Calibration with Machine Learning in Astronomy

Samuel Alan Kossoff Leeney

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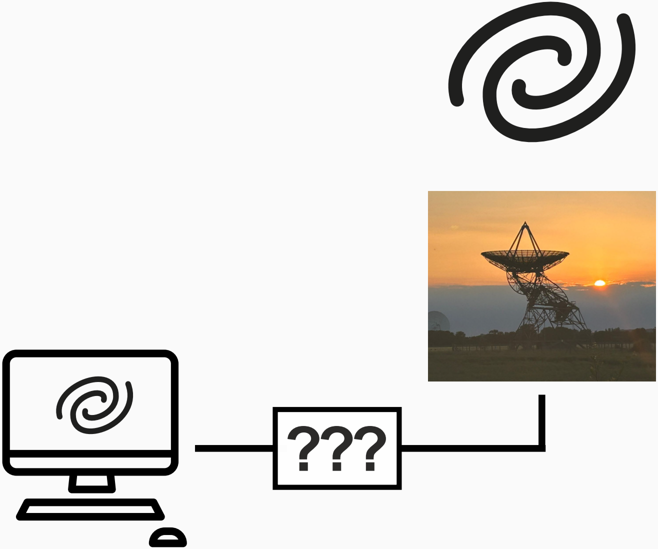
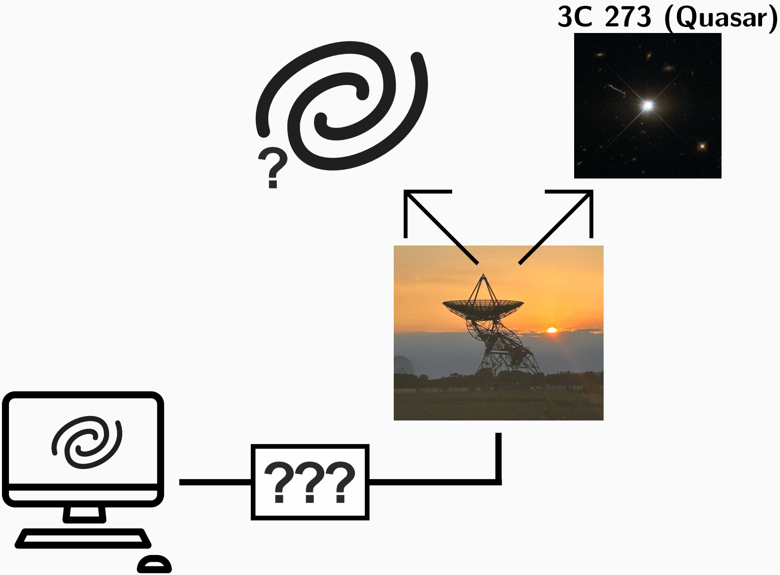
tbd

Co Authors: Harry Bevins, Will Handley, Eloy de Lera Acedo, Christian Kirkhan, Jiacong Zhu, Kaan Artuc, Daniel Molnar

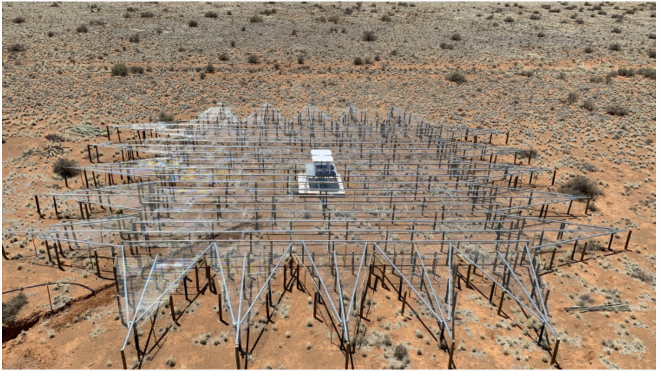
What is calibration?



How to calibrate?



Why is calibration in Global 21cm Cosmology difficult?



We measure *sky averaged* signal.

Antenna LNA impedance mismatch

Very faint signal.

Objective: Map input temperature to output power.

Key Factors:

- LNA introduces time-dependent gain, $g(t)$.
- Impedance mismatch adds noise (T_{rec}) to the system.

