nucleons (n-n, p-p, and n-p) in the nucleus. Specifying these parameters in such a way so as to obtain a good agreement between the computed and the measured values of moments for any two nuclei, one can calculate the moments of all other nuclei. If the interaction between nucleons is ignored ($\tau_{\alpha\beta} \equiv 1$), the gap between theory and experiment increases sharply (see Figs. 3.1 and 3.2).

Computations for spherical nuclei, for example, performed by Troitsky and Khodel' [927] on the basis of the theoretical expression for $\tau_{\alpha\beta}(\boldsymbol{r})$ (when $g^{nn} \cong g^{pp} \cong 1$ and $g^{np} \cong 0$) are in good agreement with experiment, as can be seen from Figs. 3.3, 3.4 and from Table 3.5. Aside from this, for the sum of the magnetic moments of the neutron

Table 3.5

Comparison of Magnetic Moments of Spherical Nuclei Computed in Migdal's Theory (μ_{theor}; [645]) with Experimental Values (μ_{exp}) and Moments Calculated by Schmidt's Method (μ_{Sch}; [820])

Nucleus	State	μ_{exp}	μ_{theor}	μ_{Sch}	Nucleus	State	μ_{exp}	μ_{theor}	μ_{Sch}
S^{33}	$d_{3/2}$	0.64	0.80	1.14	Zn^{67}	$f_{5/2}$	0.88	0.90	1.36
S^{35}	$d_{3/2}$	1.00	1.05	1.14	Rb^{85}	$f_{5/2}$	1.35	1.40	0.86
Cl^{37}	$d_{3/2}$	0.68	0.65	0.12	Rb^{87}	$p_{3/2}$	2.75	2.70	3.8
K^{39}	$d_{3/2}$	0.39	0.30	0.12	Zr^{91}	$d_{5/2}$	−1.30	−1.45	−1.91
Ca^{41}	$f_{7/2}$	−1.59	−1.71	−1.91	Mo^{95}	$d_{5/2}$	−0.93	−1.20	−1.91
Ca^{43}	$f_{7/2}$	−1.31	−1.35	−1.91	Mo^{97}	$d_{5/2}$	−0.93	−1.00	−1.91
Sc^{45}	$f_{7/2}$	4.75	4.85	5.1	Ba^{135}	$d_{3/2}$	0.84	0.85	1.14
Ti^{47}	$f_{5/2}$	−0.79	−0.75	−1.36	Ba^{137}	$d_{3/2}$	0.93	0.95	1.14
Ti^{49}	$f_{7/2}$	−1.10	−1.10	−1.91	Nd^{143}	$f_{7/2}$	−1.00	−1.05	−1.91
V^{51}	$f_{7/2}$	5.15	5.25	5.8	Hg^{201}	$p_{3/2}$	−0.61	−0.70	−1.91
Mn^{55}	$f_{5/2}$	3.47	3.60	4.14	Te^{205}	$s_{1/2}$	1.62	1.85	2.79
Mn^{57}	$f_{7/2}$	5.05	5.10	5.8	Bi^{209}	$h_{9/2}$	4.08	3.40	2.62
Co^{57}	$f_{7/2}$	4.65	4.95	5.8					

and proton in the same single-particle state the theory provides a simple solution, which accords well with the average values of magnetic moments of spherical nuclei.

Using Migdal's theory [645, 646, 648] of interacting quasi-particles, Glas [392] took into account the effect of the spin-orbit interaction between two particles on the magnitude of the nuclear magnetic moment (for values of A from 11 to 65) in addition to the spin-spin interaction. His computations improved the agreement between theory and experiment for the group of light nuclei under consideration. For most nuclei the magnitude of the spin-orbit contribution into the value of the nuclear magnetic moment is close to 7 per cent, but there are individual cases of 20-30 per cent.

6. Experimental Methods of Determining the Spin and the Magnetic Moment of Nucleons and Nuclei

Let us now consider the *experimental methods* of determining the spin and the magnetic moment of nucleons and nuclei. These methods can be divided into two groups: (1) those based on the interaction between nuclear moments and internal atomic or molecular fields, and (2) methods using the interaction with external magnetic fields. Naturally, methods of the first group depend on the knowledge of internal fields for their successful measuring of nuclear moments. Hence, the accuracy in determining the moments is limited by the degree to which we know the magnitude of these internal fields. This is not always sufficiently great. The more accurate methods use the external magnetic fields.

The following concrete methods of experimental determination of nuclear spins and their magnetic dipole moments are in fairly wide use:

I. Study of hyperfine structure (HfS) in atomic spectra.

II. Study of band rotational spectra of diatomic molecules whose two nuclei are identical, such as H_2 (band spectra measurements).

III. Analysis via microwave absorption.

IV. Analysis of nuclear magnetic resonance (NMR).

V. Analysis of nuclear resonance induction.

VI. Analysis of electron paramagnetic resonance (EPR).

VII. Studies based on atomic beam deflection experiments and molecular beam deflection experiments: nonresonant and resonant versions of these methods; the special case of the neutron beam resonance method.

VIII. Method of oriented nuclei.

IX. Analysis of angular distribution of nuclear emissions: time-differential perturbed angular correlations of radiation (DPAC) and time-integrated perturbed angular correlation of radiation (IPAC); time differential angular distribution following nuclear reactions (DPAD) and time-integrated angular distribution following nuclear reactions (IPAD).

X. Method of in-beam optical pumping.

XI. Analysis of reorientation effects in Coulomb excitation.

XII. Analysis of in-beam stroboscopic resonance.

Below we give a brief description of some of these methods. A more detailed presentation can be found in special monographs: [27, 268, 358, 359, 551, 607, 765, 766, 853, 924].

6.1. Determination of Nuclear Moments from the Hyperfine Structure of Atomic Spectra

First of all, we must mention the phenomenon of the hyperfine structure (HfS) of spectral lines in atomic spectra, first discovered by the Russian physicists Dobretsov and Terenin [257] and the German physicist Schüler [822]. They demonstrated that even in the fine structure of the spectrum (see Ch. 2) the individual lines are a combination of several different lines with very close values of frequency. In the doublet of the D-line of sodium (studied in these works) the distance between the two fine-structure elements is about 6 Å (in terms of wavelengths), whereas the distances between individual hyperfine structure elements are 0.0021 Å for the line with $\lambda \simeq 5890$ Å and 0.023 Å for the line with $\lambda \simeq 5896$ Å, which means that the distance of 6 Å exceeds the intervals of hyperfine structure by a factor of hundreds and thousands (see Fig. 2.8).

Further studies showed (see [358, 359]) that two types of the hyperfine structure can be distinguished. In the first all the spectral lines have the same number of components. Hyperfine structure in this case appears as a result of the existence of two or more stable isotopes of a given element* since the difference in mass of isotopes' nuclei modifies the energy of the stationary states of atomic electron shell.

The second type of HfS is characterized by different spectral lines having different numbers of components—a phenomenon that cannot be explained by the isotopic shift, all the more so since this type of hyperfine structure is observed also in spectra of atoms with only one stable isotope (for example, in the spectrum of Bi). This kind of HfS can be explained only if one takes into account the existence of nuclear spin and the corresponding magnetic moment. It is the interaction of this moment with the magnetic moments of the atomic electron shell that causes the splitting of the atomic energy levels. This means that equation (2.17) for the total mechanical angular momentum should be refined because the total angular momentum of an atom $\boldsymbol{F}$ will be equal to the sum of the total angular momentum of the electron shell $\boldsymbol{J}$ and the resulting nuclear spin $\boldsymbol{I}$:

$$\boldsymbol{F} = \boldsymbol{J} + \boldsymbol{I} \tag{3.19}$$

According to the rules of space quantization, the quantum number F corresponding to the total mechanical angular momentum of an atom takes on the following values:

$$F = J + I,\ J + I - 1,\ \ldots,\ |J - I| \tag{3.20}$$

As follows from (3.20), electron energy levels split into $2J + 1$ (if $I > J$) or $2I + 1$ (if $J > I$) sub-levels (multiplet) with somewhat different energies. This is what causes the appearance of hyperfine structure.

Figure 3.5 contains drawings of two vector models for the addition of mechanical angular momenta and magnetic

* It was this "isotopic" shift of all lines in the Balmer series of hydrogen spectrum that led to the discovery of deuterium, the heavy isotope of hydrogen (see [551]).

moments of the electron shell and atomic nucleus in situation when (a) the electron shell has orbital angular momentum and spin, and (b) the electron shell has only a spin (s-state with $L = 0$). One can see from the figure that the gyromagnetic anomaly of spin manifests itself in that the total magnetic moment $\boldsymbol{\mu}_F$ of an atom is not parallel to

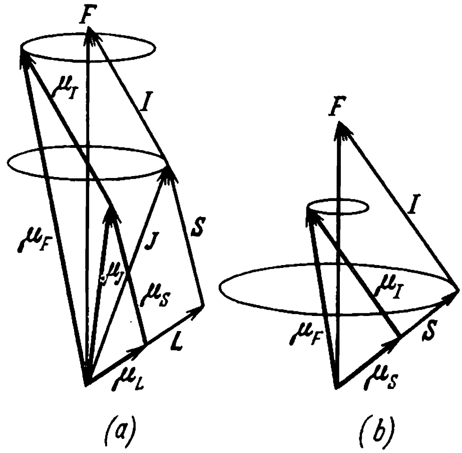

Fig. 3.5. Two vector model of the atom taking into account the nuclear angular momentum. (a) The case of an electron shell with non-zero orbital (L) and spin (S) angular momenta. (b) The case of an electron shell in the s-state ($L = 0$). (The scale for μ_I is increased by a factor of approximately 10^3.)

its total mechanical angular momentum $\boldsymbol{F}$; instead, the former is precessing around the latter. For given values of the angular-momentum quantum numbers J and I, the angular-momentum quantum number of the atom F, according to (3.20), has a series of possible values each corresponding to a certain energy level. We may say that the electron shell creates in the centre of the nucleus an effective magnetic field H_{el} (HfS field) parallel to vector $\boldsymbol{J}$. Therefore, the energy of interaction of the nuclear magnetic moment μ_I with this field is

$$\Delta\mathscr{E}_{HfS} = \mu_i H_{el} \cos(\widehat{\boldsymbol{I}, \boldsymbol{J}})$$

where

$$\cos(\widehat{\boldsymbol{I}, \boldsymbol{J}}) = \frac{F(F+1) - I(I+1) - J(J+1)}{2\sqrt{I(I+1)\,J(J+1)}},$$

$$\mu_I = g_I \sqrt{I(I+1)}\,\mu_N$$

Thus,

$$\Delta\mathscr{E}_{\mathrm{HfS}} = \frac{\mu_N H_{\mathrm{el}} g_I}{2\sqrt{J(J+1)}}\,[F(F+1) - I(I+1) - J(J+1)] \qquad (3.21)$$

where for given I and J the number F is defined by a series of values (3.20).

It is the energy interval (3.21) for given quantum numbers I and J that specifies the magnitude of hyperfine (Hf) splitting. We can easily see that the ratios between subsequent intervals in the series of the sub-levels of HfS, characterized by quantum numbers F, $F+1$, $F+2$, . . . depend only on F. As follows from (3.21), these ratios are $(F+1) : (F+2) : (F+3) : \ldots$. The same holds for the ratios of these intervals expressed through the wave numbers (or frequencies) of the components of HfS:

$$\Delta\nu_1 : \Delta\nu_2 : \Delta\nu_3 \ldots = (F+1) : (F+2) : (F+3) : \ldots$$

In addition to this interval rule, it is also possible to obtain a rule for intensities, which gives the ratios of intensities of the various components of HfS as a function of quantum number F. These two rules in combination with experimental spectroscopic data allow to determine the magnitude of the *spin* quantum number of the nucleus I (see [358, 359]).

However, this is not sufficient for determining the Lande factor or the nuclear magnetic moments. These goals can be achieved if one makes use of the Zeeman effect, i.e. studies the splitting of the components of HfS in external magnetic fields. In this case, as in the case of the electron shell, we must consider separately two extreme situations, i.e. weak and strong fields.

a) Weak fields. External magnetic fields are regarded as weak if in their presence there is still a strong coupling between vectors $\boldsymbol{J}$ and $\boldsymbol{I}$. This is equivalent to their sum $\boldsymbol{F}$ behaving as an entity in an external magnetic field.

Its projections on the direction of the field are given by

$$m_F = F, F - 1, \ldots, -F + 1, -F \qquad (3.22)$$

The possible transitions between the levels are specified by the selection rules (for dipole electric radiation)

$$\Delta m_F = 0, \pm 1 \qquad (3.23)$$

Figure 3.6 gives an example of Hf splitting of $^2S_{1/2}$ term (i.e. the level with $L = 0$, $S = 1/2$) in a weak magnetic

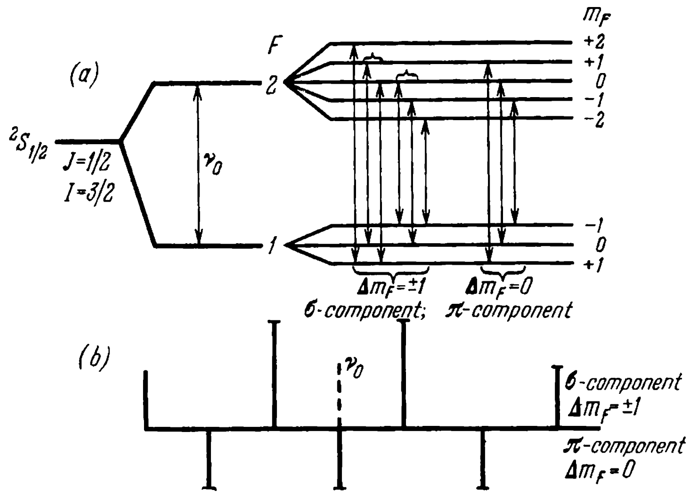

Fig. 3.6. Hf splitting of $^2S_{1/2}$ term in a weak magnetic field for $I = 3/2$. (a) The scheme of levels and transitions. (b) The intensities and order of the Zeeman π- and σ-components.

field for $I = 3/2$. Out of six possible σ-components (see Ch. 2) corresponding to transitions with $\Delta m_F = \pm 1$, only four are actually observed since transitions ($1 \to 0$, $0 \to 1$) and ($0 \to -1$, $-1 \to 0$) merge pairwise into single lines (in Fig. 3.6*b* it is shown that these lines have double intensity). Transitions with $\Delta m_F = 0$ correspond to three π-components of HfS.

By analogy with (2.30), the change of energy of the level with quantum number F in external magnetic field H is

$$\Delta \mathscr{E}_H = m_F g_F \mu_B H \qquad (3.24)$$

The value g_F of the Lande factor can be expressed through the Lande factors g_J and g_I of the electron shell and the nucleus, respectively. Indeed, the magnetic moment of the whole atom $\sqrt{F(F+1)}g_F\mu_B$ is composed of magnetic moment of the shell $\sqrt{J(J+1)}g_J\mu_B$ and magnetic moment of the nucleus $\sqrt{I(I+1)}g_I\mu_B/1836.109$ [see (3.1)], i.e.

$$\sqrt{F(F+1)}\,g_F\mu_B=\Big[\sqrt{J(J+1)}\,g_J\cos(\widehat{\boldsymbol{J},\boldsymbol{F}})$$

$$+\sqrt{I(I+1)}\,g_I\cos(\widehat{\boldsymbol{I},\boldsymbol{F}})\frac{1}{1836.109}\Big]\mu_B$$

Using an expression similar to (2.19a) for the quantum-mechanical values of $\cos(\widehat{\boldsymbol{J},\boldsymbol{F}})$ and $\cos(\widehat{\boldsymbol{I},\boldsymbol{F}})$, we obtain

$$g_F=g_J\frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)}$$

$$+\frac{g_I}{1836.109}\,\frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)} \qquad (3.25)$$

If in (3.25) the second member in the right-hand side is ignored (because of the small factor 1/1836.109), in the special case of ${}^2S_{1/2}$ term we obtain

$$g_{F=I+1/2}=\frac{g_J}{2I+1},\quad g_{F=I-1/2}=-\frac{g_J}{2I+1} \qquad (3.26)$$

Thus, in this approximation the splitting of both levels of HfS is of the same magnitude given by (3.24), whereas the dependence of the order of the sequence of the Zeeman sub-levels on m_F for $F=1$ is reverse with respect to that for $F=2$, due to the difference in signs in formulas (3.26) for $F=2$ and $F=1$. Therefore, measurements of $\nu_H = \Delta\mathscr{E}_H/h$ allow to determine the magnitude of spin I of the nucleus.

The whole structure of the Zeeman splitting is symmetric with respect to the frequency ν_0 of HfS component for $H=0$ (see Fig. 3.6*b*). The number of π- and σ-components specifies uniquely the *spin quantum number of the nucleus*. For example, when $I=1$, there are three σ-components ($\Delta m_F=\pm1$) and two π-components ($\Delta m_F=0$).

b) Strong fields. The term "strong field" is again used (see Chapter 2) in the sense of a field which violates the coupling between $\boldsymbol{I}$ and $\boldsymbol{J}$. The vectors precess in an independent manner around the direction of the magnetic field $\boldsymbol{H}$. In this case, instead of one quantum number m_F, we are dealing with two separate quantum numbers m_I and m_J. Therefore, it is impossible to introduce sub-levels of HfS corresponding to given values of F (see Fig. 3.6*a*) and to consider their splitting. Thus, the energy of the Zeeman levels should be referred to the "centre-of-gravity" of the energy $\mathscr{E}_s$ of the multiplet system as a whole and not to the levels of the original hyperfine structure. In other words, for each of the Zeeman levels (for given values of I and J) we have

$$\mathscr{E} = \mathscr{E}_s + g_J m_J \mu_B H + g_I m_I \frac{\mu_B}{1836.109} H + A m_I m_J \qquad (3.27)$$

The second member in the right-hand side of (3.27) is the energy of the electron shell magnetic moment, and the third member is the energy of the magnetic moment of the nucleus in the external field.

Although in the case of strong fields the "rigid" coupling between vectors $\boldsymbol{I}$ and $\boldsymbol{J}$ is completely "disrupted", and they precess independently around the direction of the field, there is magnetic interaction between the nucleus and the shell the magnitude of which is given by the mean values of the cosine of the angle between vectors $\boldsymbol{I}$ and $\boldsymbol{J}$:

$$[I(I+1)J(J+1)]^{1/2} \cos(\widehat{\boldsymbol{I}, \boldsymbol{J}})$$
$$= [I(I+1)J(J+1)]^{1/2} \cos(\widehat{\boldsymbol{I}, \boldsymbol{H}}) \cos(\widehat{\boldsymbol{J}, \boldsymbol{H}}) = m_I m_J$$

This is what the last term in (3.27) accounts for (it gives the average energy of interaction between the nucleus and the shell).

The coefficient A depends on quantum numbers I and J and in the special case of $^2S_{1/2}$ state it has the form $A = \frac{h\nu_0}{I+1/2}$, where ν_0 is the frequency of transition between the Hf levels (see Fig. 3.6).

Figure 3.7 gives the splitting pattern of the energy levels of the same $^2S_{1/2}$ term in a strong field for $I = 3/2$. The selection rules for this case are as follows: $\Delta m_I = \pm 1$ and

$\Delta m_J = 0$ or $\Delta m_I = 0$ and $\Delta m_J = \pm 1$. As we shall see later, for the determination of nuclear magnetic moment the transitions are used which correspond to the first group of selection rules (see Fig. 3.7).

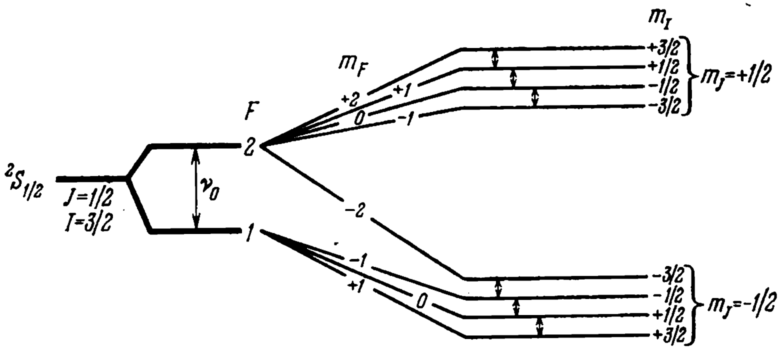

Fig. 3.7. Hf splitting of $^2S_{1/2}$ term in a strong magnetic field for $I = 3/2$ and the scheme of transitions with $\Delta m_I = \pm 1$ and $\Delta m_J = 0$.

From formula (3.27) for $\Delta m_J = 0$ we obtain the following values for the Zeeman frequencies:

$$\nu = \frac{\nu_0}{I+\frac{1}{2}} m_J \Delta m_I + \frac{\mu_B}{1836.109} \frac{Hg_I \Delta m_I}{h} \tag{3.28}$$

When $\Delta m_I = \pm 1$ and $m_J = \pm 1/2$, we have

$$\nu = \frac{\nu_0}{2l+1} \pm \frac{\mu_B}{1836.109} \frac{Hg_I}{h} \tag{3.29}$$

In contrast to the case of a weak field, the member describing the interaction between the nuclear moment and the external field also plays an important role here. For example, when $H \cong 6000$ Oe and $g_I = 2$, the second member in (3.29) is about 1×10^7 s^{-1} whereas the first one for $I = 3/2$ is approximately 20×10^7 s^{-1}.

As follows from (3.29), in strong fields the translation frequencies $\Delta m_J = 0$ and $\Delta m_I = \pm 1$ approach two values which differ by the quantity $2\mu_B g_I H/1836.109h$; this difference allows to determine the unknown quantity g_I.

Since the arithmetic mean of (3.29) is $\nu_0/(2l+1)$, from the known value of frequency ν_0 of the component of HfS corresponding to the absence of magnetic field one can find the *spin quantum number I of the nucleus.*

c) Intermediate fields. The theory also contains a possibility to obtain formulas in the case of intermediate fields, which is extremely important from the point of view of interpretation of experimental results.

An atom has simplest HfS if its electron shell possesses neither mechanical angular momentum nor magnetic moment ($^1S_0$ term). The magnetic properties of the atom are then determined by the small magnetic effect produced by the nucleus. In the external magnetic field the $^1S_0$ term splits into $2I+1$ equidistant sublevels with the magnitude of this splitting given by

$$\Delta\mathscr{E} = \mu_I H \cos(\widehat{\boldsymbol{I}, \boldsymbol{H}}) = \frac{\mu_I H m_I}{[I(I+1)]^{1/2}} \tag{3.30}$$

According to the selection rules in this case ($\Delta m_I = \pm 1$) the frequency of transition between the levels of HfS multiplet is

$$\nu_H = \frac{\mu_I H}{h[I(I+1)]^{1/2}} = \frac{g_I H}{h}\mu_N \tag{3.31}$$

As follows from (3.31), when $g_I \cong 2$ and $H = 1000$ Oe, the quantity ν_H is about 10^6 s^{-1}, and the respective wavelength $\lambda \cong 10^2$ m. In classical terminology, this frequency is simply the Larmor frequency of nuclear spin; it belongs to the interval of radio frequencies.

Thus, the measurement of ν_H allows to determine the g_I-factor of the nucleus. Unfortunately, there are very few such simple cases. If they occur, it is mainly for even-even isotopes with an even Z and an even A, whose nuclei have neither spin nor magnetic moment (see, for example, Table II in the Appendix).

6.2. Determination of Nuclear Moments from Rotational Spectra of Diatomic Molecules (Band Spectra Measurements)

Nuclear spins of certain nuclei can be found from observations of relative intensities of spectral lines in rotational spectra of diatomic molecules with identical nuclei (H_2,

N_2, O_2, etc.). The probability of realization of one or another quantum rotational molecular state in such molecules will depend on nuclear spin. Therefore the spin of the nucleus will also determine the sequence of line intensities in the rotational spectrum of a molecule. Measuring the intensities' ratios of consecutive lines, we are able to find the spin of the nucleus in a molecule. When $I \neq 0$, the ratio of intensities of consecutive spectral lines is proportional (see [268]) to the sum $I + \frac{1}{I}$. For $I = 1/2$, 1, 3/2, 2, and 5/2 the values of this binom are 3, 2, 1.67, 1.5, and 1.4 respectively. Therefore, it can be assumed that by this method one can accurately detect spin $I = 0$ (when there is no effect), and also spins $I = 1/2$, 1, and 3/2 but not above these since in the case of $I = 3/2$ the accuracy of measurement is small. Another shortcoming of this method is that it does not allow to determine magnetic moments (or the Lande factors).

6.3. The Method of Deflection of Molecular Beams and the Magnetic Resonance Method

Let us now consider other important experimental methods concerning the determination of nuclear spins and magnetic moments. At the beginning experimental techniques of detecting nuclear magnetic moments developed in the direction of improvement of the method of *deflection of molecular beams* in a non-uniform magnetic field (see [527, 551]; a detailed critical description of these techniques one can find in [268]). A real progress in this field, however, started with the advancement of the *magnetic resonance method*.

Einstein and Ehrenfest [292] pointed out that the change in the orientation of atomic magnetic moments owing to the effect produced by a magnetic field should be accompanied by radiation of electromagnetic waves in the radio-frequency range (see also [267]). On the basis of these considerations Dorfman [263] predicted the *photomagnetic effect*, which manifests itself in the change of the magnetic state of paramagnetics or ferromagnetics when they are exposed to radio-frequency radiation. Dorfman [263] also indicated that the selective absorption of radio waves in ferromagnetics, discovered and studied in details by Arkad'ev [35] and his school, can be at least partially explained by the photomagnetic effect (see also [36]).

Majorana [617] and later Gorter [405] came up with a theoretical analysis of the effect of radio-frequency magnetic fields on atoms in a magnetic field. Dorfman's idea [263] about employing radio-frequency fields for detection of atomic magnetic moments in atomic beams found its realization in the well-known works of Rabi [759] (see, for example, a review paper of Kellogg and Millman [527]). Further development of this method later allowed to determine nuclear moments not only in molecular beams, but also in condensed phase of matter (see [111, 112, 113, 267, 758, 991].

The magnetic resonance method is based on reaching the state of resonance between the frequency of precession of the nuclear magnetic moment around the direction of a constant magnetic field and the frequency of a magnetic field from the *radio-frequency* range, applied simultaneously. These techniques imply direct study of atomic transitions between HfS levels of a *given* multiplet; in contrast to this in ordinary spectroscopic analysis it is the transitions between HfS levels of different multiplets that are observed.

It is well known that the wavelengths corresponding to the transitions between Zeeman levels of HfS of one multiplet fall into the interval $1\text{-}10^4$ cm (see, for example, (3.31)). This is magnetic dipole radiation. Rabi's idea [759] was to detect this radiation when it is artificially induced by a variable external magnetic field rather than when it is in its "natural" state of a vanishingly small intensity (because of the small value of the probability of magnetic dipole radiation). The frequency of the inducing magnetic field is chosen to be in resonance with the frequency corresponding to the transitions between Zeeman levels of HfS. In the simplest case the frequency of precession of the nuclear moment around the direction of the external magnetic field is given by formula (3.31). Determining this frequency from the experimentally observed resonance, we can find the value of the nuclear Lande factor g_I.

In order to justify this method it is very important to solve the problem of the fate of a beam which is already spatially quantized if it is directed through another magnetic field. Shall we observe space "requantization" or the atoms will again precess around the direction of the field while preserving their previous state? This question was analyzed by a number of investigators [115, 759, 825] who demonstrated that if the beam passes from the area of one field to that of the other adiabatically, the atom precessing

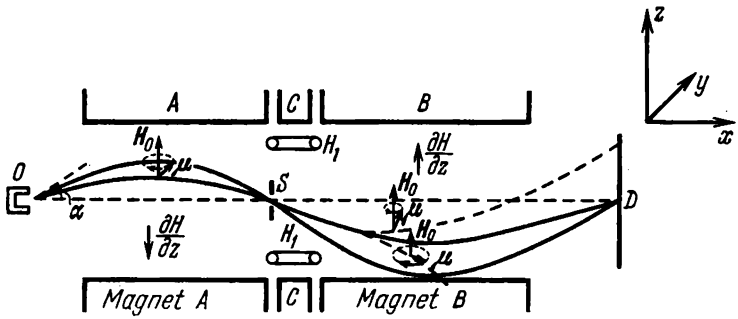

Fig. 3.8. The principal scheme of the experiment on determining nuclear magnetic moments by the magnetic resonant method.

in one field would also precess in the other. However, in the case of a non-adiabatic transition, i.e. the time during which the field changes direction is of the same order of magnitude or less than the period of the Larmor precession, there will be flipping of a certain number of atoms.

Results of all experiments based on the application of magnetic resonance method confirm this conclusion. At the present time there are several new experimental methods concerning the investigation of radio-frequency magnetic spectra of atomic nuclei.

The techniques based on the use of molecular beams are especially widespread. Figure 3.8 shows a principal scheme of the experimental set-up for determining g_I by the method of magnetic resonance in molecular beams. A narrow beam of molecules or atoms, emitted by a furnace *O* (kept at constant temperature) and subsequently shaped by means of diaphragms, passes between the poles of a magnet A and enters a space between magnets A and B; another diaphragm with a slit S is located there, and beyond the interpolar space of the magnet B a detector D is placed (it can be a photographic plate, an ionization chamber, etc.). The orifice of the furnace O, the slit S, and the detector D are all aligned along the direction of the beam in the absence of the deflecting magnetic field.

Magnets A and B create highly non-homogeneous magnetic fields in the direction of the z-axis. $\left(\frac{\partial H}{\partial z} \sim 10^5 \text{ Oe/cm.}\right)$ While these fields are parallel to each other, their gradients are antiparallel as shown in Fig. 3.8. Molecules having non-zero magnetic moments will be deflected in the direction of the field gradient if the projection of the moment on the z-axis is positive ($\mu_z > 0$), and against this direction if the projection is negative ($\mu_z < 0$). Molecules leaving the furnace O along the line OSD (with $\alpha = 0$) will be deflected from this line and will not reach the slit S if their magnetic moment is not vanishingly small and the velocity v is not too great. However, for those molecules which leave the furnace along trajectories forming other angles with the line OSD ($\alpha \neq 0$) there is a possibility of passing through the slit S due to deflection in the non-uniform magnetic field. In general, for a molecule with given μ_z and v it is always possible to find an initial angle α_0 which would ensure the passing through the slit S (see Fig. 3.8).

As follows from (1.4), the force deflecting molecules in a non-uniform field is proportional to $\mu_z(\partial H/\partial z)$. If the magnet B were absent, a beam of molecules deflected from their initial direction by the field of the magnet A after passing through the slit S would not reach the detector D; instead it would hit the screen at a distance

$$d_A = \mu_z \left(\frac{\partial H}{\partial z}\right)_A \frac{\beta_A}{2Mv^2}$$

from the detector. In the above formula M is the molecule's mass and β_A is a factor specified by the geometry of the set-up. Owing to the fact that the gradient of the field of the magnet B is antiparallel to that of the field of the magnet A, the deflection imposed on the molecular beam by the field of the magnet B will be opposite with respect to that produced by the field of the magnet A. If there is no change in the value μ_z of the projection of the magnetic moment during the passage of the molecule from the area of the field of the magnet A into that of the field of the magnet B, the deflection of the beam caused by the field of the magnet B will produce a displacement with respect to detector equal to

$$d_B = \mu_z \left(\frac{\partial H}{\partial z}\right)_B \frac{\beta_B}{2Mv^2}$$

with a sign opposite to that of d_A. Therefore, under the condition of equality of the absolute value of displacements d_A and d_B, the molecules of the beam will be *focused* on the detector D. For a given μ_z this focusing does not depend on the velocity of molecules; it only requires the validity of the following equality:

$$\beta_A \left(\frac{\partial H}{\partial z}\right)_A = \beta_B \left(\frac{\partial H}{\partial z}\right)_B$$

which can be easily ensured by an appropriate modification of dimensions of the equipment and of the magnitude of the gradients of magnetic fields. As has been observed in experiment there is practically no difference in the number of molecules reaching the detector in the presence or in the absence of fields.

Let us now place (see Fig. 3.8) into the space between the slit S and the magnet B a small magnet C, which will produce a uniform magnetic field $\boldsymbol{H}_0$ parallel to $\boldsymbol{H}_A$ and $\boldsymbol{H}_B$. In the same space, in the direction normal to that of $\boldsymbol{H}_0$, a radio-frequency magnetic field $\boldsymbol{H}_1$ is generated by means of two parallel wires located between the pole pieces of the magnet C. After reaching the zone of magnetic field H_0, the molecules with a given value of μ_z start precessing around its direction with the Larmor frequency (3.31). If there is a resonance between the frequency of the field H_1 and one of the frequencies of the transitions between the

Zeeman levels of HfS (allowed by the selection rules for magnetic dipole radiation discussed above), the transitions can take place.

As a result of such transitions (induced by the field H_1) a molecule will change its z-projection of the magnetic moment and after reaching the zone of the non-uniform magnetic field H_B it will be displaced by a distance d'_B different from (i.e. smaller or greater than) d_B. The equality between the displacement d_B in the field of the magnet B and d_A (valid in the absence of H_1) will no longer hold. This, in turn, will result in a decrease in the number of particles reaching the detector D as compared to that for $H_1 = 0$.

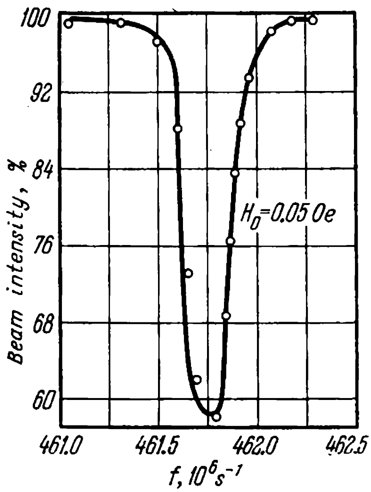

Fig. 3.9. The intensity of an atomic beam (the atoms of ${}_{19}K^{39}$) reaching the detector versus the frequency f of a variable magnetic field H_1 [561].

Studying the dependence of the intensity of the molecular flux which reaches (for a given value of H_0) the detector D via the frequency f of the magnetic field H_1, we can determine a frequency f_{min} corresponding to the minimum of intensity (the *resonance minimum*). It follows that the frequency f_{min} coincides with one of the Zeeman frequencies of HfS spectrum of a molecule for a given value of H_0. We can also do the opposite thing. Namely, to fix the frequency f of the magnetic field H_1 and to vary the intensity of the constant field H_0 in order to achieve the resonance through variation of the Larmor frequencies of molecules, i.e. until these frequencies coincide with the external frequency f.

Figure 3.9 [561] gives a typical picture of the resonance minimum obtained for a beam of ${}_{19}K^{39}$ atoms due to a modulated (by frequency f) magnetic field H_1.

A similar resonance minimum for a beam of molecules of lithium chloride LiCl in a state $^1\Sigma_0$ is depicted in Fig. 3.10 [760]. This curve corresponds to the variation of the magnetic field H_0 at a fixed frequency of the magnetic field H_1.

Substituting the experimental value of $f_{\min}$ for $\nu_H = f$ in (3.31) and solving (3.31) with respect to g_I, we find

$$g_I = \frac{4\pi Mc}{e} \frac{f_{\min}}{H_0} \tag{3.32}$$

Table 3.6 (see [527, 551]) gives the experimental values of $f_{\min}/H_0$ obtained in various series of measurements (for

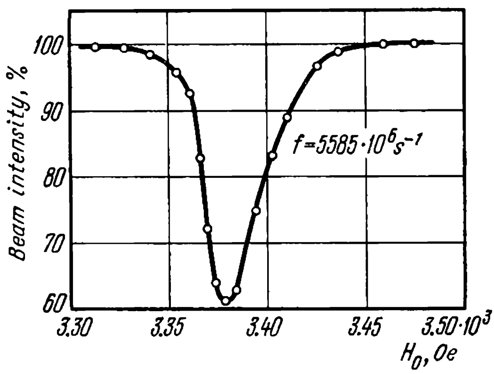

Fig. 3.10. A typical molecular beam resonance curve in nuclear magnetic moment measurements (LiCl molecules in the ground state $^1\Sigma_0$) [760].

different H_0, f, and molecules) in determining the nuclear Lande factor for the nucleus of a lithium isotope Li^7. The dispersion of the results of these measurements is less than 0.5 per cent, which is an evidence of the high accuracy of the method.

The most precise measurements of magnetic moments were those of the magnetic moments of the proton and deuteron [527, 551]. The method, however, does not differ in principle from the above one.

