Calibration with Machine Learning for REACH

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tbd

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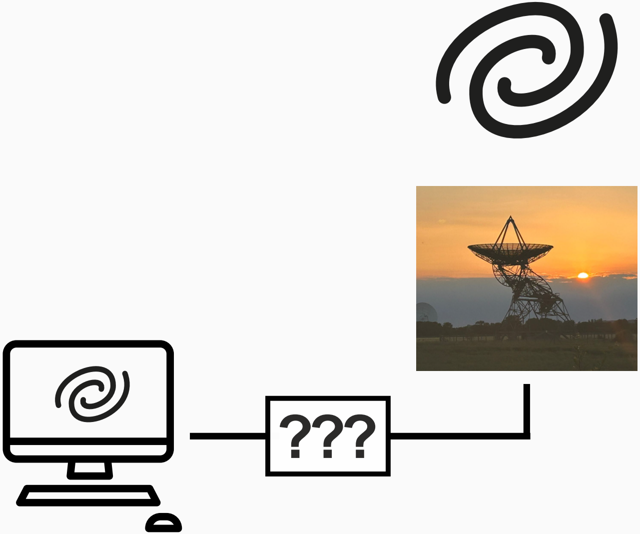
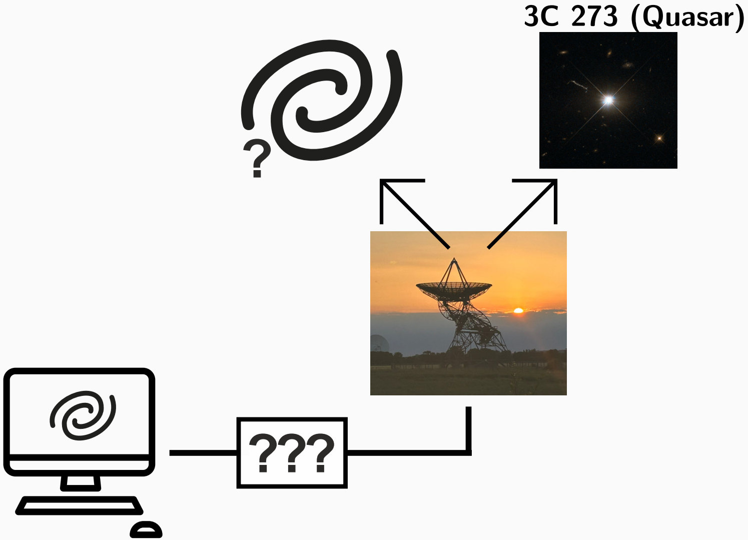
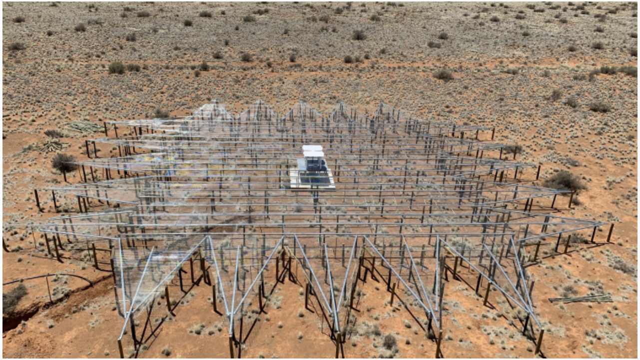
What is calibration?



How to calibrate?



Why is calibration in Global 21cm Cosmology difficult?



We measure *sky averaged* signal.

Antenna LNA impedance mismatch

Very faint signal.

Objective: Map input temperature to output power.

Key Factors:

- LNA introduces time-dependent gain, $g(t)$.
- Impedance mismatch adds noise (T_{rec}) to the system.

Link Output Power to Input Temperature:

$$P_{\mathrm{out}}^{\mathrm{src}} = gM \left(T_{\mathrm{in}}^{\mathrm{src}} + T_{\mathrm{rec}} \right) \tag{1}$$

Note: All parameters above are frequency-dependent, but the notation has been simplified here and thereafter for convenience.