Output Power Equation:

$$P_{\text{out}}^{\text{src}} = gM\left(T_{\text{in}}^{\text{src}} + T_{\text{rec}}\right) \tag{1}$$

Note: All parameters above are frequency-dependent, but the notation has been simplified here and thereafter for convenience.

Dealing with reflections...

$$P_{\mathrm{out}}^{\mathrm{src}} = gM\left(T_{\mathrm{in}}^{\mathrm{src}} + T_{\mathrm{rec}}\right) \tag{1}$$

Noise Parameter Equation:

$$P_{\mathrm{out}}^{\mathrm{src}} = gM\left(T_{\mathrm{in}}^{\mathrm{src}} + T_{\mathrm{min}} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\mathrm{src}} - \Gamma_{\mathrm{opt}}|^2}{(1 - |\Gamma_{\mathrm{src}}|^2)(1 + |\Gamma_{\mathrm{opt}}|^2)}\right) \tag{2}$$

Noise Wave Equation:

$$\begin{aligned} P_{\mathrm{out}}^{\mathrm{src}} = g \Bigg[& T_0 + T_{\mathrm{unc}} |\Gamma_{\mathrm{s}}|^2 \left| \frac{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \right|^2 \\ & + T_{\mathrm{s}}(1 - |\Gamma_{\mathrm{s}}|^2) \left| \frac{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \right|^2 + T_{\cos} \Re \left(\Gamma_{\mathrm{s}} \frac{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \right) \\ & + T_{\sin} \Im \left(\Gamma_{\mathrm{s}} \frac{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \right) \Bigg] \end{aligned} \tag{3}$$

Calibration Equation

Typically, substitute in the noise wave parameter equation here (gains cancel)

$$T_{\text{cal}}^{*} = T_{\text{NS}} \frac{P_{\text{cal}} - P_L}{P_{\text{NS}} - P_L} + T_L \tag{4}$$

Make some matching assumptions and re arrange:

$$\begin{aligned} T_{\mathrm{s}} = {} & T_{\mathrm{NS}} \left(\frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{P_{\mathrm{NS}} - P_{\mathrm{L}}} \right) \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} + T_{\mathrm{L}} \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} - T_{\mathrm{unc}} \frac{|\Gamma_{\mathrm{s}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2} + \\ & - T_{\cos} \frac{\Re\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \right) |1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}} - T_{\sin} \frac{\Im\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \right) |1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}} \end{aligned} \tag{5}$$

Note: We end up with 5 parameters that need to be estimated to calibrate the system.

Calculating the error

By partial derivatives
To find the error in T_s, we propagate the errors in Γ_s, Γ_{rec}, P_L, P_{NS}, and P_s:

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial \Gamma_s}\Delta\Gamma_s\right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta\Gamma_{rec}\right)^2 + \left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 + \left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2. \quad (6)$$

Calculating the error

By partial derivatives

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$$\frac{\partial T_s}{\partial P_L} = T_{\mathrm{NS}}\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}\cdot\frac{P_{\mathrm{s}}-P_{\mathrm{NS}}}{(P_{\mathrm{NS}}-P_{\mathrm{L}})^2}. \quad (1)$$

$$\frac{\partial T_s}{\partial P_{NS}} = -T_{\mathrm{NS}}\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}\cdot\frac{P_{\mathrm{s}}-P_{\mathrm{L}}}{(P_{\mathrm{NS}}-P_{\mathrm{L}})^2}. \quad (2)$$

$$\frac{\partial T_s}{\partial P_s} = T_{\mathrm{NS}}\left(\frac{1}{P_{\mathrm{NS}}-P_{\mathrm{L}}}\right)\frac{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{1-|\Gamma_{\mathrm{s}}|^2}. \quad (3)$$

Gamma error eqn's here

Calculating the error

Using $T_{\mathbf{NS}} \frac{P_{\mathbf{cal}}-P_L}{P_{\mathbf{NS}}-P_L} + T_L$

$$(\Delta T_s)^2 = \left(\frac{\partial T_s}{\partial P_s}\Delta P_s\right)^2 + \left(\frac{\partial T_s}{\partial P_L}\Delta P_L\right)^2 + \left(\frac{\partial T_s}{\partial P_{NS}}\Delta P_{NS}\right)^2 \tag{4}$$

$$+ \left(\frac{\partial T_s}{\partial \Gamma_s}\Delta \Gamma_s\right)^2 + \left(\frac{\partial T_s}{\partial \Gamma_{rec}}\Delta \Gamma_{rec}\right)^2 . \tag{6}$$