Table 3.6

Values of f_{min}/H_0 Obtained in Molecular Beams in the Determination of the Lande factor g_I for Nucleus Li^7

Type of molecules	$f_{min} \times 10^6$ s^{-1}	H_0, Oe	f_{min}/H_0
LiCl	5.611 6.587 2.113	3399 3992 1278	1651 1650 1654
$Li^7 - Li^7$	3.084	1879	1652
LiF	5.621	3401	1653

6.4. The Method of Microwave Absorption

Recently experiments have been performed on the determination of magnetic moments of the proton and deuteron by means of the microwave absorption method. But instead of molecular beams, the investigators studied solid and liquid samples [39, 112, 113, 268, 758]. These techniques are similar to those of Arkad'ev [36] and Zavoysky [991] (see also [481]). They are based on the analysis of the resonance absorption of the electromagnetic energy of a radio-frequency field by a sample biased by a constant magnetic field H_0. Normal to the direction of H_0 a weak variable field $H_1 \cos \omega t$ ($H_1 \ll H_0$) is imposed. When the frequency of this field ω is close to the Larmor frequency ω_L of a nucleus with magnetic moment μ, given by the relation $\hbar\omega_L = 2\mu H_0$, the radio-frequency field will cause nuclear transitions between states differing in the magnitude of the projection of the nuclear magnetic moment on the direction of the constant field. The system of nuclei in the sample will absorb the energy of the radio-frequency field because the level with the minimal energy will be occupied at most. The magnetic interaction between nuclei of the sample can be described in terms of an effective internal field with the order of magnitude μ/a^3, where a is the distance between the adjacent nuclei in the sample. As one can easily see the magnitude of the effective field is of the order of 10 Oe. It is obvious that the presence of this field causes the broadening of the resonance absorption lines, the resulting line width (in Oe) being of the same order of magnitude as the field. The theory of this phenomenon was analyzed in detail by Van Vleck [941].

Among other features of the described effect we must mention the discovery of the anisotropy of the effect and the fine structure of the spectrum, associated with the details of magnetic interaction [39].

6.5. The Nuclear Resonance Induction Method

Another way of analysing nuclear magnetism (the *nuclear resonance induction* method) consists in measuring the induced emf produced by precessing nuclear magnetic moments when the analysed sample is subjected to simultaneous action of a constant field and a variable field that is normal to the constant one and has a frequency equal to the Larmor precession frequency of the nuclei [111, 758]. For the details concerning these techniques see the monograph of Dorfman [268] and also that of Lösche [607].

6.6. Measurement of the Neutron Magnetic Moment

Of special interest is the problem of determining the magnetic moment of the neutron μ_n. The direct measurement of magnetic moment in a beam of free neutrons (see [357, 461, 755]) is associated with considerable technical difficulties. Besides, due to the fact that it is impossible to obtain a narrow, sharply delineated beam, this method is not sufficiently accurate. Because of this Bloch [109, 110] suggested another way of determining μ_n. The idea of his method is close to that of experiments with magnetic resonance in molecular beams (see Fig. 3.8), but instead of the fields A and B Bloch used a polarizer and an analyzer of neutrons.

When a neutron beam passes through a sample, it becomes subject to scattering, resulting from the interaction of neutrons with the atomic nuclei in the sample (*nuclear scattering*). In the case of fast neutrons (the energy of which is considerably greater than kT) this type of scattering is predominant. On the other hand, for slow neutrons (with thermal velocities) in addition to nuclear scattering there also appears a kind of scattering caused by interaction of the neutron magnetic moment with the magnetic field of the atomic shell. The effect of this *magnetic scattering* is greatest if the magnetic moments of the atoms in the sample are nearly parallel—a situation that takes place, for example, in ferromagnetics.

If we denote by σ_0 the effective cross section for nuclear scattering, and by p the ratio of the effective cross section for magnetic scattering to that for the nuclear one, then it follows from theory [109, 110, 642, 643] that the total effective cross section for the neutron is either

$$\sigma_0(1+p) \quad \text{or} \quad \sigma_0(1-p)$$

depending on whether the neutron spin is parallel ($m_I = +1/2$) or antiparallel ($m_I = -1/2$) to the direction of magnetization of the scattering sample (we shall identify this direction with that of the z-axis). It is assumed that the electron shell acts on a neutron as if the former were a magnetic dipole*; experimental value for p is about 0.1.

Consider an intensive non-polarized neutron beam directed along the x-axis. The beam hits a plane-parallel ferromagnetic plate P normal to it and magnetized in the direction of the z-axis (in Fig. 3.11 the direction of magnetization is shown by an arrow); the thickness of the plate is x_1. Let I_0 be the intensity of the incident beam. In a non-polarized beam half of the neutrons have $m_I = +1/2$, while the other half have $m_I = -1/2$. This means that the beam intensity after it passes through the magnetized plate is equal to

$$I_{\mathrm{m}} = \frac{1}{2} I_0 [\exp\{-nx_1\sigma_0(1+p)\} + \exp\{-nx_1\sigma_0(1-p)\}]$$
$$\cong I_0 \exp(-nx_1\sigma_0)(1+n^2\sigma_0^2 x_1^2 p^2) \tag{3.33}$$

* A detailed and exhaustive analysis of interactions and a criticism of incorrect inferences of Bloch's works [109, 110] the reader can find in articles of Migdal [642, 643].

where n is the number of scattering centres in a unit volume (it is assumed that $p \ll 1$). If the plate P were not magnetized, the intensity of the beam that had passed through it would be

$$I_{\mathrm{nm}} = I_0 \exp(-n x_1 \sigma_0) \tag{3.34}$$

Thus, magnetic scattering increases somewhat the transparency of a magnetized plate P for neutrons ($I_{nm} < I_m$).

The role of the magnetized plate P is that of a polarizer since after the beam passes through the plate P the number

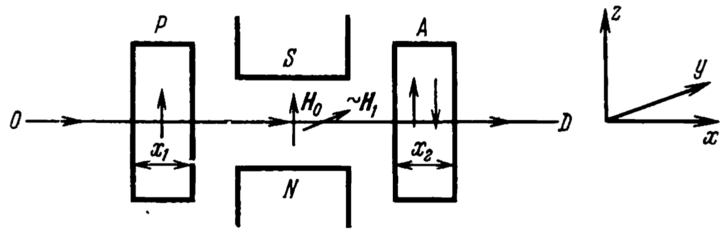

Fig. 3.11. The principal scheme of the experiment for the measurement of the neutron magnetic moment.

$N_{1/2}$ of neutrons with $m_I = +1/2$ becomes less than the number $N{-1/2}$ of neutrons with $m_I = -1/2$. Indeed, it follows from (3.33) that

$$\frac{N_{1/2}}{N_{-1/2}} \cong 1 - 2n x_1 \sigma_0 p \tag{3.35}$$

which means that the neutron beam is partially polarized. If this partially polarized beam of neutrons is transmitted now through a second ferromagnetic plate A (see Fig. 3.11) of thickness x_2, the latter will play the role of an analyzer. If the magnetization of the analyzer A is the same as that of the polarizer P (see the arrow in the direction of the z-axis in Fig. 3.11), the passage of the beam through it is equivalent to the increase in thickness of the scattering substance, with respect to the nuclear and to the magnetic scattering, i.e. the total transmitted beam intensity

$$I_{\mathrm{m}(P+A)} = I_0 \exp[-n\sigma_0 (x_1 + x_2)]\{1 + n^2\sigma_0^2 (x_1 + x_2)^2 p^2\} \tag{3.36}$$

On the other hand, if the magnetization of the analyzer A is antiparallel to that of the polarizer P (see the arrow in Fig. 3.11 in the direction opposite to the z-axis), then, as far as nuclear scattering is concerned, the effect of the plate A is, as before, equivalent to the increase of thickness while for the magnetic scattering thickness x_2 is subtracted from x_1 in the corresponding expression for the total transmitted beam intensity

$$I_{m(P-A)} = I_0 \exp[-n\sigma_0(x_1+x_2)] \{1+n^2\sigma_0^2(x_1-x_2)^2 p^2\} \quad (3.37)$$

When magnetic moment of the neutron is determined by the magnetic resonance method of Alvarez and Bloch [17], a constant magnetic field H_0 directed along the z-axis and a variable radio-frequency field H_1 parallel to the y-axis are simultaneously produced in the space between the polarizer P and the analyzer A (see Fig. 3.11). The neutrons in the partially polarized beam of intensity I_m that has passed the polarizer P will precess around the direction of the field H_0. The number of the "Zeeman levels" for a neutron is only two (since $m_I = \pm 1/2$), and there is only one possible transition between them.

The magnetic field H_1 with the frequency

$$\nu = \frac{\mu_n H_0}{h\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}}$$

will induce transitions between the two Zeeman levels in the beam of neutrons moving in the field H_0. The number of these transitions is determined by the strength of magnetic field H_1 and the probability $W_{1/2,-1/2}$ of such a transition. As a result, part of the neutrons in the beam will change the value of m_I to the opposite; this, in turn, will change the numbers $N_{1/2}$ and $N_{-1/2}$. Further scattering in the analyzer A will cause a decrease in the intensity of that part of the neutron radiation which hits the detector D. For the new value of intensity I' we shall have $I' = I_{H_1=0} - \Delta I$, where $I_{H_1=0}$ is the intensity of the beam of neutrons when $H_1 = 0$. The probability for a neutron to undergo a transition from the state with $m_I = +1/2$ into that with $m_I = -1/2$ during a time interval t is given by (see [115, 759, 825])

$$W_{1/2,\,-1/2} = \frac{\nu^2 \sin^2\theta}{\nu^2 + \nu_H^2 - 2\nu\nu_H \cos\theta}$$

$$\times \sin\{\pi t\,[\nu^2 + \nu_H^2 - 2\nu\nu_H - \cos\theta]\} \qquad (3.38)$$

where $\tan\theta = H_1/H_0$, ν is the frequency of the field H_1 and ν_H is the Larmor frequency of a neutron in the field H_0, the frequency specified by equation (3.31).

As follows from (3.38), the probability $W_{1/2,\,-1/2}$ depends on time, i.e. on the velocity of the neutron in the space

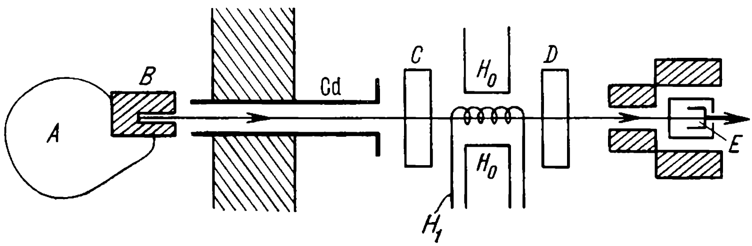

Fig. 3.12. Experimental arrangement for the measurement of the neutron magnetic moment.

occupied by the field H_1. Since a neutron beam is characterized by a wide range of neutron velocities, the square of the sine in (3.38) can be replaced by its average value (1/2). Furthermore, making use of the fact that in experiments $H_1 \ll H_0$ always, we can simplify formula (3.38) to obtain

$$\overline{W}^t_{1/2,\,-1/2} \cong \frac{1}{2}\,\frac{1}{1 + 2\dfrac{H_0}{H_1}\left(1 - \dfrac{\nu_H}{\nu}\right)^2} \qquad (3.39)$$

This expression makes it obvious that the probability $\overline{W}^t_{1/2,\,-1/2}$ has a maximum when the frequency of the field H_1 coincides with the Larmor frequency of the neutron ($\nu = \nu_H$). The resonance will be the sharper the smaller are the values of H_1/H_0.

Figure 3.12 shows the experimental arrangement for the measurement of the neutron magnetic moment by the resonance method of Alvarez and Bloch [17]. The neutron beam is obtained either through a nuclear reaction (for example,

$_4Be^9 + {}_1D^2 = {}_5B^{10} + {}_0n^1$) that results from irradiation of a certain material by a beam of accelerated particles (protons, deuterons) in an accelerator *A* (cyclotron, synchrotron) or from a nuclear reactor. A paraffin block *B* (Fig. 3.12) is used for slowing down the neutrons to thermal velocities. Then a cadmium pipe Cd (which plays the role of a diaphragm) selects a narrow beam of slow neutrons. This beam

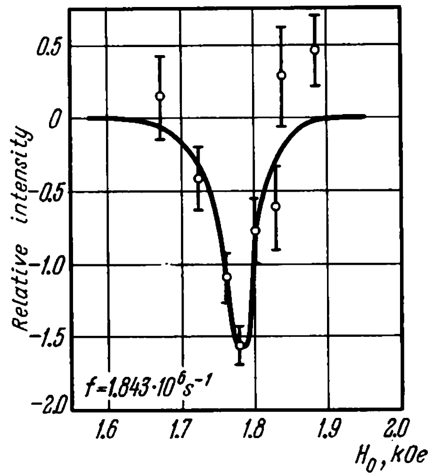

Fig. 3.13. The resonant minimum curve for neutrons. The abscissa shows the values of the static magnetic field H_0 in units of 10^3 Oe.

after passing a polarizer *C* magnetized in the plane normal to the beam (along the *z*-axis) enters the space with a strong constant field H_0 ($\cong$ 600 Oe) directed along the *z*-axis and causing precession of the neutron moments. In the same space the neutrons are subjected to the effect of the radio-frequency variable field H_1 ($\cong$ 10 Oe) induced by a solenoid. Then the beam passes an analyzer *D* and finally reaches a receiver *E* (a chamber filled with boron trifluoride and screened from "alien" neutrons) which measures the intensity of the neutron beam that has passed the whole structure.

Figure 3.13 shows a typical resonance minimum curve obtained with this equipment.

In these experiments the sign of the neutron magnetic moment was also determined [17] and which proved negative. This means that the neutron magnetic moment is antiparallel to the intrinsic angular momentum (spin) of the nuclear (see, for example, the monograph of Kopfermann [551]). In subsequent experiments of Bloch et al. [114] the method of Alvarez and Bloch [17] is combined with the measurements of nuclear induction of protons in the same field H_0 —a technique that allows to determine the neutron moment μ_n itself since the proton moment has been measured with great accuracy.

*6.7. Radioactive Methods**

At present the measurements of magnetic moments of stable isotopes and the long-lived radioactive isotopes of atomic nuclei in the ground state are practically completed. Therefore, in recent years the primary attention is focused on the development and application of the techniques of measurement of magnetic moments in very short-lived radioactive isotopes (with half-lives below 10^{-11} s) in the ground state, and also in excited states of nuclei (see Sec. 4). In both these cases a considerable effort has been made to develop new *radioactive* methods of analyzing the moments of atomic nuclei.

The sphere of application of the techniques described above (NMR, resonance in atomic beams, and the optical HfS method) for radioactive nuclei is limited by the fact that in this case we are usually dealing with extremely short lifetimes and very small number of the sample nuclei (see, for example, a review paper of Khrynkevich and Ogaza [532]). The atomic beam method allows to use microscopic quantities of the sample nuclei. Still, the smallest half-lives among those accessible to this method cannot be less than several minutes (for example, atomic beam techniques allowed to determine the magnetic moment of radioactive nitrogen $_7N^{13}$ with a half-life of 10 minutes).

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* See review papers of Steffen and Frauenfelder [874], Frauenfelder and Steffen [347], Alder and Steffen [12], and Grodzins [417].

The optical HfS method is already much less reliable as far as the microscopic quantities of the analyzed substance are concerned (the irradiation times for the lower limit of the sample mass are specified in this case by the intensity of light). The optical measurement of the magnetic moment of the radioactive isotope ${}_{81}Tl^{199}$, with $\tau_{1/2} = 7.4$ hours, is actually a limit in these techniques. The sphere of applicability of the NMR method is the smallest. The only magnetic moment that was measured by it was that of radioactive ${}_{1}H^{3}$, with $\tau_{1/2} = 12$ years.

Radioactive methods are based on the radioactivity of the analyzed nuclei themselves and on that of the products of their decay. Here we must first of all mention two types of techniques of the greatest practical value: the use of angular correlation between subsequently emitted photons when the emitting nuclei are subjected to the effect of the external, magnetic field, and the Mössbauer effect.

a) The method of angular correlation in subsequent nuclear reactions (see [8, 140]). Consider two subsequent reactions

$$A \to B + C, \quad C \to D + E$$

In the reference frame where the particle C is at rest the process of its decay is characterized by two directions:

$$\boldsymbol{n}_1 = \frac{\boldsymbol{p}_A}{|\boldsymbol{p}_A|}, \quad \boldsymbol{n}_2 = \frac{\boldsymbol{p}_D}{|\boldsymbol{p}_D|}$$

where $\boldsymbol{p}_A$ and $\boldsymbol{p}_D$ are the momenta of particles A and D respectively (in this reference frame). If the density matrices of the initial and final states are unit matrices, angular correlation depends only on $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$:

$$dN \cong W(\boldsymbol{n}_1, \boldsymbol{n}_2)\, d\Omega(\boldsymbol{n}_1)\, d\Omega(\boldsymbol{n}_2)$$

and is given by the function $W(\boldsymbol{n}_1, \boldsymbol{n}_2)$. The correlation is associated with the fact that the particle C with a non-zero spin, owing to its polarization, "remembers" the direction $\boldsymbol{n}_1$, i.e. the states corresponding to different values of projection of spin m_C on the direction of quantization $\boldsymbol{n}_1$ appear, as a rule, with different probabilities (if these conditions are not satisfied, correlation is absent). Experiment has shown that angular correlation is destroyed in those cases when the lifetime of the particle C is so small (less than 10^{-9} s) that it manages to escape the perturbing influence of various external agents (fields). It may be the intra-atomic electric or magnetic fields which interact with the electric quadrupole and magnetic dipole moments, respectively, of an excited nucleus. The resulting angular momentum causes precession of the nuclear spin and thus the change in population numbers of magnetic sub-levels.

Experiment has also revealed various perturbing effects (see [2, 346, 873]) that depend on the nature of the original source of nuclei (single- or polycrystals, liquids). Liquid sources were found to produce the smallest disturbance of angular correlation [2].

The method of detection of magnetic moments of an excited nulcear level consists in an artificial perturbation of angular correlation (PAC) by an imposed externally magnetic field H. A nucleus with magnetic momentum μ and spin I starts precession in this field with the Larmor frequency

$$\omega_L = \frac{\mu H}{\hbar I}$$

regardless of the magnetic quantum number of the nucleus. During the lifetime τ of the excited state this Larmor precession will cause a displacement of the angular correlation curve with respect to that obtained in the absence of the field by the angle of precession $\tau\omega_L$. The curve $W(\theta, H)$ in Fig. 3.14 corresponds to the presence of the external field H; the one denoted by $W(\theta)$ to its absence. These curves were obtained for two subsequent transitions accompanied by emission of photons with energy 136 keV and 492 keV in the decay of the nucleus ${}_{72}Hf^{181}$ [120]. From the measured value of this displacement $\Delta\theta = \omega_L\tau$, on the basis of the

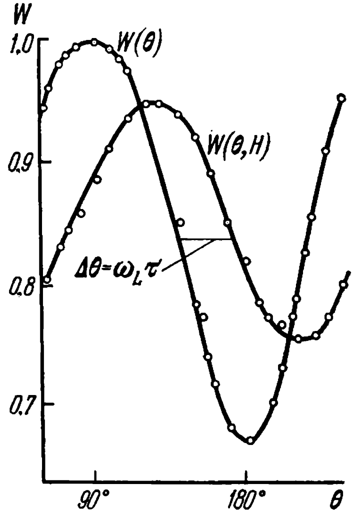

Fig. 3.14. The effect of an external magnetic field H on the angular correlation for two subsequent γ-transitions (136 keV and 492 keV) in the decay of an excited nuclei ${}_{72}Hf^{181}$.

known time τ, one can easily determine the g-factor of the excited state of a nucleus.

Thus, in this radioactive method, as well as in resonance methods described above, we determine the Larmor frequency of an atomic nucleus in an imposed external magnetic field. However, while in resonance measurements with non-radioactive nuclei the latter undergo a very large number of precession rotations during the time of measurement of the frequency of the radio-frequency magnetic field H_1, in the radioactive method the radioactive nuclei cannot even perform one complete rotation during their lifetime. Therefore, in this case the Larmor frequency is determined from the angle and time of rotation of the nucleus.

The application of this method is associated with a number of experimental difficulties, decreasing its accuracy and limiting, in general, the possibilities of its practical usage. For times $\tau \leqslant 10^{-10}$s the displacement $\Delta\theta$ becomes much too small to allow an accurate determination. The greatest troubles arise from the systematic errors caused by the perturbing influence of internal fields.

We must also note that there are two types of methods of angular correlations: the time-differential perturbed angular correlations (DPAC) and the time-integrated perturbed angular correlations (IPAC). Methods of the first group are used for analyzing states with the lifetimes greater than one nanosecond, while those of the second group for states with shorter lifetimes [417].

The reader can find more details of experimental techniques in the review papers of Bodenstedt [120] and Grodzins [417], and also in original investigations referred to in the last column of Table III (see also [215, 260]).

b) The method based on the Mössbauer effect. The term *Mössbauer effect* is used for the phenomenon of resonant absorption of photons without recoil by atomic nuclei of a solid body (nuclear gamma resonance, or NGR), discovered in 1958. In contrast to the case of photons of optical frequencies, emitted by electron shells of free atoms, the event of emission of a photon by a free nucleus is accompanied by considerable recoil of the latter. While for optical photons the ratio of the kinetic energy of recoil to the energy width of the corresponding line in the optical spectrum is of the order of 10^{-3}, the value of the same ratio in the case of nuclear photo-emission is eight orders of magnitude greater (i.e. 10^5). This is exactly the reason why NGR cannot be observed with free nuclei.

Mössbauer discovered that if the emitter nucleus is tightly bound to its neighbours through chemical bonds, the recoil will be transmitted practically to the whole crystal lattice of the body. And since in the expression for the energy of recoil

$$\mathscr{E}_R = \mathscr{E}_\gamma^2/2Mc^2$$

($\mathscr{E}_\gamma$ is the energy of the photon, c the speed of light, and M the mass of the body) the mass of the recoiling object appears in the denominator, the recoil energy $\mathscr{E}_R$ in this case will be infinitesimal as compared to that of a free nucleus. This means that there is a definite possibility of resonant absorption of a photon by the nucleus in its ground state.

This radioactive method, as well as the optical one, makes use of the Zeeman splitting of the hyperfine structure of a gamma line, caused by interaction of intra-atomic magnetic fields with the nuclear magnetic moment.

Successful applications of this method for determining the magnetic and spin characteristics of excited nuclei immediately followed the discovery of the Mössbauer effect [657]. It was used by Hanna et al. [434] for ${}_{26}Fe^{57}$, Obenshain and Wegener [691] for ${}_{28}Ni^{61}$, Hanna et al. [435] and Kistner et al. [540] for ${}_{50}Sn^{119}$, and by Bauminger et al. [69] for ${}_{66}Dy^{161}$. In all these and some other works the authors applied the usual Mössbauer techniques with a radioactive source. Wegener and Obenshain, for example, used the source

$${}_{27}Co^{61} \xrightarrow[99\,\mathrm{min}]{\beta^-} {}_{28}Ni^{61*}$$

However, this method is very limited as far as the number of nuclei is concerned; it has also some other inconveniences. In this connection a new technique was suggested, in which gamma radiation is produced by transition from excited nuclear states populated through the *Coulomb excitation.* (Of course, the nuclei excited in this way should dissipate their kinetic energy acquired in excitation and change over to stable states in the lattice during a small time interval, compared to their lifetime.) This approach leads to a considerable expansion of the sphere of applicability of this method which at present is used in combination with exactly such techniques. The first investigations based on this method were conducted by Seyboth et al. [838] with excited nuclei $_{28}Ni^{61}$, and Lee et al. [588] with $_{26}Fe^{57}$. Among later works we should mention the paper of Ritter et al. [785] with $_{26}Fe^{57}$, articles of Eck et al. [284, 285] for radioactive isotopes $_{70}Yb^{172}$, $_{70}Yb^{174}$, and $_{70}Yb^{176}$, and Wiggins and Walker [969] for an isotope $_{68}Er^{170}$.

In the application of the Mössbauer effect when the magnetic moment of the nucleus in one of the two states participating in the gamma transition is known, it is possible to determine the magnetic moment of the other state and the intensity of the magnetic field on the nucleus.

The limit of applicability of the Mössbauer method as viewed from the shortest possible half-lives τ of the analyzed excited atomic nuclei is specified by the condition that the Mössbauer line-width $2\Gamma = 2\hbar/\tau$* is smaller than the magnitude of the Zeeman splitting $\mu H/I$, i.e.

$$\tau \gg \frac{2I\hbar}{\mu H}$$

It follows from this that the lower limit of the half-life τ is actually of the order of 10^{-9}-10^{-10}s, since $2I\hbar \cong 10^{-27}$ erg·s, and $\mu H \cong 10^{-23} \times (10^{6}\text{-}10^{5}) \cong (10^{-17}\text{-}10^{-18})$ erg.

The determining of the upper limit for τ is associated with purely experimental difficulties of the observation of very narrow lines.

6.8. *Dorfman's Method*

Already some time ago Dorfman [264, 265] noticed that it is possible to observe nuclear paramagnetism in solid bodies. The necessary condition for this is that the paramagnetism of nuclei suppress the electron shell diamagnetism. Since diamagnetic susceptibility does not depend on temperature and paramagnetic susceptibility does very

* Appearance of factor 2 in this formula is a consequence of the fact that the width of an individual component of HfS is produced by the overlapping of a line of the absorber by a line of the source.

sharply increase with temperature decrease, in certain substances the above phenomenon can take place at liquid-helium temperatures.

In his experiments Lazarev (1937) [585] demonstrated the validity of this theoretical prediction of Dorfman's for solid hydrogen. Because of the small value of the Larmor frequency of nuclear precession, the nuclear diamagnetism is about 10^{13} times smaller than the electron one, and therefore the practical significance of nuclear diamagnetism is negligible.

In connection with nuclear magnetism Dorfman [267] indicated that his method of magnetic resonance, now widely used, allows to determine only the nuclear Lande factor g_I. In order to find nuclear magnetic moments it is necessary to determine also the nuclear spin, which can be done, for example, by spectral measurements. Dorfman [267] suggested a new modification of the magnetic resonance method allowing to find simultaneously the magnetic moment and the spin of a nucleus. The idea behind Dorfman's method is to measure the additional magnetic susceptibility χ_n (caused by the orientation of nuclear spins in a constant magnetic field). The susceptibility χ_n is of the order of 10^{-10}-10^{-13}. In usual situations the nuclear magnetism is suppressed by the magnetism of electrons, which exceeds the former considerably. However, if a weak variable field H_1 is imposed normally to the strong constant field H_0 the frequency of $H_1^{\cdot}$ being in resonance with the Larmor frequency ω_L of nuclear spins, the contribution of a nuclei in the paramagnetic susceptibility of the sample can be excluded.

Dorfman suggested the following experimental arrangement [267]. The substance under investigation is placed in an ampule AB located symmetrically between the poles NS of an electromagnet (Fig. 3.15). The ends of this ampule (A and B) are in the non-uniform magnetic field. Since the field is symmetric, the ampule is in equilibrium. On the other hand, if at the end A the radio-frequency field H_1 (normal to $\boldsymbol{H}_0$) is generated, the nuclear moments will precess around the direction of the field $\boldsymbol{H}_1$ and their orientation will deviate from the direction of $\boldsymbol{H}_0$. This means that the end A of the ampule will be subject to the action of a force

$$K = \chi_n v H_0 \frac{dH_0}{dz}$$

(v is the volume of the part of the ampule that is in the field H_1) tending to shift the ampule to the left. This force can be measured in principle by means of a sensitive torsion balance. It will reach its maximum value at resonance, i.e. when

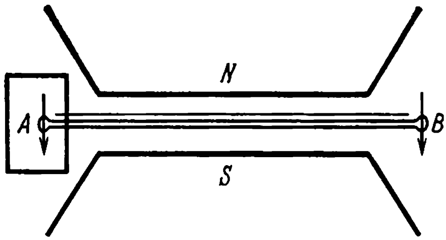

Fig. 3.15. The scheme of Dorfman's experiment [267] for the measurement of nuclear magnetic moments.

$\omega_{max} = \omega_L$. From the measurement of the frequency ω_{max} of the field H_1 at resonance we find the values χ_n and ω_L that are related to the unknown quantities g_I and I. Namely, as follows from (3.31),

$$\omega_{max} = \frac{g_I H}{\hbar} \mu_N$$

and from the formula for paramagnetic susceptibility we obtain

$$\chi = \frac{4N\mu_N^2}{3kT} I (I+1)$$

Here N is the number of atomic nuclei of a given kind in a unit volume and T is the absolute temperature. These equations allow to compute I, g_I and, therefore, μ_I using only one experimental device.

6.9. Determination of Nuclear Magnetic Moments in Liquids and Crystals

In addition to the methods of detecting nuclear magnetic moments in free atoms and molecules (spectroscopic techniques, atomic and molecular beam methods, resonance methods) in wide use now are the techniques of determining these moments in liquids and crystals. Here the investigators usually study NMR, quadrupole nuclear resonance, and paramagnetic resonance. These methods are highly accurate. Besides, in situations when the spin and the magnetic characteristics of the nuclei are known they are extremely valuable from the point of view of the analysis of the electron structure of solid bodies. For more details of the first aspect of this method (i.e. the study of magnetic moments of individual atoms and ions in condensed phases) see, for instance, the monographs of Dorfman [268], Kopfermann [551], and the review paper of Mack [614], as well as the original articles referred to in Tables II and III in the Appendix.

7. Magnetic Properties of Crystalline $_2He^3$*

In conclusion we shall discuss another manifestation of nuclear magnetism, this time not for isolated atomic nuclei but for the case of condensed phase—the nuclear magnetic properties of solid (crystalline) $_2He^3$.

As can be seen from Table II in the Appendix, the atomic nucleus of the rare isotope $_2He^3$ (under natural conditions its abundance is only 1.3×10^{-4} per cent) has spin 1/2 and a magnetic moment -2.127490 (5) μ_N. The electron shell of a helium atom is characterized by a zero orbital angular momentum (*s*-state) and a zero spin (the spins of individual electrons compensate each other).

Thus, one can expect that in condensed phase (liquid and crystalline) $_2He^3$ exhibits the magnetic properties which can be described as a combination of electron diamagnetism and nuclear paramagnetism.

Theoretical interpretation of the dynamics of atoms of helium isotopes in a liquid or a solid state can be obtained only on the basis of quantum theory, while other liquid or crystalline systems permit also a classical analysis. Thus, liquid or solid helium is the exceptional case of a quantum liquid or a quantum solid body (crystal) respectively.

* See also the collective monograph of Eselson et al. [309], Chapter 9, Section 3.

The unusual properties of crystalline $_2He^3$ result from the fact that due to the small mass of a helium atom and the weakness of the van der Waals interaction the energy of their zero oscillations is relatively large. Therefore, in order that liquid $_2He^3$ (or $_2He^4$) could solidify even at $T \cong 1$ K, it is necessary to apply a pressure no less than 30 atm. At the same time the relative rms deviation is about 30 per cent of the intra-atomic distance in equilibrium between the closest neighbours (see, for example, [83] and also a review [424]). It is these peculiarities of crystalline $_2He^3$ that were responsible for its special nuclear properties. In particular, in this crystal we find a relatively large exchange-interaction energy between nuclear spins, which allows to observe the effects of ordering of nuclear spins at extremely low but still accessible temperatures (not exceeding 10^{-3} K).

There are two kinds of terms in the effective Hamiltonian of solid $_2He^3$ that depend essentially on spin [83]. First, it is the spin-spin magnetic interaction whose energy per atom is of the order of

$$\Delta\mathscr{E}_1 \cong \frac{z}{2}\frac{\mu^2}{a^3}$$

where z is the number of closest neighbours of a given atom in a crystal, μ is the nuclear magnetic moment, and a is the distance between adjacent lattice points. Secondly, there is the exchange isotropic interaction between the nuclei of adjacent atoms that depends on the rms deviation δ of a nucleus from its lattice point. Its energy per atom is

$$\Delta\mathscr{E}_2 \cong \frac{z}{2}[\alpha_1 W(a, \delta) + \alpha_2 U(a, \delta)] \exp\left[-\frac{3}{4}\left(\frac{a}{\delta}\right)^2\right]$$

where W and U are the mean kinetic energy and the mean potential energy per atom in a crystal, respectively, and α_1 and α_2 are numerical constants with the absolute values of the order of unity.

It was assumed at first that the term $\Delta\mathscr{E}_1$ is the dominating term [751] since the exchange effects in "heavy" solid bodies are negligible due to the fact that $\delta/a \ll 1$. The energy of spin-spin interaction $\Delta\mathscr{E}_1$ guarantees the temperature of magnetic ordering ($z \cong 10$, $a \cong 3 \times 10^{-8}$ cm, $|\mu| \cong$

$\cong 10^{-23}$ erg$\cdot$Gs^{-1}

$$T_{c(\mathrm{dip})} \cong \frac{\Delta\mathscr{E}_1}{k} \cong \frac{z\mu^2}{ka^3} \cong \frac{10\times 10^{-46}}{10^{-16}\times 10^{-23}} \cong 10^{-6}\ \mathrm{K}$$

However, since the ratio δ/a in crystalline $_2He^3$ is of the order of 0.3-0.4, i.e. not small, the orbitals of adjacent nuclei overlap to a considerable degree. This leads to a much stronger spin correlation than that ensured by magnetic spin-spin interaction. If we assume [83] that $W \cong -U \cong \cong 10$ K, $\Delta\mathscr{E}_2$ guarantees a temperature of magnetic ordering

$$T_{c(\mathrm{ex})} \cong \frac{\Delta\mathscr{E}_2}{k} \cong z\alpha\,|W|\exp\left(-\frac{3}{4}\times 9\right) \cong 0.05\ \mathrm{K}$$

This leads to a conclusion that solid $_2He^3$ is indeed the only nuclear paramagnetic with a considerable nuclear spin ordering at experimentally accessible temperatures of the order of 5×10^{-2} K.

This was confirmed in measurements using NMR, spin relaxation, and spin diffusion (see, for example, a review of Meyer [639] and articles cited by him, and also the paper of Giffard et al. [382]), in measurements of pressure as a function of temperature for constant volume $p_V(T)$ [713, 714, 835], and in measurements of nuclear magnetic susceptibility (see, for example, [211, 506, 539, 744, 745, 849, 915]). Measurements of susceptibility are the only ones that allow to determine not only the magnitude but also the sign of the parameter J of exchange interaction.

It must be noted that owing to the simple structure of the body-centred cubic lattice (bcc)*, and also because of the absence of noticeable magnetic asymmetry, solid $_2He^3$ is an ideal example of a Heisenberg magnetic** (see [425] and works cited there).

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* In the pressure range 30-110 atm the crystallization of $_2He^3$ occurs in the bcc lattice. Above 110 atm the stable lattice is hcp (hexagonal close-packed lattice). The existence of a stability range of "loose" bcc lattice can be explained by a smaller mass of the lighter helium isotope and therefore by a greater amplitude of zero oscillations, which favours the formation of a less dense bcc lattice.

** The term "Heisenberg magnetic" usually means a crystal whose nodes contain spin magnetic moments with isotropic electrostatic exchange interaction between them (see, for example, [945]).

The most accurate analysis of nuclear magnetic susceptibility of pure solid $_2He^3$ (with the admixture of $_2He^4$ not exceeding 1×10^{-5}) (in the temperature range 0.4-0.04 K was conducted by Pipes and Fairbank [744, 745]. The authors demonstrated that in this interval susceptibility obeys the Curie-Weiss law $\chi = C/(T - \Theta)$, where $\Theta = -1.5 \times \times 10^{-3}$ K for molar volume $V_m = 23.3$ cm^3/mole, $\Theta = = -4 \times 10^{-3}$ K for $V_m = 23.6$ cm^3/mole, and $\Theta = = -5.4 \times 10^{-3}$ K for $V_m = 24.2$ cm^3/mole (this is in agreement with theoretical predictions to the effect that the exchange energy $J = 2k\Theta/z$ grows with the decrease in density or molar volume V_m; see [83, 368, 425, 455, 687, 688, 812, 916]). The negative sign of temperature Θ points to the fact that the ordering of spins of atomic nuclei in solid $_2He^3$ at very low temperatures (< 0.04 K) is antiferromagnetic.

This conclusion is also in agreement with results of Sites et al. [849], Kirk et al. [539], and Johnson and Wheatley [506]. However, the measurements of Osgood and Garber [706]* and especially those of Kirk and Adams [538], who were interested in pressure as a function of temperature for constant volume $p_V(T)$ in strong magnetic fields (40.3 and 57.2 kGs), do not agree with predictions of the theory [52] (based on high-temperature expansion of the distribution function $Z\,(T,\ H)$ for $T > T_c$ in powers of a small parameter J/kT). This anomaly of magnetic properties of solid $_2He^3$, expressed in the disagreement between the Heisenberg model and experiment, has not been explained yet. (Kirk and Adams [538] suggested that this could be due to taking into consideration the interaction only between the adjacent nuclei, poor convergence of the series, etc.)

