New Data Analysis Methods for Radiometer Calibration

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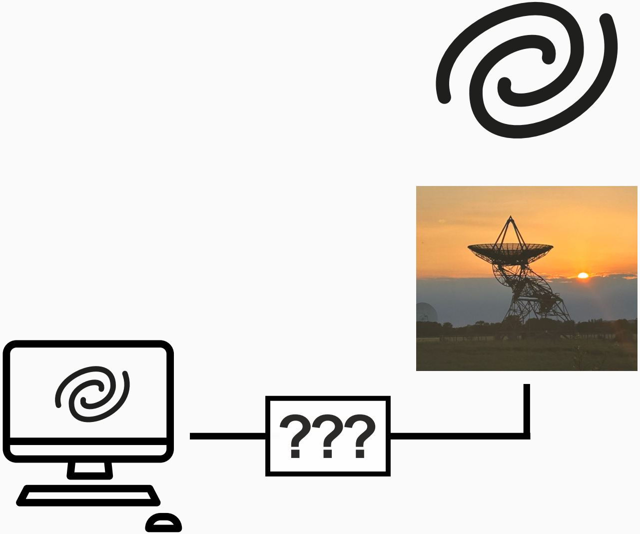
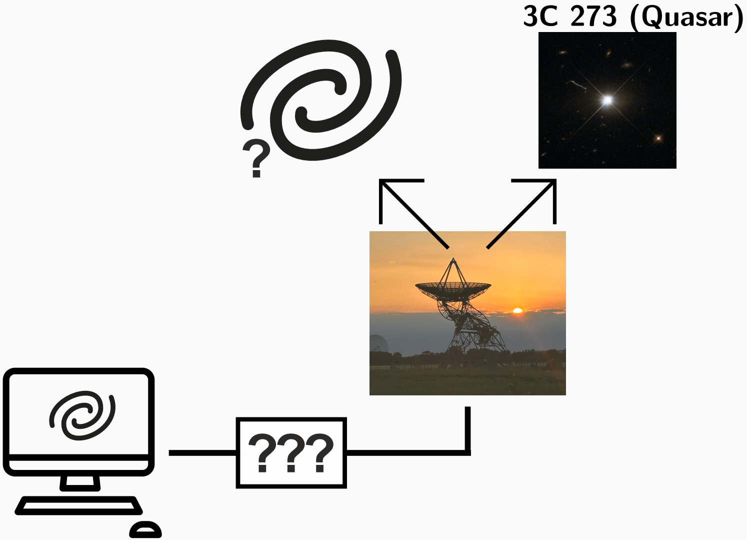
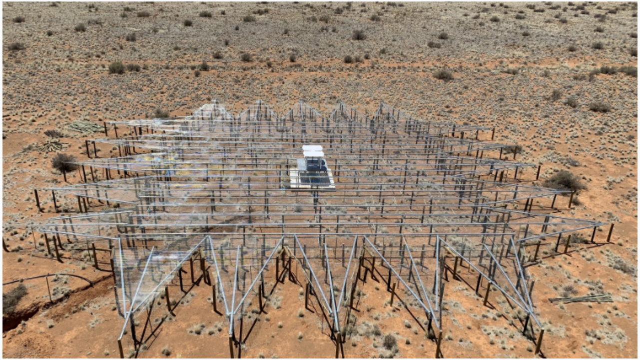
What is calibration?



How to calibrate?



Why is calibration in Global 21cm Cosmology difficult?



We measure *sky averaged* signal.

Antenna LNA impedance mismatch

Very faint signal.

Objective: Map input temperature to output power.

Key Factors:

- LNA introduces time-dependent gain, $g(t)$.
- Impedance mismatch adds noise (T_{rec}) to the system.