Using noise wave parameters only

$$(\Delta T_\mathrm{s})^2 = \left(\frac{\partial T_\mathrm{s}}{\partial P_\mathrm{s}}\Delta P_\mathrm{s}\right)^2 \tag{5}$$

$$+ \left(\frac{\partial T_\mathrm{s}}{\partial \Gamma_\mathrm{s}}\Delta \Gamma_\mathrm{s}\right)^2 + \left(\frac{\partial T_\mathrm{s}}{\partial \Gamma_\mathrm{rec}}\Delta \Gamma_\mathrm{rec}\right)^2 . \tag{6}$$

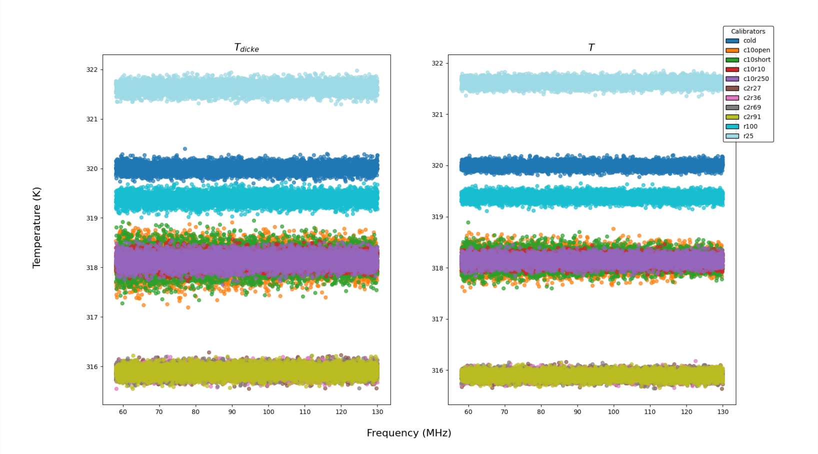

Figure 1: Caption for the image.

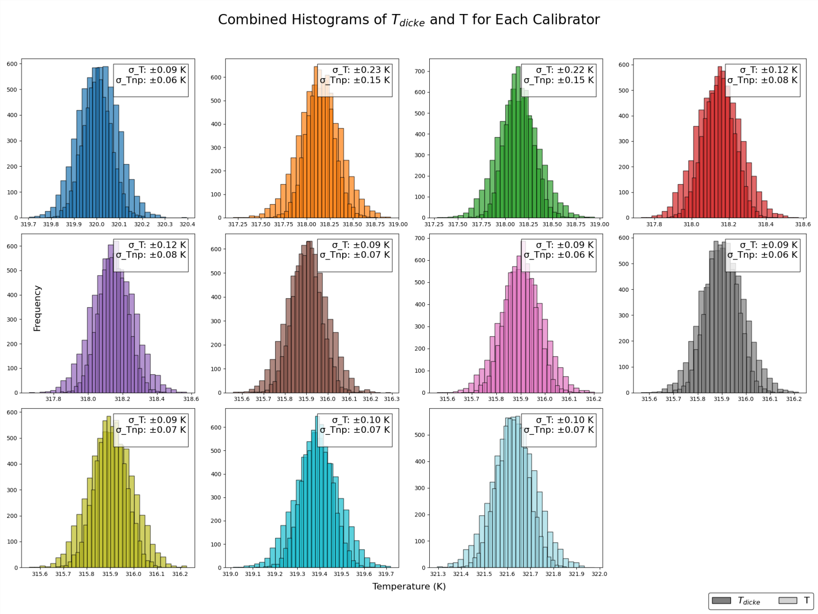

Figure 2: Caption for the image.

Why not fit noise (wave) parameters directly?