Dealing with reflections...

$$P_{\text{out}}^{\text{src}} = gM\left(T_{\text{in}}^{\text{src}} + T_{\text{rec}}\right) \tag{2}$$

Noise Parameter Equation:

$$P_{\text{out}}^{\text{src}} = gM\left(T_{\text{in}}^{\text{src}} + T_{\text{min}} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)}\right) \tag{3}$$

Noise Wave Equation:

$$\begin{aligned} P_{\text{out}}^{\text{src}} = g \Bigg[& T_0 + T_{\text{unc}}|\Gamma_{\text{s}}|^2 \left|\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right|^2 \\ & + T_{\text{s}}(1 - |\Gamma_{\text{s}}|^2)\left|\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right|^2 + T_{\cos}\Re\left(\Gamma_{\text{s}}\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right) \\ & + T_{\sin}\Im\left(\Gamma_{\text{s}}\frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{s}}\Gamma_{\text{rec}}}\right)\Bigg] \end{aligned} \tag{4}$$

Calibration Equation

Typically, substitute in the noise wave parameter equation here (gains cancel)

$$T^*_{\mathrm{cal}} = T_{\mathrm{NS}} \frac{P_{\mathrm{cal}} - P_L}{P_{\mathrm{NS}} - P_L} + T_L \tag{4}$$

Make some matching assumptions and re arrange:

$$\begin{aligned} T_{\mathrm{s}} = {} & T_{\mathrm{NS}} \left(\frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{P_{\mathrm{NS}} - P_{\mathrm{L}}} \right) \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} + T_{\mathrm{L}} \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} - T_{\mathrm{unc}} \frac{|\Gamma_{\mathrm{s}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} + \\ & - T_{\mathrm{cos}} \frac{\Re\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) |1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}} - T_{\mathrm{sin}} \frac{\Im\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) |1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}} \end{aligned} \tag{5}$$

Note: We end up with 5 parameters that need to be estimated to calibrate the system.

Calculating the error

By partial derivatives To find the error in T_s, we propagate the errors in Γ_s, Γ_{rec}, P_L, P_{NS}, and P_s:

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s}\Delta\Gamma_s\right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta\Gamma_{rec}\right)^2 + \left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 + \left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2. \quad (5)$$

Calculating the error

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s}\Delta\Gamma_s\right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta\Gamma_{rec}\right)^2 + \left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 + \left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2. \quad (6)$$

$$\frac{\partial T_s}{\partial P_L} = T_{\mathrm{NS}} \frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2} \cdot \frac{P_{\mathrm{s}} - P_{\mathrm{NS}}}{(P_{\mathrm{NS}} - P_{\mathrm{L}})^2}, \quad (7)$$

$$\frac{\partial T_s}{\partial P_{NS}} = -T_{\mathrm{NS}} \frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2} \cdot \frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{(P_{\mathrm{NS}} - P_{\mathrm{L}})^2}, \quad (8)$$

$$\frac{\partial T_s}{\partial P_s} = T_{\mathrm{NS}} \left(\frac{1}{P_{\mathrm{NS}} - P_{\mathrm{L}}}\right) \frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}. \quad (9)$$

$$\frac{\partial T_s}{\partial \Gamma_{rec}} = \frac{\partial A}{\partial \Gamma_{rec}} + \frac{\partial B}{\partial \Gamma_{rec}} + \frac{\partial D}{\partial \Gamma_{rec}} + \frac{\partial E}{\partial \Gamma_{rec}}. \quad (10)$$

$$\frac{\partial T_s}{\partial \Gamma_s} = \frac{\partial A}{\partial \Gamma_s} + \frac{\partial B}{\partial \Gamma_s} + \frac{\partial C}{\partial \Gamma_s} + \frac{\partial D}{\partial \Gamma_s} + \frac{\partial E}{\partial \Gamma_s}. \quad (11)$$

$$A = T_{\mathrm{NS}} \left(\frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{P_{\mathrm{NS}} - P_{\mathrm{L}}}\right) \frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}, \quad (12)$$

$$B = T_{\mathrm{L}} \frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}, \quad (13)$$

$$C = -T_{\mathrm{unc}} \frac{|\Gamma_{\mathrm{s}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}, \quad (14)$$

$$D = -T_{\cos} \frac{\Re\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) |1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1-|\Gamma_{\mathrm{s}}|^2)\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}, \quad (15)$$

$$E = -T_{\sin} \frac{\Im\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) |1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1-|\Gamma_{\mathrm{s}}|^2)\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}. \quad (16)$$