Link Output Power to Input Temperature:

$$P_{\mathrm{out}}^{\mathrm{src}} = gM \times (T_{\mathrm{in}}^{\mathrm{src}} + T_{\mathrm{rec}}) \qquad (1)$$

$$M = (1 - |\Gamma_{\mathrm{cal}}|^2)\left|\frac{\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}{1-\Gamma_{\mathrm{cal}}\Gamma_{\mathrm{rec}}}\right|^2$$

Note: All parameters above are frequency-dependent, but the notation has been simplified here and thereafter for convenience.

Dealing with reflections...

$$P_{\text{out}}^{\text{src}} = gM\left(T_{\text{in}}^{\text{src}} + T_{\text{rec}}\right) \tag{2}$$

Noise Parameter Equation:

$$P_{\text{out}}^{\text{src}} = gM\left(T_{\text{in}}^{\text{src}} + T_{\text{min}} + T_0\frac{4R_N}{Z_0}\frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1-|\Gamma_{\text{src}}|^2)(1+|\Gamma_{\text{opt}}|^2)}\right) \tag{3}$$

Noise Wave Equation:

$$\begin{aligned} P_{\text{out}}^{\text{src}} = g\Bigg[& T_0 + T_{\text{unc}}|\Gamma_{\text{s}}|^2\left|\frac{\sqrt{1-|\Gamma_{\text{rec}}|^2}}{1-\Gamma_{\text{s}}\Gamma_{\text{rec}}}\right|^2 \\ & + T_{\text{s}}(1-|\Gamma_{\text{s}}|^2)\left|\frac{\sqrt{1-|\Gamma_{\text{rec}}|^2}}{1-\Gamma_{\text{s}}\Gamma_{\text{rec}}}\right|^2 + T_{\cos}\Re\left(\Gamma_{\text{s}}\frac{\sqrt{1-|\Gamma_{\text{rec}}|^2}}{1-\Gamma_{\text{s}}\Gamma_{\text{rec}}}\right) \\ & + T_{\sin}\Im\left(\Gamma_{\text{s}}\frac{\sqrt{1-|\Gamma_{\text{rec}}|^2}}{1-\Gamma_{\text{s}}\Gamma_{\text{rec}}}\right)\Bigg] \end{aligned} \tag{4}$$

- g - Gain
- T_{min} - Minimum Noise Temperature
- R_N - Noise Resistance
- Γ_{opt} - Optimum Reflection Coefficient
- Γ_{s} - Source Reflection Coefficient
- Γ_{rec} - Receiver Reflection Coefficient
- T_{s} - Source Temperature
- $P_{\text{out}}^{\text{src}}$ - Power out
- $T_{\text{unc, cos, sin}}$ - Noise wave parameters
- T_0 reference temperature

Calibration Equation

Typically, substitute in the noise wave parameter equation here (gains cancel)

$$T^*_{\mathrm{cal}} = T_{\mathrm{NS}} \frac{P_{\mathrm{cal}} - P_L}{P_{\mathrm{NS}} - P_L} + T_L \tag{4}$$

Make some matching assumptions and re arrange:

$$\begin{aligned} T_{\mathrm{s}} = {} & T_{\mathrm{NS}} \left(\frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{P_{\mathrm{NS}} - P_{\mathrm{L}}} \right) \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} + T_{\mathrm{L}} \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} - T_{\mathrm{unc}} \frac{|\Gamma_{\mathrm{s}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} + \\ & - T_{\cos} \frac{\Re\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) |1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}} - T_{\sin} \frac{\Im\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) |1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}} \end{aligned} \tag{5}$$

Note: We end up with 5 parameters that need to be estimated to calibrate the system.

