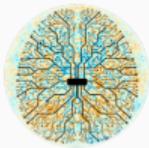


21cm Workshop Cambridge 2025

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With: Harry Bevins, Eloy de Lera Acedo, Will Handley, Christian Kirkhan, Kaan Artuc, Jiacong Zhu



REACH



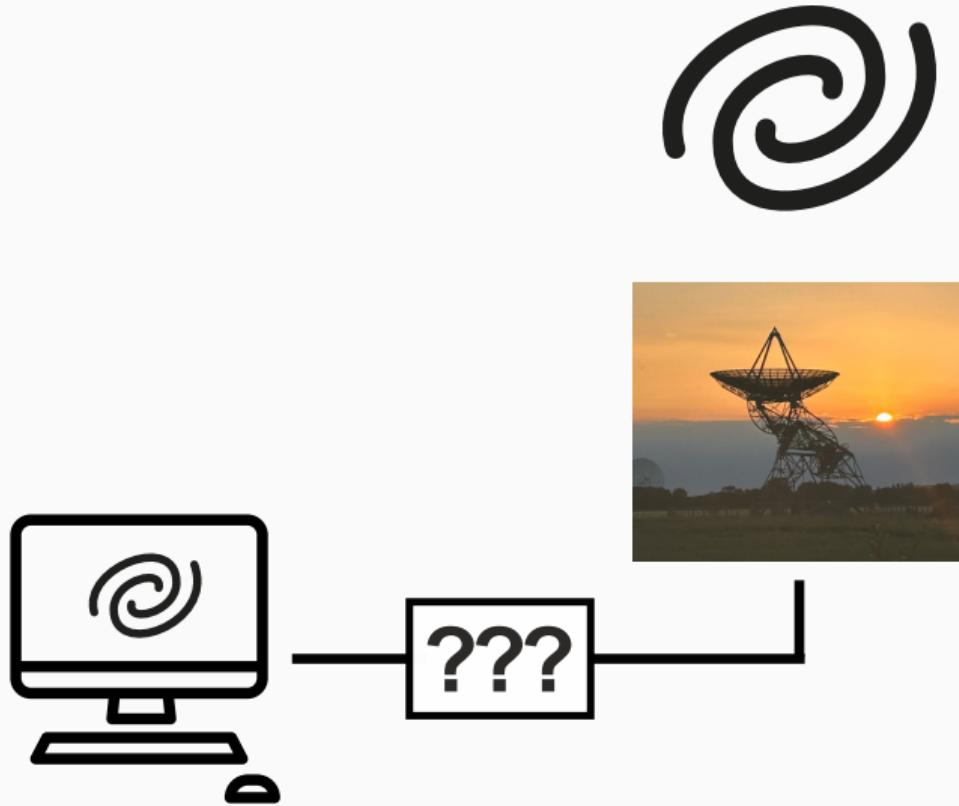
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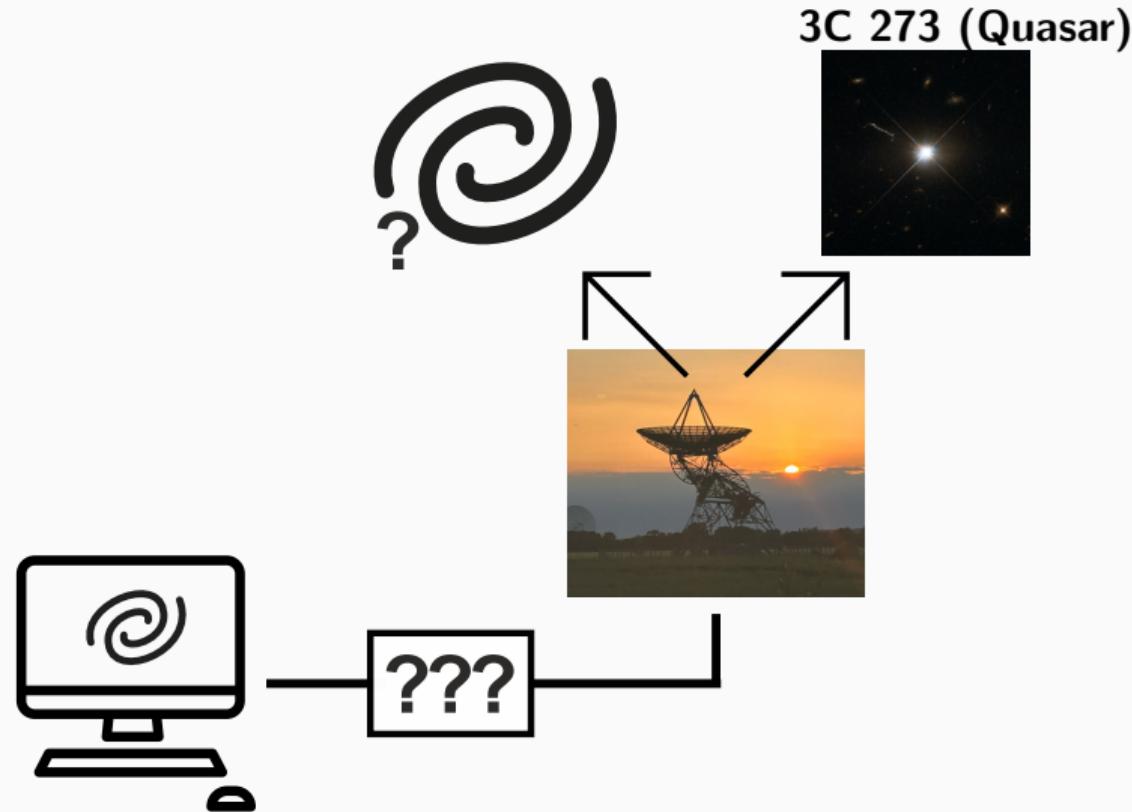
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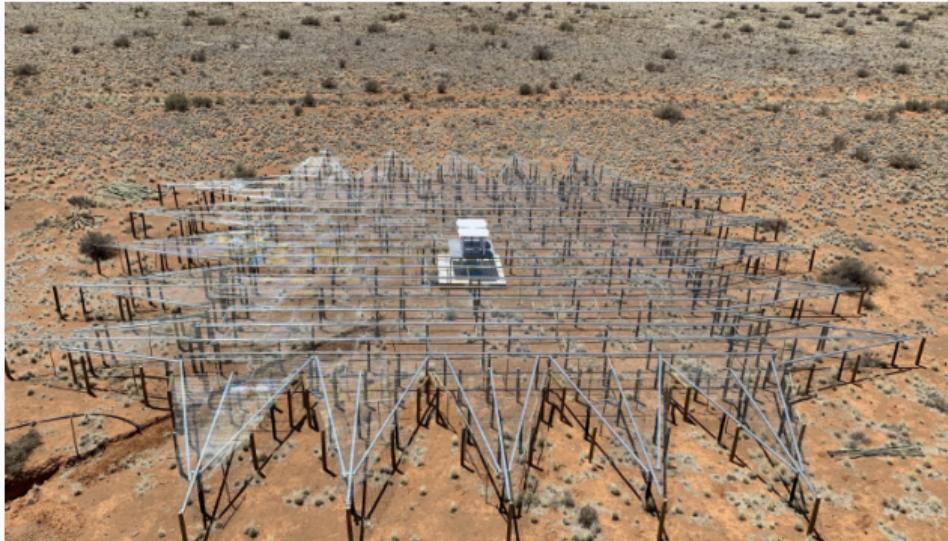
What is calibration?



