Nonlinear Dynamics

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1 Flows on the line

1.1 Introduction

Definition 1.1 (Fixed points)**.** A fixed point on a phase diagram is a point in which there is no flow, i.e. $x' = 0$. Fixed points represent equilibrium solutions.

Definition 1.2 (Phase point)**.** A phase point is an imaginary particle placed at a point x_0 from which we can observe how it is carried along with the "flow". As time increases, the phase point moves along the x-axis according to some function $x(t)$. $x(t)$ is called the trajectory based at x_0.

Theorem 1.3. *Consider the IVP*

$$x' = f(x),$$
$$x(0) = x_0.$$

If $f(x)$ and $f'(x)$ are continuous on an open interval R of the x-axis, and $x_0 \in R$, then the initial value problem has a unique solution on some time interval $-\tau, \tau$ about $t = 0$.