Calculating the error

By partial derivatives To find the error in T_s, we propagate the errors in Γ_s, Γ_{rec}, P_L, P_{NS}, and P_s:

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s}\Delta\Gamma_s\right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta\Gamma_{rec}\right)^2 + \left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 + \left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2. \quad (5)$$

Calculating the error

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s}\Delta\Gamma_s\right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta\Gamma_{rec}\right)^2 + \left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 + \left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2. \quad (6)$$

$$\frac{\partial T_s}{\partial P_L} = T_{\mathrm{NS}}\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}\cdot\frac{P_{\mathrm{s}}-P_{\mathrm{NS}}}{(P_{\mathrm{NS}}-P_{\mathrm{L}})^2}, \quad (7)$$

$$\frac{\partial T_s}{\partial P_{NS}} = -T_{\mathrm{NS}}\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}\cdot\frac{P_{\mathrm{s}}-P_{\mathrm{L}}}{(P_{\mathrm{NS}}-P_{\mathrm{L}})^2}, \quad (8)$$

$$\frac{\partial T_s}{\partial P_s} = T_{\mathrm{NS}}\left(\frac{1}{P_{\mathrm{NS}}-P_{\mathrm{L}}}\right)\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}. \quad (9)$$

$$\frac{\partial T_s}{\partial \Gamma_{rec}} = \frac{\partial A}{\partial \Gamma_{rec}} + \frac{\partial B}{\partial \Gamma_{rec}} + \frac{\partial D}{\partial \Gamma_{rec}} + \frac{\partial E}{\partial \Gamma_{rec}}. \quad (10)$$

$$\frac{\partial T_s}{\partial \Gamma_s} = \frac{\partial A}{\partial \Gamma_s} + \frac{\partial B}{\partial \Gamma_s} + \frac{\partial C}{\partial \Gamma_s} + \frac{\partial D}{\partial \Gamma_s} + \frac{\partial E}{\partial \Gamma_s}. \quad (11)$$

$$A = T_{\mathrm{NS}}\left(\frac{P_{\mathrm{s}}-P_{\mathrm{L}}}{P_{\mathrm{NS}}-P_{\mathrm{L}}}\right)\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}, \quad (12)$$

$$B = T_{\mathrm{L}}\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}, \quad (13)$$

$$C = -T_{\mathrm{unc}}\frac{|\Gamma_{\mathrm{s}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}, \quad (14)$$

$$D = -T_{\cos}\frac{\Re\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right)|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1-|\Gamma_{\mathrm{s}}|^2)\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}, \quad (15)$$

$$E = -T_{\sin}\frac{\Im\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right)|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1-|\Gamma_{\mathrm{s}}|^2)\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}. \quad (16)$$

Calculating the error

Using $T_{\mathbf{NS}}\frac{P_{\mathbf{cal}}-P_L}{P_{\mathbf{NS}}-P_L}+T_L$

$$(\Delta T_s)^2=\left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2+\left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2+\left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 \tag{17}$$

$$+\left(\frac{\partial T_s}{\partial \Gamma_s}\Delta \Gamma_s\right)^2+\left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta \Gamma_{rec}\right)^2. \tag{18}$$

