"Calculus: Early Trancendentals" Notes

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April 18, 2024

Calc 1

Definition 1.1 (Average Rate of Change)**.** The average rate of change of $y = f(x)$ with respect to x over the interval (x_1, x_2) is:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}$$

Theorem 1.1 (Limit Laws)**.** If L, M, c, k are real numbers and $\lim_{x\to c} f(x) = L$, and $\lim_{x\to c} g(x) = M$, then:

Sum rule:	$\lim\limits_{x\to c}(f(x) + g(x)) = L + M$
Difference Rule:	$\lim\limits_{x\to c}(f(x) - g(x)) = L - M$
Constant Multiple Rule:	$\lim\limits_{x\to c}(k \cdot f(x)) = k \cdot L$
Product Rule:	$\lim\limits_{x\to c}(f(x) \cdot g(x)) = L \cdot M$
Quotient Rule:	$\lim\limits_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M},\ M \neq 0$
Power Rule:	$\lim\limits_{x\to c}\left[f(x)\right]^n = L^n,\ n$ a positive integer
Root Rule:	$\lim\limits_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L},$ n a positive integer

(For the root rule, if n is even, we assume that $f(x) \geq 0$ for x in an interval containing c.)

Theorem 1.2 (Sandwich Theorem)**.** Suppose that $g(x) \leq f(x) \leq h(x)$ for all x in some neighborhood of c. Suppose also that:

$$\lim_{x\to c} g(x) = \lim_{x\to c} f(x) = L$$

Then $\lim_{x\to c} f(x) = L$.

Definition 1.2 (Limit)**.** Let $f(x)$ be defined on an open neighborhood of c not necessarily including c. We say that the limit of $f(x)$ as x approaches c is the number L if for every number $\epsilon > 0$, there exits a corresponding number $\delta > 0$ such that:

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$$