In connection with this difficulty we should also mention the fact that a number of authors [448, 451, 716, 809] have revealed that the specific heat of the bcc phase of solid $_2He^3$ does not conform to Debye T^3 behaviour at low temperatures. Through precision strain-gauge measurements in very pure $_2He^3$, Henriksen et al. [451] have removed any doubts that this anomalous behaviour could be due to impurities or "apparatus "effects.

* This paper contains some inaccuracies in thermometry (for analysis of this question see [538]).

According to Varma [942] this specific-heat anomaly arises from a phonon-mediating long-range spin interaction in solid ${}_2He^3$, which provides a contribution to the specific heat varying as T^{-2}. In bcc solid ${}_2He^3$ the results are in good agreement with the above experiments. For hcp solid He^3 the theory predicts that the specific-heat anomaly will occur at temperatures lower than have been investigated experimentally but which are easily accessible. Varma [942] points out, however, that his Hamiltonian of indirect exchange does not contribute to the zero-field susceptibility for high T.

It is also of interest to consider the effect of ${}_2He^4$ admixture on the magnetic properties of solid ${}_2He^3$. This problem was treated in a series of papers, both theoretical [54, 55, 394, 496] and experimental [84, 85, 212, 451, 654, 782].

Another specific feature of the condensed state of helium ${}_2He^3$, related in a way to its magnetic properties, is the *Pomeranchuk effect* [751]. It is associated with the fact that the exchange interaction in a Fermi system leads to magnetic correlation of nuclear spins already at temperatures of the order of Fermi degeneration for ${}_2He^3$, i.e. $T_F \cong 5$ K. At these temperatures there is no noticeable magnetic correlation yet, and the presence of magnetic ordering of nuclear spins in the liquid phase and its absence in the crystal causes the enthropy of the crystal S_{cryst} for temperatures in the range $T_{c\,(\text{ex})} < T < T_F$ to exceed the enthropy S_{liquid} of the liquid phase (S_{liquid} is proportional to $R \ln 2$), whereas in ordinary situations the relation is reverse: $S_{\text{cryst}} < S_{\text{liquid}}$. The most striking manifestation of this effect is the appearance of a minimum on the curve of phase equilibrium (in the pT plane) of helium. This phenomenon has been confirmed in experiment and is used now for obtaining extremely low temperatures (see the review of Trickey et al. [925]).

To conclude this chapter let us emphasize once more that the problem of nuclear magnetism of solid ${}_2He^3$ as well as of its liquid phase is still a current issue of magnetism, containing quite a number of unsolved and at the same time interesting questions of very general significance.

ANOMALOUS MAGNETIC MOMENTS OF ELEMENTARY PARTICLES

1. Introduction

In Chapter 3 we have already pointed out that the magnetic moments of nucleons, i.e. of the proton and the neutron, differ substantially from the value specified by the nuclear magneton (which they would have if their motion was described by Dirac's equation of motion, i.e. (1.8) and (1.9)). Interpreting the observed values of nucleon magnetic moments, μ_p and μ_n, as the sum of the nuclear magneton μ_N and a certain anomalous addition to it $\Delta\mu$, we obtain (see (3.2) and (3.3))

$$\mu_p = \mu_N + \Delta\mu_p = \mu_N + 1.792782(17)\ \mu_N$$

$$\mu_n = 0 + \Delta\mu_n = 0 + 1.913148\ \mu_N$$

These formulas make it clear that the anomalous addition terms to the "Dirac" moment are far from being small and cannot be regarded as unessential corrections to Dirac's theory.

On the other hand, relatively large anomalies did not arise in the analysis of magnetic properties of the electron and its antiparticle, the positron. Lately, however, it was discovered that these light elementary particles also exhibit anomalous discrepancies between their magnetic moments and Bohr's magneton. In contrast to the case of nucleons the additional term in the expression for the electron magnetic moment proved to be very small: it exceeded slightly one thousandth of μ_B. It would be of interest to investigate not only the nature of the anomaly of the electron magnetic moment itself but also the reason for its *relatively small** value in comparison with the anomalous additions to μ_N for nucleons.

To this end it is necessary to review in brief the general method of the modern theory of quantized fields for description of properties of elementary particles and their interactions.

The point is that already from non-relativistic quantum theory of the electron (and also from relativistic quantum theory and quantum electrodynamics) we know that every material field, because of its quantum nature, can be described as an association of microparticles, the quanta of excitation of this field. Vice versa—each collective of particles, because of its wave nature, can at the same time be interpreted as a material field. Thus, in the case of electrons and positrons we are dealing with the electron-positron quantized field, and in the case of electromagnetic fields with the association of light quanta, the photons. If no interaction between electrons and positrons through an electromagnetic field existed, their motion would be described only by Dirac's equation (free electron-positron field). These particles, however, interact with the field of photons. Therefore, a consistent microscopic theory should take these fields into account simultaneously. This is exactly what is done in modern quantum electrodynamics.

The modern version of quantum electrodynamics is based on the fact that interaction between electrons and positrons on one hand and the electromagnetic field on the other is not very strong and, therefore, theoretical computations can be carried out in the frame of the perturbation theory. The energy of interaction of the electron with the electromagnetic field is determined by the value of the electron charge e. This quantity, however, is dimensional and thus depends on the choice of physical units. Therefore, it cannot in itself specify the value of a dimensionless parameter that gives the measure of electromagnetic interaction. In order to es-

* The absolute value of these anomalous additions in the case of the electron and the positron appears to be of the same order of magnitude as the respective quantity for nucleons (see in this connection a remark at the end of Sec. 2.2 of this chapter).

timate the magnitude of the interaction we must compare it with that of some other interaction or else find a different, dimensionless parameter for its specification, which is independent of the choice of units and which can be compared to unity.

In quantum electrodynamics it appeared possible to find such a parameter. This parameter contains fundamental quantities appearing in the equations of the interacting quantum fields (i.e. in Dirac's equation and in Maxwell's equations). It is the small dimensionless fine-structure constant

$$\alpha = \frac{e^2}{\hbar c} = [137.03602\,(21)]^{-1} = 7.297351\,(11) \times 10^{-3} \qquad (4.1)$$

($c = 2.9979250(10) \times 10^{10}$ cm/s*). Therefore, the interaction of electron-positron and electromagnetic fields can be regarded as weak, which allows to employ methods of the perturbation theory for the calculation of various effects (for instance, for different kinds of scattering processes, for absorption and emission of photons by electrons, etc.).

At first there appeared considerable difficulties since only the first approximation of the perturbation theory produced results that agreed well with experimental data. The results of the second and higher approximations were divergent. It proved possible, however, to overcome this difficulty in quantum electrodynamics (at least in those cases when the major role belongs to the electromagnetic interaction, and other types of interactions, such as mesonic, can be ignored).

The elementary particles that can be described by such a well-developed field theory as quantum electrodynamics are the electron, the positron, the mu-mesons (muons), and the photon. As for other elementary particles possessing the elementary electric charge (protons, positive and negative pions, $K^{\pm}$-mesons, etc.), their electromagnetic interaction can be influenced by more strong interactions (such as mesonic forces, for which up till now no well-developed theory exists).

* For a summary of data on the values of world constants see [902].

2. Lamb-Retherford Shift of Atomic Levels and the Anomalous Magnetic Moments of the Electron and Positron

Let us first clarify the nature of the anomalous additive term in the expression for the magnetic moment of the *electron* and the *positron*. In this book, naturally, we do it in a rather approximate way. A more detailed discussion on this topic is given in monographs on quantum electrodynamics ([9]; see also [81, 94, 369, 563, 602, 683]).

As follows from the quantum theory of electromagnetic field, if the non-linear effects (such as scattering of light on light) are ignored, this field can be presented as a set of elementary excitations of certain effective oscillators, quasi-particles (light quanta, or photons) with a discrete energy spectrum. The energy of a single i-th elementary excitation (the field oscillator, a quasi-particle) can only assume the following discrete values:

$$\mathscr{E}_i = \left(n_i + \frac{1}{2}\right) \hbar\omega_i \tag{4.2}$$

where $n_i = 0, 1, 2, \ldots$ are quantum numbers specifying the number of quasi-particles of the type i. However, in the absence of field excitations, i.e. when all $n_i = 0$, the energy of field is not equal to zero; instead, it is given by

$$\mathscr{E}_0 = \sum_i \frac{1}{2} \hbar\omega_i \tag{4.3}$$

This quantity is called the *zero energy of the ground state* of the field, while the state itself is called *electromagnetic vacuum*.

From the point of view of the classical theory the absence of excitations would mean the absence of the field. This is not so in quantum electrodynamics since, as we have seen, when $n_i = 0$, the energy of the field has a non-zero value. One of the predictions of the modern theory is that $\mathscr{E}_0 \to \infty$, which is, of course, a defect of the theory. The important thing, however, is the existence of the zero energy of the field and its possible *fluctuations*. It is the interaction of the electron with these zero fluctuations of the vacuum of electromagnetic field that causes a number of specific (*radiation*) effects.

2.1. Lamb-Retherford Shift

The first of such effects observed in experiment was the shift of terms in hydrogen-like atoms (the *Lamb shift*)*. From the solution of the wave equation in the relativistic theory of the electron it follows that the terms of hydrogen-like atoms do not depend on the orbital quantum number l and are completely determined by the *total* quantum number j**. This results in the fact that the respective terms are two-fold degenerate, which corresponds to the two possible values of l for a given j:

$$l = j \pm \frac{1}{2} \tag{4.4}$$

Terms of the type $1^2S_{1/2}$, $2^2P_{3/2}$, etc., i.e. the terms with the maximal value of l for a given principal quantum number n, must be excluded (these terms correspond to parallel orientation of spin and orbital angular momentum). On the other hand, according to experiments this theoretically predicted degeneracy does not actually exist. For example, the experimental work of Lamb and Retherford [564] demonstrated that the energy interval between degenerate terms $2^2S_{1/2}$ and $2^2P_{1/2}$ in the hydrogen atom is equal to 1057 ± 0.10 MHz (in frequency units; 1 MHz corresponds to 0.33×10^{-4} cm^{-1} in wavelengths and to 4.1×10^{-9} eV in energy units).

At first this appeared to be a real catastrophe for Dirac's theory. Very soon, however, this catastrophe was completely resolved: it was found that if in the original wave equation for the electron one takes into account the interaction of the electron with the zero-oscillation field, the theory predicts the observed level shift with a remarkable accuracy (1040 MHz for hydrogen; [90]).

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* See [563, 855]; also [486, 902].

** See, for example, Sec. 44 in Schiff's monograph [817], where the energy levels for FS of the hydrogen atom are given by the following formula (accurate to the additive constant mc^2):

$$\mathscr{E}_{nj} \cong -\frac{\hbar R}{n^2}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right]$$

where R is the Rydberg constant; n is the principal quantum number; and the fine-structure constant α is given by formula (4.1).

A similar striking agreement of theoretical and experimental results was obtained for deuterium and for the helium ion. The level shift in lithium ion $(Li^6)^{++}$ was analyzed experimentally by Fan et al. [313]; the theoretical computation of this shift was performed by Erickson [303]. Erickson obtained a very good agreement with experiment. Namely, for the level shift $2S_{1/2} - 2P_{1/2}$ in $(Li^6)^{++}$

$$62739 \pm 47.1 \text{ MHz (theory)}$$

and

$$63031 \pm 32.7 \text{ MHz (experiment)}$$

The effect of the shift of electron terms in an atom yields a very simple pictorial interpretation, which was suggested by Welton [958]. The external field for the electron in an atom is the Coulomb field of the nucleus $V(\boldsymbol{r})$ ($\boldsymbol{r}$ is the radius vector of the electron). The zero oscillations of the electromagnetic field will cause a certain "Brownian" displacement $\Delta\boldsymbol{r}$ of the electron, and therefore the field acting on the electron in the atom should be computed not at the point $\boldsymbol{r}$ but at $\boldsymbol{r} + \Delta\boldsymbol{r}$. Making use of the series representation for the potential $V(\boldsymbol{r} + \Delta\boldsymbol{r})$ in powers of small displacement $\Delta\boldsymbol{r}$ and discarding terms of the second order and higher, we obtain an additional variation of the mean potential energy of the electron in an atom, which should be identified with the observed levels' shift.

Appelquist and Brodsky [30] used the case of hydrogen-like atoms as an example for carrying out computations and comparing, in detail, the obtained theoretical results with those of more recent experimental investigations (up to the accuracy of the fourth-order radiative corrections $\cong \alpha^2 (Z\alpha)^4 mc^2/n^3$). They point out that the only case when the experimental results do not coincide with the predictions of modern quantum electrodynamics is the Lamb-Retherford shift for transitions $2S_{1/2} - 2P_{1/2}$ and $2P_{3/2} - 2S_{1/2}$ in the atoms of hydrogen (H^1) and deuterium (H^2). Table 4.1 gives the results of comparison; it also contains references to the original sources. As one can see from the table, there is a tendency towards better agreement between theory and experiment.

Lamb-Retherford Shift $\Delta\nu$ for Transition $2P_{3/2}-2S_{1/2}$ in Hydrogen-Like Atoms (in MHz)

Comparison of experimental values $\Delta\nu_{exp}$ with the old theoretical values $\Delta\nu'_{theor}$ and the new theoretical values $\Delta\nu''_{theor}$ from [30]

Atom	$\Delta\nu_{exp}$	Source	$\Delta\nu'_{theor}$	$\Delta\nu_{exp}-\Delta\nu'_{theor}$	$\Delta\nu''_{theor}$	$\Delta\nu_{exp}-\Delta\nu''_{theor}$
H^1 $(n=2)$	1057.77±0.06	[926]	1057.56±0.09	0.21±0.07	1057.91±0.16	−0.14±0.08
	1057.90±0.06	[787]		0.34±0.07		−0.01±0.08
	1057.65±0.05	[526]		0.09±0.06		−0.26±0.07
	1057.78±0.07	[844]		0.22±0.08		−0.13±0.09
	1057.86±0.06	[947]		0.30±0.07		−0.05±0.08
H^2 $(n=2)$	1059.00±0.06	[926]	1058.82±0.15	0.18±0.15	1059.17±0.22	−0.17±0.09
	1059.28±0.06	[234]		0.46±0.08		+0.11±0.09
He^+ $(n=2)$	14040.2±1.8	[606]	14038.9±4.1	1.3±2.2	14044±5.2	−4.3±2.5
	14045.4±1.2	[670]		6.5±1.8		1.0±2.1
He^+ $(n=3)$	4182.4±1.0	[615]	4182.7±1.2	−0.3±1.1	4184.4±1.5	−2.0±1.1
	4184.0±0.6	Ditto		1.3±0.7		−0.4±0.8
He^+ $(n=4)$	1776.0±7.5	[443]	1768.3±0.5	−2.3±7.5	1769.0±0.6	−3.0±7.5
	1768.0±5.0	[494]		−0.3±5.0		−2.0±5.0
Li^{++} $(n=2)$	63031.0±327.0	[313]	62743.0±45.0	288.0±333.0	62771.0±50.0	260.0±333.0

In connection with the problem of comparison of experimental data on the Lamb shift with quantum-electrodynamic calculations let us mention a very interesting paper of Leventhal and Murnick [592]. In this article the authors determine the Lamb shift for hydrogen-like five-fold ionized carbon atoms $^{12}C^{5+}$. The interesting aspect of this paper is that the measurement of the Lamb shift in hydrogen-like atoms with a large value of the nucleus charge Ze allows to carry out a decisive experimental verification of modern ideas on virtual radiation processes, which include electron and photon fields.

The point is that these processes (i.e. the emission and reabsorption of virtual photons by an electron bound in an atomic orbit), resulting in the Lamb shift, are strongly influenced by the external Coulomb field of the charge of the nucleus. It is for this reason that the experimental verification of the theory of systems with $Z \gg 1$ is of special interest. For the carbon ion $^{12}C^{5+}$ $(Z = 6)$ Leventhal and Murnick [592] found that the Lamb shift is 744×10^3 MHz, whereas according to the theory (see, for example, [30]), it should be 738×10^9 MHz. This discrepancy exceeds all possible errors of experiment. This is why the authors of [592] are prone to interpret this discrepancy as an indication of some effects in radiation corrections that have not yet been accounted for.

Further improvements on the accuracy of computations of the Lamb shift one can find in [64, 158, 557, 584, 593, 670].

2.2. Corrections to the Electron Magnetic Moment

One of the by-products in formulating the relativistic theory of the term shift was the computed value of the addition to the electron *magnetic moment* [826, 827]*. In the presence of external magnetic and electric fields an additional term appears in the expression for the energy increment (respon-

* These papers appeared after the experimental work of Nafe et al. [668], where the authors found that $\mu_e \neq \mu_B$ (see material below), and after Breit's formulation of some qualitative considerations [146, 147, 148].

sible for the term shift) indicating the difference between the electron magnetic moment μ_e and the Bohr magneton μ_B (1.2a), i.e.

$$\mu_e = \mu_B\left(1+\frac{1}{2\pi}\frac{e^2}{\hbar c}\right) = \mu_B\left(1+\frac{\alpha}{2\pi}\right) \quad (4.5)$$

Substituting in the right-hand side of (4.5) the numerical value for α (see (4.1)), we obtain

$$\mu_e = \mu_B\,[1 + 0.0011591383(31)] \quad (4.6)$$

Let us note that the derivation of equation (4.5) is free from the difficulties of the infinite electron mass, and this formula, therefore, does not contain any ambiguities. In order to arrive at this result it is sufficient to compute the difference between the energies of two systems: the *first* one consists of filled levels with negative energy and a single electron occupying a level with positive energy, the system immersed in a weak magnetic field; the *second* one contains only the electrons filling the levels with negative energy, in the same field. Retaining terms linear with respect to the field, we arrive [612] at the final result (4.5).

The concepts of electron spin and its magnetic moment are not visually obvious. Therefore, it is impossible to give a qualitative description of the effect of variation of the magnitude of the electron magnetic moment, which results from the interaction with the radiation field—a description that was possible in explaining the Lamb-Retherford shift of energy levels presented above. An interpretation of this phenomenon can be obtained if one recalls (see equations (2.3)-(2.8)) that spin magnetic moment of the electron results from the existence of "circular currents" with the radius of an elementary circuit about $\hbar/mc$. These currents are to some extent influenced by the zero fluctuations of electromagnetic field, which in turn produce a fluctuation of the current induced in the "vacuum" (i.e. in states with negative energy). It is this interaction that leads to a small variation in the value of magnetic moment predicted by the theory.

Experimental discovery of the anomaly of the electron magnetic moment (4.5) occurred in the study of the Zeeman effect in hydrogen and deuterium using the magnetic resonance method (Chapter 3 and [668]). These techniques allowed to determine the magnitude of hyperfine splitting of energy levels in the electron shells of hydrogen and deuterium atoms (through the extrapolation to the zero value of magnetic field) on the basis of measurement of the resonance frequency for several transitions. If one computes the effective field H_{el} in (3.21), the theoretical value for the hyperfine level splitting (in terms of frequency) is

$$\Delta\nu = \frac{4}{3}\,\frac{2I+1}{I}\,\mu_N \mu_e\,|\psi(0)|^2 \qquad (4.7)$$

where I is the spin quantum number equal to 1/2 for the hydrogen nucleus and 1 for the deuteron; μ_N is the nuclear magnetic moment; and $\psi(0)$ is the non-relativistic wave function of the electron at the centre of the atom. Experience has shown that equation (4.7) does not agree with the observed values if in the expressions for $\Delta\nu_{H^1}$ and $\Delta\nu_{H^2}$ we substitute the usual quantity μ_B instead of μ_e. Table 4.2 contains a comparison of experimental and theoretical values of respective frequencies (see [668]).

Table 4.2

Comparison of Experimental and Theoretical Values of the Frequencies of Hyperfine Structure

Hf Splitting	Experiment (in MHz)	Theory* (in MHz)	Ratio between experimental and theoretical values
$\Delta\nu_{H^1}$	1421.3 ± 0.2	1416.90 ± 0.54	1.00242 ± 0.0004
$\Delta\nu_{H^2}$	327.37 ± 0.03	326.53 ± 0.16	1.00262 ± 0.0003

* The errors result from the inaccuracy in the determination of constants in formula (4.7).

If our purpose is to compute, on the basis of experimental data for $\Delta\nu_{H^1}$ and $\Delta\nu_{H^2}$, the electron magnetic moment from formula (4.7), where $\mu_e \neq \mu_B$, it should be borne in mind that in measuring the magnetic moments of atomic nuclei by the method of radio-frequency magnetic spectroscopy in molecular beams we actually measure the ratio of the magnetic moment of the nucleus to that of the electron. This is a consequence of the fact that in such experimental devices the constant magnetic field is calibrated by the measurement of fine structure, and computation of the magnitude of the field strength from the observed splitting is based on the value of magnetic moment of the electron. Because of this, the practical application of equation (4.7) will be easier if it is written in the following form:

$$\Delta\nu = \frac{4}{3}\,\frac{2I+1}{I}\,\mu_e^2\,\frac{\mu_N}{\mu_e}\,|\psi(0)|^2 \tag{4.8}$$

Substituting the value of μ_e from (4.5), we find for the ratio of the experimental value $\Delta\nu_{\mathrm{exp}}$ to the theoretical value $\Delta\nu_{\mathrm{theor}}$ the following:

$$\frac{\Delta\nu_{\mathrm{exp}}}{\Delta\nu_{\mathrm{theor}}} = 1 + 2\,\frac{\alpha}{2\pi} \tag{4.9}$$

From the numbers in the last column of Table 4.2 it follows that

$$\left(\frac{\alpha}{2\pi}\right)_{\mathrm{H}^1} = 0.00121\,(20) \quad \text{and} \quad \left(\frac{\alpha}{2\pi}\right)_{\mathrm{H}^2} = 0.00131\,(15) \tag{4.10}$$

The obtained values for the ratio $\alpha/2\pi$ in (4.10) are in excellent agreement with the theoretical values (see (4.5)-(4.6)).

In addition to the case of the hydrogen atom and the deuterium atom the anomaly of the electron magnetic moment was also discovered in the radio-spectroscopic analyses of other atoms. For example, Kusch and Foley [560] determined the magnetic moment of the electron in monovalent atoms of sodium, indium, and gallium using the results of experiments based on the ordinary magnetic resonance method of the Zeeman splitting of spectral lines in weak magnetic fields.

Owing to the difficulties of precise measurements of magnetic field, it is still impossible to measure the absolute value of the magnetic moment with an accuracy of 0.005 per cent necessary to catch the anomaly. In order to avoid such measurements one can employ a differential method, determining the *ratios* of the values of the Zeeman splitting in two different states of the same atom or in two states of different atoms. These ratios enable one to find the difference of deviations of gyromagnetic ratios for the orbital angular momentum from unity and for the electron angular momentum from two (using the experimental value of the ratio of Lande factors for the two states). Indeed, because of (2.21) (and under the assumption that g_L and g_S do not depend on the atomic state), the ratio of the values of g_J for these states in identical or different atoms is given by

$$\frac{g_{J_1}}{g_{J_2}}=\frac{g_L\alpha_{L_1}+g_S\alpha_{S_1}}{g_L\alpha_{L_2}+g_S\alpha_{S_2}} \tag{4.11}$$

If g_L and g_S differ from the usual values by small quantities

$$g_S = 2\,(1+\delta_S) \quad \text{and} \quad g_L = 1+\delta_L$$

instead of (4.11), we shall have

$$\frac{g_{J_1}}{g_{J_2}}=\frac{2\alpha_{S_1}+\alpha_{L_1}}{2\alpha_{S_2}+\alpha_{L_2}}+2\,\frac{\alpha_{S_1}\alpha_{L_2}-\alpha_{L_1}\alpha_{S_2}}{(2\alpha_{S_2}+\alpha_{L_2})^2}\,(\delta_S-\delta_L) \tag{4.12}$$

The next assumption of Kusch and Foley [560] is that $\delta_L = 0$, i.e. $g_L = 1$, which means that from the measured values of the ratios g_{J_1}/g_{J_2} it is possible to find the magnitude of deviation δ_S, that, according to (4.6), coincides with $\alpha/2\pi$.

In these theoretical calculations Kusch and Foley imply that they are dealing with the case of the Russell-Saunders coupling. They have demonstrated, however, that all possible deviations from this coupling scheme are negligible.

The experiment was conducted with extreme accuracy. The authors took into account various secondary effects: the time variation of magnetic field, its non-uniformity, etc.

In Table 4.3 we present the results of this experimental work of Kusch and Foley [560]. The first column indicates studied atomic terms; the second the values of the respective ratio (4.12) under the condition that $g_S = 2$ (or $\mu_e = \mu_B$); the third column gives the measured ratios specifying the accuracy of their determination; finally, the last column contains the computed values of the correction $\delta_S = \alpha/2\pi$. As follows from the table, the measured values $\alpha/2\pi$ agree sufficiently well with the theoretically predicted ones (4.6). The fact of coincidence of the two last experimental values and their differing from the first one can hardly be interpreted as real disagreement since the magnitude of this divergence is still inside the limits of experimental errors.

Table 4.3

Anomaly of the Electron Magnetic Moment

Studied ratio (atomic terms)	Theoretical values for $\mu_e = \mu_B$	Measured ratio	$\delta_S = \frac{\alpha}{2\pi}$
$\frac{g_J\,(^2P_{3/2}\,\mathrm{Ga})}{g_J\,(^2P_{1/2}\,\mathrm{Ga})}$	2	$2\,(1.00172 \pm 0.00006)$	0.00114 ± 0.00004
$\frac{g_J\,(^2S_{1/2}\,\mathrm{Na})}{g_J\,(^2P_{1/2}\,\mathrm{Ga})}$	3	$3\,(1.00242 \pm 0.00006)$	0.00121 ± 0.00003
$\frac{g_J\,(^2S_{1/2}\,\mathrm{Na})}{g_J\,(^2P_{1/2}\,\mathrm{In})}$	3	$3\,(1.00243 \pm 0.00010)$	0.00121 ± 0.00005

The mean value from three measurements: $\frac{\alpha}{2\pi} = 0.00119 \pm 0.00005$.

The anomaly of the electron g-factor in atoms H^1, H^3, and Rb^{85} was studied by Balling and Pipkin [56]; in atoms of alkali metals Na^{23}, K^{39}, Rb^{87}, and Cs^{133}, by Van den Bout et al. [938].

The existing divergencies between the theoretical formula (4.6) and experiment for the value of the electron magnetic moment anomaly have stimulated theoretical effort in the direction of taking into account higher-order approximations of the perturbation theory (up to the accuracy of α^2) and also experimentalists' endeavour to increase the accuracy in measurements of magnetic moment. Karplus and Kroll [522] were the first to compute the theoretical value for the electron g_s-factor with an accuracy up to α^2; their method was based on the Feynman diagram techniques (see [852]). Later these computations were revised by Sommerfield [870, 871] and Petermann [730, 732]. Both obtained for g_s the following expression:

$$\begin{aligned} g_s &= 2\left[1 + A_1\frac{\alpha}{\pi} + A_2\left(\frac{\alpha}{\pi}\right)^2 + A_3\left(\frac{\alpha}{\pi}\right)^3 + \dots\right] \\ &\cong 2\left(1 + \frac{\alpha}{2\pi} - 0.32847897\frac{\alpha^2}{\pi^2}\right) \\ &= 2\,(1 + a) = 2 \times 1.001159639\,(3) \end{aligned} \qquad (4.13)$$

where

$$A_1 = \frac{1}{2}$$

$$A_2 = \frac{197}{144} + \frac{1}{12}\pi^2 + \frac{3}{4}\zeta(3) - \frac{1}{2}\pi^2 \ln 2 \cong -0.32847897$$

and the term containing A_3 was not computed (see below).

It is of fundamental importance to obtain the precise value for the correction of the order of α^2 (and higher) to the normal magnetic moment (μ_B) of the electron since the comparison of theoretical predictions with results of precision measurements may prove valuable in the justification of the renormalization methods employed by the theory and also in establishing the limits of applicability of modern quantum electrodynamics. Therefore, it makes sense to carry on with further refinement of the corrections to μ_B. In this connection Terent'ev [905], making use of the unitarity of the scattering matrix and of the dispersion relations for the electron form factor (see [124)], computed the correction of the order of α^2, which coincides with the results of Sommerfield [870, 871] and Petermann [730, 732] mentioned above (their results were obtained by a different method).

More accurate measurements of the electron g-factor in experiments of Kusch and Foley [560] are based on a combination of two different experiments: that of determining the proton magnetic moment in the units of the Bohr magneton (μ_p/μ_B), and the experiment on the measurement of the ratio of the electron magnetic moment to the proton magnetic moment (μ_e/μ_p). It is the product of these two experimental results that gives us the value of g-factor for the electron.

Still more accurate data appear in the paper of Liebes and Franken [600]. Finally, Schupp et al. [824] carried out similar measurements of the electron g-factor: their techniques employ the method of polarization of high energy free electrons ($\geqslant$ 100 keV) and compared the frequency of precession of electron spin in a magnetic field $\omega_1 = geH/2mc$ with the cyclotron frequency of the orbital motion of the electron in the same field $\omega_2 = eH/mc$. The numerical results of these two experiments are:

$$g_s = 2 \times 1.001165(5) \quad [600]$$
$$g_s = 2 \times 1.0011609(12) \; [824]$$

A very accurate determination of the g-factor for the free electron can be found in the work of Wilkinson and Crane [970], who directly measured the quantity a from (4.13) using the value of the difference between the frequency of spin precession of the electron and its cyclotron frequency. Their result is

$$a_{\mathrm{exp}} = 0.001159622\,(27) = \frac{\alpha}{2\pi} - 0.327\,\frac{\alpha^2}{\pi^2}$$

if $\alpha^{-1} = 137.03602$. On the other hand, according to theoretical evaluation,

$$a_{\mathrm{theor}} = \frac{\alpha}{2\pi} - 0.328\,\frac{\alpha^2}{\pi^2}$$

Quite recently Rich [778, 779] (see also [453]) revised the data of Wilkinson and Crane [970] taking into account the relativistic corrections that were ignored before and analyzing the experimental errors more precisely. This revision resulted in the following:

$$a_{\mathrm{exp}} = 0.001159549\,(30)$$

Finally, in a still more recent paper of Wesley and Rich [960] (see also [410]) we find the following value for a:

$$a_{\mathrm{exp}} = (1159644 \pm 7) \times 10^{-9} \qquad (4.13a)$$

The computation of third order corrections ($\cong (\alpha/\pi)^3$), which calls for taking into account the terms up to the sixth order in the perturbation theory, first appeared in the paper of Drell and Pagels [277], who used dispersion techniques. According to these authors, $A_3 \cong +0.15$. Later Parson [719] came up with more accurate computations and obtained $A_3 \cong +0.13$. Finally, in the paper of Aldins et al. [13] we find a further refinement of these results:

$$a_{\mathrm{theor}} = \frac{1}{2}\,\frac{\alpha}{\pi} - 0.32848\left(\frac{\alpha}{\pi}\right)^2 + 0.55\left(\frac{\alpha}{\pi}\right)^3$$

Substituting 137.03608 ± 0.00026 for α^{-1}, we obtain

$$a_{\mathrm{theor}} = 1159644 \times 10^{-9} \qquad (4.13b)$$

The discrepancy between experimental (4.13a) and theoretical (4.13b) results is so insignificant that one can speak about a striking coincidence.

A more recent theoretical work of Levine and Wright [596] contains a somewhat different result, which is in still better agreement with the experimental result of Wesley and Rich [960], namely,

$$a_{\text{theor}} = \frac{1}{2}\frac{\alpha}{\pi} - 0.32847\left(\frac{\alpha}{\pi}\right)^2 + 1.49\left(\frac{\alpha}{\pi}\right)^3$$

Finally, in a still more recent experimental work of Wesley and Rich [961] we find a very accurate determination of the quantity a_{exp}; it has proved different from (4.13a) and is equal to

$$a_{\text{exp}} = (1159657.7 \pm 3.5) \times 10^{-9}$$

These authors believe that the difference

$$a_{\text{exp}} - \left[0.5\frac{\alpha}{\pi} - 0.32848\left(\frac{\alpha}{\pi}\right)^2\right] = (1.68 \pm 0.33)\left(\frac{\alpha}{\pi}\right)^3$$

is the experimental value of the sixth-order quantum-electrodynamic correction; its modern theoretical estimate, according to Levine and Wright [596], is 1.49 $(\alpha/\pi)^3$, which is in a sufficiently good agreement with experiment. On the calculation of the sixth-order radiation corrections see also more recent theoretical papers (see [151] and [168]); other aspects of theoretical and experimental determination of the anomalous magnetic moment of the electron and the Lamb shift are treated in [247, 536].

Let us make one more remark about the comparison of theoretical and experimental data. The point is that, beginning with the work of Schwinger [826], scientists considered only the interaction between the electron and vacuum that appears in the presence of an external field. The resulting terms in the non-relativistic approximation are linear with respect to the vector of magnetic field intensity. On the basis of the perturbation theory terms of the first, second, and third order in α were found. However, in one of the first papers on this topic Gupta [421] took into account, in the first order of the perturbation theory with respect to α, not only the linear but also the higher-order terms with respect to the dimensionless field parameter a^{-1} ($a \doteq H_0/2H$ and

$H_0 = m^2c^3/e\hbar = 4.41 \times 10^{13}$ Oe is a certain critical field; see formula (6.2) below; in the same section the physical meaning of such a field is explained). Therefore, in view of the progress in measurement there is good reason to consider more carefully the dependence of corrections on the field.

Such a calculation is presented in the paper of Ternov et al. [906] (see also [692]). They found that the anomalous magnetic moment of an electron that moves in a constant and uniform magnetic field already in the first order in the fine-structure constant α is a complex function of the field intensity and the energy of the particle. The following formula was obtained:

$$\frac{|\mu_e - \mu_B|}{\mu_B} = \frac{\alpha}{2\pi} f(n, a)$$

where f is a function of the energy of the electron (n being the principal quantum number) and of a parameter $a = H_0/2H$. In weak fields ($a \ll 1$) for energies with $n \ll a^3$ the function $f(n, a) \to 1$, which is equivalent to the usual result in the linear with respect to α approximation. However, for large values of energy ($n \gg a^3$) even in a weak field ($a \ll 1$) the magnitude of the anomalous magnetic moment decreases sharply with the energy of the electron. For instance, when $n = 12a^3$, we have $f \cong 0.53$.

Thus, even in weak fields we observe a complex dependency of the anomalous (vacuum) magnetic moment of the electron on its energy. Still more substantial deviations from the usual results are obtained in the limiting case of strong fields ($a \to 0$)*. In particular, there may be such situations when the anomalous magnetic moment even changes its sign.

It would be interesting to note that Arunasalam [41], making use of an idea of Bethe [90] about the computation of Lamb-Retherford shift and of the paper of Luttinger [612] on the calculation of the anomalous magnetic moment, applied them, using non-relativistic quantum electrodynamics in the determination of radiation corrections, to Lan-

* Authors, referring to the paper of Heisenberg and Euler [447], point out that even in such strong magnetic fields ($\gg 10^{13}$ Oe) there is every reason to believe that modern quantum electrodynamics is valid.

dau levels. His results were identical with Gupta's [421] with a slightly modified value of the numerical factor at the term, that is responsible for the dependence of the anomalous magnetic moment on the magnitude of the external magnetic field.

Chiu et al. [200] (see also [692]) consider the quantum theory of an electron gas in a strong magnetic field, allowing for the anomalous value of the electron magnetic moment. This work is based on the results of the article of Ternov et al. [906] concerned with computations of the energy levels from the solution of the Dirac-Pauli equation. Chiu et al. [200] obtain expressions for the density of the thermodynamic energy, the density of particles in the gas and the magnetic moment and density of electron-positron pairs.

Let us note in conclusion that according to the note of Strobel [885] the numerical values of the anomalous magnetic moments of the electron, the proton, and the neutron appear to be quite close. Indeed, as follows from (4.13b), (3.2) and (3.3),

$$|\Delta\mu_e| = \left[\frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{\pi}\right)^2 + 0.55\left(\frac{\alpha}{\pi}\right)^3\right]\mu_B$$
$$= 1159644 \times 10^{-9} \times 1836.109\mu_N \cong 2.129686206\mu_N;$$
$$|\Delta\mu_p| = 1.79267\mu_N; \; |\Delta\mu_n| = 1.913148\mu_N \tag{4.13c}$$

This circumstance may be of significance in future theoretical computations of nucleon anomalous magnetic moments.

