"Calculus: Early Transcendantals" Notes

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1 Section 1

1.1 Subsection 1

Theorem 1.1 (Fubini's theorem)**.** *Let $f(x, y)$ be continuous on a region R.*

(a) If R is defined by $a \leq x \leq b,\ g_1(x) \leq y \leq g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx.$$

Proposition 1.2. The area of a closed, bounded plane region R is

$$A = \iint_R dA.$$

Definition 1.3 (Path independence)**.** Let $\mathbf{F}$ be a vector field defined on an open region D in space, and suppose that for any two points A and B in D the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along a path C from A to B in D is the same over all paths from A to B. Then the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is path independent in D and the field $\mathbf{F}$ is conservative in D.

Theorem 1.4 (Fundamental theorem of line integrals)**.** *Let C be a smooth curve joining the point A to the point B in the plane or in space and parametrized by $\mathbf{r}(t)$. Let f be a differentiable function with a continuous gradient vector $\boldsymbol{F} = \nabla f$ on a domain D containing C. Then*

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r} = f(B) - f(A).$$