How to calibrate?



Why is calibration in Global 21cm Cosmology difficult?



We measure *sky averaged* signal.

Antenna LNA impedance mismatch

Very faint signal.

How to calibrate (in a bit more detail...)?

Objective: Map input temperature to output power.

Key Factors:

- LNA introduces time-dependent gain, $g(t)$.
- Impedance mismatch adds noise (T_{rec}) to the system.

Link Output Power to Input Temperature:

$$P_{\text{out}}^{\text{src}} = gM \times (T_{\text{in}}^{\text{src}} + T_{\text{rec}}) \quad (1)$$

$$M = \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{(1 - |\Gamma_{\text{cal}}|^2)} | \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_{\text{cal}} \Gamma_{\text{rec}}} |^2$$

Note: All parameters above are frequency-dependent, but the notation has been simplified here and thereafter for convenience.

Dealing with reflections...

$$P_{\text{out}}^{\text{src}} = gM (T_{\text{in}}^{\text{src}} + T_{\text{rec}}) \quad (2)$$

Noise Parameter Equation:

$$P_{\text{out}}^{\text{src}} = gM \left(T_{\text{in}}^{\text{src}} + T_{\text{min}} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)} \right) \quad (3)$$

- g - Gain
- T_{min} - Minimum Noise Temperature
- R_N - Noise Resistance
- Γ_{opt} - Optimum Reflection Coefficient

Noise Wave Equation:

$$\begin{aligned} P_{\text{out}}^{\text{src}} = g & \left[T_0 + T_{\text{unc}} |\Gamma_s|^2 \left| \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right|^2 \right. \\ & + T_s (1 - |\Gamma_s|^2) \left| \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right|^2 + T_{\text{cos}} \Re \left(\Gamma_s \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right) \\ & \left. + T_{\text{sin}} \Im \left(\Gamma_s \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right) \right] \quad (4) \end{aligned}$$

- Γ_s - Source Reflection Coefficient
- Γ_{rec} - Receiver Reflection Coefficient
- T_s - Source Temperature
- $P_{\text{out}}^{\text{src}}$ - Power out
- $T_{\text{unc}}, \cos, \sin$ - Noise wave parameters
- T_0 reference temperature

Calibration Equation

Typically, substitute in the noise wave parameter equation here (gains cancel)

$$T_{\text{cal}}^* = T_{\text{NS}} \frac{P_{\text{cal}} - P_L}{P_{\text{NS}} - P_L} + T_L \quad (4)$$

Make some matching assumptions and re arrange:

$$\begin{aligned} T_s = & \color{red} T_{\text{NS}} \left(\frac{P_s - P_L}{P_{\text{NS}} - P_L} \right) \frac{|1 - \Gamma_s \Gamma_{\text{rec}}|^2}{1 - |\Gamma_s|^2} + \color{red} T_L \frac{|1 - \Gamma_s \Gamma_{\text{rec}}|^2}{1 - |\Gamma_s|^2} - \color{red} T_{\text{unc}} \frac{|\Gamma_s|^2}{1 - |\Gamma_s|^2} + \\ & - \color{red} T_{\text{cos}} \frac{\Re \left(\frac{\Gamma_s}{1 - \Gamma_s \Gamma_{\text{rec}}} \right) |1 - \Gamma_s \Gamma_{\text{rec}}|^2}{(1 - |\Gamma_s|^2) \sqrt{1 - |\Gamma_{\text{rec}}|^2}} - \color{red} T_{\text{sin}} \frac{\Im \left(\frac{\Gamma_s}{1 - \Gamma_s \Gamma_{\text{rec}}} \right) |1 - \Gamma_s \Gamma_{\text{rec}}|^2}{(1 - |\Gamma_s|^2) \sqrt{1 - |\Gamma_{\text{rec}}|^2}} \end{aligned} \quad (5)$$

Note: We end up with 5 parameters that need to be estimated to calibrate the system.

Calculating the error

By partial derivatives To find the error in T_s , we propagate the errors in Γ_s , Γ_{rec} , P_L , P_{NS} , and P_s :

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s} \Delta \Gamma_s \right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}} \Delta \Gamma_{rec} \right)^2 + \left(\frac{\partial T_s}{\partial P_L} \Delta P_L \right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}} \Delta P_{NS} \right)^2 + \left(\frac{\partial T_s}{\partial P_s} \Delta P_s \right)^2. \quad (5)$$

Calculating the error

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s} \Delta \Gamma_s \right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}} \Delta \Gamma_{rec} \right)^2 + \left(\frac{\partial T_s}{\partial P_L} \Delta P_L \right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}} \Delta P_{NS} \right)^2 + \left(\frac{\partial T_s}{\partial P_s} \Delta P_s \right)^2. \quad (6)$$

$$\frac{\partial T_s}{\partial P_L} = T_{NS} \frac{|1 - \Gamma_s \Gamma_{rec}|^2}{1 - |\Gamma_s|^2} \cdot \frac{P_s - P_{NS}}{(P_{NS} - P_L)^2}, \quad (7)$$

$$\frac{\partial T_s}{\partial P_{NS}} = -T_{NS} \frac{|1 - \Gamma_s \Gamma_{rec}|^2}{1 - |\Gamma_s|^2} \cdot \frac{P_s - P_L}{(P_{NS} - P_L)^2}, \quad (8)$$

$$\frac{\partial T_s}{\partial P_s} = T_{NS} \left(\frac{1}{P_{NS} - P_L} \right) \frac{|1 - \Gamma_s \Gamma_{rec}|^2}{1 - |\Gamma_s|^2}. \quad (9)$$