2.3. The Anomalous Magnetic Moment of the Positron and of Positronium

The question of the anomalous magnetic moment of the positron has been analyzed to a much less extent. Rich and Crane [780] were the first to measure the anomalous value of *g*-factor of the *free positron*. The method they employed was very similar to that of Wilkinson and Crane [970] for electrons, the only difference being that with positrons there was no necessity in the initial polarization of particles through Mott scattering since polarization of the positrons occurs during their emission from the source of radioactive $_{27}Co^{58}$. The direction of this polarization, after the positrons had passed through a magnetic field (in a given interval of time), was determined by detecting the photon emitted during annihilation of a triplet state of positronium (this state arises when positrons stop). This method allowed to obtain the following values of the constant a from formula (4.13):

$$a(e^+) = 0.001168 \pm 0.000011 = \frac{\alpha}{2\pi} + (1.2 \pm 2)\frac{\alpha^2}{\pi^2}$$

Comparing this result with the value of $a\,(e^-)$ from (4.13a) we find

$$a(e^+) - a(e^-) = (9.0 \pm 5.5) \times 10^{-6}$$

As follows from more accurate measurements of Gilleland and Rich [384],

$$a\,(e^+) = 0.0011602 \pm 0.0000011$$

and for the difference we find

$$a(e^+) - a(e^-) = (6.0 \pm 5.5) \times 10^{-7}$$

We can conclude that g-factors for the electron and the positron are identical. They coincide up to the accuracy of 0.6 parts per million.

The most accurate measurements of g-factors of the positron have been obtained by Gilleland and Rich [385].

Positronium (Chapter 1), the bound state of an electron and a positron, is an ideal system for checking the validity of quantum electrodynamics. This is associated, first of all, with the absence of "alien" particles, such as nucleons, in this system. Second, a good description of the two-body problem is provided by the Bethe-Salpeter equation [94]. The study of positronium is also of interest from the point of view of the verification of this equation.

The most important experimental quantity of positronium measured is the energy difference between the ground states with the principal quantum number $n = 1$ of orthopositronium (state $^3S_1$) and of parapositronium (state $^1S_0$). This energy interval corresponds to the hyperfine splitting $\Delta\nu_{\mathrm{Hfs}}$. The magnitude of this splitting was computed by Karplus and Klein [521], who made use of the Bethe-

Salpeter equation. The first experiments were conducted by Deutsh and Brown [248], Weinstein et al. [954, 955], and also by Hughes et al. [473].

The most consistent theoretical computation of $\Delta\nu_{\mathrm{Hfs}}$ appears in the paper of Fulton et al. [363]. According to them,

$$(\Delta\nu_{\mathrm{Hfs}})_{\mathrm{theor}} = 2.03427 \times 10^5 \mathrm{MHz}$$

This shows that in such a purely quantum-electrodynamic problem as that of positronium there is still some quantitative uncertainty that requires improvement of theoretical techniques and the increase in the accuracy of experiment.

We can conclude this section by noting that the discovery and subsequent explanation of the anomaly of the magnitude of the spin magnetic moment of the electron and positron is a new striking evidence of the validity of modern quantum electrodynamics. The present status of quantum electrodynamics has been very thoroughly reviewed by Brodsky and Drell [150]; we highly recommend this paper to the reader. See also the review article of Farlev [317].

3. The Anomalous Magnetic Moment of Mu-Mesons (Muons) and of Mesic Atoms

We have mentioned before that quantum electrodynamics is applicable not only to electrons, positrons, and photons but also to *mu-mesons* (muons) (see [251]). These elementary particles were discovered by Neddermeyer and Anderson [675] as the main constituent of the hard component of cosmic rays. Later it was demonstrated that muons are produced in the decay of pions according to the scheme $\pi^\pm \to \mu^\pm + \nu$, where ν is the neutrino.

The lifetime of muons is only 2.21×10^{-6} s. They disintegrate into an electron or a positron and two neutrinos: $\mu^\pm = e^\pm + 2\nu$. The mass of muons is somewhat smaller than that of pions; the most accurate estimates give the quantity $m_\mu \cong (206.9 \pm 0.2)\, m_e$. Since the spin of a pion is zero and the spin of a neutrino is 1/2, the spin of a muon is 1/2.

From the whole set of experimental facts concerning the behaviour of muons one can draw a conclusion that they participate mainly in weak interactions (i.e. the kind that determine β-decay of radioactive nuclei) and also in electromagnetic ones. Therefore, muons can be classified as "heavy electrons and positrons" thus constituting a part of the group of elementary particles called *leptons*. The existence of such "heavy electrons" is one of the greatest mysteries of elementary particle physics.

Since the muon can be interpreted as a heavy electron, its wave function should satisfy the Dirac equation; therefore, its magnetic moment should be 207 times as small as the Bohr magneton:

$$\mu_{\text{mes}} = \frac{e\hbar}{2m_{\mu}c} \cong \frac{1}{207}\mu_{\text{B}} \tag{4.14}$$

The interaction with the zero oscillations of electromagnetic vacuum leads, as in the case of the electron, to the fact that the magnetic moment of a muon instead of being equal to meson magneton μ_{mes} (4.14) has an anomalous value.

By analogy with formula (4.13) we obtain the following theoretical expression for the anomalies of g-factor of the free positive muon and of its antiparticle, the negative muon, $a_{\mu}^{+} = a_{\mu}^{-} = (g_{\mu} - 2)/2$:

$$a_{\mu} = B_1\frac{\alpha}{\pi} + B_2\left(\frac{\alpha}{\pi}\right)^2 + B_3\left(\frac{\alpha}{\pi}\right)^3 + \ldots \tag{4.15}$$

If we assume that muon interacts with the electromagnetic field in the same way as electrons, then, due to the absence of a dependence of second-order radiation corrections on the particle mass*, the quantity B_1 equals A_1 from (4.13), i.e.

$$B_1 = \frac{1}{2} \tag{4.16}$$

Similarly, the fourth-order radiation corrections will contain a term already computed for the electron, which is mass-independent and equals $A_2 \cong -0.3285$. Besides, virtual photons emitted by the muon may lead to the polarization of the vacuum, accompanied by the formation of virtual electron-positron pairs. The magnitude of this effect, which depends on the mass ratio for the meson and the electron

* See [602].

m_μ/m_e, was computed by Elend ([295, 296]) (earlier computations appear in the papers of Suura and Wichmann [890] and Petermann [731, 733])*; his result is

$$\frac{1}{8}\left\{\ln\frac{m_\mu}{m_e}-\frac{25}{12}+\frac{3\pi^2}{4}\frac{m_e}{m_\mu}+3\left(3-4\ln\frac{m_\mu}{m_e}\right)\left(\frac{m_e}{m_\mu}\right)^2 + \mathcal{O}\left(\frac{m_e}{m_\mu}\right)^3\right\}\left(\frac{\alpha}{\pi}\right)^2 \tag{4.17}$$

Substituting the quantity 206.769(3) for m_μ/m_e (see [902]) in (4.17), we obtain

$$1.094261\,(4)\left(\frac{\alpha}{\pi}\right)^2 \tag{4.18}$$

Finally, by adding A_2 from (4.13) to (4.18), we find

$$B_2 = 0.76578 \tag{4.19}$$

The computation of sixth-order radiation corrections, which determine the coefficient B_3 in formula (4.15), is even more complex. If one limits himself to considering the interaction of a muon only with an electron and a photon, then, as shown by Kinoshita [534, 535] and also by Lautrup and de Rafael [581, 582], the difference between respective corrections for the muon and the electron with $m_\mu/m_e = 206.769(3)$ is

$$B_3 - A_3 \cong 2.819$$

Thus, if we use the value $A_3 = 0.55$ from the paper of Aldins et al. [13],

$$B_3' \cong 3.369 \tag{4.20}$$

This value for B_3, however, does not contain the contribution from processes associated with the polarization of vacuum by virtual particles characterized by strong interaction (hadrons), such as pion pairs, etc. This effect has been taken into account by Kinoshita and Oakes [537], Terazawa [904], and Gourdin and de Rafael [409] with the result that

$$B_3'' \cong 5.2 \tag{4.21}$$

* In the case of electron there is a similar contribution from muon pairs (see [581, 582]). However, it appears to be very small: $\cong 6.20 \times 10^{-7}\ (\alpha/\pi)^2$.

Thus, the total coefficient B_3 is

$$B_3'+B_3''\simeq 8.87$$

Expression (4.15) finally takes the form

$$a_\mu^{\text{theor}}\simeq\frac{1}{2}\left(\frac{\alpha}{\pi}\right)+0.76578\left(\frac{\alpha}{\pi}\right)^2$$
$$+8.87\left(\frac{\alpha}{\pi}\right)^3\simeq 0.001165652 \qquad (4.22)$$

The most accurate experimental determination of a_μ^{exp} was carried out by a group of investigators at CERN [48, 50] (see also [432, 452, 480]). It resulted in the following:

$$a_\mu^{\text{exp}}=0.00116616\,(31) \qquad (4.23)$$

This value is obtained as the average over the measurements of a_μ^+ and a_μ^-, the difference between the latter being interpreted as the statistical error of the experiment equal to $(5.0 \pm 7.5) \times 10^{-7}$. The experiment on the determination of a_μ does not differ from that of Wilkinson and Crane [970] on the determination of a_e. The details of the two experiments are, however, quite different. They can be looked up in the above-mentioned original papers and also from the earlier research of Charpak et al. [189]. There is a certain discrepancy between a_μ^{exp} and a_μ^{theor}:

$$a_\mu^{\text{exp}}-a_\mu^{\text{theor}}\simeq 51\times 10^{-8}$$

The reason for this discrepancy is still far from being clear. There exists only an assumption that it is related to the inaccuracy in the computation of coefficient B_2 and especially B_3 and also in the determination of muon masses.

It would be interesting to compare a_e with a_μ. As can be easily seen, for $\Delta a = a_\mu - a_e$ we have the following:

$$\Delta a^{\text{theor}}\simeq 601\times 10^{-8},\quad \Delta a^{\text{exp}}\simeq 661\times 10^{-8}$$

and

$$\Delta a^{\text{exp}}-\Delta a^{\text{theor}}\simeq 60\times 10^{-8}$$

Thus, here we also come across a discrepancy that has not yet been explained.

Let us mention still more recent works on determination of the muon magnetic moment and g-factor: the paper of Grayer et al. [411], Link et al. [604], Pietschmann and Stremnitzer [741], Lautrup [579], Bramon et al. [142] and Bailey et al. [49].

Of considerable importance from the point of view of checking the validity of quantum electrodynamics is the analysis of radiation corrections for *muonium*, i.e. an atom consisting of a positive muon and an electron and resembling in many respects a hydrogen atom (although its lifetime is only 2.2 s!).

For more details see the review paper of Taylor et al. [902] and also the following papers: [149, 170, 173, 195, 208, 250, 290, 346, 474, 492, 504].

Of considerable interest is also the study of magnetic properties not only of free muons and muonium but of *mesic atoms*, i.e. atoms with negative muons on an atomic orbit near the nucleus. The magnetic moment of the muon in a mesic atom differs from that of the free muon, and this difference should depend on nuclear dimensions. The measurements of the magnetic moment of the muon in a mesic atom have demonstrated the reality of the above difference [479]. Ford et al. [335] carried out a detailed theoretical analysis of various corrections to the anomalous magnetic moment of the free muon in a mesic atom (we refer the reader to this article). The most interesting is the correction that is associated with the fact that the muon "orbit" can pass through the volume of the nucleus and therefore, the muon magnetic moment can serve as a source of information about the inner structure of atomic nuclei.

One can expect that the refinement of the theory and the accumulation of more detailed and precise experimental data on magnetic moments of muons (free or bound in mesic atoms) will produce valuable information not only concerning the nature of muons themselves but also relating to the structure of atomic nuclei, nucleons, and pions.

Certain general aspects of the anomalous magnetic moments of leptons are treated from quantum-electrodynamic point of view in [246, 498] and also in the detailed review papers of Lautrup et al. [580] and Rich and Wesley [781].

In conclusion let us refer to the review article of van Hove [940], where the author points out that the good agreement of theory and experiment in determination of anomalous magnetic moments of leptons is one of the triumphs of quantum electrodynamics.

4. Anomalous Magnetic Moments of Nucleons and Other Hadrons

The problem of interpretation of magnetic properties of nucleons and other elementary particles classified as *hadrons** which are characterized by the so-called *strong*, or mesonic, interaction, and not by electromagnetic, or weak, interactions, is more complex. At the same time the detailed analysis of magnetism of nucleons and other elementary particles can provide very valuable information on their inner structure. Without considering in detail this interesting problem, let us dwell only upon the question of the anomalous magnetic moment of nucleons.

On the basis of the existing imperfect theory of nuclear (mesonic) forces there have been carried out a considerable number of computations of the anomalous magnetic moment of nucleons (see, for example, a review of these papers in Chapter 46 of [92]). However, in view of new experimental data concerning the electromagnetic structure of nucleons, obtained in measurements of the scattering cross section of high-energy (up to 1300 MeV) electrons, all these calculations have lost their meaning. The only conception that has been preserved is that the anomalous magnetic moment of the nucleons is associated mainly with the pion cloud surrounding the central part, the *core* of the nucleon. It was the experiments on scattering of fast electrons by nucleons that gave the direct proof of these assumptions.

4.1. The Charge and Magnetic Form Factors of Nucleons

It is a well-known fact of atomic physics that the structure of the atom and, in particular, the very existence and dimensions of the atomic nucleus were detected in the famous ex-

* This term was introduced by Okun [702].

periments of Rutherford (see, for example, [842]). The experiments consisted in irradiation of atoms by fast alpha-particles and determination of the scattering cross section for these particles as a function of the scattering angle and the initial energy.

The same method was also used for the analysis of the inner structure of nucleons. Here, however, electrons were used, the reason for this being that the forces acting between electron and nucleons are primarily electromagnetic ones. The theory of these interactions is developed sufficiently well. Therefore, we can hope to obtain quite definite information on the inner structure of nucleons from the observed values of the electron-nucleon cross section.

It is well known from scattering theory that in the scattering of an electron beam or X-rays by the electrons of an atomic shell the angular distribution of scattered particles is determined by the so-called atomic factor of scattering, or the *form factor* $F(\boldsymbol{q})$. The general expression for the form factor has the following form:

$$F(\boldsymbol{q}) = \int \rho(\boldsymbol{r})\, e^{i(\boldsymbol{q}\boldsymbol{r})}\, d\boldsymbol{r} \tag{4.24}$$

where $\rho(\boldsymbol{r})$ is the spatial density of the scattering charge; $\boldsymbol{r}$ the radius vector with its origin at the centre of the scattering system; and $\boldsymbol{q}$ the change in the wave vector of the scattered particle, which is a function of the initial energy of the incident particle and the scattering angle θ. One can see from equation (4.24) that the density of the scattering charge $\rho(\boldsymbol{r})$ and the form factor $F(\boldsymbol{q})$ are related through the Fourier transformation. Therefore, in principle it is possible, on the basis of experimental determination of $F(\boldsymbol{q})$, to obtain the unknown distribution of the charge density by applying the inverse Fourier transformation. This will be equivalent to determining the electromagnetic structure of the scattering system. (Here we do not take into account the essential complications introduced into the form factor when one has to take into account relativistic corrections to the trajectories of very fast scattered particles and also the possible variations in the state of the "target" due to the effect of recoil.)

In the case of nucleons* there are two form factors: F_1, which corresponds to scattering on the spatial distribution of the electric charge of the nucleon $\rho_{el}(\boldsymbol{r})$; and F_2, which is associated with the spatial distribution of the anomalous magnetic moment of the nucleon $\rho_{magn}(\boldsymbol{r})$ (F_2 is similar to the magnetic form factor arising in scattering of neutrons by atomic electron shells**). As to the spatial distribution of the regular ("Dirac") magnetic moment, it is taken into account by the "charge" form factor F_1 since, as we have noted above, according to Dirac s theory the free particle experiences a sort of "shaking" (Chapter 1). If the particle is electrically charged, such "shaking" generates a circulating current and, therefore, a certain magnetic moment. It is obvious that the density of this "Dirac" regular moment is directly related to the density of the electric charge. On the other hand, as has been already noted, the anomalous magnetic moments of the proton and the neutron are associated with the emission of virtual pions, and, therefore, the density of distribution of these moments will differ essentially from the density of Dirac's magnetic moment.

As follows from (4.24), the value of the form factor for $\boldsymbol{q} = 0$, i.e. $F(0)$, immediately gives us the total charge or the total anomalous magnetic moment. Their value is equal to the integral of the density of their distribution over the whole space. Usually these quantities are normalized in such a way that

$$F_{1p}(0) = F_{2p}(0) = F_{2n}(0) = 1 \quad \text{and} \quad F_{1n}(0) = 0$$

Expression (4.24) can also be transformed if one makes use of the expansion of the exponential function $\exp(i(\boldsymbol{qr}))$. After term-by-term integration we obtain, for example, for the first two non-vanishing terms of the series, the following:

$$F(\boldsymbol{q}) = 1 - \frac{\langle r\rangle^2}{6} q^2 + \dots \tag{4.25}$$

---

* At present there exists no electrodynamic theory of hadrons that can be regarded as complete in any sense. Therefore form factors of hadrons cannot be computed, and they have to be treated purely phenomenologically (see, for instance, Sec. 139 in [602]).

** This, however, cannot be interpreted literally in the relativistic limit (see [602]).

Here $\langle \boldsymbol{r} \rangle$ is the root-mean-square radius of the scattering system.

Equation (4.25) means that the larger the dimensions of the target, the sharper is the decrease of the form factor with the increase of q^2 (i.e. of the scattering angle). Besides, it also follows from (4.25) that in the case of a point scatterer ($\langle \boldsymbol{r} \rangle = 0$) the form factor is constant. Similarly, very slow electrons (i.e. very small values of q^2) allow to determine only the total charge or the total magnetic moment of the nucleon. Only fast electrons provide a possibility to find the spatial distribution of the charge and the magnetic moment. First we obtain the root-mean-square radius of this distribution from the second term in (4.25), and then it will be possible to get more detailed information. This clearly shows the necessity of conducting experiments with scattering of electrons of the highest possible energy.

The first successful experiments were conducted in 1953-6 in the United States by a group of investigators led by Hofstadter (see numerous references in his review [463], and also in [274]). Electrons with energies up to 600 MeV were used. The electrons were scattered by a hydrogen or deuterium target and analyzed by means of a magnetic spectrometer. These experiments showed that the proton, rather than being a point particle, represents a system characterized by a certain spatial distribution of the electric charge and magnetic moment. The root-mean-square radii $\langle \boldsymbol{r} \rangle^2$ for these distributions are very close to each other and constitute a quantity of the order of 0.8 fermi (1 fermi $= 10^{-13}$ cm is a unit of length convenient for measurements on the nuclear scale).

Determination of neutrons form factors was somewhat more difficult since it was technically impossible to study the scattering of electrons by free neutrons (i.e. to manufacture a stable neutron target). Therefore, the magnetic structure of neutrons is analyzed by less direct methods than those employed for protons. Namely, first, the inelastic (and also elastic) scattering of fast electrons of deuterons is studied, and all the effects caused by the presence of the proton in these nuclei are "subtracted". Second, investigators make use of the process of generation of a positive pion in electron-proton collisions: $e + p \rightarrow e' + n + \pi^+$. This method does not imply the necessity of subtraction of scattering by a proton (as in the case of scattering on a deuteron); there is also a possibility of the analysis of pions' electromagnetic structure.

Finally, Friedman et al. [356] suggested a new method for measuring the magnetic structure of the neutron. In this method the elastic electron-deuteron scattering is compared for large and small scattering angles. It was found that to a high degree of accuracy the ratio of the scattering cross sections is model-independent and is equal to the ratio of the anomalous magnetic form factors of the neutron and the proton. First experiments showed that the distribution of magnetic moment in the neutron is very close to that in the proton, while the neutron density of the electric charge proved to be zero.

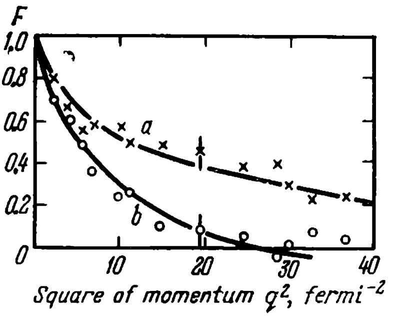

Fig. 4.1. The proton form factors [704]. a—the form factor of the electric charge distribution F_{1p}, b—the form factor of the distribution of the anomalous magnetic moment F_{2p}.

Further progress in experiments concerning the electromagnetic structure of nucleons was achieved at Stanford (see [464]) when the energy of the scattered electrons was raised up to 900 MeV; at Cornell University (U.S.A.) in the group headed by Wilson [973] the energy was increased up to 1300 MeV. These experiments allowed to measure the form factors up to the values $q^2=37$ fermi^{-2} and brought the degree of accuracy of the measurements to such a high level that it was possible to obtain the details of the distribution of the densities of charge and magnetic moment up to 0.21 fermi, which is comparable with the Compton wavelength of the nucleon $\hbar/Mc = 2.10308 \times 10^{-14}$ cm.

Figures 4.1 and 4.2 contain the curves for the proton and the neutron form factors respectively (the electric form factor and the magnetic form factor) according to the results of Olson et al. [704]. These are in perfect agreement with the results obtained at Stanford*.

* See, for example, [433, 466, 467].

As follows from the curves in Fig. 4.1 already for very small scattering angles the electric and the magnetic form factors of the proton F_{1p} and F_{2p} differ from each other, which means that the root-mean-square magnetic and electric radii of the proton are also different:

$$\langle r\rangle^2_{1p} \neq \langle r\rangle^2_{2p}$$

Analysis of the behaviour of the curve F_{1p} shows that its initial steep fall corresponds to a charged pion cloud of large radius ($\cong 0.75$ fermi). After the value $q^2 = 25$ fermi^{-2} the curve F_{1p} exhibits practically no variation with the increase of q^2. This is the constant part of the form factor corresponding to a charged core of very small diameter inside the proton. The diameter of this core can be estimated from the slope of the tangent to the nearly horizontal "tail" of the curve F_{1p}. This gives the value of the core diameter $\cong 0.2$ fermi. The location of the intersection of this tangent with the vertical axis on the plot in Fig. 4.1 allows to determine the part of the electric charge of the proton, concentrated in its core; this part constitutes approximately $0.35e$.

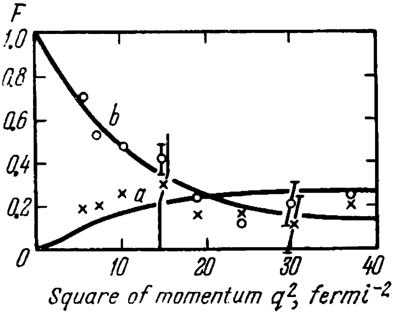

Fig. 4.2. The neutron form factors [704]. a—the form factor of the electric charge distribution F_{1n}, b—the form factor of the distribution of the anomalous magnetic moment F_{2n}.

As follows from the analysis of the form factor curves for the neutron (Fig. 4.2), the slope of the initial section of the curve corresponding to magnetic form factor of the neutron (F_{2n}) is different from that of the proton (F_{2p}), which means that their root-mean-square radii are also different. Besides, the neutron magnetic form factor F_{2n} does not tend to zero as in the case for the proton form factor F_{2p} at large values of the scattering angle. This points to the fact that the distribution of the anomalous magnetic moment of the neutron should also have a core.

Very interesting is the curve F_{1n}; it demonstrates that the neutron has a non-zero charge density such that the total charge and the root-mean-square radius of its distribution are equal to zero.

Using curves in Figs. 4.1 and 4.2, one can try to determine from equation (4.24) for the form factor the spatial distribution of charge and magnetic moment. Because of large experimental errors and also because of limitations imposed on the experimental curves (from the side of large values of energy, i.e., large scattering angles) this cannot be done exactly; another limiting factor are the relativistic corrections (see the remark above). However, it can be demonstrated qualitatively that all four form factors can be presented as sums of three partial form factors: for the core, for the vector, and for the scalar mesonic clouds (see [821]):

$$F_{ij} = F_{ij}^{\text{core}} \pm F_{ij}^{\text{vect.cl.}} + F_{ij}^{\text{scal.cl.}} \quad (i = 1,\ 2;\ j = p,\ n) \tag{4.26}$$

(the second term in the right-hand side should be taken with a sign plus for the proton and with minus for the neutron).

Such a separation into an *isoscalar* cloud and an *isovector* cloud is closely related to the experimental fact of the charge independence of nuclear forces; it corresponds to the proton

Table 4.4

Electric Charges, Magnetic Moments, and Root-Mean-Square Radii of Cores and Meson Clouds of the Nucleon (Proton and Neutron) from [821]

	Core	Isoscalar cloud	Isovector cloud
Electric charge (in fractions of the electron charge e)	0.35	0.15	+0.50 for the proton; −0.50 for the neutron
Root-mean-square radius of electric charge distribution (in fermi)	0.21	1.37	0.75
The anomalous magnetic moment (in pauli)	−0.12	0.08	+1.0 for the proton; −1.0 for the neutron
Root-mean-square radius of anomalous magnetic moment distribution (in fermi)	?	1.24	0.79

1 fermi $= 10^{-13}$ cm; 1 pauli $= \frac{1}{2}(|\mu_p| - |\mu_n|) = 1.85\ \mu_N$.

and the neutron being simply two different states of the nucleon*. Every partial term in the expression (4.26) for the form factor can be associated with two basic characteristics: (1) the contribution into the total charge of the particle and (2) the root-mean-square radius. On the whole, here

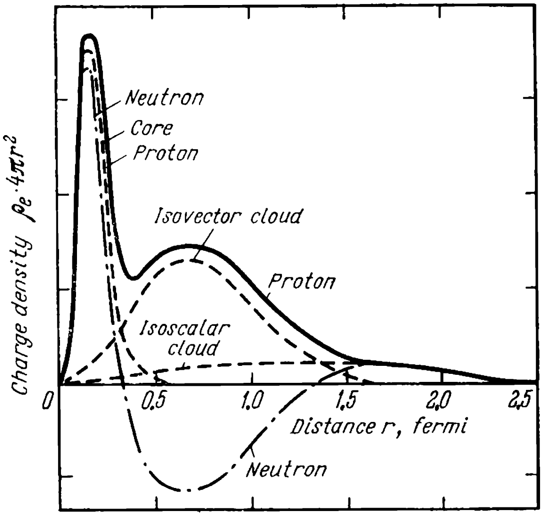

Fig. 4.3. The electric charge density $\rho_e(r) \times 4\pi r^2$ as a function of the distance r from the centre of the nucleon. The solid line gives the distribution of the total charge of the proton; the dot-broken line, the charge of the neutron. The broken lines depict the contribution of the isoscalar and isovector clouds and also of the core (in the case of the neutron the charge density of the isovector cloud enters the expression of the total charge density with a negative sign) [704].

we must perform a rather complicated procedure of fitting, making use of what is known about the fundamental characteristics of the nucleons.

The results of this qualitative description of the electromagnetic structure of the proton and the neutron are given in Figs. 4.3 and 4.4, respectively, and also in Table 4.4.

* We cannot consider this question here in detail. Let us only note that the vector cloud can be related to that of virtual pions (having opposite signs for protons and neutrons). On the other hand, the nature of the scalar cloud does not allow such a simple interpretation; it is associated with more complex virtual processes (see [821]).

As follows from these data, the proton and the neutron have cores of small dimensions (of the order of 0.21 fermi), which is 35 per cent of the proton charge. The core of the proton is surrounded by an internal isovector and an external isoscalar positively charged meson clouds. In the

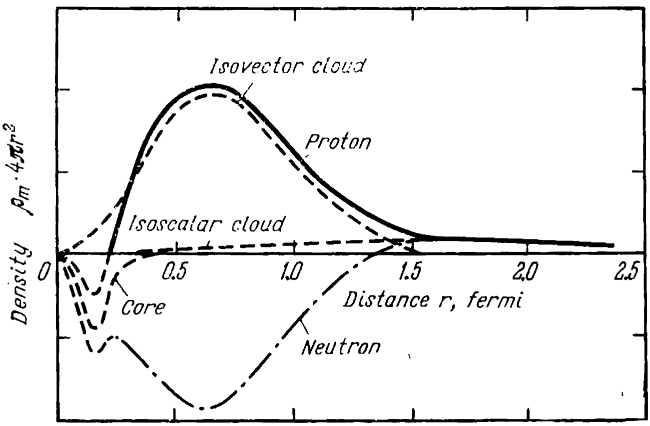

Fig. 4.4. The density of the anomalous magnetic moment $\rho_m(r) \times 4\pi r^2$ as a function of the distance r from the centre of the nucleon. The notations and the signs of the contributions are the same as in Fig. 4.3. [704].

neutron the internal cloud is charged negatively. The external cloud in the proton and in the neutron has a very low charge density and a large root-mean-square radius (of the order of 1.37 fermi).

The major portion of the anomalous magnetic moment of the nucleon is concentrated in the inner isovector cloud with the root-mean-square radius of 0.79 fermi (which is somewhat greater than the radius of the charged cloud, i.e. 0.75 fermi); for the neutron the moment density is negative while for the proton it is positive. The density of the magnetic moment is also characterized by a core whose radius has not been measured yet. Besides, the distribution of magnetic moment for the proton and the neutron contains a positive contribution from the isoscalar cloud with a very large root-mean-square radius (of the order of 1.24 fermi).

Apparently, the distribution of the isovector cloud corresponds to virtual pions discussed above. On the other hand, the nature of the core and the outer scalar cloud and also of the core of the magnetic moment has not yet been fully understood.

We can hope that further experiments and the development of quantum theory of strong (mesonic) interactions will explain the physical nature of the anomalous magnetic moments, similar to the way it was done in Dirac s quantum relativistic theory for the electron and the positron.

Measurements of magnetic form factors are now being carried out also for complex nuclei, such as the deuteron [397], nuclei $_1H^3$ and $_2He^3$ [6, 239], and also for pions [379].

A theoretical expression for the scattering cross section of the electron on a point nucleons was obtained by Rosenbluth [792], who generalized the ordinary Mott formula [659, 660] taking into account the fact that the proton has not only a charge, $+e$, but also a regular ("Dirac") magnetic moment and an anomalous magnetic moment. For more details see the paper of Rosenbluth [792] and also review articles [86, 433, 464, 465, 815]. Later experimental results on the determination of the neutron form factor can be found in [279, 356, 466, 877, 986]; that of the proton in [11, 193, 354].

There are also theoretical computations of nucleon form factors on the basis of the group-theoretic method and the quark hypothesis (Sec. 4.3 below); see [28, 185, 205, 209, 297, 416, 499, 598, 625, 640, 674, 737, 839, 847, 911, 932, 933]. We must also mention the discussion on the dipole nature of the electromagnetic form factors of the proton and the neutron between Hammer and Weber [431], on one side, and Goebel [395] and Hagen and Sudarshan [430], on the other.

In order to explain all the details of the experiments on elastic and inelastic scattering of fast electrons by nucleons Feynman [323] suggested a new theoretical model for the nucleon, according to which the latter consists of point particles, "partons", that cause the inelastic scattering of electrons. The performed analysis of experimental data did not allow to identify the partons with pions. There are certain attempts to set up a "bridge" between partons and quarks (on the quark hypothesis see Sec 4.3 of this chapter). However, this problem still contains many unclear aspects. In this connection we refer the reader to two review papers: [530] (experimental results) and [275] (theoretical results). Also of interest is the article of Tryon [928] in which the quark-parton model is considered from the point of view of application to the problem of electromagnetic mass difference.

4.2. Magnetic Moments of Hadrons

The theoretical and experimental analysis of magnetic moments of π-mesons, K-mesons, hyperons (Λ, Σ and Ξ), and resonances* is very important for further development of the theory of magnetic properties of nucleons. However, there are still very few definite experimental results on the measurement of magnetic moments of Λ^0, Σ^+, and Ξ^--hyperons.

In these works Λ^0-hyperons are obtained via the reactions

$$\pi^- + p = \Lambda^0 + K^0 \quad \text{or} \quad \pi^+ + n = \Lambda^0 + K^+$$

(K^0 and K^+ are kaons) that takes place when a hydrogen-containing (for example, polyethylene) or a beryllium (for the production of neutrons) target is irradiated by a beam of negative pions produced in a proton synchrotron. Since the spin of the Λ^0-hyperon in such reactions is polarized, after passing a beam of them through a strong magnetic field normal to the beam and then observing the decay $\Lambda^0 \to \pi^- + p$ in a chamber, we obtain the well-known asymmetry in the angular distribution of pions with respect to spin polarization because of the non-conservation of parity in this decay. From here we can find the direction of the spin of the Λ^0-hyperon and also the magnitude and the sign of the magnetic moment.

So far there appeared several such investigations. Due to considerable experimental difficulties, the results differ greatly and are of insufficient accuracy.

Cool et al. [227] using the reaction $\pi^+ + n = \Lambda^0 + K^+$ found that $\mu_{\Lambda^0} = -1.5(5)\ \mu_N$.

---

* The term "resonance", or "resonant state", of an elementary particle denotes very short-lived formations (with lifetimes $\cong 10^{-22}$ s) arising in particle interactions.

Kernan et al. [531] using the reaction $\pi^- + p = \Lambda^0 + K^0$ (with a LiH target) obtained the value $\mu_{\Lambda^0} = 0.0(6)\ \mu_N$.

Anderson and Crawford [26] using the reaction $\pi^- + p = \Lambda^0 + K^0$ found that $\mu_{\Lambda^0} = -1.39(72)\ \mu_N$.

Charrière et al. [191] using the reaction $\pi^- + p = \Lambda^0 + K^0$ (polyethylene target) obtained $\mu_{\Lambda^0} = -0.50(28)\ \mu_N$.

Hill et al. [458] found the value $\mu_{\Lambda^0} = -0.77(27)\ \mu_N$ by means of the reaction $\pi^+ + n = \Lambda^0 + K^+$; the same paper gives the mean value over all five investigations for the magnetic moment of Λ^0-hyperon equal to (see also [190])

$$\mu_{\Lambda^0} = -0.73\,(17)\,\mu_N$$

Charrière [190], analyzing the same experiments, confirms his previously obtained value

$$\mu_{\Lambda^0} = -0.50\,(28)\,\mu_N$$

Barkov et al. [58] obtained the following result:

$$\mu_{\Lambda^0} = (-0.67^{+0.31}_{-0.31})\,\mu_N$$

In the paper of Hill et al. [459] we find the value

$$\mu_{\Lambda^0} = (-0.73 \pm 0.18)\,\mu_N$$

Finally, Dahl-Jenson et al. [238] obtain a value

$$\mu_{\Lambda^0} = (-0.66 \pm 0.16)\,\mu_N$$

There are also five publications on the experimental determination of the magnetic moment of Σ^+-hyperon. In the paper of Cook et al. [225] Σ^+-hyperons were obtained in the reaction $\pi^+ + p = \Sigma^+ + K^+$ (pions were produced in a bevatron and then directed at a polyethylene target). Then the precession of polarized Σ^+-hyperons in a magnetic field was measured by observing the asymmetric decay $\Sigma^+ \to \pi^0 + p$. The resulting value for the magnetic moment was

$$\mu_{\Sigma^+} = (1.5 \pm 1.1)\,\mu_N$$

Sullivan et al. [887] (see also [637]) determined the magnetic moment of the Σ^+-hyperon from the reaction $\gamma + p = \Sigma^+ + K^0$ and by observing the decay $\Sigma^+ \to \pi^0 + p$.

Finally they obtained the quantity

$$\mu_{\Sigma^+} = (3.0 \pm 1.2)\, \mu_N$$

In the third paper (Kotelchuck et al. [553]) the authors made use of the reaction $K^- + p = \Sigma^+ + \pi^-$. They believe that this reaction is preferable to pion and photon reactions of the two preceding papers since the exothermic kaon reaction produces Σ^+-hyperons polarized to a higher degree at lower energies and larger angles (in the laboratory reference frame), and also since it is characterized by a large cross section for the production of Σ^+-hyperons. As a result they found for the magnetic moment the value $\mu_{\Sigma^+} = (3.5 \pm 1.5)\, \mu_N$.

