

Differential Equations

Samuel Lindskog

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First-Order Differential Equations

Definition 1.1 (Order). The order of a differential equation is the order of the highest derivative appearing in the equation.

Definition 1.2 (Normal form). The normal form of a first-order equation is a function f which relates a function $x = x(t)$ with its first derivative.

$$x' = f(t, x).$$

A function $x = x(t)$ is a solution of this equation on the time interval $I : a < t < b$ if it is differentiable on I and, when substituted into the equation, it satisfies the equation identically for every $t \in I$, i.e.

$$x'(t) = f(t, x(t)), \text{ for every } t \in I.$$

In other words to check if a function is a solution, substitute the function in question into the differential equation and check that it reduces to an identity.