Professor Robert A. Miller

Carnegie Mellon University

Econometrics 2 47-812

Mini 2 2018

ASSIGNMENT 1

Overview: The purpose of this assignment is to introduce you to actually doing structural econometrics. The assignment should be undertaken in groups of about three. I do not object to discussion taking place across groups in the early stages of the work either. I do insist that each group should refine its own answers, and submit a report that is unique. For example you might collectively derive the perfect set of answers and then negotiate with each other about where to inject uniquely defined errors.

Due date and assessment: Please email me a pdf file of your answers in the form of a report, and also submit a hard copy of the report only at the beginning of class Tuesday, February 6 with your code attached as an appendix. Hand written work will not be graded. The answer to each question counts equally to the perfect score. Poor grammar, unclear expression, and lack of precision, will be graded as if I have very limited expertise in this area.

The model: Consider the following model of invention. If you decide to be an inventor (perhaps only for a while) here is what happens. Choices for period t are indicated by $d_t \in \{0, 1\}$ where $d_t = 0$ means "doing something else" in period t and $d_t = 1$ means "inventing". Your lifetime choices is a sequence $\{d_t\}_{t=1}^{T}$ where T is the last period of your life.

In each period you invent there is a fixed probability ξ, your ability parameter, that you "draw" a successful new product/firm in which case you get a "prize" value of "start-up" equal to a normalized value of one. Otherwise you get zero in that period if you made that choice. These draws distributed independently over time. Let $x_t \in \{0, 1\}$ indicate whether your invention is successful $(x_t = 1)$ or not $(x_t = 0)$. The value of "doing something else" in period t is a constant w. You have a subjective discount factor of β and your lifetime utility is:

$$\sum_{t=0}^{\infty} \beta^t \left[d_t I \{x_t = 1\} + (1 - d_t) w \right]$$

If you knew ξ then this would be an easy problem to solve: choose $d_t = 1$ for all t if $\xi > w$; otherwise choose $d_t = 0$ for all t. The difficulty arises because when you start work at the beginning of your career at period don't your own ξ. You do, however know that your ability parameter ξ is drawn from a Beta probability distribution, with parameters (γ, δ) say. Thus the model is fully characterized by parameters, namely w, the outside wage, β the discount factor, and the parameters (γ, δ) defining the distribution from which your ξ is drawn.

Question 1: Prove it is never optimal to "do something else" in any period t and invent in period $t+1$. Thus the optimal career strategy is to spend no periods inventing, or spend one or more periods inventing at the beginning of your career.

Question 2: Fully characterize the theoretical solution to this model when $T = 2$. That is solve for the last period's decision given what your beliefs will in the last period. Then write down the problem solved in the first period, taking account of the fact that if you invent, then you update your information about ξ which is useful in solving the second.

Question 3: Solve the optimal decisions for the 2 period model for the gamma distribution and wage rate. What bounds must be set on the parameters to ensure everybody tries inventing in the first period?

Question 4: Solve for the probability of quitting after the first period.

Question 5: Suppose the data set consists of the choice of the individual in the second period. In other words, the data analyst does not observe whether the invention of each individual was successful or not. (If such data was available the beta distribution could be estimated from output in the first period, and an estimator of w could be obtained from the fraction of individuals choosing to invent in the second.) Explain why it reasonable to think that you can only estimate one parameter from this two period model.

Question 6: Can you show why the parameters for you model are not identified from the solution to the quit rates without imposing restrictions (such as knowing the values of some of the parameters)?

Question 7: Simulate a data set of 50, and 500 individuals from this model.

Question 8: Fixing three of the four parameters estimate the remaining one with Maximum Likelihood (ML). Try this for a couple of different choices about what is estimated. Include estimated standard errors.

Question 9: Repeat the exercise in which you misspecify the model by picking the wrong values for the parameters you fix. How sensitive is your estimate of the remaining parameter to this misspecification?

Question 10: Set $T = \infty$. (In practice set T high enough so that individuals would not change their decisions early in life if they were told they would live one period longer.) Solve the optimal decision rule for this model numerically using a recursion similar to the formula for the value function we discussed in Lecture 2.

Question 11: Generate the hazard rate of quitting, that is a sequence $\{h_t\}_{t=1}^{S}$, where S is "reasonable" date at which h_S is "close" to zero, and/or your numerical integration routine is not helpful.

Question 12: Compare the solutions of the two models, and thus show the effects of increasing the horizon from $T = 2$ to $T = \infty$.

Question 13: Simulate a data set of 50, and 500 individuals for the $T = \infty$ model.

Question 14: Estimate the $T = \infty$ model off the first period's data in the same way you did in Question 8. Now use extra periods, still estimating just one parameter.

Question 15: How well does the ML estimator perform estimating all the parameters?