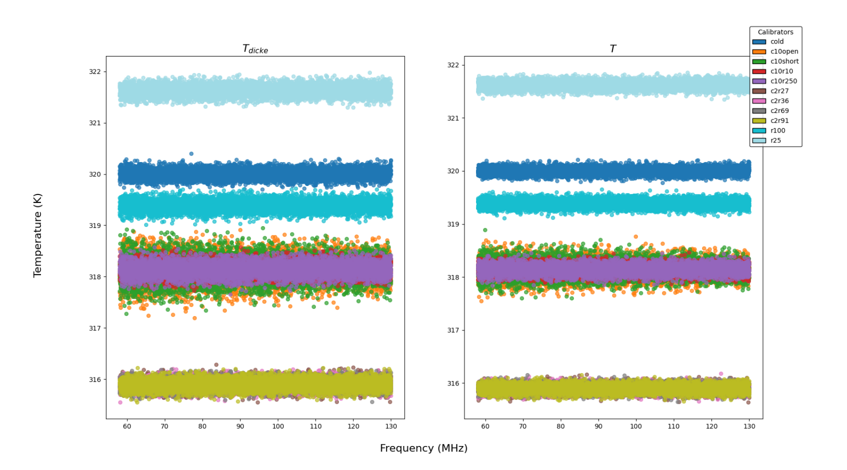
Using noise wave parameters only

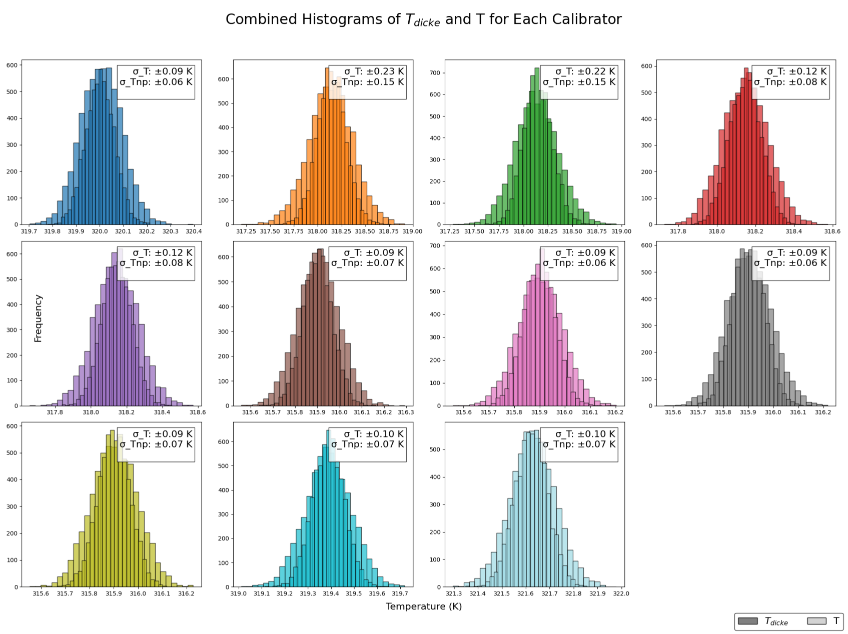
$$(\Delta T_{\mathrm{s}})^2=\left(\frac{\partial T_{\mathrm{s}}}{\partial P_{\mathrm{s}}}\Delta P_{\mathrm{s}}\right)^2 \tag{19}$$

$$+\left(\frac{\partial T_{\mathrm{s}}}{\partial \Gamma_{\mathrm{s}}}\Delta \Gamma_{\mathrm{s}}\right)^2+\left(\frac{\partial T_{\mathrm{s}}}{\partial \Gamma_{\mathrm{rec}}}\Delta \Gamma_{\mathrm{rec}}\right)^2. \tag{20}$$

Note that this argument applies for both noise parameters and noise wave parameters.

There is more noise when using $T_{\mathrm{NS}}\frac{P_{\mathrm{cal}}-P_L}{P_{\mathrm{NS}}-P_L}+T_L$





Noise amplified by **30%** when using

$$T_{\mathrm{NS}} \frac{P_{\mathrm{cal}} - P_L}{P_{\mathrm{NS}} - P_L} + T_L$$

Why not fit noise (wave) parameters directly?

Noise Parameter Equation:

$$P_{\text{out}}^{\text{src}} = gM\left(T_{\text{in}}^{\text{src}} + T_{\text{min}} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)}\right) \tag{21}$$

Noise Wave Equation:

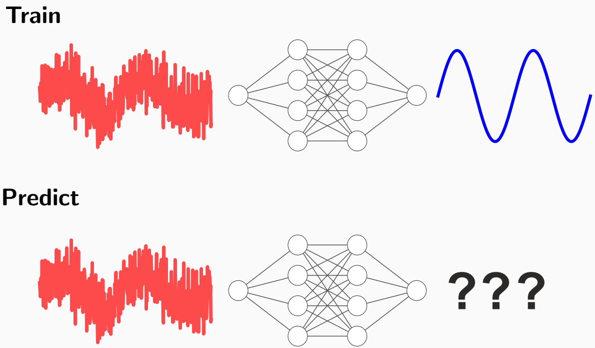
$$\begin{aligned} P_{\text{out}}^{\text{src}} = g\Bigg[& T_0 + T_{\text{unc}}|\Gamma_{\text{s}}|^2 \left|\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right|^2 \\ & + T_{\text{s}}(1 - |\Gamma_{\text{s}}|^2)\left|\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right|^2 + T_{\cos}\Re\left(\Gamma_{\text{s}}\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right) \\ & + T_{\sin}\Im\left(\Gamma_{\text{s}}\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right)\Bigg] \end{aligned} \tag{22}$$