$$\frac{\partial T_s}{\partial \Gamma_{rec}} = \frac{\partial A}{\partial \Gamma_{rec}} + \frac{\partial B}{\partial \Gamma_{rec}} + \frac{\partial D}{\partial \Gamma_{rec}} + \frac{\partial E}{\partial \Gamma_{rec}}. \quad (10)$$

$$\frac{\partial T_s}{\partial \Gamma_s} = \frac{\partial A}{\partial \Gamma_s} + \frac{\partial B}{\partial \Gamma_s} + \frac{\partial C}{\partial \Gamma_s} + \frac{\partial D}{\partial \Gamma_s} + \frac{\partial E}{\partial \Gamma_s}. \quad (11)$$

$$A = T_{NS} \left(\frac{P_s - P_L}{P_{NS} - P_L} \right) \frac{|1 - \Gamma_s \Gamma_{rec}|^2}{1 - |\Gamma_s|^2}, \quad (12)$$

$$B = T_L \frac{|1 - \Gamma_s \Gamma_{rec}|^2}{1 - |\Gamma_s|^2}, \quad (13)$$

$$C = -T_{unc} \frac{|\Gamma_s|^2}{1 - |\Gamma_s|^2}, \quad (14)$$

$$D = -T_{cos} \frac{\Re \left(\frac{\Gamma_s}{1 - \Gamma_s \Gamma_{rec}} \right) |1 - \Gamma_s \Gamma_{rec}|^2}{(1 - |\Gamma_s|^2) \sqrt{1 - |\Gamma_{rec}|^2}}, \quad (15)$$

$$E = -T_{sin} \frac{\Im \left(\frac{\Gamma_s}{1 - \Gamma_s \Gamma_{rec}} \right) |1 - \Gamma_s \Gamma_{rec}|^2}{(1 - |\Gamma_s|^2) \sqrt{1 - |\Gamma_{rec}|^2}}. \quad (16)$$

Calculating the error

Using $T_{NS} \frac{P_{cal}-P_L}{P_{NS}-P_L} + T_L$

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial P_s} \Delta P_s \right)^2 + \left(\frac{\partial T_s}{\partial P_L} \Delta P_L \right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}} \Delta P_{NS} \right)^2 \quad (17)$$

$$+ \left(\frac{\partial T_s}{\partial \Gamma_s} \Delta \Gamma_s \right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}} \Delta \Gamma_{rec} \right)^2. \quad (18)$$

