

Linear Algebra

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1 Fundamentals

Definition 1.1 (Vector space). A vector space V over a field F consists of a set on which two operations are defined so that for all $x, y \in V$ there is a unique element $x + y$ in V , and for each $a \in F$ and $x \in V$ there is a unique element $ax \in V$ such that

1. For all $x, y \in V$, $x + y = y + x$.
2. For all $x, y, z \in V$, $(x + y) + z = x + (y + z)$.
3. There exists $0 \in V$ such that for all $x \in V$, $x + 0 = x$.
4. For each element $x \in V$ there exists an element $y \in V$ such that $x + y = 0$.
5. For each element $x \in V$, $1x = x$.
6. For all $a, b \in F$ and for all $x \in V$, $(ab)x = a(bx)$.
7. For all $a \in F$ and for all $x, y \in V$, $a(x + y) = ax + ay$.
8. For all $a, b \in F$ and for all $x \in V$, $(a + b)x = ax + bx$.