Multivariable and Vector Calculus Notes

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A **Finite State Machine** or **Finite Automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

- Q is a finite set called the **states**,
- Σ is a finite set called the **alphabet**,
- $\delta : Q \times \Sigma \to Q$ is the **transition function**,
- $q_0 \in Q$ is the **start state**, and
- $F \subseteq Q$ is the set of **accept states** (also known as **final states**)

If A is the set of all strings that M accepts, we say that A is the **language of machine** M and denote it $L(M) = A$. We say that M **accepts** A or M **recognizes** A (we prefer the latter term to avoid confusion when referring to strings and languages). Machines may accept many strings, but recognize only one language.

A language is called a **regular language** if some finite automaton recognizes it.

Let A and B be languages. We define the regular operations union, concatenation, and star as follows:

- **Union**: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Concatenation**: $A \circ B = \{xy \mid x \in A \text{ and } x \in B\}$
- **Star**: $A^* = \{x_1 x_2 \dots x_k \mid k \geq 0 \text{ and each } x_k \in A\}$