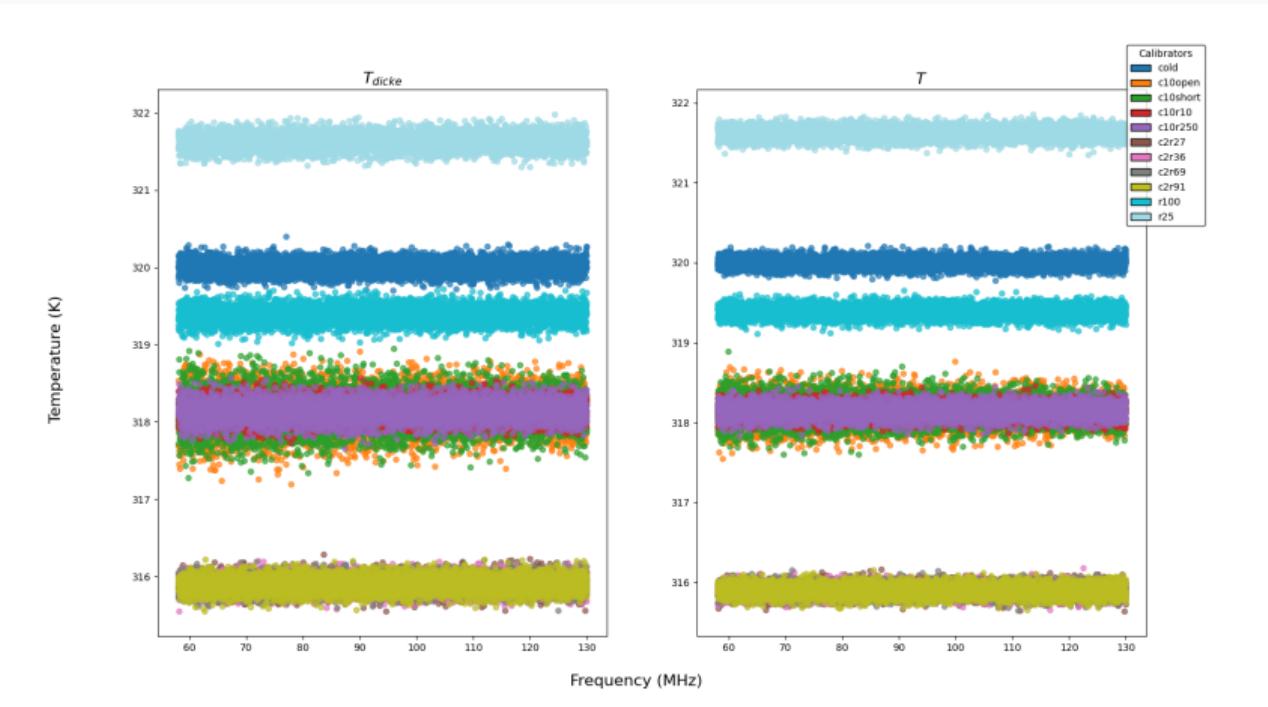
Using noise wave parameters only

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial P_s} \Delta P_s \right)^2 \quad (19)$$

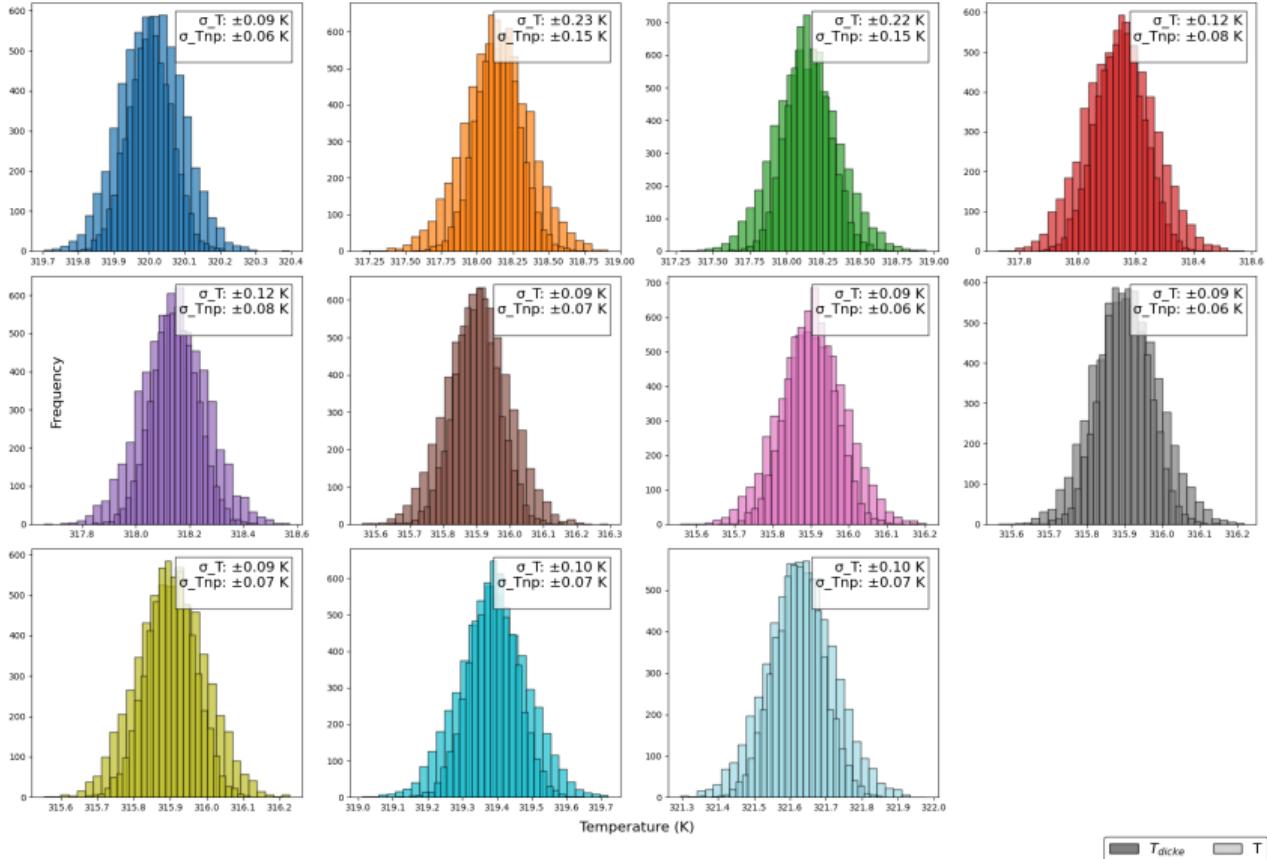
$$+ \left(\frac{\partial T_s}{\partial \Gamma_s} \Delta \Gamma_s \right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}} \Delta \Gamma_{rec} \right)^2. \quad (20)$$

Note that this argument applies for both noise parameters and noise wave parameters.

