Toffoli gate implementation with 4 CNOT gates

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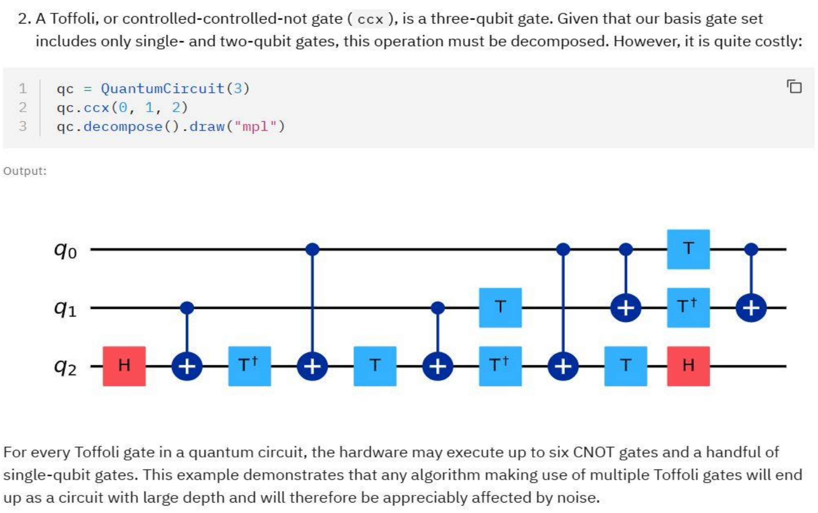
Abstract

This paper presents a minor advancement on Problem 4.4b from the Nielsen and Chuang textbook, demonstrating the construction of the Toffoli gate using single-qubit gates and four CNOT gates.

Introduction

Theorem 1 in reference [2] states that a circuit consisting of CNOT gates and one-qubit gates that implements an n-qubit Toffoli gate without ancillae requires at least 2n CNOT gates. For n=3, that would mean at least 6 CNOT gates. Problem 4.4b from the Nielsen and Chuang textbook asks about the minimal Toffoli gate construction. As per reference [3], there are at least two different ways to construct the 3-qubit Toffoli gate out of one-qubit gates and CNOT gates. One construction requires 8 CNOT gates while a more efficient construction requires 6 CNOT Gates. The current IBM qiskit implementation of the 3-qubit Toffoli gate [4] uses 6 CNOT gates.

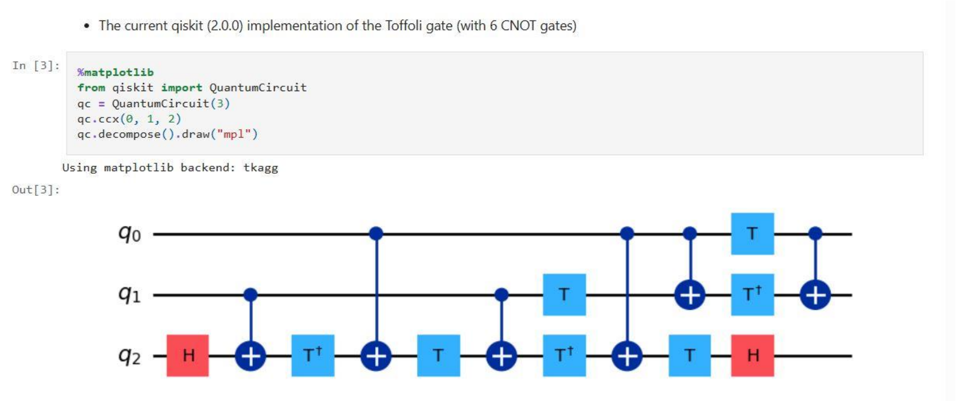


2. A Toffoli, or controlled-controlled-not gate (ccx), is a three-qubit gate. Given that our basis gate set includes only single- and two-qubit gates, this operation must be decomposed. However, it is quite costly:

```
qc = QuantumCircuit(3)
qc.ccx(0, 1, 2)
qc.decompose().draw("mpl")
```

Output:

For every Toffoli gate in a quantum circuit, the hardware may execute up to six CNOT gates and a handful of single-qubit gates. This example demonstrates that any algorithm making use of multiple Toffoli gates will end up as a circuit with large depth and will therefore be appreciably affected by noise.



- The current qiskit (2.0.0) implementation of the Toffoli gate (with 6 CNOT gates)

In [3]:

```
%matplotlib
from qiskit import QuantumCircuit
qc = QuantumCircuit(3)
qc.ccx(0, 1, 2)
qc.decompose().draw("mpl")
```

```
Using matplotlib backend: tkagg
```

Out[3]:

References

[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information,

Cambridge University Press (2000).

[2] [0803.2316] On the CNOT-cost of TOFFOLI gates

[3] Mermin ND. Frontmatter. In: *Quantum Computer Science: An Introduction*. Cambridge University Press; 2007:i-vi.

[4] https://docs.quantum.ibm.com/guides/transpiler-stages#translation-stage