The value averaged over the results of Sullivan et al. [887] and Kotelchuck et al. [553] is

$$\mu_{\Sigma^+} = (3.2 \pm 0.9)\, \mu_N$$

The authors of the fourth paper (Combe at al. [220]) measured the magnetic moment by observing the precession of the polarization vector in a pulsed magnetic field. Polarized hyperons were produced in a liquid hydrogen target by a beam of positive pions, and then the decay $\Sigma^+ \to p + \pi^0$ was analyzed. The result of the measurements was

$$\mu_{\Sigma^+} = (3.5 \pm 1.2)\, \mu_N$$

Thus, the quantity averaged over all five experiments with Σ^+-hyperons proved to be

$$\mu_{\Sigma^+} = (2.6 \pm 0.5)\, \mu_N \tag{4.27}$$

Quite recently Alley et al. [15] also measured the magnetic moment of Σ^+ and found that

$$\mu_{\Sigma^+} = (2.7 \pm 1.0)\, \mu_N$$

The paper of Walsh [951] presents also the result of measurement of the magnetic moment of Ξ^-

$$\mu_{\Xi^-} = (-0.1 \pm 2.1)\, \mu_N \tag{4.28}$$

The value should be regarded only as very preliminary (see [104]).

More accurate measurements of magnetic moment of Ξ^- were performed by Cool et al. [226], who made use of the following chain of reactions between elementary particles:

$$
\begin{array}{l}
K^- + p \rightarrow \Xi^- + K^+ \\
\qquad\qquad\quad \hookrightarrow \Lambda^0 + \pi^- \\
\qquad\qquad\qquad\quad \hookrightarrow p + \pi^-
\end{array}
$$

Their result was

$$\mu_{\Xi^-} = (-2.2 \pm 0.8)\ \mu_N$$

Some indirecting information on electromagnetic moments of charged ρ-mesons was obtained by Glück and Wagner [393]; in [194] one can find a discussion concerning the anomalous magnetic moment of charged vector bosons (see also [197]).

4.3. Group-Theoretic Analysis of the Structure of Hadron Multiplets and the Quark Hypothesis

A new impetus was given to the theory of the magnetic moments of the strongly interacting particles known as hadrons when there appeared a new general group-theoretic approach to systematizing these particles, which was first introduced in 1961-2 by Gell-Mann [373, 374] and Ne'eman [676] (unitary symmetry, "the eightfold-way", SU(3) and SU(6) groups).

Without going into the details of the mathematical apparatus of the group-theoretic method—one can find its description in a number of review papers [81, 375, 856, 891]—let us use its more popular presentation given by Gell-Mann [375] and independently by Zweig [998] and by Zel'dovich [994]; the review "for pedestrians" of the latter we shall take as the basis for our presentation.

In order to introduce this scheme let us review briefly the set of quantum numbers specifying a hadron state.

As is known from experiment, hadrons may have positive or negative *electric charges* (for example, $+e$ for the proton and $-e$ for the negative pion π^-) or be electrically neutral (for example, the neutral pion π^0, hyperons Λ^0, Σ^0, etc.). Hadrons also possess *spin* (spin of nucleons p and n is 1/2, pions have zero spin, spin of ρ-mesons is equal to 1, spin of Ω^--hyperons is 3/2, etc.); they are also characterized by *parity* and *rest masses*.

Among hadrons one can identify groups of particles having the same (or nearly the same, up to the accuracy of weak or electromagnetic interactions) properties such as spin, parity, mass, but different values of electric charge. These are the so-called *charge multiplets*: the pair of nucleons p and n, the triplet of pions π^+, π^0, π^-, the triplet of hyperons Σ^+, Σ^0, Σ^-, etc. The particles in a charge multiplet are regarded as different charge states of the same particle; this is possible if an additional charge variable and the corresponding quantum number—*isotopic spin* I—are introduced. The number of particles in a charge multiplet is equal to $2I + 1$.

Further, hadrons are divided into two groups of particles, *baryons* and *mesons*, according to the value of another special quantum number, the *baryon charge* (or *baryon number*), which characterizes conservation of the number of baryons in all known reactions. Nucleons and hyperons (including resonances) have a baryon charge $A = 1$, their antiparticles, respectively, have $A = -1$. Mesons have no baryon charge, i.e. for them $A = 0$.

If we limit ourselves to the analysis of processes in which weak interactions could be neglected, we can formulate two more conservation laws for hadrons: (1) the law of conservation of the electric charge of hadrons themselves (instead of the total charge) Q and (2) the law of conservation of *strangeness* S or *hypercharge* $Y = S + A$* (S can take on only the values 0, ± 1, ± 2, ± 3, . . .).

Following the scheme of Gell-Mann and Zweig, we shall interpret all hadrons as constructed of three still more

* If the reaction occurs only through strong interaction, then ou of the two reactions

$$\pi^+ + p = \Sigma^+ (S = -1) + K^+ (S = +1)$$

and

$$\pi^- + n = \Lambda^0 (S = -1) + K^- (S = -1)$$

only the first one is possible since it corresponds to the conservation of strangeness. Weak interaction can remove this forbiddenness and, for instance, permit (with a small probability) the reaction $\Lambda^0 (S = -1) = p + \pi^-$.

elementary particles, quarks $\mathfrak{p}$, $\mathfrak{n}$, λ*. The properties of these quarks are given in Table 4.5. In addition, we also assume the existence of *antiquarks* $\bar{\mathfrak{p}}$, $\bar{\mathfrak{n}}$, $\bar{\lambda}$ with opposite signs for Q and A.

On the basis of group-theoretic considerations (according to the eightfold scheme SU(3)) hadrons can be grouped in

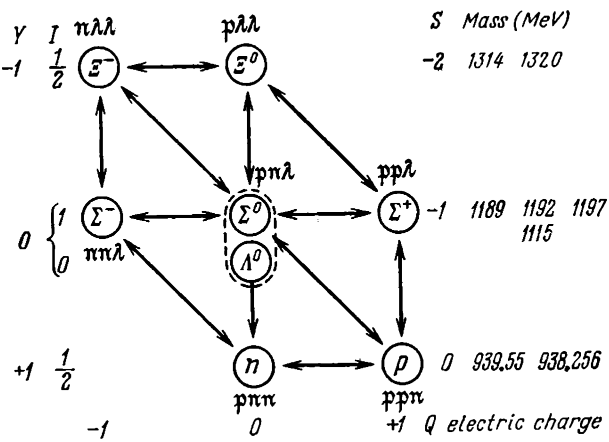

Fig. 4.5. The baryon octet: spin 1/2, parity +, baryon charge 1. Notations: Y—hypercharge; I—isotopic spin; S—strangeness; $\mathfrak{n}$, $\mathfrak{p}$, λ —quarks.

supermultiplets, such as the baryon octet and the baryon decuplet; these supermultiplets are depicted in Figs. 4.5 and 4.6, respectively; their quark structure is also given there. In each supermultiplet the lower row of particles has the smallest mass while the upper one the largest.

Fundamental characteristics of some hadrons are given in Table 4.6.

The simplest scheme is that of the baryon decuplet since out of three quarks $\mathfrak{p}$, $\mathfrak{n}$, and λ it is possible to obtain

* The name *quark* is a reference to a remark in *Finnegan's Wake*: "Three Quarks for Mister Mark". (James Joyce, *Finnegan's Wake*, Viking, New York, 1939, p. 383.)

Table 4.5

The Fundamental Characteristics of Quarks

Type of quark	Electric charge (in e units)	Strangeness	Baryon number	Spin
p	+2/3	0	1/3	1/2
n	−1/3	0	1/3	1/2
λ	−1/3	−1	1/3	1/2

Table 4.6

Fundamental Characteristics of Hadrons (Baryons and Mesons)

Type of hadron	Spin	Parity	Mass (in MeV)	Isospin I and the number of elementary particles in charge multiplet	Hypercharge Y	Strangeness S
			Baryons			
n	1/2	+	939	1/2 (2)	1	0
Λ	1/2	+	1115.4	0 (1)	0	−1
Σ	1/2	+	1193.2	1 (3)	0	−1
Ξ	1/2	+	1317.6	1/2 (2)	−1	−2
Δ	3/2	+	1236	3/2 (4)	1	0
Σ*	3/2	+	1382	1 (3)	0	−1
Ξ*	3/2	+	1529	1/2 (2)	−1	−2
Ω^-	3/2	+	1675	0 (1)	−2	−3
			Mesons			
π	0	−	138	1 (3)	0	0
K	0	−	496	1/2 (2)	1	1
η	0	−	549	0 (1)	0	0
X	0	−	959	0 (1)	0	0
ρ	1	−	765	1 (3)	0	0
ω	1	−	782	0 (1)	0	0
K*	1	−	891	1/2 (2)	1	1
φ	1	−	1019.5	0 (1)	0	0

only 10 different combinations given in Fig. 4.6. Furthermore, if we assume that the masses of quarks p and n are nearly the same and the mass of λ-quark is 146 MeV greater, then, as can be seen from Figs. 4.5 and 4.6 and Table 4.6, the difference of the particle masses in the adjacent rows of the octet and the decuplet will be approximately equal to this value. In particular, this was how the mass of Ω^--hyperon (equal to 1675 MeV) was predicted; the very existence

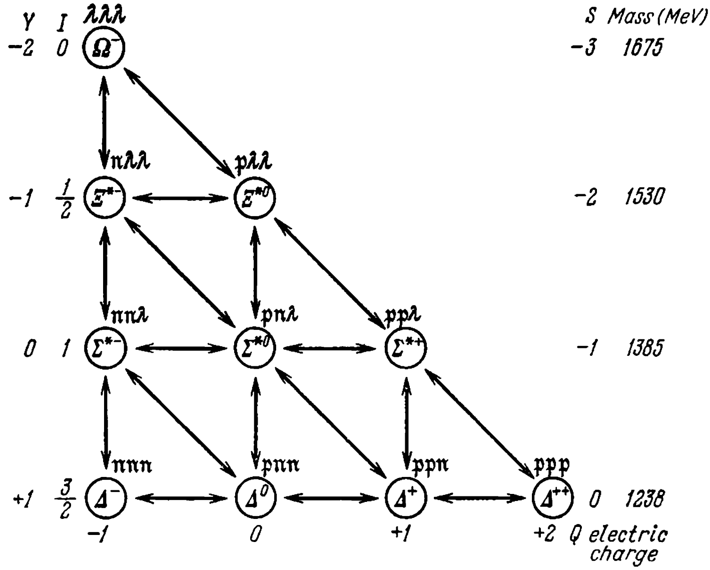

Fig. 4.6. The baryon decuplet; spin 3/2; parity +; baryon charge 1; n, p, λ—quarks.

of this particle was also the result of theoretical prediction and as such appeared as a triumph of the theory of unitary symmetry (see, for example, [338]).

Further, it can be easily shown (see [994]) that there are no combinations ppp, nnn, and λλλ (the "corner" elements) in the baryon octet with spin 1/2, while there is a double state in the middle corresponding to the combination pnλ.

In a similar way one can construct octets for pions, ρ-mesons, etc. Since the baryon charge of mesons $A = 0$, in the simplest case each meson consists of one quark and one antiquark. There may be nine such combinations possible formed by three quarks and three antiquarks. If in these combinations quarks' spins are antiparallel, we obtain a pion octet with spin zero; if they are parallel, there is a ρ-meson

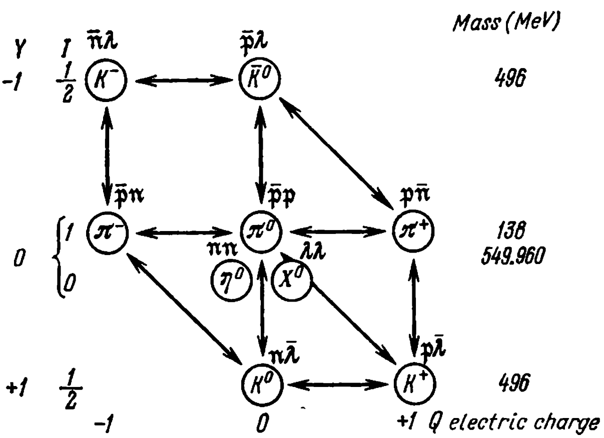

Fig. 4.7. The pion octet: spin 0; parity—; baryon charge 0; n, p, λ—quarks; $\bar{n}$, $\bar{p}$, $\bar{\lambda}$—antiquarks.

octet with spin unity. Figure 4.7 gives an example of the pion octet.

4.4 Determination of Magnetic Moments of Hadrons on the Basis of the Quark Hypothesis

Of greatest interest for us is the determination of magnetic moments of hadrons.

In the baryon decuplet magnetic moment of each hadron is equal simply to the sum of magnetic moments of three quarks. The latter, obviously (see, for example, eq. (1.2) for μ_B) are proportional to their charges:

$$\mu_p = \frac{2}{3}\mu_1,\ \mu_n = -\frac{1}{3}\mu_1,\ \text{and}\ \mu_\lambda = -\frac{1}{3}\mu_1 \quad (4.29)$$

where μ_1 is at this moment the unknown quantity. From Fig. 4.6 it immediately follows that

$$\begin{gathered}\mu_{\Delta^{++}} = 2\mu_1,\quad \mu_{\Delta^+} = \mu_1,\quad \mu_{\Delta^0} = 0,\quad \mu_{\Delta^-} = -\mu_1\\ \mu_{\Sigma^{*-}} = -\mu_1,\quad \mu_{\Sigma^{*0}} = 0,\quad \mu_{\Sigma^{*+}} = \mu_1,\quad \mu_{\Xi^{*-}} = -\mu_1,\\ \mu_{\Xi^{*0}} = 0,\quad \mu_{\Omega^-} = -\mu_1\end{gathered} \quad (4.30)$$

These equations show that in the decuplet the magnetic moment is simply proportional to the baryon charge.

In order to compute the magnetic moments of hadrons from the baryon octet let us consider first the difference, for example, between combination $\mathfrak{ppn}$, which in the decuplet corresponds to Δ^+ (Fig. 4.6), and the same combination giving p in the octet (Fig. 4.5). In the first case the spin quantum number $s = 3/2$ and the spin projection $s_z = 3/2$, i.e. $\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\uparrow$ is the only possible state. On the other hand, when $s_z = +1/2$ we have two states: $\mathfrak{p}\uparrow\mathfrak{p}\downarrow\mathfrak{n}\uparrow$ and $\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\downarrow$. Out of these two combinations can be formed: one of them corresponds to the particle Δ^+ with the direction of spin forming an angle with the z-axis ($s = 3/2$ and $s_z = +1/2$), while the other represents a state with $s = 1/2$ and $s_z = +1/2$ which is nothing but the proton p from the octet. The total orbital angular momentum of quarks is assumed to be zero: $L = 0$.

All other cases can be analyzed in the same way.

Now, if we consider again the resonance Δ^+ from the decuplet with $s_z = +3/2$ ($\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\uparrow$) and transform it into the state with $s_z = +1/2$, the probability to obtain $\mathfrak{p}\uparrow\mathfrak{p}\downarrow\mathfrak{n}\uparrow$ will be twice as great as that of obtaining $\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\downarrow$ since there are two $\mathfrak{p}$-quarks and only one $\mathfrak{n}$-quark. In other words, the state Δ^+ with $s = 3/2$ and $s_z = +1/2$ can be presented as

$$\frac{2}{3}(\mathfrak{p}\uparrow\mathfrak{p}\downarrow\mathfrak{n}\uparrow)+\frac{1}{3}(\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\downarrow)$$

and the corresponding eigenfunction will be

$$\sqrt{\frac{2}{3}}(\mathfrak{p}\uparrow\mathfrak{p}\downarrow\mathfrak{n}\uparrow)+\sqrt{\frac{1}{3}}(\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\downarrow)$$

The wave function of a hadron from the octet, which is described by the same combination $\mathfrak{ppn}$, is orthogonal to the wave function of a baryon from the decuplet, i.e. it is equal to

$$-\sqrt{\frac{1}{3}}(\mathfrak{p}\uparrow\mathfrak{p}\downarrow\mathfrak{n}\uparrow)+\sqrt{\frac{2}{3}}(\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\downarrow)$$

This means that the respective probabilities change places; for the proton p we shall have

$$\frac{1}{3}(\mathfrak{p}\uparrow\mathfrak{p}\downarrow\mathfrak{n}\uparrow)+\frac{2}{3}(\mathfrak{p}\uparrow\mathfrak{p}\uparrow\mathfrak{n}\downarrow)$$

Now the magnetic moment of the proton can be easily computed:

$$\mu_p=\frac{1}{3}\left(+\frac{2}{3}\mu_1-\frac{2}{3}\mu_1-\frac{1}{3}\mu_1\right)$$
$$+\frac{2}{3}\left[+\frac{2}{3}\mu_1+\frac{2}{3}\mu_1-\left(-\frac{1}{3}\mu_1\right)\right]=\mu_1 \qquad (4.31)$$

Similarly, for the neutron (with $s = 1/2$ and $s_z = +1/2$) we have

$$\frac{1}{3}(\mathfrak{p}\uparrow\mathfrak{n}\uparrow\mathfrak{n}\downarrow)+\frac{2}{3}(\mathfrak{p}\downarrow\mathfrak{n}\uparrow\mathfrak{n}\uparrow)$$

Therefore

$$\mu_n=\frac{1}{3}\left(+\frac{2}{3}\mu_1-\frac{1}{3}\mu_1+\frac{1}{3}\mu_1\right)$$
$$+\frac{2}{3}\left(-\frac{2}{3}\mu_1-\frac{1}{3}\mu_1-\frac{1}{3}\mu_1\right)=-\frac{2}{3}\mu_1 \qquad (4.32)$$

For the ratio of these moments the theory gives

$$\left(\frac{\mu_n}{\mu_p}\right)_{\text{theor}}=-\frac{2}{3}\cong-0.667 \qquad (4.33)$$

while from experiment (Chapter 3) we obtain

$$\left(\frac{\mu_n}{\mu_p}\right)_{\text{exp}}=-0.685 \qquad (4.34)$$

The agreement is surprisingly good up to the accuracy of 2 per cent. This is a strong argument in favour of the quark hypothesis.

It would be very desirable to measure the magnetic moments of all hadrons since the theory, as we have seen in (4.30), is capable of certain predictions. Namely, from (4.30), (4.31), and (4.32) and from computations according to SU(3) scheme it follows:

$$\mu_{\Sigma^+}=\mu_p, \quad \mu_n=\frac{3}{2}\mu_{\Lambda^0}-\frac{1}{2}\mu_{\Sigma^0},$$
$$\mu_{\Sigma^-}=\mu_{\Xi^-}=[\mu_p+\mu_n], \quad \mu_{\Xi^0}=\mu_n$$

and if the quark model is valid, we have two more relations

$$\mu_{\Sigma^0} = \mu_{\Lambda^0}, \quad \mu_{\Lambda^0} = \mu_{\Sigma^-}$$

Hence,

$$\mu_n = 2\mu_{\Lambda^0} \quad \text{and} \quad -\mu_n = \mu_{\Sigma^+} + \mu_{\Sigma^-}$$

Thus, all magnetic moments can be expressed through those of any two hadrons, for example, μ_p and μ_n (or even one of them if we use the scheme SU(6)).

The problem of magnetic moments of hadrons is also discussed in the end of Sec. 2 of Chapter 5 in connection with the question of relation between the Dirac monopoles and the "magnetic" quarks.

Let us mention some theoretical papers on the magnetic moments of elementary particles.

The first article on this subject belongs to Coleman and Glashow [219]. The authors were the first to obtain on the basis of the quark hypothesis the above relations for the baryon magnetic moments.

In an earlier paper of Marshak et al. [623] the relation

$$\mu_{\Sigma^0} = \frac{1}{2} (\mu_{\Sigma^+} + \mu_{\Sigma^-})$$

was found, which is independent of the unitary symmetry scheme of Gell-Mann and follows only from the fact of isospin invariance.

In [703] (which lies outside the quark model) Okun and Rudick, instead of using the indirect method of Marshak et al. [623] based on the determination of momenta of Σ^+- and Σ^--hyperons, conducted a theoretical analysis of the possibilities of direct determination of magnetic moment of Σ^0-hyperon from comparison of two decay reactions

$$\Sigma^0 \to \Lambda^0 + 2\gamma \quad \text{and} \quad \Sigma^0 \to \Lambda^0 + \gamma$$

Okonov [697], making use of experimental data concerning the creation of K^0-meson, gave an estimate of the upper limit for the possible value of its magnetic moment: $\mu_{K^0} \leqslant \leqslant 0.04\mu_0$, where $\mu_0 = e\hbar/cM_{K^0}$.

For those readers who are specially interested in the question of hadron magnetic moments let us mention some subsequent papers in which relations between magnetic moments of hadrons are derived on the basis of the unitary symmetry scheme. These are the following: [4, 10, 19, 43, 46, 65, 67, 72, 73, 164, 196, 207, 277, 342, 343, 344, 360, 370, 436, 495, 512, 627, 628, 629, 630, 685, 689, 698, 699, 711, 720, 764, 770, 774, 776, 791, 798, 917].

As far as the determination of moments μ_p and μ_n (or one of them, or the moment of quark μ_1) is concerned, the theory of unitary symmetry of hadrons does not provide an adequate answer. A deeper microscopic theory of strong interactions, which is not yet constructed, may be necessary. We can only say that the determination of magnetic properties of elementary particles is of great importance for the formulation of a consistent microscopic theory of hadrons. Among theoretical works concerned with the computations or estimates of the ratio μ_n/μ_p we should mention the following [1, 5, 60, 165, 276, 277, 342, 482, 595, 609, 655, 717, 777, 806, 962].

Apart from magnetic moments the essentially electromagnetic property of elementary particles is the mass difference between the particles belonging to a charge multiplet (between n and p, between π^+, π^0, and π^-, etc.).

This problem was discussed in many papers mentioned in this section. In addition see [152, 231, 380, 436, 797, 932].

Bogoliubov et al. [123, 124] have shown that the magnetic moment of a "Dirac" particle in a bound state in a strong scalar field can increase and become inversely proportional to the energy of the bound state instead of the particle's mass.

Lately there were investigations concerning the measurement of magnetic moments of nucleon antiparticles. Button and Maglic [159] conducted experimental determination of the magnetic moment of the antiproton by means of measuring the asymmetry of double scattering of these particles in a hydrogen bubble chamber. These measurements are still very inprecise. The result is $\mu_{\bar{p}}=(-1.8 \pm 1.2)\mu_N$ instead of the value $\mu_{\bar{p}} = -2.792\mu_N$ (a theoretical prediction based on the so-called CPT theorem; see [20]).

A more recent paper of Fox et al. [339] presents the results of more accurate measurements of the magnetic moment of the antiproton. In these studies (performed at the Brockhaven National Laboratory, U.S.A.) the magnetic moment was determined from the measurement of the fine structure splitting in the X-ray spectrum of atoms formed by antiprotons with nuclei of lead and uranium-238. Since the fine structure (see Chapter 2) results from the interaction between the antiproton magnetic moment and the electrostatic field of the nucleus, the observed fine structure splitting $\Delta\mathscr{E}_{FS}$ allows to determine the antiproton magnetic moment, which according to the CPT theorem should be equal in magnitude to and have an opposite sign with respect to the magnetic moment of the proton:

$$\mu_{\bar{p}} = -\mu_p = -\mu_N - \Delta\mu_p$$

where $-\Delta\mu_p = -1.792782(17)\ \mu_N$. From measurements of $\Delta\mathscr{E}_{FS}$ we obtain

$$\Delta\mu_{\bar{p}} = (-1.83 \pm 0.10)\mu_N$$

which agrees with the prediction of the CPT theorem.

THE MAGNETIC MONOPOLE*

Nothing is too wonderful to be true, if it be consistent with the laws of nature, and in such things as these, experiment is the best test of such. consistency.
M. Faraday**

1. Introductory Remarks

For a long time it was believed there was no real magnetic charge, that this was a purely auxiliary concept. This concept was introduced at the birth of electromagnetic theory by analogy with the concept of the electric charge. But Gilbert in his famous monograph *De Magnete* (1600) refused to support the concept of a magnetic charge. The decisive blow to this concept was dealt by the well-known hypothesis of Ampère, who regarded electric current as the only source of magnetism (permanent magnets included).

In 1873 Maxwell, the father of the electromagnetic field theory, which has remarkable features of symmetry with respect to electric and magnetic fields, supported Ampère's hypothesis on molecular currents [633] and even rejected the limited concept of magnetic charges being two magnetic liquids bound within each individual molecule. He gave preference to the idea of intramolecular currents, although in the eighteen seventies there was no conclusive experimental confirmation of their existence. Proof was obtained after the discovery of the nuclear structure of the atom and the electron spin. These discoveries turned Ampère's hypothesis into a concrete physical statement of experimental facts.

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* See the review papers [132, 252, 256, 321, 334, 513, 547, 832].
** Quoted from Schwinger [832].

It seemed that the concept of a magnetic charge was doomed to play an auxiliary role in science. However, in 1931 Dirac [254] with his usual boldness of thought challenged the dominant idea of the absence of a magnetic charge in nature (the Bohr magneton μ_B and the nuclear magneton μ_N are magnetic moments) and rehabilitated this concept. Dirac made use of the symmetry contained in Maxwell's fundamental differential equations for an electromagnetic field:

$$\operatorname{div} \boldsymbol{E} = 4\pi\rho_e, \quad -\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \operatorname{rot} \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j}_e,$$

$$\operatorname{div} \boldsymbol{B} = 0, -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} - \operatorname{rot} \boldsymbol{E} = 0 \tag{5.1}$$

In the first line of equations (5.1) in the right-hand sides there are the density of electric charge ρ_e, the source of the electric field $\boldsymbol{E}$, and the density of electric current $\boldsymbol{j}_e$, the source of magnetic field $\boldsymbol{B}$. In the second line of (5.1) the right-hand sides are zeros. The symmetry would be complete if instead of the zero in the first equation of the second line of (5.1) we could write the "density of magnetic charges" $4\pi\rho_m$ and in the second the density of their "magnetic currents" $\frac{4\pi}{c}\boldsymbol{j}_m$. In his paper [254] Dirac wrote: "Under these circumstances one would be surprised if Nature had made no use of it.", i.e. if it would not admit magnetic charge.

The classical theory of an electromagnetic field would seem to offer no objections to the establishment of symmetry by the introduction of the quantities ρ_m and $\boldsymbol{j}_m$. However, apropoe of the symmetry of Maxwell's equations with respect to electric and magnetic charges we can quote Bolotovsky, author of the supplement to the translation of the Devons review [252]: "The very fact of this symmetry long served as an argument in favour of the existence of magnetic charges. This symmetry, however, cannot be preserved in formulating the variational principle that would give both the field equations and the equations of motion of particles with the electric and magnetic charges coexisting. In formulating a variational principle, Dirac [255] introduced for the description of a magnetic charge a certain non-physical concept ("filaments"—S.V.), which is not necessary for an electric charge. To this day there has been no variational principle formulated that treats electric and magnetic charges equally. Some attempts in this direction [700] emphasize the difficulty of the problem rather than contribute to its solution. This difficulty may also indicate the incompleteness of our conceptions about a magnetic charge. It may also mean that the existence of monopoles is prohibited."

It is clear from this statement that the problem of a magnetic charge is still far from being solved. At the same time this problem can lead to extremely interesting experimental conclusions. For this reason it has in recent years attracted growing attention among physicists, both theoreticians and experimenters.

2. Dirac's Quantization of Magnetic Charges

Up to this point we have limited ourselves to the strictly classical approach to the problem. Now, if we consider the quantum theory, we encounter a special situation, which Dirac discussed in his fundamental study "Quantised Singularities in Electromagnetic Field" [254]. We have no opportunity here to examine this study in detail. We shall consider it in the simplified form suggested in Fermi's well-known lectures on atomic physics [321].

The main conclusion of the above referenced paper of Dirac [254] was quite unexpected: the magnetic charge of the monopole g appeared to be quantized, i.e. it was a multiple of a certain minimal quantity. Dirac also obtained a relation containing the magnitude of the elementary electric charge e:

$$g = n\frac{\hbar c}{2e} \quad (n \text{ is an arbitrary integer}) \tag{5.2}$$

In order to understand the meaning of equation (5.2) let us present three different ways of its derivation. First of all, we assume, as Fermi did, that there is a magnetic charge g fixed in a certain point P of space. It generates a Coulomb magnetic field

$$\boldsymbol{H} = \frac{g\boldsymbol{r}}{r^3} = -g \times \operatorname{grad}\left(\frac{1}{r}\right) \tag{5.3}$$

similar to the electric field $\boldsymbol{E}$ in the case of electric charge e. Obviously, the divergence of this type of magnetic field is not equal to zero (div $\boldsymbol{H} \neq 0$) because of the existence of the field source at point P. We know that the field $\boldsymbol{H}$ can be expressed through a vector potential $\boldsymbol{A}$:

$$\boldsymbol{H} = \operatorname{rot} \boldsymbol{A} \tag{5.4}$$

where

$$\operatorname{div} \boldsymbol{A} = 0 \tag{5.5}$$

Equations (5.4) and (5.5) completely determine the vector potential $\boldsymbol{A}$. However, in order to avoid explicit computations let us make use of a certain electric analogy.

Let us introduce an equivalent system of currents with the density $\boldsymbol{j}'$ generating a magnetic field $\boldsymbol{H}'$ satisfying the following equations:

$$\operatorname{div} \boldsymbol{H}' = 0, \quad \operatorname{rot} \boldsymbol{H}' = 4\pi \boldsymbol{j}' \tag{5.6}$$

Comparing (5.6) with (5.4) and (5.5), we see that the problem of determining the vector potential $\boldsymbol{A}$, which is responsible for the magnetic field $\boldsymbol{H}$, is equivalent, from the mathematical point of view, to that of determining a magnetic field generated by the system of currents with density $\boldsymbol{j}'$. Since the methods of the determination of a magnetic field ($\boldsymbol{H}'$) from a given distribution of currents ($\boldsymbol{j}'$) are well known in electrodymanics, the initial problem of determining $\boldsymbol{A}$ from a given $\boldsymbol{H}$ can be also, in principle, regarded as solved.

In the case under consideration the system of currents $\boldsymbol{j}'$ with the origin at point P should be such that at a distance $\boldsymbol{r}$ from the source g the following equality is satisfied:

$$4\pi \boldsymbol{j}' = \frac{g\boldsymbol{r}}{r^3} \tag{5.7}$$

Thus, in the equivalent problem from point P where the monopole that creates a stationary magnetic flux is located there flows the constant current of density $\boldsymbol{j}'$. This appears possible if the point P is continuously supplied with an electric current I by means of a conducting "filament" of an arbitrary shape. The current intensity I should be equal to that of the current that leaves point P symmetrically in all directions. This is due to the fact that at point

P we have div $\boldsymbol{H}' \neq 0$. Therefore, in the equivalent problem the condition div $\boldsymbol{j}' \neq 0$ should be satisfied. As a result, because of (5.7), we obtain

$$I = j' \times 4\pi r^2 = \frac{g}{4\pi r^2} \times 4\pi r^2 = g \tag{5.8}$$

Here $j' \times 4\pi r^2$ is the flux of vector $\boldsymbol{j}'$ through a closed surface surrounding point P. Due to the arbitrary choice of the "filament" leading the current I to point P, the value of I can be chosen in a number of ways, which gives a number of solutions of the initial problem.

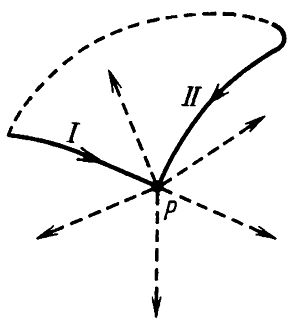


Fig. 5.1. Concerning the derivation of the quantization rule for the charge of the Dirac monopole according to Fermi [321].

Consider two such solutions $\boldsymbol{A}_{\mathrm{I}}$ and $\boldsymbol{A}_{\mathrm{II}}$ corresponding in the equivalent problem to two currents in the filaments I and II (see Fig. 5.1). Then the rotor of the difference of vector potentials $\boldsymbol{A}_{\mathrm{I}}$ and $\boldsymbol{A}_{\mathrm{II}}$ will be equal to the magnetic field $\boldsymbol{H}'$ that results from the distribution of equivalent currents (which correspond to the two filaments I and II leading to the point P) equal to the difference $j_{\mathrm{I}} - j_{\mathrm{II}}$. These two cases differ: in the first case the current flows to point P through the filament I; in the second, through the filament II. Therefore, if $\boldsymbol{H}'$ (I) is the magnetic field created by the current flowing to point P through the filament I and $\boldsymbol{H}''$ (II) is the field generated by the current flowing to point P through the filament II, we shall have

$$\text{rot}\,(\boldsymbol{A}_{\mathrm{I}} - \boldsymbol{A}_{\mathrm{II}}) = \boldsymbol{H}'\,(\mathrm{I}) - \boldsymbol{H}''\,(\mathrm{II}) \tag{5.9}$$

In other words the rotor of the difference of vector potentials $\boldsymbol{A}_{\mathrm{I}}$ and $\boldsymbol{A}_{\mathrm{II}}$ corresponds to the magnetic field generated by the current flowing from infinity to point P through the filament I and leaving this point for infinity through the filament II. As we know from magnetostatics, such a magnetic field can be described by a many-valued potential f whose value in a given point of space changes by a quantity $4\pi I$ or $4\pi g$ after describing the contour formed by the filaments (according to (5.8)). Thus,

$$A_{\mathrm{I}} - A_{\mathrm{II}} = \operatorname{grad} f \tag{5.10}$$

Now, let us turn to quantum mechanics, which specifies (see, for example, Chapter 4 in [571]) that a wave function ψ characterizing the motion of a particle with a charge e changes its phase when the vector potential of the electromagnetic field changes its value by a gradient of a scalar function of coordinates. Therefore, the transformation

$$\boldsymbol{A} \to \boldsymbol{A} + \operatorname{grad} f$$

corresponds to the following transformation of the wave function ψ:

$$\psi \to \psi_{\exp}\left(\frac{ie}{\hbar c} f\right)$$

In describing the contour formed by the filaments I—II the function ψ is multiplied by a phase factor $\exp\left(\frac{ie}{\hbar c} 4\pi g\right)$. For an arbitrary g this is an arbitrary imaginary number. However, as it follows from the basic requirement of quantum mechanics, that the wave function be single-valued, this factor should be equal to unity:

$$\exp\left(\frac{ie}{\hbar c} 4\pi g\right) = 1 \tag{5.11}$$

This leads immediately to the condition for the argument of the exponent, which should be a multiple of $2\pi i$:

$$\frac{ie}{\hbar e} \times 4\pi g = n \times 2\pi i$$

or

$$g = n \frac{\hbar c}{2e}$$

This is the Dirac quantization rule (5.2) for the magnetic charge g.

The last formula can be written in a somewhat different way if one makes use of the fine-structure constant $\alpha = e^2/\hbar c = 1/137$ (see (4.1)):

$$g = n \frac{\hbar c}{2e^2} e = n \frac{c}{2\alpha} = n \frac{137}{2} e = n \times 68.5 e \tag{5.12}$$

Thus, the value of magnetic charge can only be a multiple of the product of the elementary electric charge and the number $137/2 = 68.5$. As it follows from (5.12), the elementary magnetic charge (corresponding to $n = 1$) exceeds the observed electric charge e nearly by two orders of magnitude.

In Schwinger's paper [828] for a model with two filaments n is replaced by $2n$. Therefore, the minimal magnetic charge is equal to $2 \times 68.5e = 137e$ instead of $68.5e$. In a more recent paper Usachev [935] has shown that the concept of

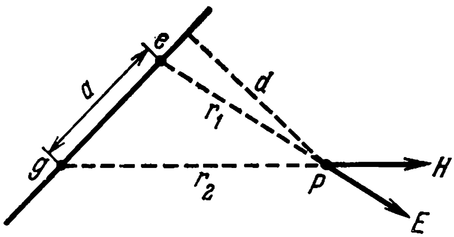


Fig. 5.2. Concerning the determination of the coupling between electric and magnetic charges of the Dirac monopole according to Fermi [321].

a potential containing s filaments leads to an arbitrary scheme of quantization of the magnetic charge $s\,\frac{137}{2}\,e$. According to Bolotovsky and Usachev [132], this circumstance emphasizes the weak points of any theory based on non-observable filaments.