We still end up with 5 unknowns, as before.

Machine learning

How/why?

- Can predict noise wave parameters using machine learning.
- Could apply method to noise parameters.
- Predict **directly** on noise parameters on frequency by frequency basis.
- Maleable to environmental changes.



How to Calibrate with Machine Learning?

1. Define the Loss Function

Regress over measured power and predicted power.

$$\mathcal{L} = \frac{1}{n}\sum_{i=1}^{n}\left(\mathcal{P}_{\text{measured},i} - \mathcal{P}_{\text{pred},i}\right)^2 \tag{23}$$

2. Write Down the Equation for $\mathcal{P}_{\text{pred}}$

Using the noise wave formalism, relate $\mathcal{P}_{\text{pred}}$ to T_{src}.

$$\mathcal{P}_{\text{pred}} = g \cdot M \Big(T_{\text{in}}^{\text{src}} + T_{\min} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)} \Big) \tag{24}$$

3. Minimise the Loss Function

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}) \tag{25}$$

parameter vector $\boldsymbol{\theta}$ includes all tunable parameters in the model:

$$\boldsymbol{\theta} = \{g, T_{\min}, R_N, \Gamma_{\text{opt}}^{\phi}, |\Gamma_{\text{opt}}|) \tag{26}$$

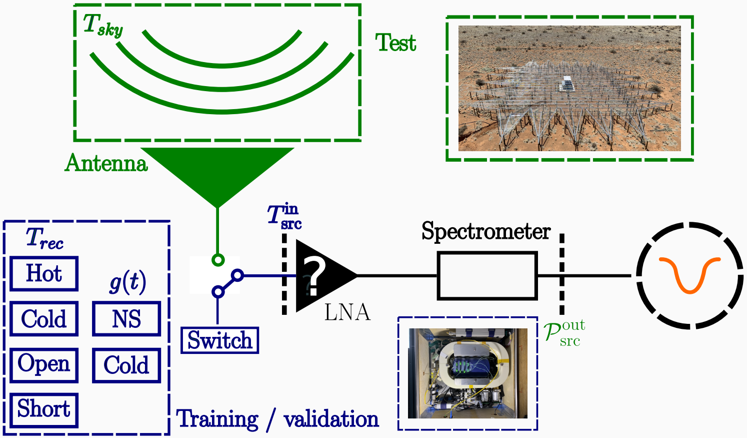
4. Rearrange and Predict (T_{src}) using $\boldsymbol{\theta}^*$

$$T_{\text{src}} = \frac{\mathcal{P}_{\text{pred}}}{g^* \cdot M} - \left(T_{\min}^* + T_0 \frac{4R_N^*}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}^*|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}^*|^2)} \right) \tag{27}$$

The physical system

Process:

