Homework 4

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Problem 1

Suppose $n \in \mathbb{N}$. Suppose to the contrary that there exists $n \geq 2$ such that $(a_n > a_{n-1} \geq 4)$ and $(a_{n+1} \leq a_n \vee a_n, a_{n+1} < 4)$. It follows

$$3(a_n - 1) \leq a_n$$
$$a_n \leq \frac{3}{2}$$

and a contradiction arises because $a_n > 4$. Therefore

$$\forall n \geq 2 \in N, \ (a_n > a_{n-1} \geq 4) \Rightarrow (a_{n+1} > a_n \geq 4)$$

and because $a_1 = 4$ and $a_2 = 9$, for all $n > 2$ we have $a_n > a_{n-1}$. Let $n, m \in \mathbb{R}$, and suppose to the contrary that $a_n = a_m$ with $n \neq m$. But if $n \neq m$, then wlog $n < m$ and $a_n < a_m$, a contradiction because $<$ is transitive.

Problem 2