Let us now demonstrate (see [321]) that the quantization rule for the magnetic charge of the monopole can also be derived from still more elementary considerations. Consider a monopole with charge g and an elementary electric charge e (Fig. 5.2). At a point P at a distance $\boldsymbol{r}_1$ from the charge e the electric field will be $\boldsymbol{E} = e\boldsymbol{r}_1/r_1^3$, and the magnetic field of the monopole at a distance $\boldsymbol{r}_2$ at the same point P will be equal to $\boldsymbol{H} = g\boldsymbol{r}_2/r_2^3$. The vector product of these fields, up to the accuracy of the factor $(4\pi c)^{-1}$, is equal to the electromagnetic momentum. Let us denote the distance between the charges g and e by a and evaluate the order of magnitude of the fields and the momentum at a point located at a distance of the order of a from the charges. We obtain

$$E \propto \frac{e}{a^2}, \quad H \propto \frac{g}{a^2} \left| \frac{[\boldsymbol{EH}]}{4\pi c} \right| \propto \frac{ge}{ca^4}$$

The effective arm d in the expression for the angular momentum is also of the order of a while the effective volume of integration is of the order of a^3. As we can easily see, the integral of motion, equal to the total angular momentum of the field, is independent of a and equal to $2eg$. According to quantum mechanics, the value of this angular momentum is a multiple of $\hbar$ ($2eg/c = n\hbar$), which means that

$$g = n\frac{\hbar c}{2e} = n\frac{\hbar c}{2e^2}e \cong n \times 68.5e \tag{5.13}$$

Thus, we have again demonstrated that the magnitude of the monopole charge of the monopole can only be equal to the multiple of the product of the elementary electric charge and half the fine-structure constant $\alpha^{-1} = 137$. And vice versa, it follows from (5.13) that

$$e = n\frac{\hbar c}{2g}$$

Therefore, as was pointed out by Dirac, since we know that there exists a quantized elementary electric charge, the monopole charge should be such that the electric charge would take on the value e known from experiment. In the case of the monopole the quantity similar to the fine-structure constant is equal to

$$\frac{g^2}{\hbar c} \cong 84$$

i.e. its value is not small, and, therefore, it would not be correct to use an approximation in which the field of the monopole is separated from that of electromagnetic radiation (which is possible to do with an electric charge).

Finally, according to Efinger [288], the quantization rule (5.2) or (5.12) can be obtained by another method in a very elegant way using the quantum-mechanical analysis of the problem of motion of the monopole in a uniform electric field. Thus, let us imagine a uniform electric field $\boldsymbol{E}$ between the condenser plates, as shown in Fig. 5.3. It would be natural to assume that a monopole with charge g moves along the orbit AA' lying in the plane normal to vector $\boldsymbol{E}$ (by analogy with an electron in a magnetic field; according to Landau,. this is the diamagnetism of free electrons). The Lorentz force acting on the magnetic charge g moving in a circular orbit with linear velocity v will be equal to

$$F_E = g\,|\boldsymbol{E}|\,\frac{v}{c} \tag{5.14}$$

In equilibrium this force is compensated by the centrifugal force $F_c = Mv^2/R$, where M is the mass of the monopole

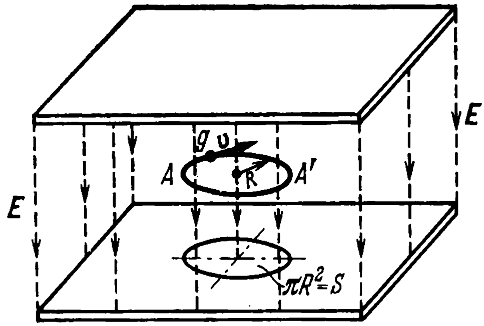


Fig. 5.3. Concerning the derivation of the quantization rule for the charge of the Dirac monopole according to Efinger [288].

and R is the radius of the orbit (see Fig. 5.3). From equality $F_E = F_c$ we find the following:

$$|\boldsymbol{E}| = \frac{Mvc}{Rg} \tag{5.15}$$

Let us now make use of the fundamental result from the quantum theory of electron diamagnetism formulated by Landau [565] according to which the angular momentum J_z of a charge in a static, uniform field satisfies the quantization rule

$$J_z = (2n+1)\,\hbar \quad (n = 0, 1, 2, \ldots) \tag{5.16}$$

which takes into account the zero motion (when $n = 0$). Since $J_z = MvR$, equation (5.16) yields

$$MvR = (2n+1)\,\hbar \tag{5.17}$$

From (5.15) and (5.17) we easily obtain

$$|E| = \frac{4}{R^2} \frac{\hbar e}{2g} \left(n + \frac{1}{2}\right) \tag{5.18}$$

Let us note now that the electric field can be expressed through the surface density of the electric charge on the condenser plates according to the formula

$$|E| = \frac{4\pi Q}{S} \tag{5.19}$$

where $S = \pi R^2$ is the orbit's area and Q is the charge distributed over this area on the condenser plates. Substituting (5.19) for the quantity E from (5.18), we find

$$Q = \frac{\hbar c}{2g} \left(n + \frac{1}{2}\right) \tag{5.20}$$

It follows from (5.20) that, when $n = 0$, there is still a finite charge, which is related to the zero motion of the monopole. The subtraction of the vacuum effect leads to the following value for the observed charge on the plate:

$$Q_{\text{obs}} = n \frac{\hbar c}{2g} \quad (n = 0,\ 1,\ 2,\ \ldots) \tag{5.21}$$

Thus, we have obtained the quantization rule for the electric charge on the condenser plate with the minimal quantum $\hbar c/2g$. This agrees with the condition that the monopole moves under the effect of electric field produced by the electric charge. Equating this minimal value of the quantized electric charge to the known charge of the electron e, we obtain a definite value for the charge of magnetic monopole. Namely,

$$g = \frac{\hbar c}{2e} = \frac{137}{2} e$$

which is in complete agreement with the Dirac formula (5.12).

At present there is already a vast literature on the theory of the Dirac monopole with a tendency for a fast growth in recent years (beginning approximately with 1963-5). Nearly simultaneously with the paper of Dirac [254] there appeared

Tamm's article [895] that treated the problem of the monopole-electron system and demonstrated the absence of bound states in such a system, which are possible in the presence of an additional electric field, for instance, the field of an atom (see [618]). There are also papers on the scattering of electrons by fixed monopoles; the authors also tried (unsuccessfully) to clarify the question of the possibility to observe the Dirac filaments [57, 398]. There have been attempts at constructing theories without singular filaments (see, for example, [727, 805, 972]). A shortcoming is their semiclassical character (in this connection see the remarks to Appendix A in [22]).

According to Bolotovsky and Usachev [132], the most convincing although not final demonstration of the validity of relation (5.2) is the derivation of Dirac himself [254, 255]. On the other hand, the question of whether this relation remains valid in the theory of magnetic charge should be still regarded as open. These authors believe that the final solution of this problem can be found in a variant of the theory (if it is at all possible) with a potential having no singularities in space except at the point where the charge itself is located.

In some theoretical works there are conclusions, obtained on the basis of group considerations, of the impossibility in principle of the existence of Dirac's monopole. For example, Tomil'chik [920] states that the existence of monopole is forbidden by the conservation of parity in electromagnetic interactions. Pintacuda [743], however, argues against such a conclusion. According to Zwanziger [995] the search of monopoles is useless since their existence is incompatible with the analytic properties of the scattering matrix. Therefore, the group-theoretic approach has not yet disproved Dirac's hypothesis of the existence of monopole.

Among work on the theory of the monopole in the light of the principles of symmetry we should mention the following: [327, 428, 724, 753, 767, 803, 807, 814, 816, 857, 881, 882, 953].

Let us also mention the articles in which the problem of the monopole is considered from the point of view of the quark hypothesis [178, 819] and also in connection with the phenomenon of superconductivity [244, 728, 901].

Finally, let us also list the papers treating the problem from the electrodynamic point of view: [70, 99, 163, 181, 182, 236, 261, 262, 269, 270, 271, 272, 280, 281, 288, 289, 294, 322, 325, 326, 333, 366, 371, 418, 438, 440, 462, 496, 497, 507, 509, 523, 524, 525, 591, 611, 652, 653, 665, 679, 693, 701, 712, 726, 754, 761, 790, 793, 800, 804, 829, 830, 831, 883, 903, 908, 909, 910, 921, 930, 931, 944, 959, 996, 997].

In conclusion we shall formulate, following the paper of Dirac [254], three main reasons for the possible existence of monopoles, reasons that are still valid.

(1) Monopoles would lend symmetry to Maxwell's equations by allowing the magnetic charge density to appear along with the electric charge density.

(2) They are forbidden by no law of physics.

(3) They would explain the quantization of electric charge e through the relation (5.2).

What is commonly referred to as a "Dirac monopole" has $n=1$ in (5.2); clearly larger integers are permitted by Dirac.

Schwinger [828, 829, 830, 831, 832] has given a proof (invariant from the relativistic point of view) that if condition (5.2) is satisfied, then on the basis of compatibility of the concept of a magnetic charge and the principles of relativistic quantum field theory we can obtain the following: $n = 2, 4, \ldots$, i.e. the quantum numbers should be even. Thus, Schwinger's condition is more rigid that of Dirac (with $n = 1, 2, 3, 4, \ldots$).

In addition, Schwinger [832] has suggested that the existence of unpaired magnetic poles as *dyons*, particles having both electric and magnetic charges, would answer the origin of the bewildering array of "elementary" particles and their groupings. They could also explain the observed weak violations of CP symmetry (see also [65]).

Schwinger [832] suggests to replace quarks (see Sec. 4.4, Chapter 4) by dyons, which are regarded as the fundamental constituents of hadrons. However, Chang [188] has found that this hypothesis of dyons leads to difficulties associated with the explanation of electric and magnetic dipole moments of hadrons.

He then formulated a very interesting idea on the relation between the problem of a magnetic monopole and modern concepts concerning the inner structure of elementary particles, hadrons, based on the quark hypothesis (see Secs. 4.3 and 4.4, Chapter 4). Chang admitted a possibility for existence of the Dirac magnetic monopoles inside hadrons and generalized the concept of monopoles: he interpreted monopoles as the magnetic version of quarks. According to Chang, magnetic monopoles together with ordinary electric quarks constitute the inner structure of hadrons. While the *electric quarks* Q_e are fermions, the *magnetic quarks* Q_m introduced by Chang are bosons. Both types of fermion and boson quarks are similar with respect to strong interactions and differ from each other from the point of view of electrodynamics. Chang uses the Q_e and Q_m quarks to construct the *electromagnetic* composite *quarks* Q_{em} in the form of tightly bound pairs of an electric quark q and a magnetic antiquark $\bar{q}'$, i.e. $q\bar{q}'$, and also *electromagnetic antiquarks* $\bar{Q}_{em}$ in the form of pairs of an electric antiquark $\bar{q}$ and a magnetic quark q', i.e. $\bar{q}q'$. As in the case of electric quarks and antiquarks when we usually have six types of these particles, $\mathfrak{p}$, $\mathfrak{n}$, λ, $\bar{\mathfrak{p}}$, $\bar{\mathfrak{n}}$, and $\bar{\lambda}$ (Sec. 4, Chapter 4), in Chang's paper three types of magnetic quarks $\mathfrak{p}'$, $\mathfrak{n}'$, and λ' are introduced, together with their antiparticles $\bar{\mathfrak{p}}'$, $\bar{\mathfrak{n}}'$, and $\bar{\lambda}'$. Their basic properties are given in Table 5.1.

Chang's model of electromagnetic quarks allowed him to analyze the symmetry of the hadron wave functions, to obtain relations between hadron masses, and also to establish the connection between magnetic dipole moments of hadrons. Besides, this model helped to remove some difficulties associated with statistics, which arise in the usual, electric, model of quarks (for details, see [188]).

Since we cannot dwell on the details of this interesting article, let us just present its results concerning relations between dipole magnetic moments of hadrons and compare them with similar relations obtained in the usual model of quarks (Sec. 4.4, Chapter 4). Chang gives these relations for magnetic dipole moments of electric quarks $\mathfrak{p}$, $\mathfrak{n}$, and λ:

$$\mu_{\mathfrak{p}} = -2\mu_{\mathfrak{n}} = -2\mu_{\lambda}$$

Table 5.1

Properties of Electric Quarks q (p, n, λ) and Antiquarks $\bar{q}$ ($\bar{p}$, $\bar{n}$, $\bar{\lambda}$), and also of Magnetic Quarks q' (p′, n′, λ′) and Antiquarks $\bar{q}'$ ($\bar{p}'$, $\bar{n}'$, $\bar{\lambda}'$) from [188]

Symbols	Quarks and antiquarks	Strangeness	Baryon number	Spin	Parity	Electric (in e units) or magnetic (in g units) charges
q	p	0	1/3	1/2	+	2/3
	n	0	1/3	1/2	+	−1/3
	λ	−1	1/3	1/2	+	−1/3
$\bar{q}$	$\bar{p}$	0	−1/3	1/2	−	−2/3
	$\bar{n}$	0	−1/3	1/2	−	1/3
	$\bar{\lambda}$	1	−1/3	1/2	−	1/3
q'	p′	0	1/3	0		2/3
	n′	0	1/3	0		−1/3
	λ′	−1	1/3	0		−1/3
$\bar{q}'$	$\bar{p}'$	0	−1/3	0		−2/3
	$\bar{n}'$	0	−1/3	0		1/3
	$\bar{\lambda}'$	1	−1/3	0		1/3

and also the equations relating these to the moment of hadrons:

$$\mu_p = \mu_{\mathrm{p}} + \mu_x, \quad \mu_n = \mu_{\mathrm{n}} - \mu_x, \quad \mu_{\Lambda^0} = \mu_\lambda,$$

$$\mu_{\Sigma^+} = \mu_{\mathrm{p}} + \mu_x, \quad \mu_{\Sigma^0} = \frac{2}{3}\mu_{\mathrm{p}} + \frac{2}{3}\mu_{\mathrm{n}} - \frac{1}{3}\mu_\lambda,$$

$$\mu_{\Sigma^-} = \mu_{\mathrm{n}}, \quad \mu_{\Xi^0} = \mu_\lambda - \mu_x, \quad \mu_{\Xi^-} = \mu_\lambda$$

where μ_x is the so-called cooperative (or exchange) contribution to the magnetic dipole moment of hadrons, which can be in principle of arbitrary sign or even equal to zero.

From the above relations one can express the quantities μ_{p}, μ_{n}, and μ_λ through the magnetic moments of two nucleons, the proton and the neutron, μ_p and μ_n, measured with sufficient accuracy. Namely,

$$\mu_{\mathrm{p}} = 2(\mu_p + \mu_n) = 1.760\mu_N,$$

$$\mu_\lambda = \mu_{\mathrm{n}} = -(\mu_p + \mu_n) = -0.880\mu_N,$$

$$\mu_x = -(\mu_p + 2\mu_n) = 1.033\mu_N$$

This makes it clear that as with the ordinary electric model of quarks, in the case of the magnetic model the magnetic dipole moments of all other hadrons from the baryon octet can be expressed through those of the proton (μ_p) and the neutron (μ_n). Chang's model of magnetic quarks then gives

$$\mu_{\Lambda^0} = \mu_{\Sigma^-} = \mu_{\Xi^-} = -(\mu_p + \mu_n) = -0.880\mu_N,$$
$$\mu_{\Sigma^+} = \mu_p = 2.793\mu_N,$$
$$\mu_{\Sigma^0} = \mu_p + \mu_n = 0.880\mu_N,$$
$$\mu_{\Xi^0} = \mu_n = -1.913\mu_N$$

The respective results of the ordinary electric model of quarks (see, for example, [859]) have the form

$$\mu_{\Lambda^0} = \mu_{\Xi^-} = \mu_{\Sigma^-} = -0.931\mu_N, \quad \mu_{\Sigma^+} = \mu_p = 2.793\mu_N,$$
$$\mu_{\Sigma^0} = 0.53\mu_N, \quad \mu_{\Xi^0} = -1.863\mu_N$$

According to experimental data (see Sec. 4.2, Chapter 4 and also the summary on elementary particles in *Physics Letters* **39B**, 1, 1972) we have

$$\mu_{\Lambda^0} = (-0.67 \pm 0.06)\,\mu_N, \quad \mu_{\Sigma^+} = (2.59 \pm 0.46)\,\mu_N$$

From comparison of theoretical formulas with each other and with available experimental data one can see that agreement between Chang's model and experiment is not worse than that between experiment and the model of electric quarks; moreover, Chang's model has certain advantages over the electric one.

3. Interaction of a Magnetic Charge with Matter

Let us now assume that the monopole exists in nature. Immediately other questions arise: how are monopoles created and absorbed, what are their physical properties, how do they interact with matter, and how can they be detected?

As far as production and annihilation processes are concerned, they must satisfy the law of conservation of the magnetic charge of monopoles. In other words, monopoles should be created in pairs, i.e. a positive and a negative monopole, and annihilate also in pairs. In this case there must be a complete analogy with electron-positron pairs. On the basis of this analogy one can assume that a pair of monopoles is produced in strong collisions between other particles, for example, protons, or between photons and protons. In these processes the energy of the photon should exceed the double monopole rest mass multiplied by c^2. The rest mass of the monopole carrying a large ($\cong 137e$) magnetic charge g is also large, i.e. of the order of two and a half proton masses (the respective rest energy $\cong$ 2.4 GeV). If the mass of the monopole is known, one can compute the minimal energy of particles that is sufficient for the creation of a pair of monopoles. For the above masses the energy of protons should reach the value of $\cong$ 30 GeV, and that of the photons $\cong$17 GeV. With modern acceleration techniques these are quite attainable values. They are also present in cosmic rays.

Let us now consider the monopole interacting with an external electromagnetic field. Obviously, the electric field does not affect a fixed monopole, as a motionless electric charge does not affect by a constant magnetic field. A monopole moving with a velocity $\boldsymbol{v}$ in an electric field $\boldsymbol{E}$ is subjected to the action of the magnetic Lorentz force

$$\boldsymbol{F}_E = -\frac{g}{c}[\boldsymbol{v}\boldsymbol{E}] \tag{5.22}$$

On the other hand, a magnetic field acts on the monopole with the force

$$\boldsymbol{F}_H = g\boldsymbol{H} \tag{5.23}$$

As it follows from (5.23), the monopole can be accelerated by placing it, for example, in a solenoid with an electric current. Such a solenoid could be an ideal accelerator. A magnetic field of moderate intensity ($\cong 10^4$ Oe), which is easily attainable in simplest devices, could increase the energy of the monopole by 200×10^6 eV per each centimeter of the path. This type of an accelerator, not more than 2 m long, would surpass by its action the most powerful modern equipment. Besides, in order to operate with monopoles,

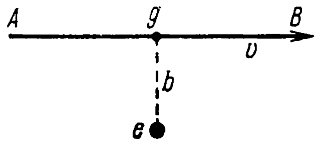


Fig. 5.4. Concerning the determination of the ionization power of the Dirac monopole [321].

this accelerator does not need many particles since these can be extracted from the targets and fed into the accelerator (solenoid) again.

How can the monopole be detected? For this one should know how it interacts with matter, i.e. its ionization loss or the losses due to the Čerenkov radiation. Let us give an approximate estimate of the ionization power of the monopole (see [321]).

Let the monopole move past an electric charge e along a line AB (see Fig. 5.4). When the monopole is at the shortest distance b from the charge, the monopole produces a magnetic field $H \propto g/b^2$ at the point of location of the electric charge e. But the fixed charge e will be also subject to the action of an electric field $E \propto gv/b^2c$ generated by the moving magnetic particle of charge g (if the electric charge were moving, its field would be equal to $E \propto e/b^2$). The ionization effect is proportional to the square of the electric field created by the moving particle. Therefore,

$$\frac{\text{Ionization effect of the monopole}}{\text{Ionization effect of the electric charge}} \propto \left(\frac{g}{e}\right)^2 \left(\frac{v}{c}\right)^2 \qquad (5.24)$$

When $v \cong c$, this ratio approximately equals $(g/e)^2 \cong \cong 4692 \cong (68.5)^2$. Hence, the ionization power of the monopole will be equivalent to that of an atomic nucleus with the atomic number $Z \cong 68$. From (5.24) it also follows that the appearance (in the nominator) of the factor v^2 in the expression for ionization power excludes the possibility of a velocity-dependence of this quantity (since at small values of velocity there should be a factor $1/v^2$). Therefore, one can expect that the track left in an ionization chamber by the Dirac monopole should not exhibit considerable narrowing towards the end contrary to the track of an electrically charged atomic nucleus. The latter, when slowed down, starts capturing electrons, which decreases the charge and, therefore, the ionization power.

The large value of the ionization power of monopoles is due to their large charge ($\cong 68.5e$) since ionization loss is proportional to the square of the charge. Therefore, the monopole track in the photographic emulsion must be very thick (as a fat caterpillar) and cannot pass unnoticed. The ionization properties of the monopole in a solid body and especially at small velocities depend on its interaction with atoms (because of the possibility of formation of bound states). These issues are considered in greater detail in [22, 618] (see also [68, 133, 218, 336, 558, 575, 667]).

The monopole having such a large charge (as compared to the electric charge), it is not only the ionization loss that is great but also the losses due to other types of interaction. These could serve as good indicators of the presence of monopoles. One of these effects is the Čerenkov radiation. This question was analyzed in detail in [549] and in [922, 923]. If a monopole is moving in a body with dielectric permeability $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$ (both depend on the frequency of light ω) and if the velocity of the monopole exceeds the phase velocity of light in the sample, equal to $c[\varepsilon(\omega)\mu(\omega)]^{-1}$, the monopole starts emitting photons. The intensity of the Čerenkov radiation of frequency ω will be proportional to g^2, i.e. is $(68.5)^2$ times greater than the respective quantity in the "electric case". Therefore, the radiation can be easily observed, using the existing Čerenkov counters.

Mergelyan [638] studied another radiation phenomenon—the transition radiation of a monopole hitting a flat boundary that separates a refracting medium. This radiation is also proportional to g^2, i.e. the intensity is very large. One also has to have in mind that both Čerenkov and transition radiations of the monopole differ sharply from the nature of radiation in the electric case, a phenomenon that can help in the detection of monopoles.

Devons [252] states that between a complex atom and a monopole there should exist a short-range diamagnetic repulsive force and a long-range paramagnetic attraction force (from magnetically active d- and f-shells of the electron shell of the atom). One should observe especially strong absorption of monopoles in ferromagnetics, where, generally speaking, they should be looked for.

Martem'yanov and Khakimov [624] performed computations on slowing down monopoles in non-magnetic metals and ferromagnetics. They demonstrated that a ferromagnetic placed in a strong external magnetic field (of the order of 10^4 Oe) is an effective monopole trap.

In conclusion let us enumerate the properties of monopoles, which are most important for the subsequent description of ways of experimental detection of these mysterious particles.

(1) The magnetic charge is subject to strong magnetostatic coupling in ferromagnetic (see [408]) or paramagnetic [331] bodies and, therefore, it can be trapped by these bodies and remain there for a long time. Here we come across a complete analogy with the trapping of electric charges by dielectrics.

(2) The magnetic charge in a magnetic field $\boldsymbol{B}$ is subject to the action of the accelerating force $g\boldsymbol{B}$, which (see equation (5.2)) increments its kinetic energy by a quantity $20n$ MeV/kGs·cm per unit length (cm) per unit field strength (kilogauss). Therefore, fields of 100 kGs acting at atomic distances ($\cong 10^{-8}$ cm) can accelerate a monopole (with $n = 1$) up to the energy of the order of 45 eV, which is quite sufficient to displace the atoms of a crystal from their lattice points. This leads to the possibility of magnetic "extraction" of monopoles from their "stores" in magnetic bodies by means of sufficiently strong external fields [757].

(3) Fast monopoles are strongly ionizing particles. In this sense they resemble relativistic atomic nuclei with

atomic number 137 $n^2/2$ and specific energy loss in a substance per unit length (1 cm) equal to $8n^2 \frac{\text{GeV}}{\text{g/cm}^3}$ [68, 218].

It is these three fundamental properties of monopoles that enable us to understand the mechanism of their slowing down and to select appropriate ways of their detection.

However, up to now there is still no rigorous theoretical predictions concerning the rest mass of the monopoles; we have only approximate hypothetical estimates (see, for example, Appendix B in [22]). The monopole mass m_g is usually (see, for example, Item 2, Sec. 2, [22]) related to the electron mass m_e through the following equation

$$m_g = \left(\frac{g}{e}\right)^2 m_e = \left(\frac{g}{e}\right)^2 \frac{m_e}{m_p} m_p = 2.56 m_p$$

where m_p is the proton mass, since in this case the classical radius of the monopole $r_g = g^2/m_g c^2$ equals that of the electron (see [68]).

In Dirac's paper [255] it was supposed that the monopole is a fermi-particle with the spin quantum number equal to 1/2 (it can be assumed on the same footing that it is a bose-particle with spin zero). Then the monopole should possess a spin electric dipole moment.

4. Experimental Search for Monopoles

The experimental search for monopoles in nature has been elaborated in more than two dozen studies. The most interesting is the latest review by Fleischer et al. [329], which we will largely follow here.

Malkus [618] pioneered in the experimental detection of the monopole. Twenty years after the publication of Dirac's study [254] he performed the first experiment, which gave no results, and there have been no results since. The experiment was simple in the extreme. A long solenoid draws monopoles moving at a low terminal velocity (because of a large ionization loss) along the earth's field lines through a thin mica window into its evacuated core (see Fig. 5.5). The monopoles are then accelerated to several hundred

MeV* and pass through a second mica window to strike a photographic emulsion. On passing through the second window the monopole will lose less than 50 MeV, while its loss in the photographic emulsion is roughly 1 MeV per micron. Careful scanning of the emulsions exposed during the two weak period of operation showed no heavy tracks left by monopoles.

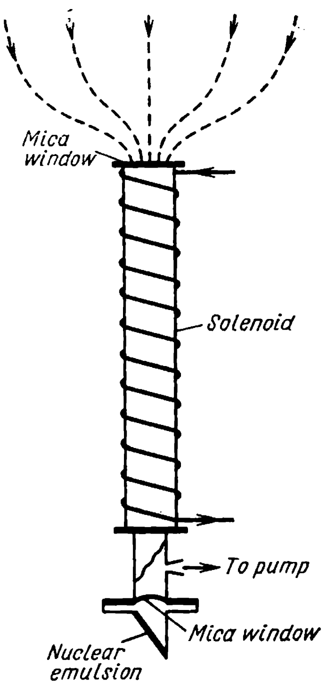


Fig. 5.5. Schematic diagram of an instrument to detect magnetic monopoles arriving at the earth's surface [618].

The upper statistical limit of the monopole arrival rate set as a consequence of this negative result is 10^{-10} monopoles per cm^2 per second. The corresponding limit for the monopole production cross section $\sigma_{max} \leqslant 3 \times \times 10^{-35}$ cm^2 **.

Lower values for the upper limit for the cross section were later obtained in the work of Bradner and Isbell [139], who were the first to perform experiments using the internal beam of the Bevatron of the L.R.L. A pulsed magnetic field of 200 kGs was used to pull monopoles out of a polyethylene target. Because of the rather low energy of the incident protons ($\leqslant$7 GeV), the maximum value of the mass of the monopoles that could have been produced was 1 GeV. In this case the upper limit for the cross section for production of pairs of monopoles in proton-proton collisions is less than 10^{-40} cm^2.

* The monopole, of charge (137/2) e emu, gains (137/2) $300B$ eV/cm in free fall in a field of B Gs. Hence, in a field of 250 Gs, one metre long, the monopole gains 500 MeV.

** If monopoles are produced in the atmosphere by high-energy cosmic rays.

Thus, monopole experiments have been built around the assumption that sufficiently high-energy interactions of particles with matter would produce monopole pairs, which could either be directly observed in flight or slowed down and later accelerated into a detector system. This general hypothesis has many variants.

Accelerator searches are the most direct searches involving interactions. In some cases (see, for instance, [324]) the target is irradiated for some time in the beam of a proton synchrotron. The monopoles produced in the target by proton-nucleon collisions or by secondary gamma-rays must quickly lose energy on ionization and come to rest inside the target itself. After this the target is removed from the proton beam and placed in a pulsed magnetic field ($\cong$ 150 kGs) sufficient to extract tightly bound monopoles.

In a different type of experiment (see [22]) a magnetic field of several hundred gauss was produced near a target in an accelerator chamber with every pulse of irradiation. The monopoles extracted from the target by this field were next accelerated in a vacuum by solenoids (as in Malkus's experiment [618]) and, as they emerged from it through a thin mica window, they were to leave typical tracks in a photographic emulsion. Aside from this, the target after irradiation was removed from the accelerator and, as in the first type of experiments, a strong-pulse magnetic field extracted from the target the tightly bound monopoles (which were unable to move because of the weak field in the accelerator)*.

At a definite stage in both experiments the monopoles were slowed down to thermal velocities (thermalized), and then an attempt was made to extract them. Such thermalization was avoided in the third type of experiments, in which monopoles produced in a target of very light matter (e.g., beryllium) several grams per cm^2 thick were expected to leave the target too at very high velocities, change their direction because of the pulsed field of 20 kGs, and following a magnetic channel two metres in length enter the detector

* The work of Sivers [850] indicates that monopoles are tightly bound to the nuclei of matter. For this reason the magnetic fields needed for their extraction must be much stronger than the fields which experimenters have at their disposal today.

(a photographic emulsion) with an energy of at least about 15 GeV.

A summary of some of the results of such experiments is given in Table 5.2. This Table indicates that extremely low cross sections ($\leqslant 10^{-40}$ cm^2) have been set for monopole production, but that the available energies ($\leqslant$ 30 GeV) of accelerator particles limit the monopole mass (in terms of the proton mass m_p) to $\leqslant 3m_p$. If the true mass were greater than $3m_p$, the accelerators used so far could not have produced a monopole pair. Similarly the charge region to which the cross section limits apply has been restricted by the detection systems. The limits are good for $n = 1$, $n = 2$, and in some cases for $n = 3$ but not at all for the higher values $n = 4$, 6, or 12 that might obtain if quarks exist and if Schwinger's ideas apply (see [177], also [828, 829, 831, 832]).

Table 5.2

The Upper Limits for Production Cross Sections of Pairs of Magnetic Monopoles in Proton Accelerators

Particles' energy in the beam of the accelerator (in GeV)	Number of protons per pulse	Maximal mass of monopole (in proton masses m_p)	Maximal value of the unknown charge (in $\hbar c/2e$)	Cross section for the production of pairs of monopoles (in cm^2)	Source
6.3	5×10^{12}	1.1	3	2×10^{-40}	[139]
25-28	4.5×10^{15}	3.0	3	6×10^{-41}	[21, 22]
27.5	4.5×10^{14}	3.0	3	10^{-39}	[324]
30	6×10^{15}	3.0	3	1.4×10^{-40}	[757]

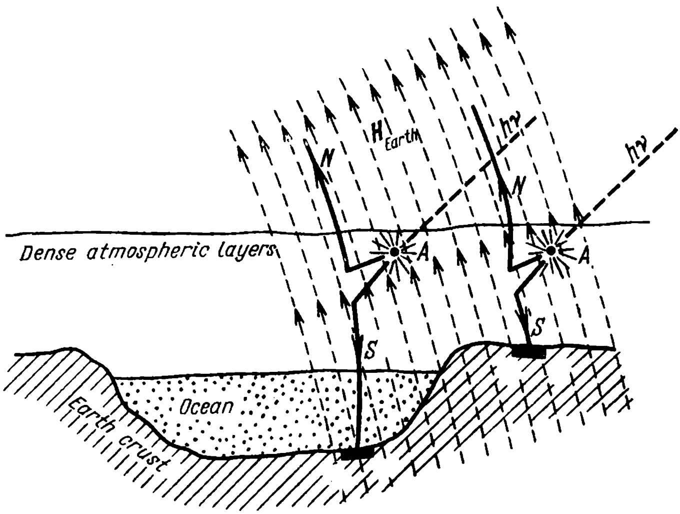
Searches for monopoles in nature are the other possible route to finding monopoles. The studies in this category utilize the particle energies of cosmic radiation, which extend nearly ten orders-of-magnitude above those used in accelerator studies. Particles of energies extending up to 2×10^{20} eV have now been observed. Hence, in principle monopoles of rest mass $10^5 m_p$ can be produced by nuclear interactions. The problem is to locate the products of such extremely rare interactions.

The processes that should be utilized are indicated in Fig. 5.6. In Fig. 5.6(a) high-energy cosmic rays that have entered the earth's atmosphere interact with nuclei of the atmosphere to produce monopole pairs, which subsequently slow down and drift along the geomagnetic field lines until either ejected into space or trapped in the solid matter of the earth's crust.

An alternative [752] is Porter's suggestion (see Fig. 5.6(b)) that part of the cosmic rays themselves (most likely those mysterious particles above 10^{17} eV) are monopoles that are accelerated to high energies by galactic magnetic fields (see [406]). Depending on the energies, masses, and the sign of magnetic charges involved, such monopoles would either be ejected into space or slowed to thermal velocities and trapped. In slowing to thermal velocities the part played by ocean depths could be great (see [406, 408]). If the monopoles have a high energy, they may bury themselves deep in the earth's crust, where they would be inaccessible for observation (see Fig. 5.6(b)).

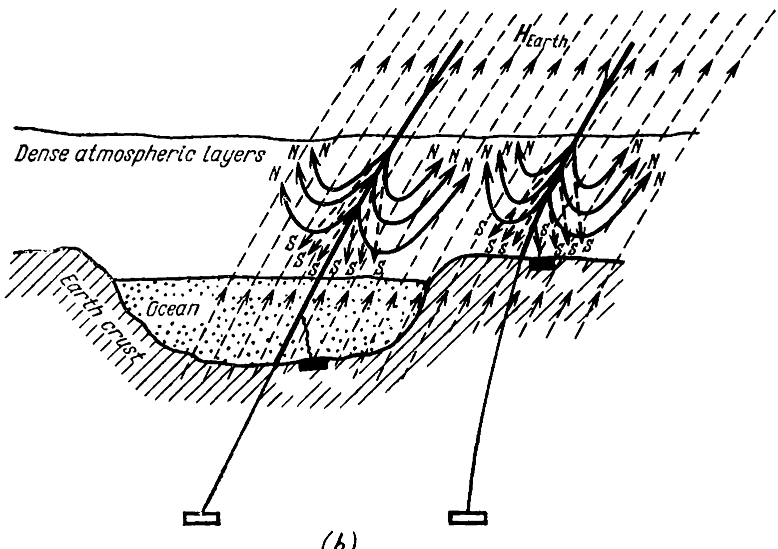
In the very first experiment of this kind performed by Malkus [618], described above, a solenoid of the proper polarity was used to "gather in" the earth's magnetic field lines, along which the thermalized monopoles would drift. In this way the effective area of the solenoid for receiving monopoles could be greatly enhanced over the geomagnetic area (in a more recent, similar, but scaled-up experiment [177] a 1/7 m^2 solenoid was used to collect field lines from an area of 1600 m^2).

Experiments by Goto et al. [408] used a powerful solenoid set at a magnetic outcrop (believed to be accumulators of monopoles) in the Adirondack Mountains in the state of New York, U.S.A. The peak extraction field was 170 kOe at the centre of the solenoid and 60 kOe at the surface of the rock. If monopoles had accumulated in the rock, they would have easily been extracted by this field and then, accelerating in the solenoid, they would have entered the detector (photographic emulsion) and left a distinct track. Simultaneously, Petukhov and Yakimenko [735] performed an experiment in which the target was the Sikhote-Alin meteorite (which fell on Feb. 12, 1947), which had been constantly irradiated by cosmic rays for about 5×10^8



H Earth
hν
hν
N
N
A
A
S
S
Dense atmospheric layers
Ocean
Earth crust

(a)



H Earth
Dense atmospheric layers
N
S
Ocean
Earth crust

(b)

years. The upper limit of the cross section for the production of monopoles in proton-nucleon collisions was found to be about 2×10^{-40} cm^2.

Two complementary sets of searches were done by Fleischer et al. [328, 330, 331, 332]. The first group of experiments utilized the earlier rationale [406, 408] for searching the ocean bed for monopoles. But the work done by Fleischer et al. [328, 330] has three new features of importance:

(1) The object of study was well-dated ferromanganese deposits from the ocean bed. Relevant magnetic properties of these deposits were measured. It was known too that they had been in direct contact with the ocean bed for 16 million years and, hence, had stored all the monopoles that had come to thermal velocities before reaching the bed.

(2) The experimenters used high field magnets that could go to fields extending from 100 to 265 kOe. Such fields should have been more than adequate to extract monopoles from solids.

(3) Solid-state track detectors were utilized for these experiments. These detectors have the virtue of responding only to highly ionizing particles such as magnetic monopoles.

The negative results of these studies have made it possible to fix the following upper limits for cross sections of monopole production in proton-nucleon collisions: $\leqslant 10^{-42}$ cm^2 when the monopole mass is equal to the proton mass m_p,

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Fig. 5.6. Hypotheses for the interaction process between magnetic monopoles and the atmosphere, the ocean, or the earth's crust.

(a) Pairs of magnetic monopoles (N and S) are produced during the interaction of photons $h\nu$ with nuclei A in the upper atmosphere. After that monopoles drift along the geomagnetic field lines until either ejected into space (for the direction of the field shown by broken lines these are the N-monopoles) or trapped at the bottom of the ocean or on the land surface.

