Programming assignment

4-Layer Neural Network (1-6-6-1)

Method:

1.activation function: $\sigma(x) = \tanh(x)$

2.Network:

(1)Input layer: 1 neuron

(2)Inside layer: 2 layers, 6 neurons

(3)Output layer: 1 neuron

Hypothesis:

$$h_{ heta}(x) = W_3 \, anh \Big(W_2 \, anh ig(W_1 x + b_1ig) + b_2\Big) + b_3$$

3.Choosen Points:

Choose at [-1,1]

(1)Training: 10000

(2) Validation: 1500

4.loss function:

$$\mathcal{L}(\theta) = \underbrace{\mathrm{MSE}(h_{\theta}(x), R(x))}_{\text{function loss}} + \lambda_{\text{deriv}} \underbrace{\mathrm{MSE}\big(\frac{d}{dx}h_{\theta}(x), R'(x)\big)}_{\text{derivative loss}}.$$

5.optimizer: Adam

6.learning rate: 0.001

7.Epoch: 7000 times

Differential way:

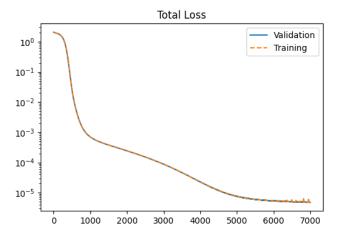
Runge function use $R'(x) = -\frac{50x}{(1+25x^2)^2}$.

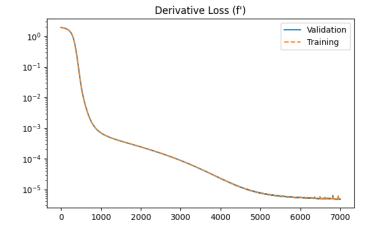
```
Training data use f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}, where h = (1-(-1)/10000
```

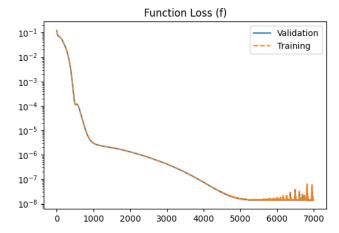
Result:

Loss(lambda = 1):

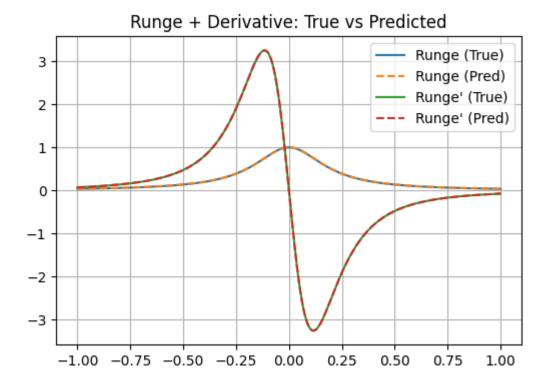
```
Train Loss: 0.026777 (f: 0.000119, df: 0.026658)
                                                                 Val Loss: 0.026266 (f: 0.000118, df: 0.026148)
                                                                 Val Loss: 0.000701 (f: 0.000003, df: 0.000698)
Epoch 1000
             Train Loss: 0.000703 (f: 0.000003, df: 0.000700)
                                                                 Val Loss: 0.000379 (f: 0.000002, df: 0.000377)
Epoch 1500
             Train Loss: 0.000380 (f: 0.000002, df: 0.000378)
Epoch 2000
             Train Loss: 0.000244 (f: 0.000001, df: 0.000243)
                                                                 Val Loss: 0.000244 (f: 0.000001, df: 0.000243)
Epoch
      2500
             Train Loss: 0.000152 (f: 0.000001, df: 0.000151)
                                                                 Val Loss: 0.000152 (f: 0.000001, df: 0.000151)
Epoch 3000
             Train Loss: 0.000088 (f: 0.000000, df: 0.000088)
                                                                 Val Loss: 0.000088 (f: 0.000000, df: 0.000088)
Epoch 3500
             Train Loss: 0.000046 (f: 0.000000, df: 0.000046)
                                                                 Val Loss: 0.000046 (f: 0.000000, df: 0.000046)
Epoch 4000
             Train Loss: 0.000023
                                  (f: 0.000000, df: 0.000023)
                                                                 Val Loss: 0.000023 (f: 0.000000, df: 0.000023)
Epoch 4500
             Train Loss: 0.000012 (f: 0.000000, df: 0.000012)
                                                                 Val Loss: 0.000012 (f: 0.000000, df: 0.000012)
                                                                 Val Loss: 0.000008 (f: 0.000000, df: 0.000008)
             Train Loss: 0.000008 (f: 0.000000, df: 0.000008)
Epoch 5000
Epoch 5500
             Train Loss: 0.000006 (f: 0.000000, df: 0.000006)
                                                                 Val Loss: 0.000006 (f: 0.000000, df: 0.000006)
             Train Loss: 0.000005 (f: 0.000000, df: 0.000005)
                                                                 Val Loss: 0.000005 (f: 0.000000, df: 0.000005)
Epoch 6000
             Train Loss: 0.000005 (f: 0.000000, df: 0.000005)
                                                                 Val Loss: 0.000005 (f: 0.000000, df: 0.000005)
Epoch 6500
             Train Loss: 0.000005 (f: 0.000000, df: 0.000005)
                                                                 Val Loss: 0.000005 (f: 0.000000, df: 0.000005)
Epoch 7000
```







Result:



MSE loss: 0.000000, Max Error: 0.000218