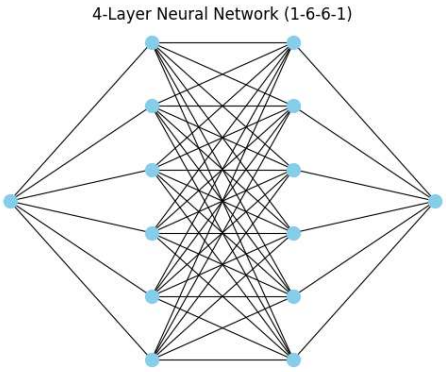
Programming assignment

Method:



1.activation function: σ(x) = tanh(x)

2.Network:

(1)Input layer: 1 neuron

(2)Inside layer: 2 layers, 6 neurons

(3)Output layer: 1 neuron

Hypothesis:

$$h_\theta(x) = W_3 \tanh\Big(W_2 \tanh\big(W_1 x + b_1\big) + b_2\Big) + b_3$$

3.Choosen Points:

Choose at [-1,1]

(1)Training: 10000

(2)Validation: 1500

4.loss function:

$$\mathcal{L}(\theta) = \underbrace{\mathrm{MSE}(h_\theta(x), R(x))}_{\text{function loss}} + \lambda_{\text{deriv}} \underbrace{\mathrm{MSE}\big(\tfrac{d}{dx} h_\theta(x), R'(x)\big)}_{\text{derivative loss}}.$$

5.optimizer: Adam

6.learning rate: 0.001

7.Epoch: 7000 times

Differential way:

Runge function use $R'(x) = -\dfrac{50x}{(1+25x^2)^2}.$

Training data use $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h},$ where h = (1-(-1)/10000

Result:

Loss:

```
Epoch  500 | Train Loss: 0.026777 (f: 0.000119, df: 0.026658) | Val Loss: 0.026266 (f: 0.000118, df: 0.026148)
Epoch 1000 | Train Loss: 0.000703 (f: 0.000003, df: 0.000700) | Val Loss: 0.000701 (f: 0.000003, df: 0.000698)
Epoch 1500 | Train Loss: 0.000380 (f: 0.000002, df: 0.000378) | Val Loss: 0.000379 (f: 0.000002, df: 0.000377)
Epoch 2000 | Train Loss: 0.000244 (f: 0.000001, df: 0.000243) | Val Loss: 0.000244 (f: 0.000001, df: 0.000243)
Epoch 2500 | Train Loss: 0.000152 (f: 0.000001, df: 0.000151) | Val Loss: 0.000152 (f: 0.000001, df: 0.000151)
Epoch 3000 | Train Loss: 0.000088 (f: 0.000000, df: 0.000088) | Val Loss: 0.000088 (f: 0.000000, df: 0.000088)
Epoch 3500 | Train Loss: 0.000046 (f: 0.000000, df: 0.000046) | Val Loss: 0.000046 (f: 0.000000, df: 0.000046)
Epoch 4000 | Train Loss: 0.000023 (f: 0.000000, df: 0.000023) | Val Loss: 0.000023 (f: 0.000000, df: 0.000023)
Epoch 4500 | Train Loss: 0.000012 (f: 0.000000, df: 0.000012) | Val Loss: 0.000012 (f: 0.000000, df: 0.000012)
Epoch 5000 | Train Loss: 0.000008 (f: 0.000000, df: 0.000008) | Val Loss: 0.000008 (f: 0.000000, df: 0.000008)
Epoch 5500 | Train Loss: 0.000006 (f: 0.000000, df: 0.000006) | Val Loss: 0.000006 (f: 0.000000, df: 0.000006)
Epoch 6000 | Train Loss: 0.000005 (f: 0.000000, df: 0.000005) | Val Loss: 0.000005 (f: 0.000000, df: 0.000005)
Epoch 6500 | Train Loss: 0.000005 (f: 0.000000, df: 0.000005) | Val Loss: 0.000005 (f: 0.000000, df: 0.000005)
Epoch 7000 | Train Loss: 0.000005 (f: 0.000000, df: 0.000005) | Val Loss: 0.000005 (f: 0.000000, df: 0.000005)
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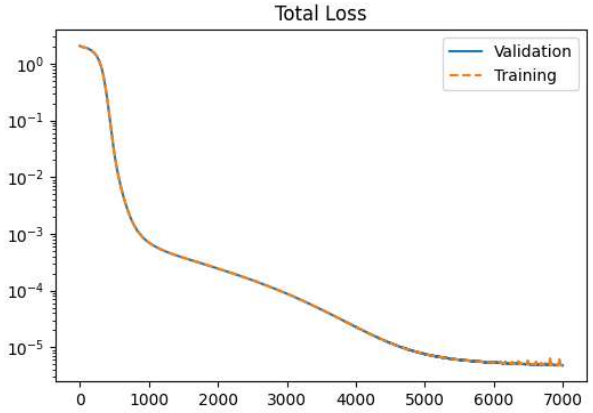
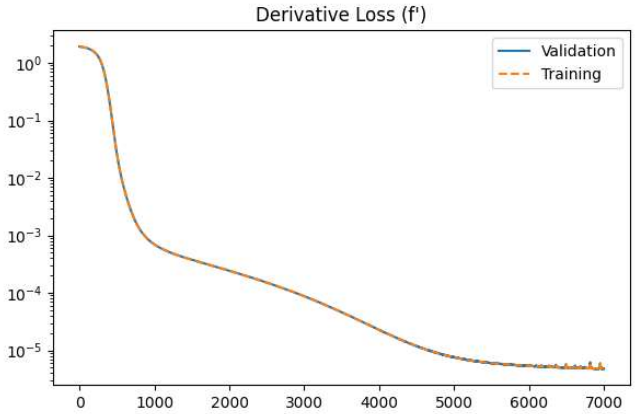
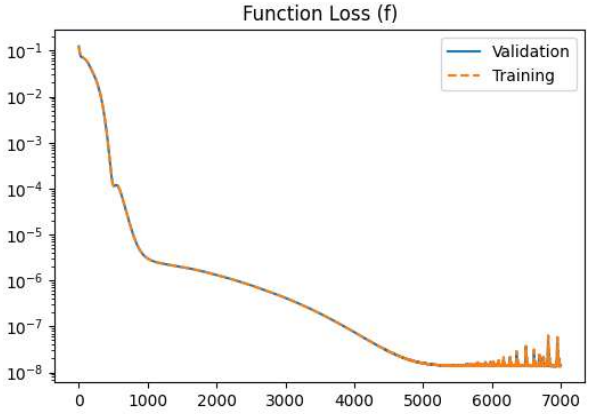
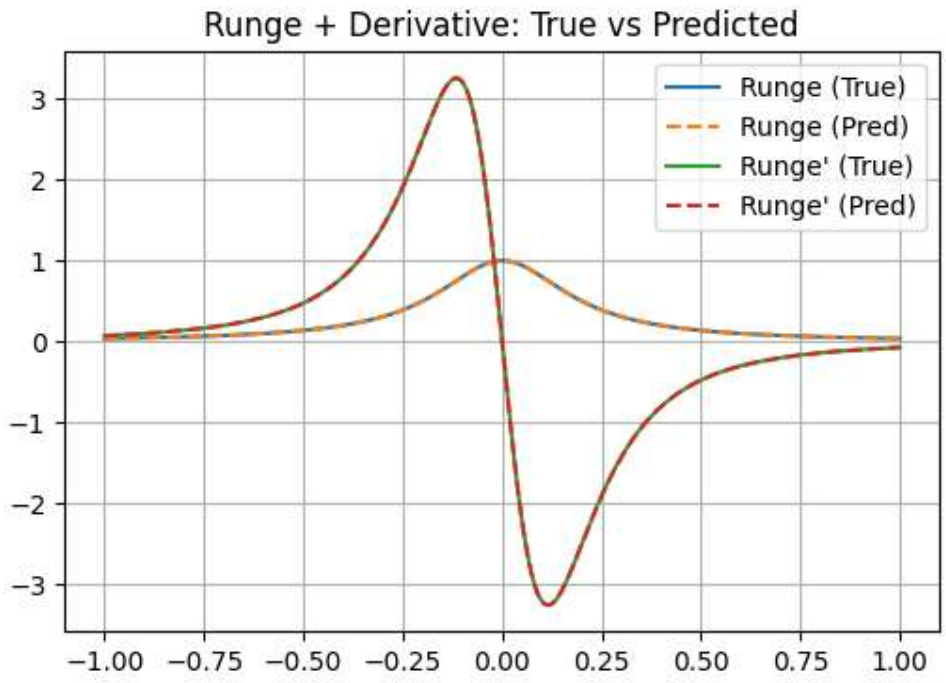






Result:



MSE loss: 0.000000, Max Error: 0.000218