Calculating the error

Using $T_{\mathbf{NS}} \frac{P_{\mathbf{cal}} - P_L}{P_{\mathbf{NS}} - P_L} + T_L$

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial P_s} \Delta P_s \right)^2 + \left(\frac{\partial T_s}{\partial P_L} \Delta P_L \right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}} \Delta P_{NS} \right)^2 \tag{17}$$

$$+ \left(\frac{\partial T_s}{\partial \Gamma_s} \Delta \Gamma_s \right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}} \Delta \Gamma_{rec} \right)^2 . \tag{18}$$

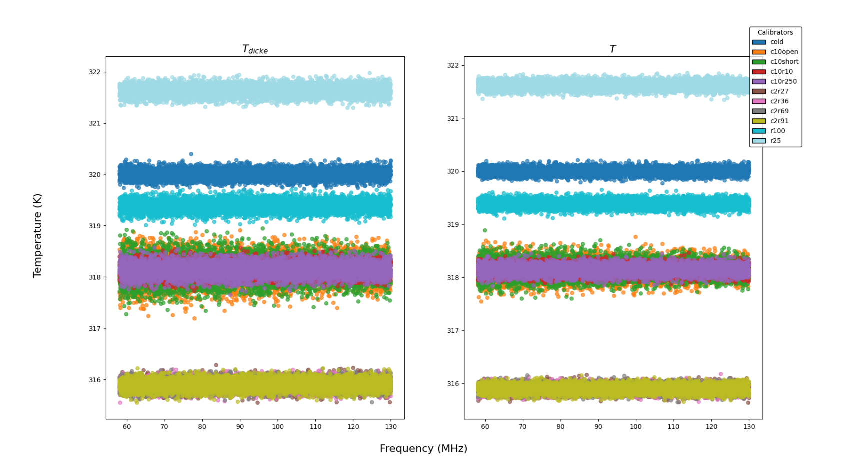
Using noise wave parameters only

$$(\Delta T_{\mathrm{s}})^2 = \left(\frac{\partial T_{\mathrm{s}}}{\partial P_{\mathrm{s}}} \Delta P_{\mathrm{s}} \right)^2 \tag{19}$$

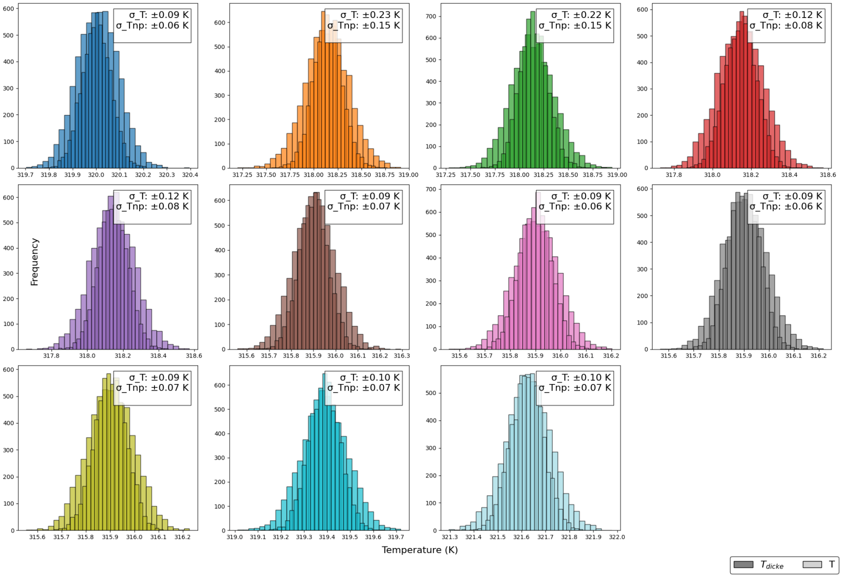
$$+ \left(\frac{\partial T_{\mathrm{s}}}{\partial \Gamma_{\mathrm{s}}} \Delta \Gamma_{\mathrm{s}} \right)^2 + \left(\frac{\partial T_{\mathrm{s}}}{\partial \Gamma_{\mathrm{rec}}} \Delta \Gamma_{\mathrm{rec}} \right)^2 . \tag{20}$$

Note that this argument applies for both noise parameters and noise wave parameters.