There is more
noise when using
 $T_{\text{NS}} \frac{P_{\text{cal}} - P_L}{P_{\text{NS}} - P_L} + T_L$



Combined Histograms of T_{dicke} and T for Each Calibrator



Noise amplified
by **30%** when
using

$$T_{NS} \frac{P_{cal} - P_L}{P_{NS} - P_L} + T_L$$

Why not fit noise (wave) parameters directly?

Noise Parameter Equation:

$$P_{\text{out}}^{\text{src}} = \textcolor{red}{g} M \left(T_{\text{in}}^{\text{src}} + \textcolor{red}{T}_{\min} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)} \right) \quad (21)$$

Noise Wave Equation:

$$\begin{aligned} P_{\text{out}}^{\text{src}} = & \textcolor{red}{g} \left[\textcolor{red}{T}_0 + \textcolor{red}{T}_{\text{unc}} |\Gamma_s|^2 \left| \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right|^2 \right. \\ & + T_s (1 - |\Gamma_s|^2) \left| \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right|^2 + \textcolor{red}{T}_{\cos} \Re \left(\Gamma_s \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right) \\ & \left. + \textcolor{red}{T}_{\sin} \Im \left(\Gamma_s \frac{\sqrt{1 - |\Gamma_{\text{rec}}|^2}}{1 - \Gamma_s \Gamma_{\text{rec}}} \right) \right] \end{aligned} \quad (22)$$

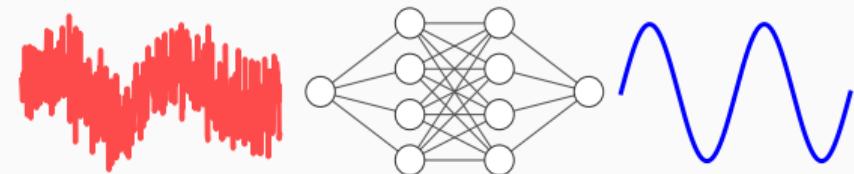
We still end up with 5 unknowns, as before.

Machine learning

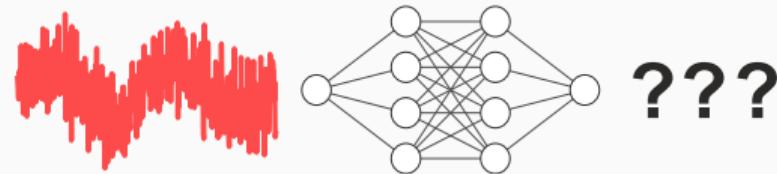
How/why?

- Can predict noise wave parameters using machine learning.
- Could apply method to noise parameters.
- Predict **directly** on noise parameters on frequency by frequency basis.
- Malleable to environmental changes.

Train



Predict



How to Calibrate with Machine Learning?

1. Define the Loss Function

Regress over measured power and predicted power.

$$\mathcal{L} = \frac{1}{n} \sum_{i=1}^n (\mathcal{P}_{\text{measured},i} - \mathcal{P}_{\text{pred},i})^2 \quad (23)$$

2. Write Down the Equation for $\mathcal{P}_{\text{pred}}$

Using the noise wave formalism, relate $\mathcal{P}_{\text{pred}}$ to T_{src} .

$$\begin{aligned} \mathcal{P}_{\text{pred}} &= \mathbf{g} \cdot M(T_{\text{in}}^{\text{src}} \\ &+ T_{\text{min}} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)} \Big) \end{aligned} \quad (24)$$

3. Minimise the Loss Function

$$\theta^* = \arg \min_{\theta} \mathcal{L}(\theta) \quad (25)$$

parameter vector θ includes all tunable parameters in the model:

$$\theta = \{\mathbf{g}, T_{\text{min}}, R_N, \Gamma_{\text{opt}}^\phi, |\Gamma_{\text{opt}}|\} \quad (26)$$

