Seperable Differential Equations

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1 Form of Differential Equations

Differential equations will be of the form:

$$\frac{dy}{dx} = y' = G(y)$$

Where y is a function of x

2 Seperable Differential Equations

Most differential equations are not readily solvable. A common solvable form is the seperable differential equation:

$$\frac{dy}{dx} = y' = G(y)F(x)$$

Method of finding solutions for this:

$$\int \frac{dy}{G(y)} = \int F(x)dx \text{ i.e.} \int G(y)^{-1}dy = \int F(x)dx$$

An example of a seperable differential equation:

$$y' = \frac{x+1}{y^2+y+2}$$

This can be solved via:

$$\begin{aligned} y' = \frac{dy}{dx} &= (x+1)(y^2+y+2)^{-1} \\ &= dy(y^2+y+2) = dx(x+1) \\ &= dy(poopypants) \end{aligned}$$

Interesting Thoughts

$$y = f(x) \text{ and } y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$dx : h \mapsto h \text{ and } dy : h \mapsto h$$

dx and dy represent the instantaneous rate of change of x or y with respect to themselves. This is why they are identity functions mapping $h \mapsto h$. They form the bridge between the infinitesimal(instantaneous rate of change) and what is algebraically maniputable.

$$\text{With } y = f(x), \, dy \text{ can be defined by } dy : x \mapsto f'(x)dx$$
$$\frac{dy(h;x)}{dx(h)} = \frac{f'(x)dx(h)}{dx(h)} = f'(x)$$