There is more noise when using $T_{\mathrm{NS}} \frac{P_{\mathrm{cal}} - P_L}{P_{\mathrm{NS}} - P_L} + T_L$



Combined Histograms of T_{dicke} and T for Each Calibrator



Why not fit noise (wave) parameters directly?

Noise Parameter Equation:

$$P_{\mathrm{out}}^{\mathrm{src}} = gM\left(T_{\mathrm{in}}^{\mathrm{src}} + T_{\mathrm{min}} + T_0 \frac{4R_N}{Z_0}\frac{|\Gamma_{\mathrm{src}} - \Gamma_{\mathrm{opt}}|^2}{(1-|\Gamma_{\mathrm{src}}|^2)(1+|\Gamma_{\mathrm{opt}}|^2)}\right) \tag{21}$$

Noise Wave Equation:

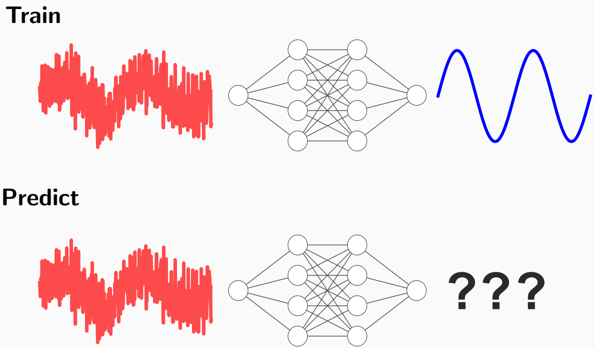
$$\begin{aligned} P_{\mathrm{out}}^{\mathrm{src}} = g\Bigg[T_0 + T_{\mathrm{unc}}|\Gamma_{\mathrm{s}}|^2\left|\frac{\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right|^2 \\ + T_{\mathrm{s}}(1-|\Gamma_{\mathrm{s}}|^2)\left|\frac{\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right|^2 + T_{\mathrm{cos}}\Re\left(\Gamma_{\mathrm{s}}\frac{\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right) \\ + T_{\mathrm{sin}}\Im\left(\Gamma_{\mathrm{s}}\frac{\sqrt{1-|\Gamma_{\mathrm{rec}}|^2}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}}\right)\Bigg] \end{aligned} \tag{22}$$

We still end up with 5 unknowns, as before.

Machine learning

How/why?

- Can predict noise wave parameters using machine learning.
- Could apply method to noise parameters.
- Predict **directly** on noise parameters on frequency by frequency basis.
- Maleable to environmental changes.



How to Calibrate with Machine Learning?

1. Define the Loss Function

Regress over measured power and predicted power.

$$\mathcal{L} = \frac{1}{n}\sum_{i=1}^{n}(\mathcal{P}_{\text{measured},i} - \mathcal{P}_{\text{pred},i})^2 \tag{23}$$

2. Write Down the Equation for $\mathcal{P}_{\text{pred}}$

Using the noise wave formalism, relate $\mathcal{P}_{\text{pred}}$ to T_{src}.

$$\mathcal{P}_{\text{pred}} = g \cdot M\Bigg(T_{\text{in}}^{\text{src}} + T_{\text{min}} + T_0\frac{4R_N}{Z_0}\frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1-|\Gamma_{\text{src}}|^2)(1+|\Gamma_{\text{opt}}|^2)}\Bigg) \tag{24}$$

3. Minimise the Loss Function

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}) \tag{25}$$

parameter vector $\boldsymbol{\theta}$ includes all tunable parameters in the model:

$$\boldsymbol{\theta} = \{g, T_{\text{min}}, R_N, \Gamma_{\text{opt}}^{\phi}, |\Gamma_{\text{opt}}|) \tag{26}$$