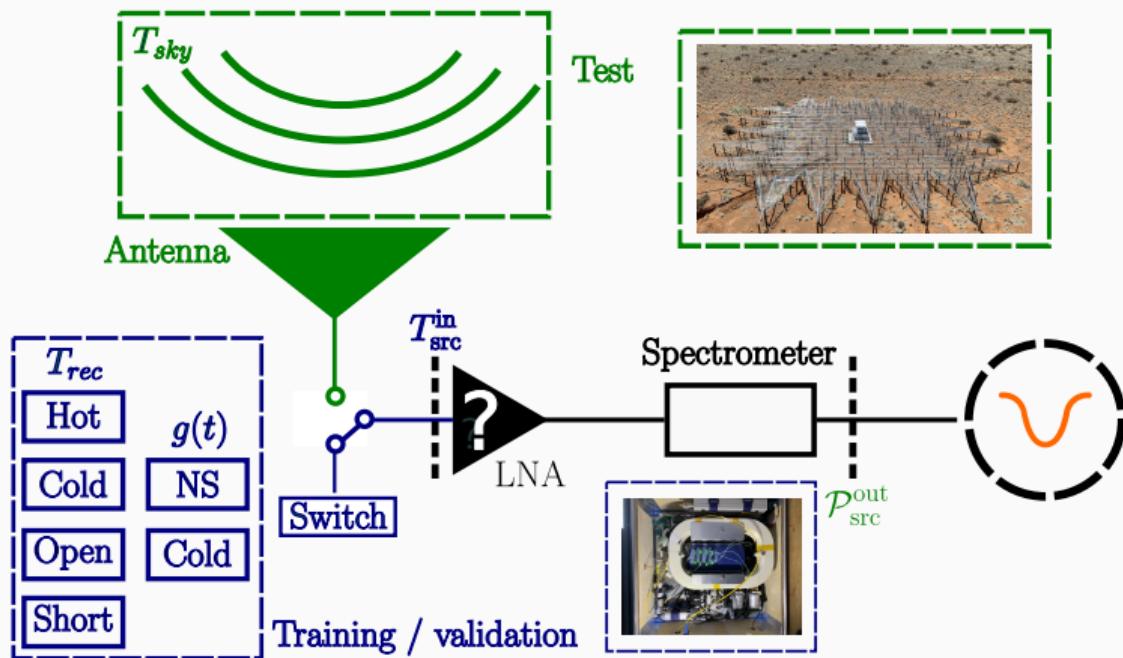
4. Rearrange and Predict (T_{src}) using θ^*

$$\begin{aligned} T_{\text{src}} &= \frac{\mathcal{P}_{\text{pred}}}{\mathbf{g}^* \cdot M} \\ &- \left(T_{\text{min}}^* + T_0 \frac{4R_N^*}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}^*|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}^*|^2)} \right) \end{aligned} \quad (27)$$

The physical system

Process:

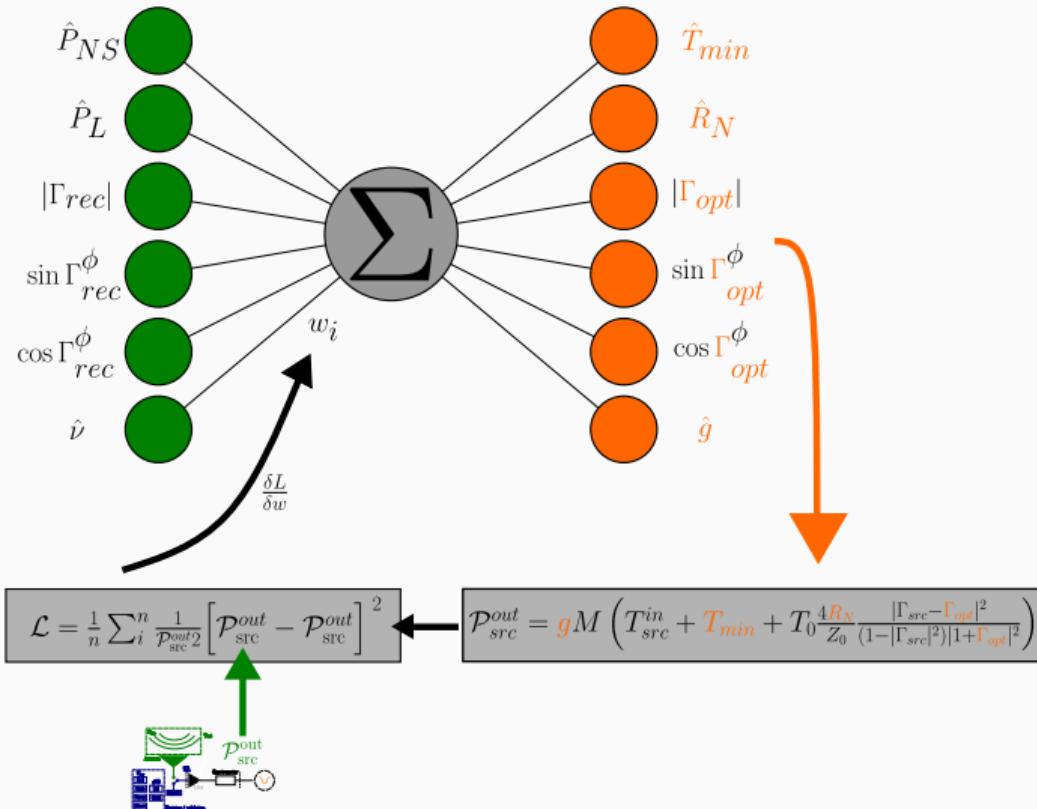
- Switch between sources to generate training data.
- Calibration sources with known temperature train neural net.
- Predict T_{src} of antenna.