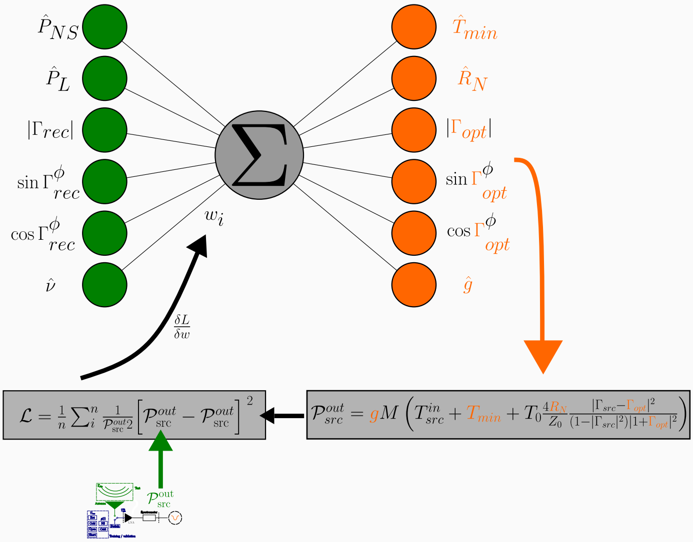
- Switch between sources to generate training data.
- Calibration sources with known temperature train neural net.
- Predict $T_{\rm src}$ of antenna.



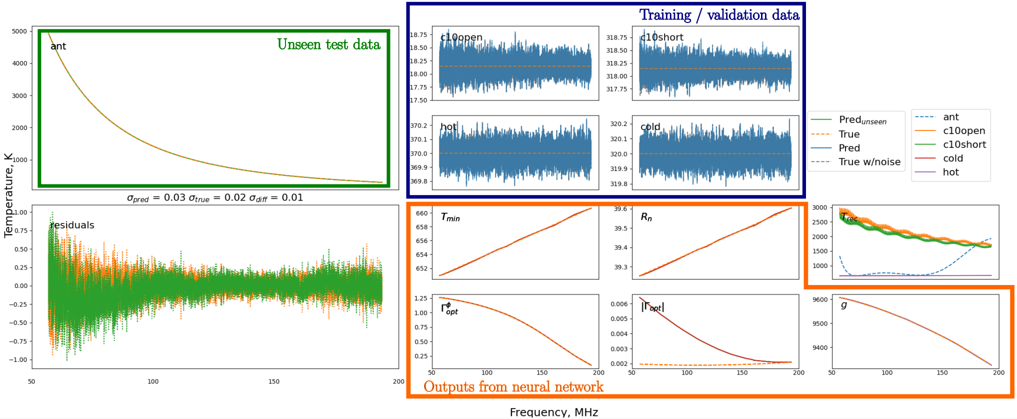
Network Architecture

Structure:

- Input thermistor and VNA measurments.
- Also input frequency.
- Predict noise parameters.
- Regress over loss function.



Testing on simulated data with realistic antenna



Thank you!

SCAN ME

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github.com/samleeney/Talks

APPENDIX A: Fudge Factors

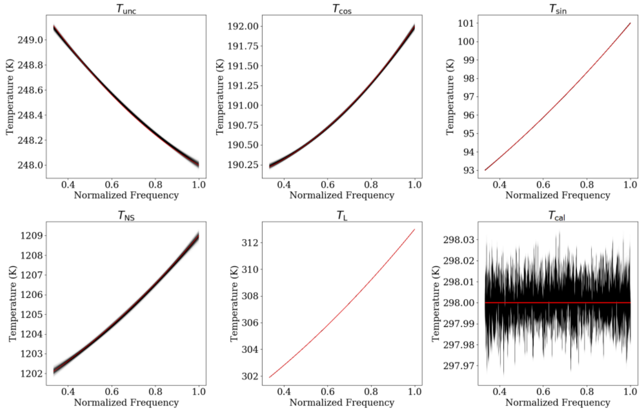

Figure 9. Results from 1000 samples using data generated with our more realistic noise model (shown in black). The second-order noise wave parameters shown in red are used to generate the data inputted to our pipeline. The polynomial order and values of the noise wave parameters that best suit the data according to our algorithm match that of the empirical model. This solution is applied to an ambient-temperature load, shown in the bottom right panel as our predictive $\hat{y}$ from equation (27), and calibrates it to within 1σ of ambient temperature.

T_L should be spectrally flat, however it has structure because we are including a fudge factor in the fit to account for other uncertain approximations in the derivation.

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