(b) The case when the upper atmosphere is reached by the monopole component of cosmic rays containing monopoles of both signs. In the atmosphere the monopoles slow down, drift along (for N-monopoles) and against (for S-monopoles) the direction of the geomagnetic field lines and eventually either go out into space again or are trapped at the bottom of the ocean or on the land surface. The hardest component is trapped only at very large depths of the earth's crust.

and $\leqslant 2 \times 10^{-34}$ cm^2 when the monopole mass equals $1000\, m_p$. Monopoles with a charge of up to $n = 60$ could have been found in these experiments. The flux of monopoles (if they exist) that reaches the ocean bed must be less than 4×10^{-18} cm^{-2}sec^{-1}.

The second group of searches (see [332]) tested the logical alternative shown in Fig. 5.6(b): that monopoles penetrate the earth's crust to great depths. Here we cannot hope to collect the monopoles themselves, but we can make use again of the unique properties of solid-state track detectors—track detectors that exist in nature, such as mica and obsidian. The two properties are (1) (as noted before) that they ignore lightly ionizing radiation and, hence, only events of interest can be seen, and (2) that they store tracks over long periods of time.

The following conclusions can be drawn from the negative result of this work:

(1) No fraction of the cosmic rays up to 3×10^{19} eV consists of highly penetrating magnetic monopoles.

(2) Cosmic-ray interactions with the earth's atmosphere produce less than 3×10^{-19} penetrating magnetic monopoles per cm^2 per second. This is equivalent to fewer than two monopoles per second over the entire surface of the earth.

(3) If monopoles are uniformly dispersed, there are fewer than one monopole per 4000 m^3 of the earth.

☛ Regarding the estimates of the monopole flux in cosmic rays we can also refer to the work of Osborne [705] and Carrigan and Nezrick [179]. We can likewise mention the experiments of Fleischer et al. [330].

☛ The research done by Newmeyer and Trefil [677] estimates the suppression of production due to superstrong attractive forces that exist between the constituents of the monopole pair. The same work places the following limits on the monopole rest mass: $M \geqslant 3.25$ GeV for $\frac{eg}{\hbar c} = \frac{1}{2}$ and $M \geqslant$ $\geqslant 2.25$ GeV for $\frac{eg}{\hbar c} = 2$.

Berrondo and McIntosh [88] investigated the symmetry and degree of degeneracy of the relativistic Dirac equation for a Coulomb potential with a fixed centre bearing both an electric and a magnetic charge. McIntosh and Cisneros [636] examined the general problem of degeneracy in the presence of a magnetic monopole. Schatten [813] researched the possibility of a magnetic charge of the moon. He established that the upper limit of the average difference in the number of monopoles within the moon is 7×10^{-32} per nucleon.

We can also mention experimental searches for magnetic monopoles (so far unsuccessful) and theoretical studies connected with these searches by Alwarez et al. [18], Gurevich et al. [422, 423], Kolm et al. [548], Miller [650], Joseph [508], and Newmeyer and Trefil [679].

We may say that magnetic monopoles have not been found. Perhaps they are not to be found in nature. This conclusion does not conflict with the Dirac theory either since the integer in the relationship (5.2) can equal zero as well! But until this negative assumption becomes a theoretical prohibition following from the fundamental laws of physics, it is no more convincing than the assertion that monopoles exist but for some reason cannot be found. The second explanation for the lack of success in the monopole search may be that magnetic charges are rare in nature. Finally, it may be that the theory of magnetic monopoles is incomplete and for this reason the instructions on which experimenters base their searches for monopoles are incorrect. For this reason we must wait patiently for progress in theory and experiment in this exciting riddle of nature.

NON-LINEAR QUANTUM-ELECTRODYNAMIC EFFECTS IN A MAGNETIC FIELD

1. General Considerations

This chapter will deal briefly with some general aspects of quantum electrodynamics connected with non-linear interactions of fields (the electromagnetic and the electron-positron) in the presence of a constant magnetic field, including a very strong field. External magnetic fields, in the same way as external electric fields, can serve as good catalyzers for electromagnetic conversion processes such as bremsstrahlung, pair production, and photon splitting. The only significant distinction between an electric and a magnetic polarization of vacuum arises from the circumstance that the symmetry between electric and magnetic fields is disturbed by the apparent absence of magnetic monopoles (Chapter 5). A direct consequence of this asymmetry of electricity and magnetism is the asymmetry of the stability conditions for arbitrarily strong constant electric and magnetic fields.

Back in 1929 O. Klein [543] pointed to an interesting corollary of the basic principles of quantum electrodynamics. An electric field maintained near the critical level, $E_{cr} = m_0^2 c^3/eh \approx 10^{18}$ V/m, has an appreciable probability of spontaneously disintegrating into electron-positron pairs. In such a field an electron, accelerating on the path of about the Compton wave-length

$$\lambda_C \cong \frac{h}{m_0 c} \cong \frac{10^{-27}}{10^{-27}\times 10^{10}} = 10^{-10}\text{ cm}$$

would acquire the energy

$$eE_{cr}\lambda_C \cong m_0 c^2 \cong 10^6\text{ eV}$$

* See reviews by Erber [303, 304] and J. J. Klein [542].

sufficient for pair production ("Electric Klein Paradox"; see [869]). Thus, a fairly strong electric field ($E > E_{cr}$) proves to be unstable with respect to spontaneous production of electron-positron pairs. Significantly, a magnetic field has no such instability. This circumstance characterizes the fundamental asymmetry between magnetic and electric fields in quantum electrodynamics (see [71]).

Practically speaking, however, electromagnetic conversion processes occurring in external electric fields are far more familiar since the intense Coulomb fields surrounding atomic nuclei provide a readily accessible means for experimentally studying bremsstrahlung and pair production. In the natural environment there are no such favourable conditions for a magnetic field, largely because of the absence of magnetic monopoles. For this reason the magnetic conversion process that has received detailed attention to date is the magnetic bremsstrahlung appearing in particle accelerators (the so-called *synchrotron radiation*).

Before detailing the electrodynamic processes in the magnetic field, let us define the natural unit for quantum mechanical measurement of strong magnetic fields. We can do this by using the "cyclotron quantum" $\hbar\omega_H$, where ω_H is the usual cyclotron frequency: $\omega_H = eH/m_0c$. The cyclotron quantum is equal in magnitude to the rest energy of the electron, i.e.

$$\hbar\omega_H = m_0c^2 \tag{6.1}$$

If we replace ω_H with its expression in terms of the field, we get an estimate of the critical value of the magnetic field corresponding to this cyclotron frequency:

$$H_{cr} \equiv \frac{m_0^2c^3}{e\hbar} = 4.414 \times 10^{13}\ \text{eV} \tag{6.2}$$

Then in the simplest case the probability of radiation processes or pair production in a magnetic field H will be determined by the dimensionless parameter

$$\eta = \frac{\mathscr{E}}{m_0c^2} \times \frac{H}{H_{cr}} \tag{6.3}$$

where $\mathscr{E}$ is the energy typical for the process. It follows that for a process to have a considerable probability the magnetic field H must be comparable with the critical field H_{cr} defined in formula (6.2). Then the parameter η will not be small.

In natural conditions on Earth the magnetic fields H do not exceed 10^6-10^7 Oe. Thus, according to (6.2) the ratio H/H_{cr} has a value of the order of 10^{-7}-10^{-6} Oe, which is an infinitely small value.

There is reason to assume that in the vicinity of neutron stars we have to do with magnetic fields $H \simeq 10^{12}$ Oe (see the Ginzburg review on pulsars [388]; their discovery was reported in the work of Hewish et al. [456]). It is here, apparently, that intensive magnetic synchrotron radiation takes place and electron-positron pairs are produced.

The possibility of the existence of neutron stars was hypothesized in 1934 by Baade and Zwicky [44]. Stars of this kind are at the same time sources of strong magnetic fields. However, the way in which these fields are formed is not sufficiently clear as yet.

There have been assumptions that the matter of the neutron star is ferromagnetic (see [194, 845]). However, Pearson and Saunier [725] have shown that nuclear ferromagnetism in neutron-star matter is not possible for densities of $\rho < < 5 \times 10^{14}$ g/cm^3. Landau orbital ferromagnetism (LOFER) is possible in principle because of the formation of Landau levels of the electron in a strong magnetic field. This possibility was examined in the work of Lee [586] and Lee et al. [587]. But this effect is non-equilibrium since it depends on the kinetics of the formation of the star and on the relaxation time of the magnetic moment of the star. Hence, there is a need for a detailed study of the possibility of the existence of superfluidity and (or) superconductivity (see [206, 388, 460]).

After the discovery of pulsars (stellar sources of radiation with a highly stable and very small period P of the order of 3×10^{-2}-4 s (see, for instance, [387]) there appeared grounds for identifying them with neutron stars (see [388, 390, 456]). For more about magnetic stars we can suggest a comprehensive review by Pikel'ner and Khokhlov [742].

On the whole, the origin of the magnetic fields of astrophysical objects (stars, planets, interstellar space, etc.) is a problem of great significance to science. A survey done by Weinstein and Zel'dovich [956] offers a fine picture of the present range of man's knowledge in this field.

2. Magnetic Bremsstrahlung (Synchrotron Radiation)*

As we know (see, for instance, [176, 200, 201, 907]), in a magnetic field the energy of a relativistic electron in a plane perpendicular to the magnetic field H (the field is taken in the direction of the z-axis) is quantized according to the equation (Chapter 4)

$$\mathscr{E}(n, s, p_z, H) = \pm m_0c^2 \left\{ \left(\frac{p_z}{m_0c} \right)^2 + \left[\left(1 + \frac{H}{H_{cr}} (2n+s+1) \right)^{1/2} + s\frac{\alpha}{4\pi} \times \frac{H}{H_{cr}} \right]^2 \right\}^{1/2} \tag{6.4}$$

Here p_z is the z component of the momentum of the electron; $\alpha = e^2/\hbar c$ is the fine structure constant; H_{cr} is the critical value of the magnetic field, given in (6.2); n is the principal quantum number (enumerating the Landau levels), which characterizes the size of the electron orbit R_H in a magnetic field: $R_H^2 = 2\,(H_{cr}/H)\,(\hbar/m_0c)\,n$; $s = \pm 1$ characterizes the polarization of the electron spin with respect to the direction of the magnetic field ($s = 1$ along the field, $s = -1$ against the field). Equation (6.4) takes account of the anomalous magnetic moment of the electron (in the linear with respect to α approximation, see (4.13c)).

Thus, the formula (6.4) considers the following relativistic effects: the contribution of the Landau diamagnetism due to the quantization of orbital motion in the plane perpendicular to the z-axis, the contribution of the Pauli paramagnetism due to the "normal" magnetic moment, and, finally, the contribution due to the anomalous magnetic moment.

The non-relativistic limit of the formula (6.4) is

$$\mathscr{E}'(n, s, p_z, H) = \frac{p_z^2}{2m} + \frac{m_0c^2}{2}\left[(2n+s+1) + \frac{s}{2\pi}\alpha \right] \frac{H}{H_{cr}} \tag{6.5}$$

* For more details see [304, 542].

This means that a "free" electron has only one degree of freedom (along the z-axis) when it is in a magnetic field. The electron undergoes three fundamental electromagnetic-radiation processes in intense magnetic fields:

(a) Spontaneous radiation. An electron can spontaneously make a transition from one state (n, s) to another (n', s') with $n' < n$ and, generally, different z momenta p_z. A transition of this kind corresponds to the classical synchrotron radiation, which gives rise to photons of energies at multiples of $m_0c^2 (H/H_{cr}) \cong 1.16 \times 10^{-8}$ eV. No continuum emission is possible because of the Landau orbital quantization. The emitted radiation in fields of the order of 10^9 Oe will have a finite width, usually of the order of $\Delta\lambda/\lambda \cong 10^{-6}$. This type of transition is called the "magnetic transition". However, transitions between two states with $n' = n + 1$, $s' = -1$ and $n, s = \pm 1$ (we may call these fine-structure magnetic transitions) will give rise to photons of energies $\alpha e\hbar H/2\pi m_0 c \cong 0.00135 \times 10^{-8} H$ eV (see [201]).

(b) Coulomb de-excitation through magnetic transition. This is similar to (a) except that the emission takes place in the Coulomb field of a nucleus (Z, A):

$$e^- + (Z, A) \to e^- + (Z, A) + \gamma \tag{6.6}$$

The initial state of the electron (n, s, p_z) differs from the final state (n', s', p_z').

(c) One-dimensional bremsstrahlung. The reaction is similar to (6.6) except that the transition takes place between two electron states with the same n and s but different p_z. This type of transition gives rise to a continuum emission in the radio region (see [176]).

At first the interest in synchrotron radiation was connected largely with the radiation emitted in electron accelerators. Hence, the term synchrotron radiation. (The first studies of radiation in accelerators belong to Ivanenko and Pomeranchuk [488] and Arzimovich and Pomeranchuk [40]; also see Ivanenko and Sokolov [490]). But lately this type of radiation has been attracting astrophysicists. Polarization measurements have shown that non-thermal radio emission, optical radiation, and coherent radiation reaching the earth from space have a magnetic bremsstrahlung (synchrotron) origin. At present a study of cosmic synchrotron radiation is one of the important sources of information about relativistic particles and magnetic fields in remote parts of space. For more details concerning magnetic bremsstrahlung we suggest the works of Noerdlinger [684], Goldman and Oster [399], and Leventuev et al. [594].

3. Pair Production in a Magnetic Field

A beam of light passing through a magnetic field in a vacuum undergoes a specific kind of absorption because of electron-positron pair production. Attention was drawn to this phenomenon first by Pomeranchuk [748, 749] and then

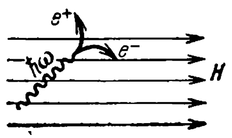

Fig. 6.1. Production of an electron-positron pair (e^-, e^+) by a photon $\hbar\omega$ in a strong magnetic field.

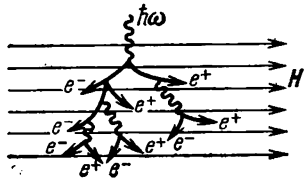

Fig. 6.2. Formation of electron-positron cascades (e^-, e^+) by an initial photon $\hbar\omega$ in a strong magnetic field.

by Tzu [934]. This and related effects were calculated by Erber [301, 303, 304], Klepikov [545], Roebl [789], and Toll [919] (see also [542, 692, 694]). This effect is totally absent in Maxwell's classical electrodynamics, in which the principle of superposition of fields is dominant.

Figure 6.1 shows the Feynman diagram of the photon disintegration into an electron-positron pair. For this process to be possible the energy $\hbar\omega$ of the photon must be high enough to produce the pair ($\hbar\omega > 2m_0c^2 \cong 10^6$ eV). Also, the magnetic field in which the process takes place must satisfy certain conditions so that the probability of pair production will sharply increase. As we have noted before, the calculations show (see [304, 545]) that there is a critical field H_{cr}, defined in the formula (6.2), that plays an important role in such phenomena. All correction terms for computed values, e.g., the refraction index in a vacuum, contain $(H/H_{cr})^2$ as a factor, where H is the external magnetic field, which, consequently, must not be too small in comparison with H_{cr} (although there may be non-linear phenomena also in weak fields $H \ll H_{cr}$, as can be seen below).

Certain "magneto-optical" consequences of this non-linear effect are examined in the works of Erber [302, 304].

Let us note, following Erber [303], that the pair production effect with the help of light in a magnetic field may be of considerable practical significance. There is no substance, in fact, that can be used as a shield from ultra high energy cosmic rays (10^{12} electron volts and higher). However, if we could produce a one-millimeter "layer" of magnetic field at the 100 million oersted level, it would provide reliable protection from penetrating gamma-radiation of such strength. As shown in Fig. 6.2, any incident gamma ray would instantaneously fragment into an electron-positron cascade, which would be so degraded in energy that ordinary shielding (lead, for instance) could be used to stop the gamma ray.

A more detailed study of this question (see [175]) showed that spontaneous electron-positron pair production cannot take place in a constant magnetic field. An analysis of the energy eigenstates of an electron in a constant magnetic field [502, 503, 957] leads to the conclusion that the separation of positive and negative energy states is always at least $2m_0c^2$. However, as O'Connell has pointed out [692], this analysis does not take account of the energy of the electron due to its anomalous magnetic moment (Chapter 4). O'Connell took this specific contribution into consideration and showed that spontaneous pair production is possible in the presence of a constant magnetic field.

O'Connell proceeded from the Dirac equation for an electron (see equations (1.8) and (1.9)), which in an external electromagnetic field with a vector potential A and in the presence of a constant homogeneous magnetic field $\boldsymbol{H}$ takes the form:

$$i\hbar \frac{\partial \psi(\boldsymbol{r}, t)}{\partial t} = \{c\boldsymbol{\alpha}(\boldsymbol{p} - e\boldsymbol{A}) + \beta m_0 c^2 - \Delta\mu_e \beta \boldsymbol{\sigma} \boldsymbol{H}\} \psi(\boldsymbol{r}, t) \quad (6.7)$$

where $\psi(\boldsymbol{r}, t)$ is the wave function, $\boldsymbol{p} = -i\hbar\nabla$ is the momentum operator, $\boldsymbol{\alpha}$ and β are 4-by-4 matrices (see (1.10) and (1.10a)), $\boldsymbol{\sigma}$ is the Pauli matrix (1.11), and $\Delta\mu_e$ is the anomalous magnetic moment of the electron (see, for instance, (4.13c)). The term that introduces a substantial change in the energy spectrum of the relativistic electron is the term in (6.7) containing $\Delta\mu_e$, which takes the form of the so-called Pauli anomalous-moment interaction term. O'Connell [692] used the standard techniques, rederiving the result obtained, for one, by Ternov et al. [907], and found the energy eigenvalues for an electron in a constant homogeneous magnetic field oriented along the z-axis that are given in (6.4). Instead of $\Delta\mu_e$ this formula contains the first term of the extension of $\Delta\mu_e$ in increasing powers of the fine structure constant $\alpha = e^2/\hbar c$, i.e.

$$\Delta\mu_e \cong \frac{\alpha}{2\pi}\mu_B$$

For values of $p_z = 0$, $n = 0$, and $s = -1$ we find from (6.4) a minimum value for $|\mathscr{E}|$

$$|\mathscr{E}_{\min}| = m_0c^2\left(1 - \frac{\alpha}{4\pi}\times\frac{H}{H_{cr}}\right) \tag{6.8}$$

Thus, we see that the allowance for the anomalous magnetic moment of the relativistic electron leads to the conclusion that in a constant homogeneous magnetic field H the minimum separation between positive and negative energy states, $\Delta\mathscr{E}_\pm$ say, is given by

$$\Delta\mathscr{E}_\pm = 2m_0c^2\left(1 - \frac{\alpha}{4\pi}\times\frac{H}{H_{cr}}\right) \tag{6.9}$$

Using the formula (6.9), we can now state the main result of O'Connell [692]: for values of H satisfying the equation

$$H = \frac{4\pi H_{cr}}{\alpha} \tag{6.10}$$

$\Delta\mathscr{E}_\pm$ may be zero and, thus, spontaneous pair production may occur. It is interesting to note that the value of H of this order represents the well-known maximum value of H beyond which classical electrodynamics breaks down (see, for instance, Chapter 9 in [570]).

O'Connell points out that the expression for $|\mathscr{E}_{\min}|$ (6.8) and the corresponding conclusions refer to none but the lowest "orbit" with the Landau quantum number $n = 0$.

Clearly, for orbits with $n > 0$ the value of $|\mathscr{E}_{\min}|$ will be more than m_0c^2, particularly for large values of H. The total possible number of electrons having the Landau quantum number $n = 0$ is given by the level degeneracy number, g' say, as follows (see, for instance, [469], Eq. (11.74), p. 240):

$$g' = \frac{V^{2/3} m_0^2 c^2 H}{\pi \hbar^2 H_{\text{cr}}} \tag{6.11}$$

where V is the total volume. Hence, the large values of H that are necessary, according to (6.10), to obtain zero values

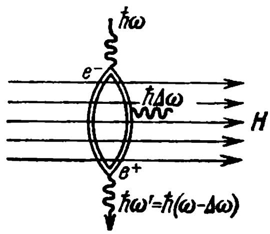

Fig. 6.3. The diagram of production of a virtual electron-positron pair by a photon $\hbar\omega$; $\hbar\Delta\omega$ is the red shift in a magnetic field.

of $|\mathscr{E}_{\min}|$ in (6.8) help to increase the number of particles capable of possessing the quantum numbers appropriate to these zero energy values. The probability that each of the g' levels corresponding to $n = 0$ are occupied depends, of course, on the temperature and density of the system and the properties of the vacuum. For more details we refer the reader to Adler et al. [7].

There is another non-linear effect of higher order when an electron-positron pair is produced virtually, i.e. it annihilates itself very fast and recreates a photon that continues to travel in the same direction as the original photon (see Fig. 6.3). For this process to occur the light beam does not have to consist of high energy rays; it may be a low-frequency electromagnetic radiation, i.e. visible light or even radio waves. Since here the virtual electron-positron pair exists only an exceedingly short time $\Delta t \cong 10^{-22}$ s, the uncertainty principle of quantum mechanics, $\Delta\mathscr{E} \times \Delta t \gg \hbar$ allows us even for a low-frequency photon ($\hbar\omega \ll 2m_0c^2$) to obtain the energy

$$\Delta\mathscr{E} \geqslant \frac{\hbar}{\Delta t} \cong \frac{10^{-27}}{10^{-22}} = 10^{-5}\ \mathrm{erg} \simeq 10^{7}\ \mathrm{eV}$$

needed for the production of the pair. In this sense even a low energy light beam may have a breaf encounter involving pair production.

The practical importance of such a virtual process is that the electron-positron pair, even during its short lifetime, may interact with the ambient magnetic field and, for instance, emit a real (not virtual) photon whose frequency ω' is perceptibly lower than the frequency ω of the original photon. We can expect the cross section for this process to be small. However, the process does not require any kind of photon-threshold energies or excessively high magnetic fields, as is the case of Fig. 6.1. Therefore, even weak fields, of the order of 10^{-5} Oe, at interstellar and intergalactic distances can lead to a peculiar red shift ($\omega' - \omega < 0$) which will combine with a similar effect predicted by Einstein's general theory of relativity.

To measure the magnetic red shift, we can take advantage of the Mössbauer effect, which in principle permits the measurement of a very small frequency shift $\Delta\omega/\omega \cong 10^{-15}$. There is hope that using high magnetic fields of about 10^6 Oe and lasers as a source of light will permit measuring the magnetic red shift over distances of about 10 m (see Erber [301] and Oertel [694]).

APPENDIX

Table I

Electron Configurations and Principal Terms of Atoms of Elements from Transition Groups d and f and Atoms of Adjacent Normal Elements

a) The Group of Iron (3d)

The inner shell has the electron configuration of the filled shell of argon: $1s^22s^22p^63s^23p^6$

Atomic number Z	Element	Electron configuration above the argon shell	Principal term	Lande factor g_J	$g_J J$ Maximal projection of the total magnetic moment (in units μ_B)
20	Ca	$4s^2$	$^1S_0$	0	0
21	Sc	$3d4s^2$	$^2D_{3/2}$	4/5	1.2
22	Ti	$3d^24s^2$	$^3F_2$	2/3	4/3
23	V	$3d^34s^2$	$^4F_{3/2}$	2/5	0.6
24	Cr	$3d^54s$	$^7S_3$	2	6
25	Mn	$3d^54s^2$	$^6S_{5/2}$	2	5
26	Fe	$3d^64s^2$	$^5D_4$	3/2	6
27	Co	$3d^74s^2$	$^4F_{9/2}$	4/3	6
28	Ni	$3d^84s^2$	$^3F_4$	5/4	5
29	Cu	$3d^{10}4s$	$^2S_{1/2}$	2	1

b) The Group of Palladium (4d)

The inner shell has the electron configuration of the filled shell of krypton: $1s^22s^22p^63s^23p^63d^{10}4s^24p^6$

Atomic number Z	Element	Electron configuration above the krypton shell	Principal term	Lande factor g_J	$g_J J$
38	Sr	$5s^2$	$^1S_0$	0	0
39	Y	$4d5s^2$	$^2D_{3/2}$	4/5	1.2
40	Zr	$4d^25s^2$	$^3F_2$	2/3	1/3

Table Ib (continued)

Atomic number Z	Element	Electron configuration above the krypton shell	Principal term	Lande factor g_J	$g_J J$
41	Nb	$4d^45s$	$^6D_{1/2}$	10/3	5/3
42	Mo	$4d^55s$	$^7S_3$	2	6
43	Tc	$4d^55s^2$	$^6S_{5/2}$	14/9	7
44	Ru	$4d^75s$	$^5F_5$	7/5	7
45	Rh	$4d^85s$	$^4F_{9/2}$	4/3	6
46	Pd	$4d^{10}$	$^1S_0$	0	0
47	Ag	$4d^{10}5s$	$^2S_{1/2}$	2	1

c) The Group of Platinum (5d)

The inner shell has the electron configuration of the filled shell of ion Yb^{2+}:

$$1s^22s^22p^63s^23p^63d^{10}4s^24p^64d^{10}4f^{14}5s^25p^6$$

Atomic number Z	Element	Electron configuration above the shell of ion Yb^{2+}	Principal term	Lande factor g_J	$g_J J$
70	Yb	$6s^2$	$^1S_0$	0	0
71	Lu	$5d6s^2$	$^2D_{3/2}$	4/5	1.2
72	Hf	$5d^26s^2$	$^3F_2$	2/3	1/3
73	Ta	$5d^36s^2$	$^4F_{3/2}$	2/5	0.6
74	W	$5d^46s^2$	$^5D_0$	0	0
75	Re	$5d^56s^2$	$^6S_{5/2}$	2	5
76	Os	$5d^76s$	$^5D_4$	3/2	6
77	Ir	$5d^76s^2$	$^4F_{9/2}$	4/3	6
78	Pt	$5d^96s$	$^3D_3$	4/3	4
79	Au	$5d^{10}6s$	$^2S_{1/2}$	2	1

d) The Group of Rare Earths (Lanthanides, 4f)

The inner shell has the electron configuration of the filled shell of the xenon atom:

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$

Atomic number Z	Element	Electron configuration above the xenon shell	Principal term	Lande factor g_J	$g_J J$
57	La	$5d6s^2$	$^2D_{3/2}$	4/5	1.2
58	Ce	$4f5d6s^2$	$^3H_4$	4/5	3.2
59	Pr	$4f^3 6s^2$	$^4I_{9/2}$	8/11	3.29
60	Nd	$4f^4 6s^2$	$^5I_4$	3/5	2.4
61	Pm	$4f^5 6s^2$	$^6H_{5/2}$	24/35	1.72
62	Sm	$4f^6 6s^2$	$^7F_0$	0	0
63	Eu	$4f^7 6s^2$	$^8S_{7/2}$	2	7
64	Gd	$4f^7 5d6s^2$	$^9D_2$	8/3	5.35
65	Tb	$4f^8 5d6s^2$	$^8H_{17/2}$	24/17	12
66	Dy	$4f^{10} 6s^2$	$^5I_8$	10/8	10
67	Ho	$4f^{11} 6s^2$	$^4I_{15/2}$	18/15	9
68	Er	$4f^{12} 6s^3$	$^3H_6$	7/6	7
69	Tm	$4f^{13} 6s^2$	$^2F_{7/2}$	8/7	4
70	Yb	$4f^{14} 6s^2$	$^1S_0$	0	0

e) The Group of Actinides (5f and 6d)

The inner shell has the electron configuration of the filled shell of the radon atom:

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$

Atomic number Z	Element	Electron configuration above the radon shell	Principal term	Lande factor g_J	$g_J J$
89	Ac	$6d7s^2$	$^2D_{3/2}$	4/5	1.2
90	Th	$6d^2 7s^2$	$^3F_2$	2/3	4/3
91	Pa	$5f^2 6d7s^2$ (?)	$^4K_{11/2}$	10/13	4.23
92	U	$5f^3 6d7s^2$	$^5L_6$	5/7	4.28

Table Ie (continued)

Atomic number Z	Element	Electron configuration above the radon shell	Principal term	Lande factor g_J	$g_J \cdot J$
93	Np	$5f^4 6d 7s^2$ (?)	$^6L_{11/2}$	8/13	3.24
94	Pu	$5f^6 7s^2$	$^3F_0$	0	0
95	Am	$5f^7 7s^2$	$^8S_{1/2}$	2	7
96	Cm	$5f^7 6d 7s^2$ (?)	$^9D_2$	8/3	5.33
97	Bk	$5f^8 6d 7s^2$ (?)	$^8H_{17/2}$	24/17	12
98	Cf	$5f^{10} 7s^2$ (?)	$^5I_8$	10/8	10
99	Es	$5f^{11} 7s^2$ (?)	$^4I_{15/2}$	18/15	9
100	Fm	$5f^{12} 7s^2$ (?)	$^3H_6$	7/6	7
101	Md	$5f^{13} 7s^2$ (?)	$^2F_{7/2}$	8/7	4
102	No	$5f^{14} 7s^2$ (?)	$^1S_0$	0	0

Table II

Spins and Dipole Magnetic Moments of Nuclei of Some Radioactive and Stable Isotopes in the Ground State

In compilation of this table we made use of Smith's review [853] (in this case the column "Source" contains no reference). In addition, Fuller and Cohen's tables [362] were used (in such cases we refer to this review). Data concerning some isotopes contain references to original papers obtained in part from [576], and for years 1966-70 we refer the reader to the original investigations the results of which did not enter the above-mentioned review tables. Data reported at the International Conference on Nuclear Moments and Nuclear Structure (Osaka, Japan, September 1972) are marked with a star near the author's name.

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
0	n	1		12 min	1/2	-1.913148 (66) (μ_n)	[216]
1	H	1	99.985		1/2	$+2.79267$ (12) (μ_p)	[622]
		2	0.015		1	$+0.8574073$ (2)	
		3	$< 10^{-10}$	12.262 years	1/2	$+2.97884$ (1)	
2	He	3	1.3×10^{-4}		1/2	-2.127490 (5)	[971]
		4	~ 100		0	0	
3	Li	6	7.30		1	$+0.822008$ (22)	
		7	92.70		3/2	$+3.256310$ (85)	
4	Be	9	100		3/2	-1.17737 (41)	
5	B	8		0.77 s	2	$+1.0355$ (3)	Minamisono * et al.
		10	18.83		3	$+1.80081$ (49)	
		11	81.17		3/2	$+2.68852$ (4)	

6	C	11		20.4 min	3/2	|1.027 (10)|	[427[
		12	98.892		0	0	
		13	1.108		1/2	+0.702381	
		14		5.6×10^{3} years	0	0	
7	N	13		10.08 min	1/2	|0.32212 (36)|	[87]
		14	99.635		1	+0.403562 (10)	[53]
		15	0.365		1/2	−0.283049 (7)	[53]
8	O	15		123.95 s	1/2	+0.7189 (8)	[222]
		16	99.7575		0	0	
		17	0.0392		5/2	−1.89370 (9)	
		18	0.2033		0	0	
9	F	17		66 s	5/2	+4.7224 (12)	[886]
		18		111 min	1	+0.8	
		19	100		5/2	+2.628353 (5)	[53]
10	Ne	19		17.7 s	1/2?	−1.886 (1)	[221]
		20	90.51		0	0	
		21	0.28		3/2	−0.5 (1)	
		22	9.21		0	0	
11	Na	21		23 s	3/2	+2.38612 (10)	[24]
		22		2.58 years	3	+1.7469 (22)	
		23	100		3/2	+2.21753 (10)	
		24		14.97 h	4	+1.688 (5)	[76]

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
12	Mg	24	78.60		0	0	
		25	10.11		5/2	−0.85532 (14)	
		26	11.29		0	0	
13	Al	26		10^6 years	5	+2.8 (2)	
		27	100		5/2	+3.641421 (30)	
14	Si	28	92.28		0	0	
		29	4.67		1/2	−0.55492 (4)	
		30	3.05		0	0	
15	P	30		2.49 min	1	+0.6 (1)	
		31	100		1/2	+1.131621 (31)	
16	S	32	95.060		0	0	
		33	0.742		3/2	+0.64342 (13)	
		34	4.182		0	0	
		35	<0.002	86.3 days	3/2	+1.0 (1)	
		36	0.016		0	0	
17	Cl	34		1.53 s	3	+1.4 (4)	
		35	75.53		3/2	+0.821808 (71)	
		36		3.1×10^5 years	2	+1.28538 (6)	
		37	24.47		3/2	+0.68409 (6)	
18	Ar	35		1.8 s	3/2	+0.632 (2)	[166]
		36	0.35		0	0	
		37		34 days	3/2	+0.95 (20)	[786]
		38	0.08		0	0	
		40	99.57		0	0	

19	K	36		245 ms	2	+0.547 (2)	Schweickert * et al.
		37		1.2 s	3/2	+0.2036 (9)	[89]
		38			3	+1.3735 (10)	[738]
		39	93.260		3/2	+0.39146 (7)	
		40	0.011	1.32×10^9 years	4	−1.2981 (4)	
		41	6.729		3/2	+0.215173 (84)	
		42		12.52 h	2	−1.1424 (2)	[184]
		45		20 min	3/2	|0.1734 (4)|	[367]
20	Ca	40	96.92		0	0	
		42	0.64		0	0	
		43	0.129		7/2	−1.31720 (12)	
		44	2.13		0	0	
		46	0.003		0	0	
		48	0.178		0	0	
21	Sc	41		590 ms	7/2	+5.43 (2)	Sugimoto * et al.
		43		3.92 h	7/2	+4.61 (4)	[232]
		44		3.92 h	2	+2.56 (3)	[232]
		44 *m*		2.44 days	6	+3.87 (1)	[232]
		45	100		7/2	+4.7563	[355]
		46		84 days	4	+3.03 (2)	[734[
		47		3.43 days	7/2	+5.33 (2)	[232]
22	Ti	45		3.1 h	7/2	|0.095 (2)|	[233]
		46	7.95		0	0	
		47	7.75		5/2	−0.788130 (84)	
		48	73.45		0	0	
		49	5.51		7/2	−1.10377 (12)	
		50	5.34		0	0	

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
23	V	50	0.25		6	+3.34702 (94)	
		51	99.75		7/2	−5.1470 (57)	
24	Cr	50	4.31		0	0	
		52	83.76		0	0	
		53	9.55		3/2	−0.74391 (42)	
		54	2.38		0	0	
25	Mn	52 *m*		21.3 min	2	|0.0077 (4)|	[739]
		53		2×10^6 years	7/2	|5.050 (7)|	[258]
		55	100		5/2	+3.46766 (14)	
						+3.4438 (20)	[651]
26	Fe	54	5.81		0	0	
		56	91.64		0	0	
		57	2.21		3/2	+0.05	
		58	0.34		0	0	
27	Co	56		77.3 days	4	|3.855 (7)|	
		57		267 days	7/2	|4.65 (5)|	
		58		9.2 h	2	+3.5 (3)	
		59	100		7/2	+4.6488 (5)	
		60		5.24 years	5	+3.80 (2)	

28	Ni	58	69.76		0	0	
		60	26.16		0	0	
		61	1.25		3/2	−0.74868 (4)	[273]
		62	3.66		0	0	
		64	1.16		0	0	
29	Cu	60		24 min	2	+1.219 (3)	[740]
		61		3.3 h	3/2	+2.13 (4)	[259]
		62		10 min	1	−0.380 (4)	[740]
		63	69.04		3/2	+2.22664 (17)	
		64		12.8 h	1	−0.216 (2)	[259]
		65	30.96		3/2	+2.38473 (45)	
		66		5.2 min	1	|0.283 (5)|	[788]
30	Zn	63		38 min	3/2	−0.28156 (5)	[577]
		64	48.89		0	0	
		65		245 days	5/2	+0.7692 (2)	[161]
		66	27.81		0	0	
		67	4.11		5/2	+0.87571 (10)	
		68	18.57		0	0	
		70	0.62		0	0	
31	Ga	68		68.33 min	1	+0.01176	
		69	60.16		3/2	+2.01605 (51)	
		71	39.84		3/2	+2.56158 (26)	

Table II (continued)

Z	Symbol of element	*A*	Abundance in %	$\tau_{1/2}$	*I*	μ (in units of μ_N)	Source
32	Ge	70	21.2		0	0	
		71		12 days	1/2	+0.65 (20)	[198], [199]
		72	27.3		0	0	
		73	7.9		9/2	−0.87914 (12)	
		74	37.1		0	0	
		76	6.5		0	0	
33	As	75	100		3/2	+1.43896 (16)	
34	Se	74	0.87		0	0	
		75		121 days	5/2	?	
		76	9.02		0	0	
		77	7.58		1/2	+0.534058 (14)	
		78	23.52		0	0	
		79		0.5×10^4 years	7/2	−1.015 (15)	[437]
		80	49.82		0	0	
		82	9.19		0	0	
35	Br	79	50.53		3/2	+2.10555 (30)	
		80		18 min	1	+0.5138 (6)	[967]
		80 *m*		4.5 h	5	+1.3170 (6)	[967]
		81	49.47		3/2	+2.26958 (3)	