Noise Parameter Equation:

$$P_{\text{out}}^{\text{src}} = gM \left(T_{\text{in}}^{\text{src}} + T_{\text{min}} + T_0 \frac{4R_N}{Z_0} \frac{|\Gamma_{\text{src}} - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_{\text{src}}|^2)(1 + |\Gamma_{\text{opt}}|^2)} \right) \tag{2}$$

Noise Wave Equation:

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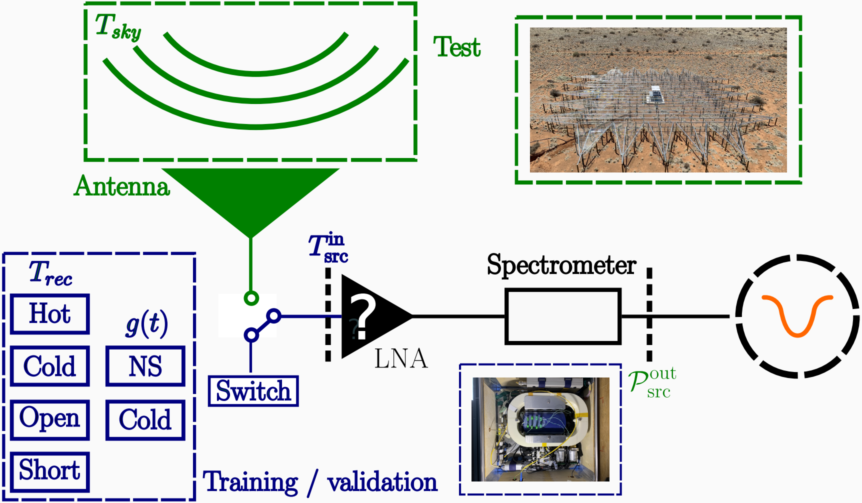
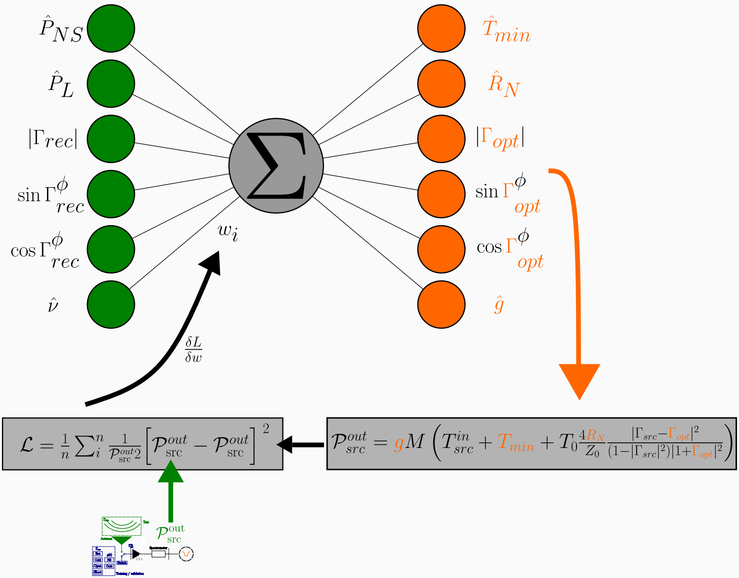
We still end up with 5 unknowns, as before.

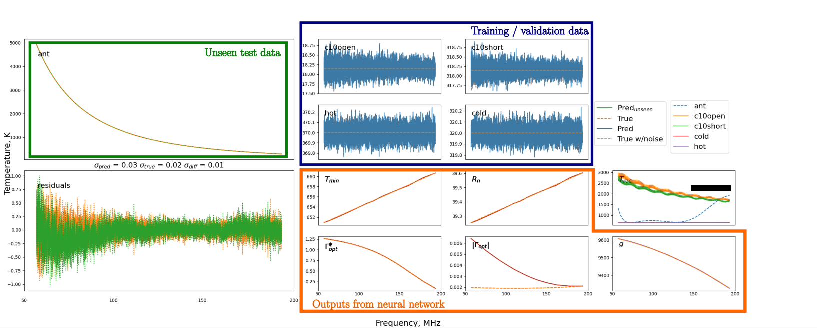
Machine learning for radiometer calibration in global 21cm cosmology

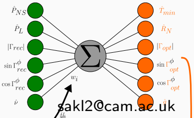


Machine learning for radiometer calibration in global 21cm cosmology



Machine learning for radiometer calibration in global 21cm cosmology





Sam Leeney sakl2@cam.ac.uk github.com/samleeney

Thank you!

SCAN ME