# 3-qubit Toffoli gate implementation with 4 CNOT gates

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**Abstract**

This paper presents a minor advancement on Problem 4.4b from the Nielsen and Chuang textbook, demonstrating the construction of the Toffoli gate using single-qubit gates and four CNOT gates.

**Introduction**

Theorem 1 in reference [**[2]**](https://arxiv.org/abs/0803.2316) states that a circuit consisting of CNOT gates and one-qubit gates that implements an n-qubit Toffoli gate without ancillae requires at least 2n CNOT gates. For n=3, that would mean at least 6 CNOT gates. Problem 4.4b from the Nielsen and Chuang textbook asks about the minimal Toffoli gate construction. As per reference [3], there are at least two different ways to construct the 3-qubit Toffoli gate out of one-qubit gates and CNOT gates. One construction requires 8 CNOT gates while a more efficient construction requires 6 CNOT Gates. The current IBM qiskit implementation of the 3-qubit Toffoli gate [4] uses 6 CNOT gates.

A diagram of a circuit

AI-generated content may be incorrect.

**A screenshot of a computer

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As a counterexample to the Theorem 1 in reference [**[2]**](https://arxiv.org/abs/0803.2316), in this paper we present a small variation on this 6 CNOT gates construction of the 3-qubit Toffoli gate that uses only 4 CNOT gates.

**3-qubit Toffoli gate implementation with 4 CNOT gates**

Figure 1 displays the 3-qubit Toffoli gate implementation consisting of 4 CNOT gates and one-qubit gates.

A diagram of a circuit

AI-generated content may be incorrect.

Figure 1

**References**

[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information,

Cambridge University Press (2000).

[2] [[0803.2316] On the CNOT-cost of TOFFOLI gates](https://arxiv.org/abs/0803.2316)

[3] Mermin ND. Frontmatter. In: *Quantum Computer Science: An Introduction*. Cambridge University Press; 2007:i-vi.

[4] <https://docs.quantum.ibm.com/guides/transpiler-stages#translation-stage>