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36	Kr	78	0.342		0	0	
		80	2.228		0	0	
		82	11.50		0	0	
		83	11.48		9/2	−0.969	
		84	57.02		0	0	
		85		10.76 years	9/2	−1.004	
		86	17.43		0	0	
37	Rb	81		4.7 h	3/2	+2.05	[362]
		81 *m*		32 min	9/2	?	
		82		6.3 h	5	+1.51 (1)	
		83		83 days	5/2	+1.43 (1)	
		84		33 days	2	−1.32 (1)	
		85	72.15		5/2	+1.35268 (11)	
		86		18.66 days	2	−1.6912 (1)	
		87	27.85	5×10^{10} years	3/2	+2.750529 (38)	
		88		17.8 min	2	|0.508 (5)|	[937]
38	Sr	84	0.55		0	0	
		86	9.75		0	0	
		87	6.96		9/2	−1.09302 (13)	
		88	82.74		0	0	
39	Y	89	100		1/2	−0.137314 (29)	
		91		58 days	1/2	|0.1634 (8)|	[734]
40	Zr	90	51.46		0	0	
		91	11.23		5/2	−1.9 (2)	
		92	17.11		0	0	
		94	17.40		0	0	
		96	2.80		0	0	

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
41	Nb	93	100		9/2	+6.16719 (35)	
42	Mo	92	15.84		0	0	
		94	9.04		0	0	
		95	15.72		5/2	−0.93270 (18)	
		96	16.53		0	0	
		97	9.46		5/2	−0.95229 (10)	
		98	23.78		0	0	
		100	9.63		0	0	
43	Tc	99		2.12×10^5 years	9/2	+5.68048 (35)	
44	Ru	96	5.68		0	0	
		98	2.22		0	0	
		99	12.81		5/2	−0.63 (15)	[666]
		100	12.70		0	0	
		101	16.98		5/2	+1.09 (3)	
		102	31.5		0	0	
		104	18.5		0	0	
45	Rh	102		206 days	1/2	−0.11	
		103	100		1/2	−0.08790 (7)	

46	**Pd**	102	0.8		0	0	
		104	9.3		0	0	
		105	22.6		5/2	−0.57	[105]
		106	27.1		0	0	
		108	26.7		0	0	
		110	13.5		0	0	
47	**Ag**	105		40 days	1/2	+0.101	
		107	51.92		1/2	−0.113042 (13)	[860]
		108		2.4 min	1	+4.2 (5)	[788]
		109	48.08		1/2	−0.129955 (13)	[860]
		110 *m*		{ 253 days	6	{ +3.55 (4)	[283]
				260 days		+3.587 (4)	[862]
		111		7.6 days	1/2	−0.145 (1)	
		112		3.2 h	2	| 0.0545 (5) |	[186]
		113		5.3 h	1/2	| 0.158 (2) |	[186]
48	**Cd**	105		55 min	5/2	−0.7385 (2)	[578]
		106	1.215		0	0	
		107		6.7 h	5/2	−0.6162 (8)	[160]
		108	0.875		0	0	
		109		470 days	5/2	−0.8286 (15)	[634]
		110	12.39		0	0	
		111	12.75		1/2	−0.59499 (8)	
		111 *m*		49 min	5/2	−1.1040 (4)	[578]
		112	24.07		0	0	
		113	12.26		1/2	−0.62243 (8)	
		113 *m*		14 days	11/2	+1.08885 (13)	[162]
		114	28.86		0	0	
		115		2.3 days	1/2	−0.6469 (3)	[635]
		115 *m*		43 days	11/2	−1.0437 (10)	[635]
		116	7.58		0	0	

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
49	In	111		2.81 days	9/2	+5.53	
		113	4.33		9/2	+5.52317 (54)	
		113 *m*		1.73 h	1/2	−0.21053	
		114		49 days	5	+4.7	
		115	95.67	6×10^{14} years	9/2	+5.53441 (66)	
		115 *m*		4.5 h	1/2	−0.24371 (5)	[171]
		116		54 min	5	+4.218	[172]
		117		1.9 h	1/2	−0.25146 (3)	[663]
50	Sn	111			7/2	?	
		112	0.94		0	0	
		113		118 days	1/2	| 0.875 (9) |	[756]
		114	0.65		0	0	
		115	0.33		1/2	−0.917798 (76)	
		116	14.36		0	0	
		117	7.51		1/2	−0.9990 (19)	
		118	24.21		0	0	
		119	8.45		1/2	−1.04611 (84)	
		120	33.11		0	0	
		121		27 h	3/2	| 0.695 (7) |	[756]
		122	4.61		0	0	
		124	5.83		0	0	
51	Sb	115		31 min	5/2	+3.46 (1)	[493]
		117		28 h	5/2	+2.67 (1)	[493]

		118		3.5 min	1	|2.46 (7)|	[493]
		119		38 h	5/2	+3.45 (1)	[493]
		120		16 min	1	|2.34 (22)|	[493]
		121	57.25		5/2	+3.335892 (19)	
		123	42.75		7/2	+2.54653 (3)	
		125		2.7 years	?	|2.59 (3)|	[880]
52	Te	120	0.89		0	0	
		122	2.46		0	0	
		123	0.87	$>10^{13}$ years	1/2	−0.73188 (4)	[952]
		124	4.61		0	0	
		125	6.99		1/2	−0.88715	
		126	18.71		0	0	
		127		9.4 h	3/2	+0.66 (5)	Silverance* et al.
		128	31.79		0	0	
		129		69 min	3/2	+0.67 (5)	Silverance* et al.
		130	34.49		0	0	
53	J	126		13 days	2	?	
		127	100		5/2	+2.80897 (23)	
		129		1.72×10^{7} years	7/2	+2.617266 (12)	
		131		8.08 days	7/2	+2.738	
54	Xe	124	0.095		0	0	
		126	0.088		0	0	
		128	1.916		0	0	
		129	26.235		1/2	−0.776786 (53)	
		130	4.051		0	0	
		131	21.24		3/2	+0.690635 (85)	
		132	26.925		0	0	
		134	10.52		0	0	
		136	8.93		0	0	

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
55	Cs	127		6.3 h	1/2	+1.42 (2)	
		129		30.7 h	1/2	+1.48 (8)	
		130		30 min	1	+1.33 (10)	
		131		9.6 days	5/2	+3.517 (2)	[978]
		132		6.5 days	2	+2.22	
		133	100		7/2	+2.57887 (30)	
		134		2.07 years	4	+2.95 (1)	
		134 *m*		2.9 h	8	+1.10 (1)	
		135		3×10^6 years	7/2	+2.7382 (19)	
		137		26.6 years	7/2	+2.8502 (25)	
		138		32.2 min	3	|0.48 (10)|	[876]
56	Ba	130	0.102		0	0	
		132	0.098		0	0	
		134	2.42		0	0	
		135	6.59		3/2	+0.832293 (24)	
		136	7.81		0	0	
		137	11.32		3/2	+0.9324 (27)	
		138	71.66		0	0	
57	La	138	0.089	3.2×10^{11} years	5	+3.6844 (4)	[861]
		139	99.911		7/2	+2.77807 (61)	

58	Ce	136	0.193		0	0	
		138	0.250		0	0	
		140	88.48		0	0	
		141		33 days	7/2	+0.16 (6)	
		142	11.07		0	0	
59	Pr	141	100		5/2	+4.09 (60)	[773]
		142		19.2 h	2	+0.30	[362]
		143		13.76 days	7/2	?	[362]
60	Nd	142	26.80		0	0	
		143	12.12		7/2	−1.063 (5)	[854]
		144	23.91	5×10^{15} years	0	0	
		145	8.35		7/2	−0.654 (4)	[854]
		146	17.35		0	0	
		147		11.9 days	9/2	+0.22 (5)	[23]
		148	5.78		0	0	
		150	5.69		0	0	
61	Pm	147		2.64 years	7/2	+2.58 (7)	[772]
		148		5.3 days	1	+2.07 (21)	[14]
		151		27.5 h	5/2	| 1.8 (?) |	[156]
62	Sm	144	2.95		0	0	
		147	14.62	1.13×10^{11} years	7/2	−0.83	
		148	10.97		0	0	
		149	13.56		7/2	−0.85	[362]
		150	7.27		0	0	
		152	27.34		0	0	
		153		47.1 h	3/2	−0.03	[362]
		154	23.29		0	0	
		155		23 min	3/2	?	[308]

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
63	En	151	47.77		5/2	+3.419	[362]
		152		12.7 years	3	+1.912	[362]
		153	52.23		5/2	+1.501	[362]
		154		16 years	3	|2.0|	[362]
		156		15.4 days	3	+1.971	[362]
64	Gd	152	0.20		0	0	
		153		236 days	3/2	?	[362]
		154	2.15		0	0	
		155	14.78		3/2	+0.242	[362]
		156	20.59		0	0	
		157	15.71		3/2	+0.3225	[362]
		158	24.78		0	0	
		159		18 h	3/2	?	[362]
		160	21.79		0	0	
65	Tb	156		5.6 days	3	+1.45	[362]
		157		280 years	3/2	+2.0 (1)	[282]
		159	100		3/2	+1.52	[362]
		160		72.3 days	3	+1.685 (8)	[282]
		161		6.88 days	3/2	?	[362]
66	Dy	156	0.05	2×10^{14} years	0	0	
		158	0.05		0	0	
		160	0.1		0	0	
		161	21.1		5/2	−0.46	[362]

		162	26.6		0	0	
		163	24.8		5/2	+0.65	[362]
		164	27.3		0	0	
67	Ho	165	100		7/2	+4.1	[362]
68	Er	162	0.1		0	0	
		163		75 min	5/2	?	[878]
		164	1.5		0	0	
		165		10 days	5/2	| 0.65 (3) |	[14]
		166	32.9		0	0	
		167	24.4		1/2	−0.5647 (24)	[854]
		168	26.9		0	0	
		169		9.4 days	1/2	+0.513	[362]
		170	14.2		0	0	
		171		7.52 days	5/2	| 0.70 (5) |	[157]
69	Tu	166		7.7 h	2	+0.05	[362]
		167		9.6 days	1/2	?	[362]
		169	100		1/2	−0.2310 (15)	[383]
		170		129 days	1	+0.245	[362]
		171		1.9 years	1/2	| 0.227 (5) |	[157]
70	Yb	168	0.06		0	0	
		170	4.21		0	0	
		171	14.26		1/2	+0.43 (5)	
		172	21.49		0	0	
		173	17.02		5/2	−0.60 (5)	
		174	29.58		0	0	
		176	13.38		0	0	

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
79	Au	190		39 min	1	$\lvert 0.065 \rvert$	[183]
		191		3 h	3/2	$\lvert 0.137\,(7) \rvert$	[311]
		192		4.7 h	1	+0.0079	[362]
		193		18 h	3/2	$\lvert 0.139\,(7) \rvert$	[311]
		194		39 h	1	$\lvert 0.073\,(4) \rvert$	[187]
		195		183 days	3/2	$\lvert 0.146\,(7) \rvert$	[187]
		196		6.2 days	2	+0.58 (3) or −0.62 (3)	[187]
		197	100		3/2	+0.14349 (9)	[237]
		198		2.697 days	2	+0.58	[362]
		199		3.14 days	3/2	+0.265	[362]
80	Hg	183		8.8 s	1/2	+0.513 (9)	Bonn* et al.
		185		50 s	1/2	+0.499 (4)	Bonn* et al.
		187		2.4 min	3/2	−0.580 (6)	Bonn* et al.
		196	0.15		0	0	
		197		65 h	13/2	−1.033	[362]
		198	10.12		0	0	
		199	17.04		1/2	+0.504117 (41)	
		199			1/2	0.4979	Bonn* et al.
		200	23.25		0	0	
		201			3/2	−0.5513	Bonn* et al.
		201	13.18		3/2	−0.556701	[362]
		202	29.45		0	0	
		203		46.9 days	5/2	+0.830 (20)	[775]
		204	0.72		0	0	
		205		5.5 min	1/2	+0.5911 (5)	Bonn* et al.

81	Tl	197		2.7 h	1/2	?	[362]
		198		53 h	2	+0.0017	[362]
		199		7.4 h	1/2	+1.59	[362]
		200		26.1 h	2	?	[362]
		201		73.5 h	1/2	$\leqslant$ +1.59	[362]
		202		12 days	2	+0.15	[362]
		203	29.50		1/2	+1.616 (14)	
		204		3.56 years	2	+0.0893	[362]
		205	70.50		1/2	+1.62734 (42)	
82	Pb	204	1.54		0	0	
		206	22.62		0	0	
		207	22.62		1/2	+0.58954	[117]
		208	53.22		0	0	
83	Bi	199		25 min	9/2	?	[362]
		201		1.85 h	9/2	?	[362]
		202		95 min	5	?	[362]
		203		11.8 h	9/2	+4.59	[362]
		204		11.2 h	8	+4.25	[362]
		205		15.3 days	9/2	+5.5	[362]
		206		6.24 days	6	+4.56	[362]
		209	100		9/2	+4.0802	[117]
		210		2.6×10^{10} years	1	+0.0442	[362]
84	Po	201		17.5 min	3/2	?	[362]
		202		44.5 min	0	0	[362]
		203		42 min	5/2	?	[362]
		204		3.54 h	0	0	[362]
		205		1.8 h	5/2	+0.26	[362]
		206		8.8 days	0	0	[362]
		207		6.2 h	5/2	+0.27	[362]
		209		103 years	1/2	?	[362]

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
71	Lu	175	97.41		7/2	+2.236	[362]
		176	2.59	2.4×10^{10} years	7	+3.14	[362]
		176 *m*		3.71 h	1	+0.318 (3)	[966]
72	Hf	174	0.18		0	0	
		176	5.30		0	0	
		177	18.47		7/2	+0.5	
		178	27.10		0	0	
		179	13.84		9/2	−0.47	[362]
		180	35.11		0	0	
73	Ta	180	0.0123	$>10^{12}$ years	0	0	
		181	99.9877		7/2	{ +2.35	[362]
						{ $+2.361\pm0.01$	[305]
		183		5 days	1/2	?	[362]
74	W	180	0.16		0	0	
		182	26.35		0	0	
		183	14.32		1/2	+0.11846 (13)	
		184	30.68		0	0	
		185		76 days	3/2	?	[362]
		186	28.49		0	0	
		187		24 h	3/2	?	[362]

75	Re	183		71 days	5/2	+2.88 (12)	Vanneste* et al.
		184		38 days	3	+2.67 (16)	
		185	37.07		5/2	+3.17156 (34)	Vanneste* et al.
		186		88.9 h	1	+1.728 (3)	[37]
		187	62.93	6×10^{10} years	5/2	+3.17591 (34)	
		188		16.7 h	1	+1.777 (5)	[37]
76	Os	184	0.018		0	0	
		186	1.582		0	0	
		187	1.64		1/2	+0.12 (4)	
		188	13.27		0	0	
		189	16.14		3/2	+0.655914 (78)	
		190	26.38		0	0	
		192	40.97		0	0	
77	Ir	191	38.5		3/2	+0.1440 (6)	[671, 672]
		193	61.5		3/2	+0.1568 (6)	[671, 672]
78	Pt	192	0.8		0	0	
		194	30.2		0	0	
		195	35.2		1/2	+0.60596 (21)	
		196	26.6		0	0	
		198	7.2		0	0	

Table II (continued)

Z	Symbol of element	A	Abundance in %	$\tau_{1/2}$	I	μ (in units of μ_N)	Source
85	At	211		7.2 h	9/2	?	[362]
86	Rn				?	?	
87	Fr				?	?	
88	Ra				?	?	
89	Ac	227		21.6 years	3/2	+0.38	[362]
90	Th	229		7340 years	5/2	+0.39	[362]
		232	100	39×10^{10} years	0	0	[362]
91	Pa	231	100	32480 years	3/2	+1.98	[362]
		233		27 days	3/2	+3.4	[362]

92	U	233		62×10^5 years	5/2	+0.52	[362]
		234	0.0056	2×10^{16} years	0	0	[362]
		235	0.7205	7.1×10^8 years	7/2	+0.34	[362]
		237		6.75 days	1/2	?	[362]
		238	99.2739	1×10^{16} years	0	0	[362]
93	Np	237		2.2×10^6 years	5/2	6.0 (25)	
		238		2.1 days	2	?	
		239		2.34 days	5/2	?	
94	Pu	239		24 360 years	1/2	|0.27|	[362]
		241		13 years	5/2	−0.73	[362]
95	Am	241		458 years	5/2	+1.58 (3)	[37]
		242		16.01 h	1	+0.3808 (15)	[37]
		243		7.95×10^3 years	5/2	+1.4	[362]

Table III

Spins, Magnetic Moments, and *G*-Factors of Excited Nuclear States

Abbreviations used to indicate experimental method:

IB-DPAD	In-beam time-differential perturbed angular distribution
IB-IPAD	In-beam time-integrated perturbed angular distribution
$\gamma\gamma$-DPAC	γ-γ time-differential perturbed angular correlation
$\gamma\gamma$-IPAC	γ-γ time-integrated perturbed angular correlation
IB-STROB	In-beam stroboscopic resonance
IB-NMR/β	In-beam NMR detected by β-decay asymmetry
IB-NMR/γ	In-beam NMR detected by γ-ray anisotropy
IB-OPUMP	In-beam optical pumping
LT-NO	Low temperature nuclear orientation
LT-NMR/ON	NMR of oriented nuclei at low temperature
REOR	Reorientation effect in Coulomb excitation

The absence of reference means that the respective data are taken from the review of Bodenstedt [120]. Data reported at the International Conference on Nuclear Moments and Nuclear Structure (Osaka, Japan, September 1972) are marked with a star near the author's name.

Isotope	Level in keV	$\tau_{1/2}$	I	μ (in units of μ_N)	g	Method	Source
N^{14} F^{18}	5830 1.131	12.5×10^{-12} s 234×10^{-9} s	3 5	$1.5 < \lvert\mu\rvert < 2.55$ $\lvert 2.840(65) \rvert$	$0.5 < \lvert g \rvert < 0.85$ $\lvert 0.568(13) \rvert$	IB-IPAD Reaction $O^{16}(He^3, p)F^{18}$ and the differential time-delay technique	Berant* et al. [747]
F^{19}	197	8.5×10^{-8} s	5/2	$+3.50(24)$	$+1.4(1)$	Inelastic scattering of protons	

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Ne^{19}	275	$<0.3\times10^{-9}$ s	1/2			PAD	[107]
Ne^{19}	238	$17.7\,(7)\times10^{-9}$ s	5/2		$-0.296\,(3)$	PAD	[107]
Sc^{44}	69	$153\,(1)\times10^{-9}$ s	2	$+0.70\,(4)$	$+0.35\,(2)$	PAC	[82]
Sc^{47}	760	$274\,(10)\times10^{-9}$ s	3/2	$+0.35\,(5)$	$+0.24\,(4)$	Reaction Ca^{44} (α, p) Sc^{47} and the differential time-delay technique	[337]
Fe^{54}	1409	1.4×10^{-9} s			$1.43\,(28)$	PAC	Habber* et al. (1972)
Fe^{56}	845	7.3×10^{-12} s	2	$+1.06\,(10)$	$+0.53\,(16)$	Resonance scattering of γ-rays	
Fe^{57}	14.4	1.0×10^{-7} s	3/2	$-0.153\,(4)$	$-0.102\,(3)$	Mössbauer effect	
Ni^{61}	71	5.0×10^{-9} s	5/2	$\lvert 0.14\,(3)\rvert$	$\lvert 0.056\,(12)\rvert$	Mössbauer effect	
Cu^{64}	1594	2.04×10^{-8} s	6	$\lvert 1.06\,(3)\rvert$		IB-DPAD	Bleck* et al.
Cu^{66}	1154	5.96×10^{-7} s	6	$\lvert 1.038\,(3)\rvert$		IB-DPAD	Ditto
Zn^{67}	605	3.40×10^{-7} s	9/2	$-1.094\,(20)$	$-0.243\,(4)$	IB-DPAD	Bertschat* et al.
Ge^{67}	734	7.0×10^{-8} s	9/2	$-0.945\,(30)$	$-0.210\,(7)$	IB-DPAD	Ditto

Isotope	Level in keV	$\tau_{1/2}$	I	μ (in units of μ_N)	g	Method	Source
As^{72}	215	8.0×10^{-8} s	3	1.575 (18)	| 0.525 (6) |	IB-DPAD	Ditto
As^{75}	265	1.2×10^{-11} s	3/2	0.93 (24)	| 0.62 (16) |	$\gamma\gamma$-IPAC	Chopra* et al.
As^{75}	280	2.4×10^{-10} s	5/2	+1.05 (10)	+0.42 (13)	PAC	
Br^{78}	181	120 s	4	| 4.10 |	| 1.025 |	IB-NMR/γ	Brauer* et al.
Kr^{83}	9.3	212×10^{-9} s	7/2	−0.99 (8)		Mössbauer effect	[412]
Kr^{83}	9.3	212×10^{-9} s	7/2	−0.939 (2)	−0.283	Mössbauer effect	[174]
Rb^{82}	280	6.2 h	5	| 1.50 (2) |	| 0.300 (4) |	Atomic beam	
Sr^{86}	2958	4.60×10^{-7} s	8	−1.93 (12)	−0.241 (15)	IB-DPAD	Hashimoto* et al.
Zr^{91}		2.9×10^{-8} s	15/2		| 0.71 (1) |	IB-DPAD	Baba* et al.
Nb^{91}	2378	1.0×10^{-8} s	17/2	| 10.63 (34) |	| 1.25 (4) |	IB-DPAD	Ditto
Mo^{94}	2953	9.8×10^{-8} s	8	| 10.54 (16) |	| 1.317 (20) |	IB-DPAD	Faestermann* et al.
Tc^{99}	181	3.5×10^{-9} s	5/2	| 3.6 (10) |	| 1.44 (13) |	PAC	
Rh^{103}	298	6×10^{-12} s	3/2	| 0.71 (21) |	| 0.47 (14) |	IB-PAD	Miller* et al.
Rh^{103}	360	6×10^{-11} s	5/2	| 0.95 (33) |	| 0.38 (13) |	IB-PAD	Ditto

Pd^{100}	511.6	$12.0\,(8)\times 10^{-10}$ s	2	$\|0.80\,(7)\|$	$\|0.40\,(34)\|$	PAC	[501]
Ag^{104}	?	27 min	2	$+3.7\,(2)$	$+1.85\,(10)$	Atomic beam	
Ag^{107}	325	6×10^{-12} s	3/2	$\|0.62\,(21)\|$	$\|41\,(14)\|$	IB-PAD	Miller* et al.
Ag^{107}	423	3.4×10^{-11} s	5/2	$\|0.88\,(20)\|$	$\|0.35\,(13)\|$	IB-PAD	Ditto
Ag^{109}	309	6×10^{-12} s	3/2	$\|0.68\,(23)\|$	$\|0.45\,(15)\|$	IB-PAD	Ditto
Ag^{109}	414	3.3×10^{-11} s	5/2	$\|0.68\,(23)\|$	$\|0.27\,(9)\|$	IB-PAD	Ditto
Cd^{111}	247	8.5×10^{-8} s	5/2	$-0.783\,(23)$	$-0.31\,(1)$	PAC	
Cd^{111}	558.5	$1.32\,(9)\times 10^{-11}$ s	2	$\|0.88\,(12)\|$	$\|0.44\,(6)\|$	PAC	[97]
In^{113}	392	105 min	1/2	$-0.21050\,(2)$	$-0.42100\,(4)$	Atomic beam	
In^{114}	191	50 days	5	$+4.7\,(1)$	$+0.94\,(2)$	Atomic beam	
In^{116}	70	54 min	5	$+4.21\,(8)$	$+0.88\,(2)$	Atomic beam	
In^{117}	660	60×10^{-9} s	3/2	$+0.95\,(8)$	$+0.63\,(5)$	PAC	[715]
Sn^{114}	3091	7.26×10^{-7} s	7		$-0.081\,(3)$	IB-DPAD	Borsaru* et al.
Sn^{118}	2580	2.30×10^{-7} s	7	$-0.69\,(8)$	$-0.099\,(11)$	IB-DPAD	Ditto
Sn^{119}	24	1.9×10^{-8} s	11/2	$+0.672\,(25)$	$+0.1220\,(45)$	Mössbauer effect	
Sb^{122}	61	1.8 s	3	$\|2.964\,(12)\|$	$\|0.988\,(4)\|$	IB-STROB	Heubes* et al.
Te^{120}	560	13.4×10^{-12} s	2		$\|0.438\,(92)\|$	PAC	[134]

Table III (continued)

Isotope	Level in keV	$\tau_{1/2}$	I	μ (in units of μ_N)	g	Method	Source
Te^{122}	564	111×10^{-12} s	2		$\lvert 0.541\ (36) \rvert$	PAC	[134]
Te^{123}	248	117 days	11/2	$-1.00\ (5)$		LT-NO	Vanneste* et al.
Te^{124}	603	$9.5\ (5) \times 10^{-12}$ s	2		$\lvert 0.585\ (35) \rvert$	PAC	[134]
Te^{125}	145	58 days	11/2	$-0.93\ (5)$		LT-NO	Vanneste* et al.
Te^{126}	667	5.82×10^{-12} s	2		$\lvert 0.872\ (144) \rvert$	PAC	[134]
Te^{127}	89	109 days	11/2	$-0.91\ (5)$		LT-NO	Vanneste* et al.
Te^{128}	743	4.36×10^{-12} s	2		$\lvert 0.932\ (146) \rvert$	PAC	[134]
Te^{129}	106	34 days	11/2	$-1.15\ (5)$		LT-NO	Vanneste* et al.
Cs^{131}	133	9.3×10^{-9} s		$\lvert 0.74\ (3) \rvert$		$\gamma\gamma$-DPAC	Aoki* et al.
Cs^{133}	81	6.3×10^{-9} s	5/2	$+3.4\ (2)$	$+1.36\ (5)$	PAC	
Cs^{134}	137	3.2 h	8	$+1.10\ (1)$	$+0.138\ (10)$	Atomic beam	
Nd^{150}	131	1.56×10^{-9} s	2		$+0.26\ (3)$	Inelastic scattering of protons	

Pm^{147}	92	2.44×10^{-9} s	5/2		$+1.69\,(24)$	PAC	
Pm^{149}	114	3.64×10^{-9} s	5/2	$\vert 2.20\,(21) \vert$	$\vert 0.92\,(8) \vert$	PAC	[95]
Pm^{149}	270	3.72×10^{-9} s	7/2	$\vert 2.17\,(21) \vert$	$\vert 0.62\,(6) \vert$	PAC	[95]
Sm^{152}	125	1.4×10^{-9} s	2		0.351 (25)	Inelastic scattering of protons	
Sm^{152}	122	$2.12\,(7) \times 10^{-9}$ s	2		$+0.277\,(28)$	Coulomb excitation and DPAC	[975]
Sm^{152}	122	$2.04\,(3) \times 10^{-9}$ s	2	$+0.832\,(50)$	$+0.416\,(25)$	Mössbauer effect	[42]
Sm^{154}	84	2.78×10^{-9} s	2		$+0.30\,(3)$	Inelastic scattering of protons	
Sm^{154}	82	$4.37\,(7) \times 10^{-9}$ s	2		$+0.288\,(29)$	Coulomb excitation and DPAC	[975]
Sm^{154}	82	3×10^{-9} s	2	$+0.778\,(36)$		Mössbauer effect with Coulomb excitation	[965]
Gd^{154}	123	1.2×10^{-9} s	2		$+0.365\,(35)$	PAC	
Gd^{155}	86.5	$6.35\,(9) \times 10^{-9}$ s	5/2	$-0.53\,(5)$		Mössbauer effect	[118[
Gd^{155}	105.3	$1.14\,(3) \times 10^{-9}$ s	3/2	$+0.13\,(4)$ or $-0.38\,(6)$		Mössbauer effect	[118]

Isotope	Level in keV	$\tau_{1/2}$	I	μ (in units of μ_N)	g	Method	Source
Gd^{156}	89	2.0×10^{-9} s	2		$+0.35\ (6)$	PAC	
Gd^{156}	89	$321\ (8) \times 10^{-9}$ s	2		$+0.296\ (18)$	Coulomb excitation and DPAC	[975]
Gd^{158}	79.5	$3.69\ (8) \times 10^{-9}$ s	2		$+0.315\ (25)$	Coulomb excitation and DPAC	[975]
Gd^{160}	75.3	$3.92\ (8) \times 10^{-9}$ s	2		$+0.303\ (26)$	Coulomb excitation and DPAC	[975]
Dy^{160}	87	1.8×10^{-9} s	2		$+0.282\ (34)$	PAC	
Dy^{160}	86.8	$296\ (3) \times 10^{-9}$ s	2		$\lvert 0.38\ (2) \rvert$	Mössbauer effect	[695]
Dy^{161}	26	2.7×10^{-8} s	3/2	$+0.61\ (12)$	$+0.41\ (8)$	Mössbauer effect	
Dy^{164}	73.4	$3.444\ (54) \times 10^{-9}$ s	2		$\lvert 0.336\ (14) \rvert$	Mössbauer effect	[664]
Er^{164}	91.5	1.4×10^{-9} s	2		$\lvert 0.353\ (10) \rvert$	Mössbauer effect	[664]
Er^{166}	80.6	$2.696\ (42) \times 10^{-9}$ s	2		$\lvert 0.320\ (8) \rvert$	Mössbauer effect	[664]
Er^{166}	80.6	$2.696\ (42) \times 10^{-9}$ s	2	$\lvert 0.61\ (3) \rvert$	$\lvert 0.305\ (15) \rvert$	Mössbauer effect	[214]

Er^{166}	80	1.82×10^{-9} s	2	+0.61 (12)	+0.300 (36)	PAC	
Er^{166}	265	1.2×10^{-10} s	4	| 1.20 (7) |	| 0.299 (7) |	γγ-IPAC	Miyokawa* et al.
Er^{168}	79.8	1.9×10^{-9} s	2		| 0.333 (8) |	Mössbauer effect	[664]
Er^{170}	84.2	2.734 (42)	2		| 0.319 (11) |	Mössbauer effect with Coulomb excitation	[969]
Tm^{169}	8.42		3/2	+0.533 (8)		Mössbauer effect	[516]
Tm^{169}	118	6.2×10^{-11} s	5/2		+0.21 (6)	PAC	
Yb^{170}	84.3	2.28 (7) $\times 10^{-9}$ s	2		| 0.335 (6) |	Mössbauer effect	[664]
Yb^{171}	66.7	1.25 (15) $\times 10^{-9}$ s	3/2	| 0.351 (3) |	| 0.234 (2) |	Mössbauer effect	[420]
Yb^{171}	75.88	2.46 (43) $\times 10^{-9}$ s	5/2	+1.01 (1)		Mössbauer effect	[449]
Yb^{172}	78.7	2.46 (7) $\times 10^{-9}$ s	2		| 0.332 (8) |	Mössbauer effect	[664]
Yb^{174}	76.5	2.59 (7) $\times 10^{-9}$ s	2		| 0.337 (8) |	Mössbauer effect	[664]
Yb^{174}	76.5	2.54 (7) $\times 10^{-9}$ s	2		| 0.338 (15) |	Mössbauer effect with Coulomb excitation	[285]

Table III (continued)

Isotope	Level in keV	$\tau_{1/2}$	I	μ (in units of μ_N)	g	Method	Source
Yb^{176}	82.1	$2.54(7) \times 10^{-9}$ s	2		$\|0.381(16)\|$	Mössbauer effect with Coulomb excitation	[284]
Lu^{175}	114	6.6×10^{-11} s	9/2		$+0.5(2)$	PAC	
Hf^{177}	113	4.2×10^{-10} s	9/2		$+0.22(6)$	PAC	
Hf^{178}	93	$1.50(3) \times 10^{-9}$ s	2		$+0.29(2)$	PAC	[520]
Hf^{180}	93	1.53×10^{-9} s	2		$+0.371(32)$	PAC	
Hf^{180}	309	8.1×10^{-11} s	4		$+0.5(1)$	PAC	
Ta^{181}	482	1.08×10^{-8} s	5/2	$+3.45(8)$	$+1.36(3)$	PAC	
W^{182}	100	1.37×10^{-9} s	2		$+0.404(27)$	Inelastic scattering of protons	
W^{182}	1289	1.12×10^{-9} s	2	1.70	$\|0.85(11)\|$	$\gamma\gamma$-IPAC	Seo* et al.
W^{182}	1289	$1.44(6) \times 10^{-9}$ s	2	$+1.04(24)$	$+0.52(12)$	IPAC	[97]
W^{182}	100	$1.98(2) \times 10^{-9}$ s	2		$\|0.266(9)\|$	Mössbauer effect	[729]

W^{184}	111.1		2		$\vert 0.295 (10) \vert$	Mössbauer effect	[729]
W^{184}	111	1.28×10^{-9} s	2		$+0.38 (5)$	PAC	
W^{186}	122.5		2		$\vert 0.312 (11) \vert$	Mössbauer effect	[729]
Re^{184}	188	169 days	8	$\vert 2.77 (14) \vert$		LT-NO	Vanneste* et al.
Os^{186}	137	0.84×10^{-9} s	2		$+0.316 (28)$	PAC	
Os^{188}	155	5.8×10^{-10} s	2		$+0.361 (43)$	PAC	
Os^{189}	36.2	$0.72 (4) \times 10^{-9}$ s	1/2	$\vert 0.226 (29) \vert$		Mössbauer effect	[949]
Os^{192}	205.79	3.03×10^{-10} s	2		$\vert 0.28 (4) \vert$	PAC	[590]
Ir^{191}	129	$144 (10) \times 10^{-12}$ s	5/2	$+0.55 (5)$	$+0.22 (2)$	PAC Mössbauer effect	[708]
Ir^{191}	82.4	$5.5 (5) \times 10^{-9}$ s	1/2	$+0.52 (3)$	$+1.03 (5)$	PAC Mössbauer effect	[708]
Ir^{191}	82.4		1/2		$+1.083 (9)$	Mössbauer effect	[949]
Ir^{193}	73		1/2		$+0.9400 (19)$	Mössbauer effect	[949]
Pt^{192}	316.49	$5.05 (43) \times 10^{-11}$ s	2		$\vert 0.28 (3) \vert$	PAC	[590]

Table III (continued)

Isotope	Level in keV	$\tau_{1/2}$	I	μ (in units of μ_N)	g	Method	Source
Pt^{192}	612.43	$3.4\,(19) \times 10^{-11}$ s	2		$-0.13\,(7)$	PAC	[590]
Pt^{192}	316	2.7×10^{-11} s	2	$\|0.550\,(32)\|$	$\|0.275\,(11)\|$	γγ-IPAC	Roughny* et al.
Pt^{192}	612	3.0×10^{-11} s	2	0.618 (88)	$\|0.309\,(44)\|$	γγ-IPAC	Roughny* et al.
Pt^{194}	329	3.5×10^{-11} s	2	0.596 (36)	$\|0.298\,(18)\|$	γγ-IPAC	Ditto
Pt^{194}	622	3.5×10^{-11} s	2	0.562 (94)	$\|0.281\,(47)\|$	γγ-IPAC	Ditto
Pt^{196}	356	3.0×10^{-11} s	2	0.646 (40)	$\|0.323\,(20)\|$	γγ-IPAC	Ditto
Au^{196}	596	9.7 h	12	5.35 (20)		LT-NO	Bacon* et al.
Hg^{197}	297	24 h	13/2	$-1.037\,(20)$	$-0.160\,(3)$	HfS in optical spectra	
Hg^{199}	158	2.33×10^{-9} s	5/2		$+0.413\,(32)$	PAC	
Tl^{202}	950	560 s	7	0.896 (42)	$\|0.128\,(6)\|$	IB-DPAD	Hashimoto* et al.
Tl^{203}	279	0.283×10^{-9} s	3/2	$+0.4\,(3)$	$+0.27\,(20)$	Resonance scattering of γ-rays	
Pb^{204}	1274	2.6×10^{-7} s	4		$+0.054\,(5)$	PAC	
Pb^{207}	570	1.1×10^{-10} s	5/2	0.179 (3)		γγ-IPAC	Schroeder* et al.
Pb^{208}	3198	$4.30\,(25) \times 10^{-10}$ s	5	$-0.11\,(14)$	$-0.021\,(27)$	PAC	[500]
Pb^{208}	2615	2.1×10^{-11} s	3	1.89		γγ-IPAC	Schroeder* et al.

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