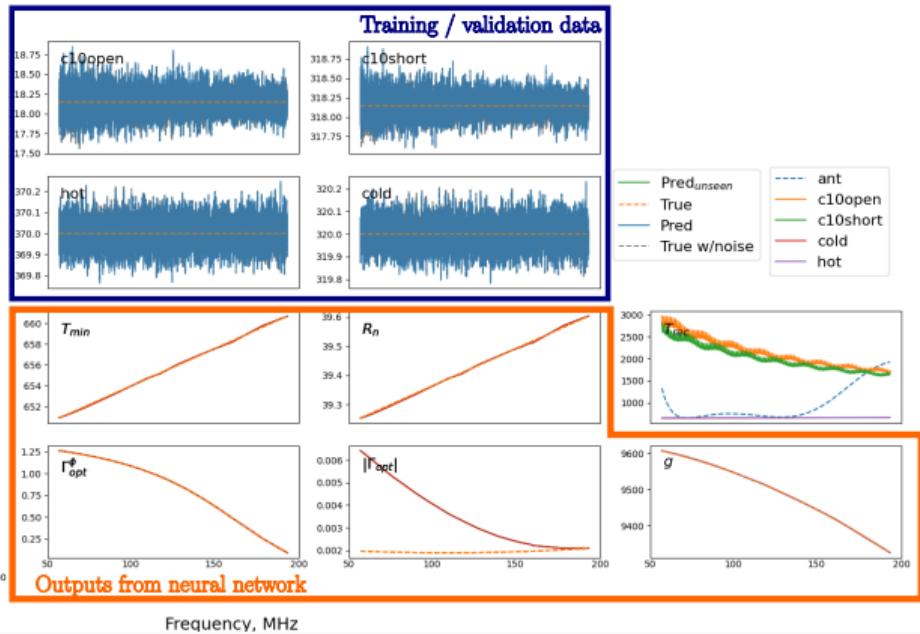
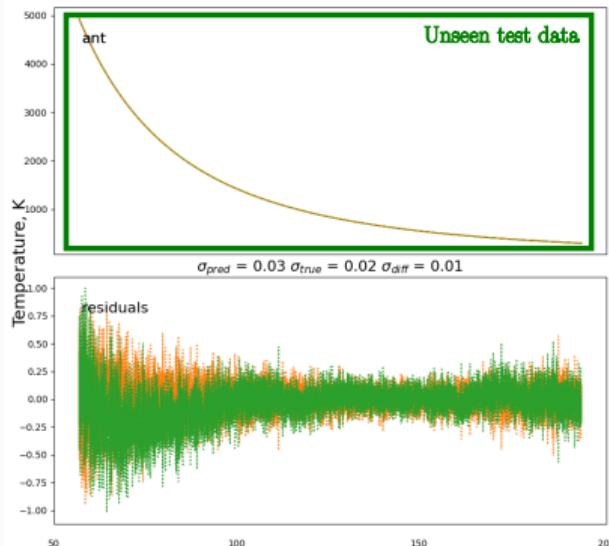
Network Architecture

Structure:

- Input thermistor and VNA measurements.
- Also input frequency.
- Predict noise parameters.
- Regress over loss function.



Testing on simulated data with realistic antenna



Thank you!



SCAN ME

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