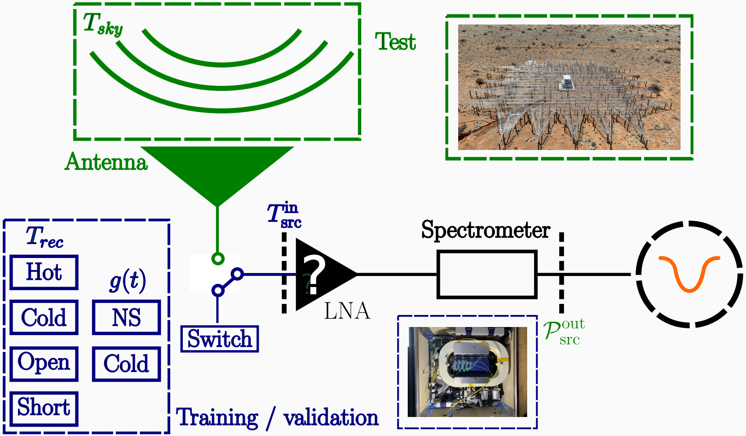
4. Rearrange and Predict (T_{src}) using $\boldsymbol{\theta}^*$

$$T_{\text{src}} = \frac{\mathcal{P}_{\text{pred}}}{g^* \cdot M} - \Bigg(T_{\text{min}}^* + T_0\frac{4R_N^*}{Z_0}\frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}^*|^2}{(1-|\Gamma_{\text{src}}|^2)(1+|\Gamma_{\text{opt}}^*|^2)}\Bigg) \tag{27}$$

The physical system

Process:

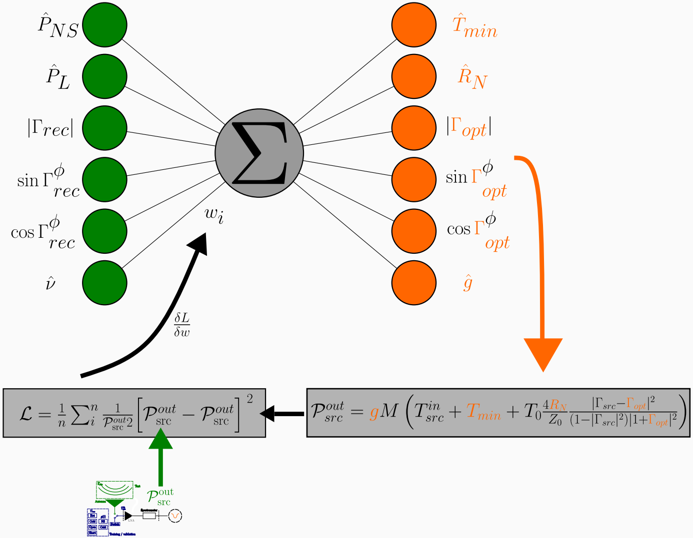
- Switch between sources to generate training data.
- Calibration sources with known temperature train neural net.
- Predict T_{src} of antenna.



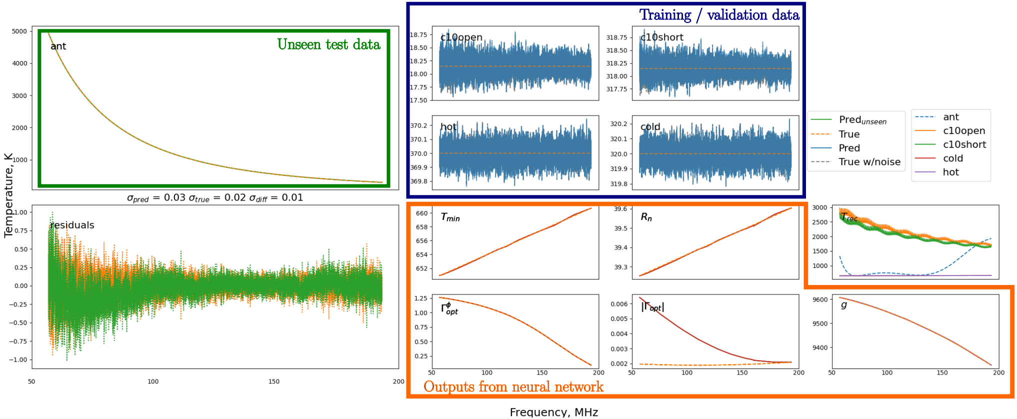
Network Architecture

Structure:

- Input thermistor and VNA measurments.
- Also input frequency.
- Predict noise parameters.
- Regress over loss function.



Testing on simulated data with realistic antenna



General thoughts on REACH calibration

- We have tried 5 different 'data analysis' techniques.
- There are advantages and disadvantages but results are not drastically different.
- So what are we limited by?
- The conclusion of Ians thesis is that we are limited by the quality of specific Physical measurments of the system (namely cable measurments and switches).
- I do not think Dicke switching is needed at all - can someone convince me otherwise?

Thank you!

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